SPONS'

DICTIONARY OF ENGINEERING.

DIVISION V.

SPONS'

DICTIONARY OF ENGINEERING,

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WITH TECHNICAL TERMS

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DIVISION V.

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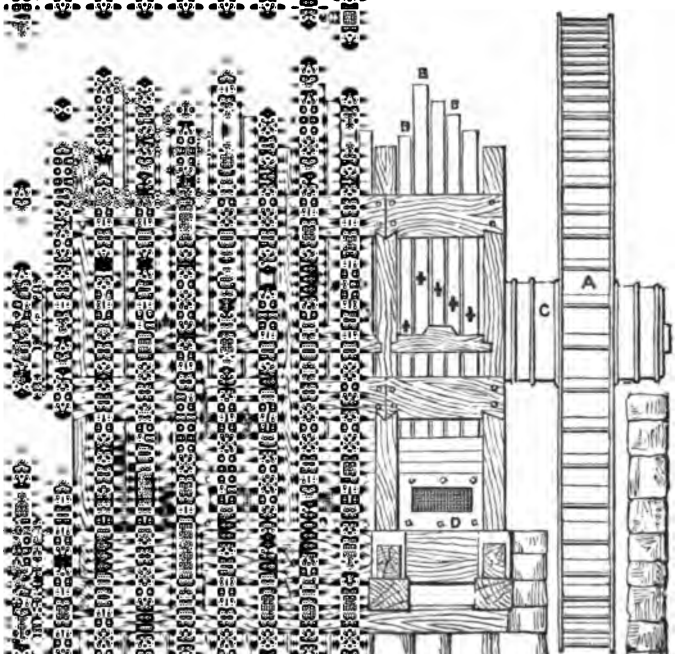
composition have an influence on the results
sed for sandstone is applied here. Good clay
m, after exposure to a strong heat, a compact
and tempering of clay has a decided influence
good furnaces are required to form good fire-
moulded with great facility into any shape,
quality is caused by the presence of more or
is not plastic. However valuable this quality
rcelain, it is of little use to the metallurgist;
substance which resists fire; the coarse forms
egree of tenacity. In order to test clay, it is
of half an inch in thickness, which are gently
, and then exposed to a strong heat. When
necessary to mix it with sufficient fine, pure,

naces; it is, however, used in some smelting
n mixed with a large quantity of sand; also
hers. Its chief use is for mortar.
durable fire-proof stone, particularly when
heat in the native rock. This kind of slate
it is cemented by heat it is extremely hard.
s in puddling furnaces, for which it is adapted
exceedingly well.
d in many instances it is very doubtful if the
ly a modification of it in form, characterized
These slates resist fire well, if not too much
ica. The quantity of quartz determines the
venient, because in most instances it is easily
slate, gneiss, porphyry, granite, and similar
ut their quality depends entirely on a peculiar
refractory, and are all liable to be broken by

s cannot be obtained, or the purpose requires
composed, such as clay and silex, are pounded,
any form that may be desired. Quartz, which
ce besides clay, is pounded in stamping mills,
either performed dry, which causes much dust

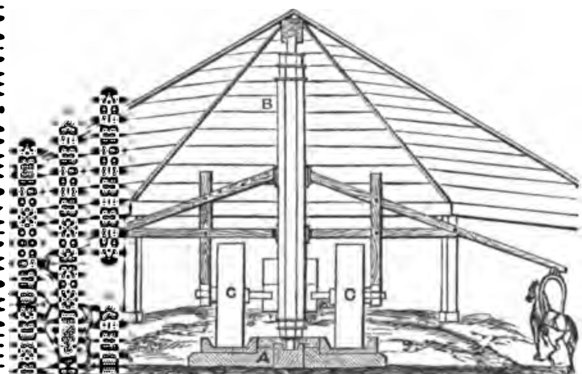


one by passing a current of water through the
n which it settles, and the water flows off. If
quartz, it may be exposed to a red heat in a
easily.

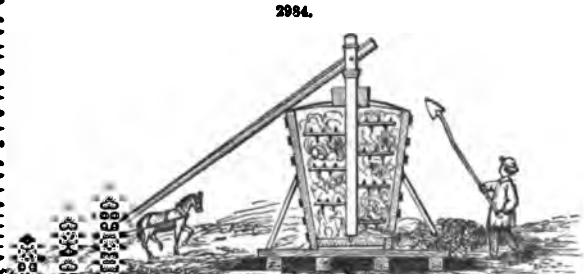
…e of a grain of wheat, or smaller, three parts are
…hole well soaked with water, and diligently mixed,
…erely air-dried Of this mixture, bricks and slabs
…in reverberatory, puddling, reheating, and all such
…ne on the surface of the brick; for though they are
…bed by solid matter. Bricks of this kind may be
…o to make them compact, they are not generally;
…e artificial sandstones, or fire-brick, are in many
…y are cheap where the materials are close at hand,
…ding and drying causes hardly any expense. An
… secured with remarkable facility, for the brick is
…idly, which causes the latter to dry quickly; this
…y of mortar; and as the mortar itself is but the
… work is done very cheaply. In this case, as in all
…e mixing of the clay and sand; too much labour
…In mixing plastic clay with sand, it is the object to
… particle of silex, and produce by that means a
…time adhesive, and free from friable spots. This
…d for forming boshes, and even hearths, in furnaces.
…what is the best, rammed down in a moist condition,

…quired, such as are made of slaty clay, or of kaolin,
… the Alleghanies, it is necessary that the materials
…y to the slate clay. The clay of the coal regions,
…xposed for some time to the atmosphere, under the
…ces; and when exposed for a season to frost and the
… converted into a fine meal, which is easily ground.
…se of the refractory quality of the clay, it is mixed
…by itself, and then mixed in due proportions. The
…ed by applying scientific principles; this must be
…e by mixing various quantities, and exposing them
… for these purposes must be taken either from pure
…ttoms. Sand obtained from pounded sandstone, or
… pure for fire-brick, or for retorts or crucibles. Clay

2983.



2984.



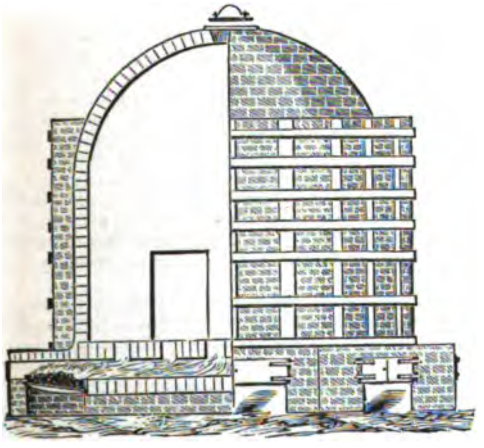
…forms an inverted cone, so as to admit of being
… this clay-mill is a vertical shaft, provided with
… of wood, but is better when made of iron; the

knives must be in all cases of iron. The latter are a little twisted, so as to cause the clay to move downward. The tempered clay is thrown in at the top, and the mill always kept full. At the lower end of the cylinder, close to the bottom, is a square hole, through which the clay is pressed, and issues continually. This square hole is provided with a gate, so as to regulate the quantity of clay which is permitted to pass. If the clay is not sufficiently mixed by passing it once through the mill, the process is repeated; in some cases this is required five or six times. In some instances the knives are provided with projecting points, so as to keep the clay in constant motion, as shown in the engraving; this may be advantageous, but it requires more power than plain knives, and a stronger machine than can be made of wood. This mill, of course, may be driven by horse-power, as shown, or by a water-wheel, or a steam-engine. When circumstances admit, it is advantageous to temper the clay when warm; this causes the air or gas in the pores of the clay to expand and escape, so that a close contact of the particles may be accomplished. It has been proposed to mix carbon, either in the form of graphite, or anthracite dust, or coke-dust, with the clay of which fire-bricks are to be made, but we are not aware that it has been put in practice to any extent. For crucibles, such a mixture is used; the black-lead pot is one of the kind, and the pots in which cast steel is melted are another kind; the latter are generally a composition of clay and coke-dust. For thin pots, and similar articles, we perceive no objection to coal, but in bricks and other heavy masses there are serious objections, which have been confirmed by experience. Coal, no matter in what form, causes always the formation of gas when in contact with oxides, such as clay and iron. If the substance is thin, such as a crucible, this gas may escape on the unglazed side; but if the mass is thick, it must escape at the hottest, or glazed, surface, and is the cause of a premature destruction of the fire-brick. Coal diminishes the shrinkage of clay, and thus far it is advantageous in the clay of crucibles, in preventing their fracture when in fire.

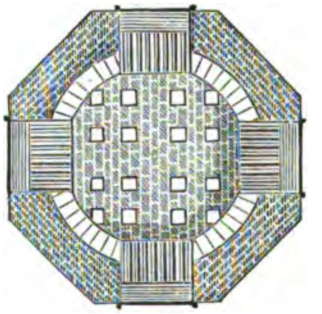
Fire-bricks are not generally manufactured from raw clay, at least not wholly of it; and there is no doubt but that a twice-burnt brick is superior to a brick made of fresh clay. The prepared and ground clay is subjected to one fire, either in the form of brick or in lumps, then ground and mixed with about one-third or one-fourth of fresh clay; this mixture is formed into bricks and baked. Some of our manufacturers do not follow this method, but there is no doubt, if their bricks are good now, they would be far better if baked twice. For this reason, brickbats, ground and mixed with a little fresh clay, will form a superior brick to the original brick made of raw clay.

Fire-bricks, in order to be baked, are generally subjected to a strong heat, in ovens built in a peculiar manner; this is not necessary if the bricks are not to be transported far, and if too much clay is not used in the mixture. In the latter case the brick is subject to much shrinkage, and when exposed to the heat in a furnace the joints between the various layers will separate and allow the heat to penetrate, which now acts on many sides and soon destroys it. All that kind of fire-proof material which must be transported, or in the composition of which a large amount of clay is necessary, must be baked; but those bricks which are manufactured and used on the spot, and which contain a large amount of silex, do not require baking previous to their use. In Fig. 2985 is represented a vertical section of an oven in which fire-bricks are baked. It is in appearance similar to a porcelain kiln, only not so large. The diameter is generally from 10 to 15 ft., and

2985.



2986.



equally as high, according to the quantity of bricks to be made. One cubic foot of space will contain eight bricks of 10 by 5 in. The capacity of an oven is thus easily calculated. One charge will take a week's time—three days for baking and three for cooling. The oven is built wholly of fire-brick, secured by iron tires and vertical binders. The floor is also formed by fire-brick with draft-holes or flues, as shown in Fig. 2986, wherein four fire-places are indicated. This oven may be operated by one or two fire-places, but there is no harm done in having more of them. The fire-places may be without grate-bars in case wood is used as fuel; but when stone-coal is burned there must be grate-bars, which are withdrawn and the stock-holes shut with ashes when the baking is finished. At the top of the oven is a round aperture of about 20 in. in diameter, through which the hot gases escape; when the heat is at the highest degree this top is shut by an iron plate. At the floor there is an entrance of 3 ft. in height and 2 ft. in width, through which the oven is set, or

...ricks, which are removed when the heat is finished ... are also discharged. There are various forms of ... represent those most frequently found and to all

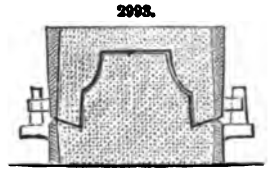
...rdinary plan of moulding with odd-side boxes, the ... imbedded partly in the sand of the top box, or in ...he bottom box is then placed upon it and rammed ...he boxes are then turned over, and the top box or ... in the bottom box; parting sand is then applied, ...ern still remaining between; the boxes are then ...m box turned over, and the pattern left upon the ... the sand of the two boxes has been completed, by ...ern, the top box is again placed upon the bottom

... first partially imbedded in the sand of the bottom ... parting surface been accurately formed, the top ... of Paris, or other similar material, to which the ...he boxes are turned over, the sand carefully taken ...ted with it, Fig. 2988, using clay wash to prevent ...s a corresponding plaster mould of the lower por-...ay be called the *waste blocks*, as they are not used ...sequently destroyed.

2989.

2990.

2993.



Finished Sand Mould.

... are now made from these waste blocks, Figs. 2987, ...g upon the bottom box a second top box, an exact ...p with plaster (having used clay wash as before), ...ed moulds are thus obtained, from which the final ...h as *ramming blocks*, upon which the sand forming ...ate top box, Fig. 2992, upon the ramming block, ... 2991, upon the ramming block, Fig. 2989.
...ned previously in the original sand mould, and are ...Figs. 2989, 2990, by corresponding projections or ribs ...he other (which are then stopped up with plaster), ...d moulds, Figs. 2991, 2992; these last, therefore, ...plete mould for casting, just like an ordinary sand

...de from the original ramming blocks by the simple ...e pattern or turning over the boxes, both top and ...d at the same time if desired. The parting being ...the succeeding ones are necessarily correct, without ...y trimming the original and slightly paring down ...e, the faces of the final sand moulds have a corre-... the first trial, to fit so closely together that prac-...bour of forming the gits and runners afresh for each ...letely imprinted upon each mould in the process of ...voided of imperfect castings arising from want of ... gits, &c., by the moulder in the ordinary process. ...ult castings, where several trials may be required ...ertained so as to ensure sound, good castings; and ...e plan is ensured, without requiring any further

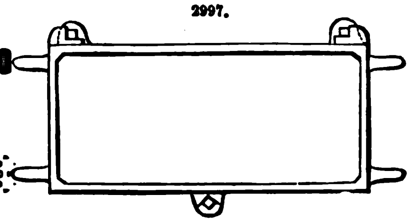
...g block for the top box, into which the plug for ...med, and by this means the git is ensured being ...he part of the moulder.
...mple and certain that ordinary labourers are quite

nothing to do but ramming the sand upon the
ht of the pattern, and putting them together
gits or runners; and also it is much easier to
n to pick out the pattern from the face of the
in one solid mass in the new plan, it can be
nd mould.
ricate (as in the case of an ornamental fender
nding, as in Figs. 2994 to 2996, the difficulty

2995.

Ramming Block for Bottom Mould.

2997.



Plan of Boxes.

t as to require the most skilful workman; and
of the mould, causes about eight sets of fender
r that can be moulded by each man and boy.
in the ordinary way (if it is arranged to draw
on the labour is very little greater than with
that as many as thirty a day are moulded on the
the number that the best moulders can produce

to be broken in the frequent handling to which
and the expense and delay caused by breakage
ental work, where the patterns are often very
voided, as the pattern is never handled at all
he ramming blocks.
rticularly well finished (as in the case of orna-
c, and is dressed up and finished to the degree
ng or other additional ornament put upon it;
box by a plaster-cast from the pattern in the
itself is made to form the permanent face of
by leaving it in the mould when the plaster is
ting face, and a solid back to the pattern. In
s of the box by several small bolts screwed up
plaster is poured in, filling up the whole vacant
ts and over these nuts, the iron pattern becomes
ving it is subjected to afterwards has any risk

ting is formed from the original metal pattern,
ecured in the plaster bed, so that however thin
e pattern in moulding any number of castings.
his article is taken, that as many as 3000 have
pattern.
er of Paris is generally employed, as the most
found to be sufficiently durable for general
e sand in the box, and do not fall directly upon
with ordinary care in ramming. When a greater
one pattern, or when the size or nature of the
is employed for the ramming block of the bottom
s. This is formed simply by running into the
tion of metal, consisting of zinc hardened with
form a strong plate for the surface of the ram-
filled with plaster as usual. In practice it is
running this metal for the face of the mould, by
r, full of sand, then lifting it off, and paring off
d to such depth (about ¼ of an inch) as may be
in its former position the metal is run in, fill-
ay. The sand in the upper box at the back of

the metal face is then all removed, without moving the box (part at a time if requisite) and plaster poured in above to fill up the box and make a solid back as before.

The metal face is firmly secured to the plaster back by several small dovetail blocks cast upon the back of the metal, by cutting out corresponding holes in the sand mould before the metal is run in. Various modifications of this plan of construction are employed, according to circumstances, for economy or convenience, and sometimes the face of the ramming block is partially covered by separate pieces of metal; but in every case the entire face of the two ramming blocks forms a perfect counterpart of the intended casting (half being represented upon each), surrounded by parting faces which exactly fit one another, because the one has been moulded from the other.

Where the pattern is long, and a metal face is employed, a narrow division is made, subdividing the metal face into two or more lengths, to allow for the shrinking of the metal forming the face, the effect of which is then found to be imperceptible. The plaster ramming blocks are varnished when dried, to preserve them from damp; and in moulding from them, the faces of the blocks are dusted with rosin, to prevent adhesion of the sand.

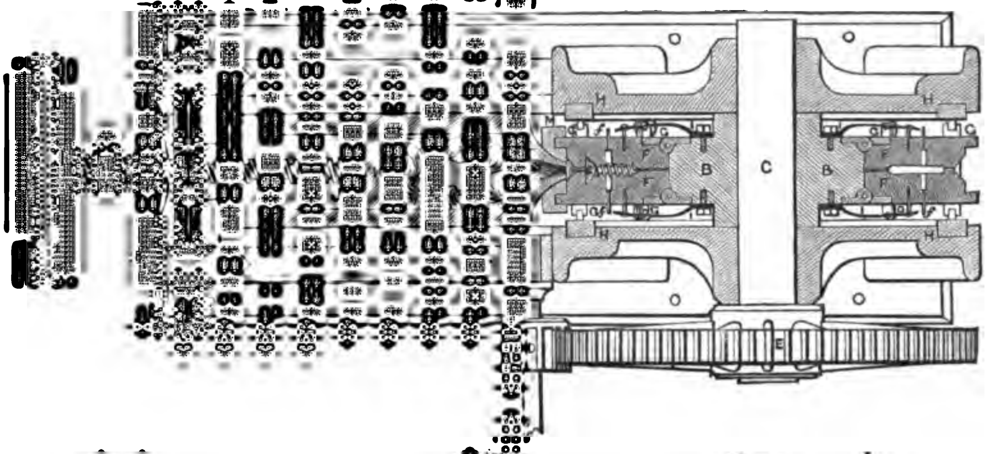
This process of producing blocks, though somewhat complicated in description, involves practically but little increase of work over the process of moulding required for the first casting produced by the ordinary method; but every subsequent casting, instead of requiring a repetition of the whole process of the first moulding, as in the ordinary method, is moulded by simply ramming the boxes upon their respective blocks. The ordinary odd-side boxes are used for this purpose, all that is requisite being that every top box fits steadily and securely upon every bottom box, so that they may be interchanged in the process of forming the ramming blocks, without disturbance of the relative position of the pattern. An improved form of the steady pins for connecting the top and bottom boxes has been adopted, as shown in Figs. 2993, 2997, which is easier to construct with accuracy. Instead of four or more round pins fixed on the bottom box, and fitting into corresponding holes in lugs cast upon the top box, vertical angular studs are cast on each bottom box, and fit against corresponding projections on the edge of the top box (as shown in the plan, Fig. 2997, and the section, Fig. 2993); the only fitting required in making the boxes is to file the touching angles of the pins so as to fit one standard top box, and the projections on the top boxes to be all fitted to one standard bottom box.

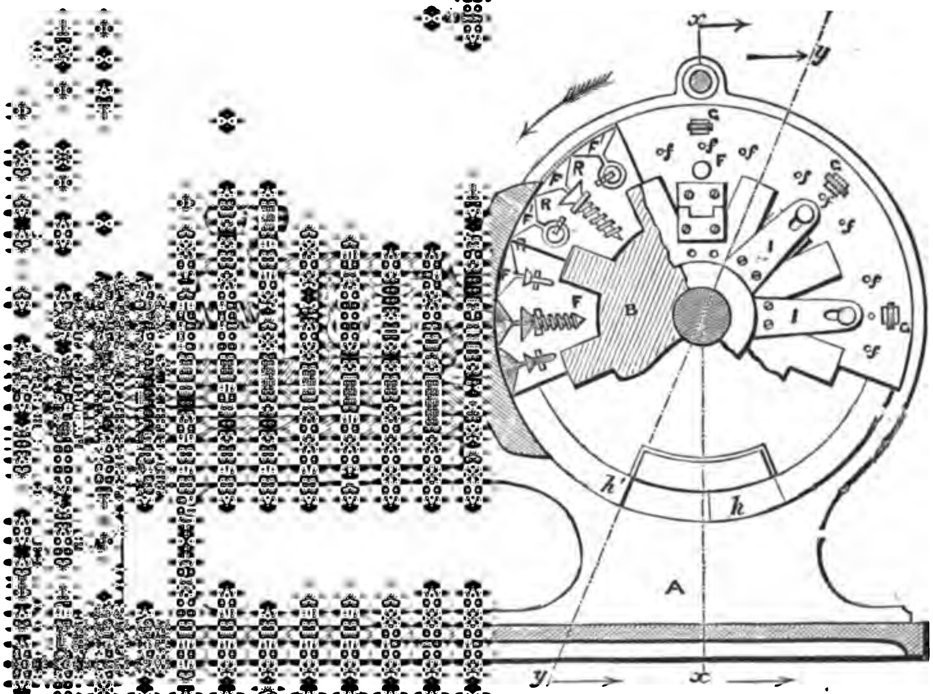
It has to be noticed that in the ordinary plan of moulding, and by the odd-side and plate methods, one side of a pattern is not available while the other is in use; by the process of Jobson each pattern is equal to two, as it will be evident that both blocks may be worked from at the same time.

Casting Metals under Pressure, Smith and Locke's process for.—The apparatus which is here selected for illustration, but to which the invention is not necessarily restricted, consists of a rotary wheel or cylinder, Fig. 2999, on the periphery of which are arranged a series of moulds, each formed of a pair of hinged metallic plates, which while the casting is being performed are held firmly together between stationary housings, suitable rollers being interposed between the rotary mould-plate and the stationary housings to reduce the friction. Each of the mould-plates is provided with one or more springs, which, as the rotation brings the moulds successively opposite recesses in the housings, cause the mould-plates to separate with a sudden movement, causing a jar which effectually detaches and discharges the casting. This effect is assisted by pins which are driven inward as the mould-plate is separated, and in their retracted position form parts of the mould. The continued rotation of the wheel carries the mould-plates between converging planes, which gradually close the moulds and conduct their rollers between the parallel parts of the housings by which the plates are held to receive the molten metal which is injected into the moulds from a cylinder by a sliding piston or plunger. A detachable non-conducting lining is applied to the interior of the cylinder, K L, each time before it is filled, and serves the combined purposes of preventing the molten metal setting or adhering to the metallic cylinder, and effectually packing the joint between the moving piston and the cylinder. From this reservoir the metal is forced through a contracted aperture directly into the moulds, the gates of the latter being funnel-shaped, so as to cause no interruption to the passage of the metal. A continuous pressure is applied to the metal within the cylinder during the casting operation, and the partitions between the moulds being tapered to an edge cause no perceptible interruption to the flow as they pass the discharge orifice of the reservoir. The reservoir is mounted upon a sliding bed provided at its forward end with a segmental plate or standard which fits around the periphery of the cylinder, so as to tightly close the gates of the moulds, and serves also as a mouthpiece for the injecting cylinder. The sliding bed is held up to the cylinder by a screw and spring, the latter permitting it to yield, so as to avoid danger of breakage in the event of any hard matters getting between the mouthpiece-plate and the cylinder. The injecting and pressing apparatus may be used with equally good effect in connection with stationary moulds, the said moulds being arranged either separately or in any number together, and placed either horizontally or vertically; or in cases where it is desirable to cast a number of small articles at one time, any number of moulds, therefore, may be placed together within a single flask or casing, and a single injecting reservoir may be used for all. The molten metal may be made to enter the moulds horizontally, or may be forced upward from the lower part, or introduced at top, as convenience or various circumstances may dictate. For casting a large number of small articles simultaneously, the stationary moulds are preferably arranged in a vertical nest or series, and the metal injected at bottom, the separate moulds or parts of moulds being provided with suitable grooves to form, when united, gates or sprues for the admission of the fluid metal. A cluster or connected series of these moulds being placed together in their flask or casing are supported and firmly compressed by a plate, which is forced against the moulds by set screws tapped into the top, bottom, or side of the flask, in order that the moulds may resist the pressure of the metal when injected, and not part or open at the joints, and thereby cause the formation of fins or seams in casting. The entire flask may then be enclosed in a tight box from which the air can be exhausted by an air-pump of large capacity,

...lask and in the body of the moulds to permit ...r. Whenever the air and gas exhausting pro... ...he outer chest are all tightly closed, and the ... luted with wet clay or other material to make

...ther metals may be used to good advantage, ...ch as is represented in Figs. 2998 to 3000.





...s chill and become set causes difficulty in the ... may be heated prior to the introduction of the ... time for applying pressure to the metal, unless ... we here describe, and even with the improved ... on the surface of the metal, which with elabo... ...d, seamed, or otherwise defective appearance. ...vent to a certain extent the sudden chilling, the ... the alloys of copper and tin; and even if metal ... of making them would forbid their use in cases ... of one and the same pattern, and in some cases

ize of the flasks or mould-boxes or other circum-
moulds of a composition which will combine the
tibility to heat; second, sufficient strength, den-
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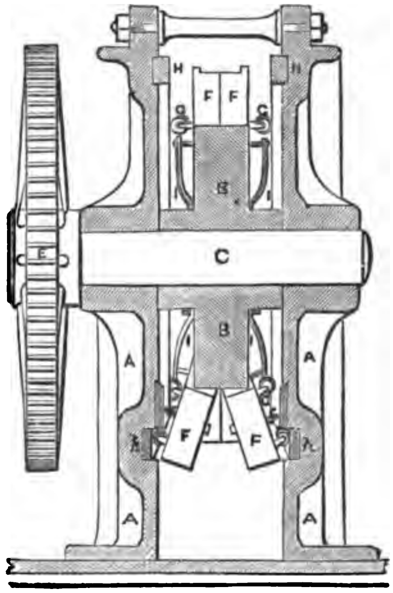


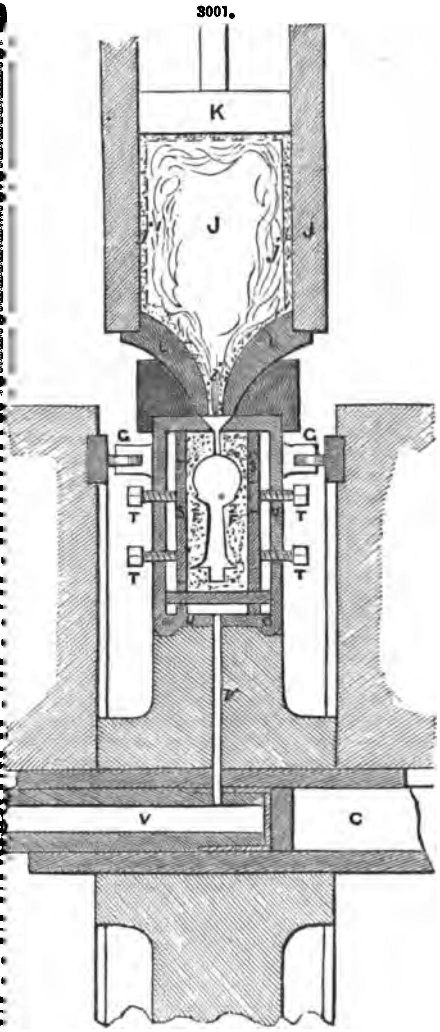
r, or follower, and the flask is filled with the moist
plunger or follower is placed in position, and the
cted to a slowly-applied but high pressure, say
y packing and pressing the loose powder into a
ure is applied it is allowed to stand under the
ture or water in the slip may be absorbed by the
to unite the loose particles of the clay powder in
ogeneous mass. The flask may then be removed
one, the pattern is easily taken out, and a mould
wax or gypsum. If a mould is to be made con-
coated on the surface coming in contact with the
coat of collodion or thin shellac varnish, so as to
they may be separated without difficulty. When
e flask or frame and handled and carried without
dered dry by exposure to the influence of air or a
rated, the moulds are placed into furnaces similar
t gradually raised until the moulds are red hot,
ck or clay-stone. When cooled off they are ready
oulds in the same shape as when first made, and
warp during the process of drying and burning,
e, for the prevention of these evils it is found best
r new clay, another half of clay powder made of
ay moulds which have been used for casting, the
der is very beneficial, as it prevents the warping
large size.

ratus; Fig. 2999 is a longitudinal section thereof,
part of it in section, and part entirely removed;
ionary parts being represented in the plane under-
el in the plane of *y*, *y*, Fig. 2999; Fig. 3001 is a
e, illustrating a mode of withdrawing air and gas
al.

y frame B is a wheel or cylinder attached to a
means: F, F, are mould-plates hinged in pairs
that each pair will join, when closed, a matrix to
f, represent pins employed to aid in the detach-
. as previously explained. The inner ends of the
moulds when the latter are closed. The outer
with friction rollers G which bear against the
ed condition to receive the metal; *h*, *h*, represent
be opened by the springs I, I, when the castings

... to permit the moulds to open suddenly, and ...ly up to the parallel parts of the housings H. ...er J provided with a piston K, which may be ... the metal out through the detachable thimble

3001.



... and set more suddenly than those who are not ...n if those surfaces should have been previously ...hat the casting will show streaks and an uneven ...e injecting cylinder (avoiding the passing of the ... the strong pressure is brought to bear on the ...t has had time to form a hard skin or surface, a ...ron and steel moulds used in this manner ought ...tent. These rusted or oxidized surfaces are some... ...adhering of the injected metal to the moulds. ...it in a metal box or flask, and an iron follower ...ether by means of the set screws, as illustrated ... be, so as to resist the pressure of the injected ...s done the cylinder or injecting vessel J is pro...

...ws;—Take some fine fire-proof clay or *kaolin* free ...h about one-half the quantity of good plumbago; ... cylinder is heated to about 200° of Fahrenheit, ... the inside of the cylinder to a uniform thickness ...er drying the paste as applied. This done, the ...te from the lining, and is then ready for further

operation. The surface of the piston K coming in contact with the fluid metal is coated in a similar manner. This lining of the cylinder serves several useful purposes; it serves to pack the space or joint between the cylinder and piston, and prevents the intrusion of the metal between the two, which intrusion would certainly and rapidly clog their parts and stop the operation, as the heat of the fluid metal will at once expand the cylinder more than the piston, and admit the metal between them at the joint. It should be explained that the plunger or piston K fits the cylinder, and is therefore larger than the internal diameter of the lining, and that at every operation the cylinder must be relined or recoated, as such lining is detached from the cylinder J, and pushed in advance of the piston K at every forward movement of the latter, the effect of which is to close the joint, and prevent the intrusion of the metal between the piston and side of the cylinder, as above stated. No matter how high the pressure is on the fluid metal, not a drop can leak out, the detached lining will pack the joint so much the more closely. The importance of this detachable lining can scarcely be over-estimated, since it forms by its non-conducting property the only practicable means for preventing the chilling or rapid setting, and also the adhering of the molten metal which would arrest the forward motion of the piston, and preclude any successful operation. Those acquainted with the nature of refractory metals know how rapidly they chill and set if they come in contact with another metal surface, even if said surface is made red hot, and it requires several minutes' time to fill the cylinder and perform the operation of injecting. This lining serves also to prevent the direct contact of the fluid metal with the surface of the cylinder, which would soon spoil the cylinder by the excessive heat of a mass of molten metal, even if the other aforesaid advantages were not of such great importance. The nozzle or thimble L (if made of metal) is lined in the same manner as the cylinder itself. The discharge orifice of the said nozzle is then stopped with a clay plug or tamp *l* capable of resisting a pressure of 6 or 8 lbs. to the square inch. The office of the plug or tamp *l* is to prevent the gradual passage of the metal into the moulds. All being in readiness, and the cylinder J charged with molten metal in quantity somewhat in excess of the capacity of the moulds to be filled and placed in proper position, a pressure of from 30 to 60 lbs. to the square inch, and sometimes more, may be applied to the metal by either a screw, lever, or other means; but experience will soon indicate to the practical operator the proper amount of pressure for various kinds of casting. When pressure is applied to the metal the plug will yield and pass along in the main gate, giving the metal free passage into the moulds. When the casting is performed and the metal is set, the cylinder is readily detached by breaking the metal in the gate at the junction of the thimble and the moulds. This ought to be done immediately after the casting is done, and before the metal in the gate has acquired its full strength by cooling off. The remaining head of metal in the cylinder is easily removed after cooling.

The operation of the rotary apparatus, represented in Figs. 2998 to 3000, may be described as follows;—Any number of the reservoirs J, K, L, may be prepared for use by coating their inner surfaces with a suitable non-conducting paste, and filling them with molten metal, the nozzle L being closed with a clay plug to prevent the escape of metal. The cylinder J being placed in the position shown in the drawings, the wheel B is set in motion, and then pressure is applied to the piston K so as to cause a continuous discharge of molten metal through the nozzle L. The small clay plug or stopper in the nozzle being driven out with the first discharge of metal may pass into one of the moulds and cause the production of a single imperfect casting, but after this a continuous jet of pure molten metal is kept up. The metal is thus injected and compressed into each mould as the rapid rotation of the wheel carries it in front of the nozzle. The guard-plate M prevents the escape of any of the metal until it has had time to become set, and as the bearing rollers G of each mould reach the shoulders h^1 of the recesses *h*, the springs I cause the mould-plates to separate instantaneously with a concussion which discharges the casting from the mould, or if the jar should be insufficient the driving of the pins *f* inwards through the mould-plates ensures the detachment of the casting.

The continued rotation of the wheel carries the rollers up the converging faces of the recesses *h* until they pass between the parallel faces of the housings H so as to effect the tight reclosure of the moulds in readiness for filling. For casting articles of larger weight than a quarter of a pound the motion of the wheel may preferably be intermittent instead of continuous. Where, from the character or size of the castings to be produced, the particular metal or alloy used therein, or other circumstances, it is found desirable to employ moulds of the clay composition in connection with the rotary apparatus, as illustrated in Fig. 3001, the said composition moulds may be arranged and secured in the several flasks or mould-chambers completely around the periphery of the wheel before the casting operation begins. The injecting reservoir J being then charged and set in position, and pressure applied to the piston K, a number of large compressed castings may be produced by a single revolution of the wheel, or in some cases it will be practicable to employ the composition moulds in continuous or repeated operation in the manner first described. While selecting to illustrate various parts of the invention the preferred styles or types thereof, it is not proposed to restrict it thereto so long as the same results are obtained by means substantially equivalent. For an example, the pressure of steam or condensed air may be applied to act on the piston, or steam or condensed air may even be used to exercise direct pressure on the fluid metal if the cylinder is in vertical position and the end of the cylinder closed, although practical experience has proved that the mechanical pressure is the most reliable and least complicated. It is also proposed in some cases to use a form of double flask, or a flask with two or more chambers connected by suitable gates, so that the molten metal may be placed in one and the moulds in the other or others, and the compression casting can be performed by forcing the metal out of the first chamber or the reservoir into the moulds, which are arranged and secured in the other chambers.

Another modification consists in the employment of a horizontal reservoir with its discharge aperture at or near the highest part, as the metal need not entirely fill the reservoir; when the piston is retracted no metal will be discharged until the piston moves, even if the plug or tamp be dispensed with, but to prevent cooling and oxidation it is preferable to use the plug in this case

also. As a substitute for the clay plug or tamp a sliding rod or other form of valve may be employed, being adjusted to resist any pressure below a certain degree, and where the pressure exceeds that degree to yield and open the way between the reservoir and moulds.

It will be apparent that some of the principal parts of the invention, as for example the injecting process and apparatus, the material for the moulds, the manner of securing the moulds within the flask, and the exhausting process are not in any manner restricted in their use to the rotary apparatus which has been more particularly described in connection with Figs. 2998 to 3000, but may be used equally well with stationary or detached flasks. In practice it is rarely found necessary to employ the atmospheric exhaustion in connection with the compression of the metal, but its use is found advantageous in cases where it is especially necessary to produce castings of the most compact and solid character with entire freedom from blow-holes throughout. In these cases it is preferable to use the stationary rather than the rotary moulds, so that the exhaustion may be effected through a simple pipe connected to the air-tight chest within which the flask is enclosed. When the injecting reservoir is placed in a vertical position the use of a thimble L separate from the reservoir J, facilitates the removal of the reservoir by slipping it off vertically when the metal has become set. The superincumbent mass of metal remaining in the reservoir might render this difficult or impossible if the lower end of the reservoir itself were contracted or formed with an inwardly projecting flange or shoulder. If said thimble is made of iron or other metal it ought to be divided longitudinally into two parts, so that the same may be readily detached from the metal in the central gate, and the head left in the cylinder after casting. Continued practice has proved that the best manner of making said thimble is to form or make it by pressing moist clay into a suitable mould, and after forming to burn it hard in the manner that brick or pottery is burnt, and to use a new thimble at every operation of casting. Such mineral thimbles are strong enough, cheaply made, and easily detached by breaking them in pieces after casting. Practical founders know that if a mould could be used of a material more dense than sand or sand loam, a more perfect and sharp mould could be made, and there would not be so great liability of the sharp lines of the mould being washed by the inflowing metal; but they are also aware that if a mould is made so dense by ramming and stamping, or by the use of dense material, the gases generated and the air in the moulds will not be ejected by the mere pressure obtained by the weight of the metal itself; the consequence is that the metal is blown out of the mould and the casting is useless. The employment of the compression process applied to refractory metals, just described, enables the use of moulds of great density and of materials which will take a very fine and sharp impression of the patterns by great pressure applied to the material formed into moulds. By providing separate injecting vessels for the reception of the molten metal previous to its introduction into the mould, and having a high pressure to bear on the fluid metal in the very act of its rapid filling of the moulds, the elimination of the gases generated and the air in the moulds is compelled to take place through very fine pores and the small orifices made at the joints of the moulds; and as the pressure is kept on the metal until it is well set and solid, such a thing as blowing will never, or very seldom take place, but the metal is compelled to fill every cavity of the mould, producing a perfect, sharply defined, and smooth surface.

Moulds made of clay composition are especially well adapted to be used with this compressing casting apparatus. Moulds to be used for another more simple method of casting metal under pressure, but applicable only to the casting of larger pieces of one face, are made in the same manner, only it is preferable to use instead of ordinary clay the clay composition used for black-lead crucibles or for good fire-tiles. The only necessary difference in the manner of forming the mould is as follows;—The iron box or frame is first filled with clay powder, and the pattern is then placed with face down. The reason for doing so is that the clay mould is left in the iron box and the bottom of the box is not removable, therefore the pattern must be placed on the top of the powder that it may be taken out after the impression is produced. The iron box or flask used for the clay mould ought to be of the outside size and shape of the die or mould to be cast in metal, and the bottom ought to be provided with a hole or slot for the purpose presently to be seen. When the pattern is removed the clay mould is made dry; after drying and while still in the iron box it is placed in a muffle furnace, and the heat gradually raised until the mould and iron box are brought to a temperature nearly equal to that of the metal to be cast in it; while this is done the metal to be used is melted in another furnace. Then the clay mould is taken out of the furnace and filled with so much of the fluid metal as is necessary to make the articles, allowing a small surplus to be acted on by the follower. The surplus of metal is kept from overflowing by an iron collar or frame which extends above the clay moulds. When so far filled the follower is put on top of the fluid metal and then the whole is placed under a screw or lever press. During the time required to perform the filling of the mould and placing under the press, the fluid metal becomes mushy, or semi-fluid, and if pressure is applied the metal will take a perfectly sharp impression of the mould. When the whole is cooled off, the casting is removed out of the iron collar or box by driving something through the hole left for that purpose in the bottom of the box. The alloys of copper and tin are most favourable for this process of casting, but it may also be employed for other refractory metals and metal compositions with good success. The use of clay in this method of producing moulds for dies and other single-faced castings greatly reduces the care and skill required and the risk of loss involved in making such castings. In forming metal moulds with metal patterns, there is great danger of the injury or destruction of the pattern to be copied by the molten metal adhering to it. It is well known that moulds have been made of clay when in a plastic state for different purposes, and also that clay mixed with sand is employed for making moulds to be used for casting large bells and other heavy bronze and brass castings so as to give strength to the sand that it may resist the pressure of a great mass of metal. Now, those acquainted with the art of forming moulds and other things of plastic clay, know that it is very difficult and almost impossible to press plastic clay into deep cavities of a pattern, especially if the pattern is made of metal or other material of great density, because the air will always be more or less confined

in the cavities and prevent the admission of the clay to produce a sharp and perfect impression. Another disadvantage, and that not the least, is that clay used in the plastic kneadable state will shrink very much, thereby reducing the size and changing the form of the article to be moulded; further, an article formed or moulded of plastic clay is very liable to warp and get out of shape by handling, and during the process of drying. But by using a moist clay powder and forming it into a compact mass by high pressure, those difficulties are entirely obviated; by using first a coating of slip applied over the pattern with a brush, the clay is brought into every cavity of the pattern at the commencement. Then the moist clay powder, being first in a loose state, attaining its strength and hardness gradually by the pressure slowly applied, permits of the easy escape of the air from the cavities of the pattern and throughout the whole mass, and produces a perfect and sharp impression. Further, by using moist clay powder, and making it a compact block by means of high pressure, the mould is, when formed comparatively to a mould made of plastic clay, in a greatly advanced state of dryness, may be handled and carried about without getting out of shape, and as it contains not one-fourth part as much water as plastic clay it is almost entirely relieved from shrinkage and adapted to retain the shape and size first given in making. Further, by mixing with fresh new clay powder one-half of powder made of burnt clay the mould is relieved from liability to warp and crack during the process of drying and burning. The application of slip made of clay as a facing closes the pores and unites the clay powder into a fine and plastic mass where it comes in direct contact with the pattern. The slip is rendered quickly plastic by absorption of the surplus of water it contains by the comparatively dry clay powder in the rear. Now, as the application of slowly applied pressure is believed to be new in this connection, it is proper to explain its importance. If an attempt is made to form the loose moist clay powder into a compact and solid mass, as a good mould must be by means of stamping and ramming as it is done in the ordinary way of moulding, it would naturally take a great deal of labour, but after removing the pattern the mould would be of insufficient and unequal compactness and imperfect impression; and further, when the mould was removed from the flask it would not properly hold together in burning, but fall to pieces as if the mould had been made of a number of layers of clay. If high pressure should be applied by means of machinery in a sudden manner, or with a momentum, the same results would occur. But, if pressure is applied slowly and gradually, the loose particles of the clay powder are gradually and firmly united into a solid compact mass of uniform density entering perfectly and sharply into every cavity of the pattern. The labour required to form a mould in the manner described is but little more than in forming a mould in the ordinary way of moulding.

Brass moulding is carried on by means of earthen or sand moulds. The formation of sand moulds is by no means so simple an affair as it would at first sight appear to be, as it requires long practical experience to overcome the disadvantages attendant upon the material used. The moulds must be sufficiently strong to withstand the action of the fluid metal perfectly, and, at the same time, must be so far pervious to the air as to permit of the egress of the gases formed by the action of the metal on the sand. If the material were perfectly air-tight, then damage would ensue from the pressure arising from the rapidity of the generation of the gases, which would spoil the effect of the casting, and probably do serious injury to the operator. If the gases are locked up within the mould, the general result is what moulders term a *blown* casting; that is, its surface becomes filled with bubbles of air, rendering its texture porous and weak, besides injuring its appearance.

Plaster of Paris is often used for a number of the more fusible metals. This material, however, will not answer for the more refractory ones, as the heat causes it to crumble away and lose its shape. Sand, mixed with clay or loam, possesses advantages not to be found in gypsum, and is consequently used in place of it, for brass and other alloys. In the formation of brass moulds, old damp sand is principally used in preference to the fresh material, being much less adhesive, and allowing the patterns to leave the moulds easier and cleaner. Meal dust or flour is used for facing the moulds of small articles; but for larger works, powdered chalk, wood-ashes, and so on, are used, as being more economical. If particularly fine work is required, a *facing* of *charcoal* or *rottenstone* is applied. Another plan for giving a fine surface is to dry the moulds over a slow fire of *cork shavings*, or other carbonaceous substance, which deposits a fine thin coating of carbon. *This*, when good fine facing-sand is not to be obtained. As regards the proportions of sand and loam used in the formation of the moulds, it is to be remarked that the greater the quantity of the former material, the more easily will the gases escape, and the less likelihood is there of a failure of the casting; on the other hand, if the latter substance predominates, the impression of the pattern will be better, but a far greater liability of injury to the casting will be incurred from the impermeable nature of the moulding material. This, however, may be got over without the slightest risk, by well drying the mould prior to casting, as you would have to do were the mould entirely of loam.

For some works, where easily fusible metal is used, metallic moulds are adopted. Thus, where great quantities of one particular species of casting is required, the metallic mould is cheaper, easier of management, and possesses the advantage of producing any number of exactly similar copies. The simplest example which we can adduce is the casting of bullets. These are cast in moulds constructed like scissors, or pliers, the jaws or nipping portions being each hollowed out hemispherically, so that when closed a complete hollow sphere is formed, having a small aperture leading into the centre of the division line, by which the molten lead is poured in.

Pewter pots, inkstands, printing types, and various other articles, composed of the easily fusible metals, or their compounds, are moulded on the same principle. The pewterer generally uses brass moulds: they are heated previous to pouring in the metal. In order to cause the casting to leave the mould easier, as well as to give a finer face to the article, the mould is brushed thinly over with red ochre and white of an egg; in some cases, a thin film of oil is used instead. Many of the moulds for this purpose are extremely complex, and, being made in several pieces, they require great care in fitting. With these peculiar cases we have, at present, little to do and,

we therefore shall conclude with a few observations on the method of filling the moulds. The experienced find that the proper time for pouring the metal is indicated by the wasting of the zinc, which gives off a lambent flame from the surface of the melted metal. The moment this is observed, the crucible is to be removed from the fire, in order to avoid incurring a great waste of this volatile substance. The metal is then to be immediately poured. The best temperature for pouring is that at which it will take the sharpest impression and yet cool quickly. If the metal is very hot, and remains long in contact with the mould, what is called *sand-burning* takes place, and the face of the casting is injured. The founder, then, must rely on his own judgment as to what is the lowest heat at which good, sharp impressions will be produced. As a rule, the smallest and thinnest castings must be cast the first in a pouring, as the metal cools quickest in such cases, while the reverse holds good with regard to larger ones.

Complex objects, when inflammable, are occasionally moulded in brass, and some other of the fusible metals, by an extremely ingenious process; rendering what otherwise would be a difficult problem a comparatively easy matter. The mould, which it must be understood is to be composed of some inflammable material, is to be placed in the sand-flask, and the moulding sand filled in gradually until the box is filled up. When dry, the whole is placed in an oven sufficiently hot to reduce the mould to ashes, which are easily removed from their hollow, when the metal may be poured in. In this way small animals, birds, or vegetables may be cast with the greatest facility. The animal is to be placed in the empty moulding box, being held in the exact position required by suitable wires or strings, which may be burnt or removed previous to pouring in the metal.

Another mode which appears to be founded on the same principle, answers perfectly well when the original model is moulded in wax. The model is placed in the moulding box in the manner detailed in the last process, having an additional piece of wax to represent the runner for the metal. The composition here used for moulding is similar to that employed by statue founders in forming the cores for *statues, busts*, and so on, namely, two parts brick-dust to one of plaster of Paris. This is mixed with water, and poured in so as to surround the model well. The whole is then slowly dried, and when the mould is sufficiently hardened to withstand the effects of the molten wax, it is warmed, in order to liquefy and pour it out. When clear of the wax, the mould is dried and buried in sand, in order to sustain it against the action of the fluid metal.

We shall examine one or two cases which come more or less within the province of the engineer. One of these is the founding of bells, a subject of much interest, as works of this kind are often of very considerable magnitude, and demand the skilful attention of the engineer. Large bells are usually cast in loam moulds, being *swept* up, according to the founder's phraseology, by means of wooden or metal patterns, whose contour is an exact representation of the inner and outer surfaces of the intended bell. Sometimes, indeed, the whole exterior of the bell is moulded in wax, which serves as a model to form the impression in the sand, the wax being melted out previous to pouring in the metal. This plan is rarely pursued, and is only feasible when the casting is small. The inscriptions, ornaments, scrolls, and so on, usually found on bells, are put on the clay mould separately, being moulded in wax or clay, and stuck on while soft. The same plan is pursued with regard to the ears, or supporting lugs, by which the bell is hung.

Brass Guns are another important branch of this manufacture. They are moulded in a manner quite distinct from any other work of this nature. The exterior surface of the gun is produced by wrapping gaskin or soft rope round a tapered rod, of a length slightly greater than that of the gun. Upon this foundation of rope the moulding loam is then applied; the surface being turned to the exact shape and proportions of the gun. A long fire is used by the founder in this process, in order to dry the mould as he proceeds in its manufacture. When perfectly dry, the surface of the mould is black-washed over, and again covered with loam to a depth of 2 or 3 in. This exterior coat of loam is secured and strengthened by a number of iron bands, and the whole is well dried. The primary mould is now completely withdrawn from the outer shell, the formation of which renders it an easy matter, as the timber rod leaves the rope with great facility, when the latter may be withdrawn, and the clay covering picked out afterwards.

The trunnions of the gun are formed separately, and attached to the shell in the ordinary way. When finished, the moulds are sunk perpendicularly in a sand-pit, near a reverberatory furnace, a vertical runner being made, leading to each mould, which it enters near the bottom. A suitable channel communicates with the furnace containing the brass intended for the guns. The metal being introduced at the bottom of the mould, no air can possibly be detained by its entrance, as each mould is full open to the atmosphere at the top.

T. V. Morgan's apparatus, or machine for making large and small crucibles, is illustrated by Figs. 3002 to 3004. This important and peculiar mechanical arrangement consists in fitting the "former," or forming tool employed in the apparatus, Fig. 3002, so that in addition to being capable of an up-and-down movement, the former is free to be moved and adjusted horizontally as the crucible is being moulded, and according to the required size or thickness of the crucible; it also consists in the employment of a lever to prevent all vibration or movement of the former or shaper, when at its final position in the crucible.

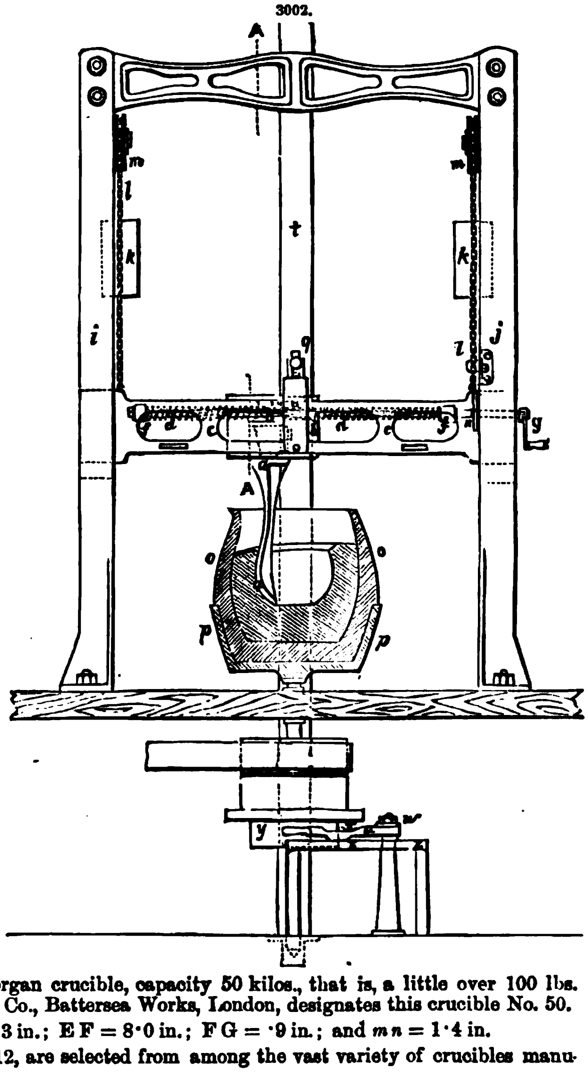
The forming tool is fitted to a block free to be moved horizontally in a frame by means of a horizontally threaded rod, which takes into a corresponding female thread in the block; the ends of this rod work in fixed nuts on the frame, and one end is provided with a handle which is turned according as the former and its block are required to be moved. The frame before mentioned is free to move up and down in slots formed in two uprights, and its weight is counterbalanced. The bottom of the slots limits the distance to which the frame with the former can be lowered. When a crucible is to be made the frame is pulled down to cause the former to enter the plastic material, which is placed in a mould, on a revolving lathe, or *jigger*, as usual, and when the former reaches the bottom of its course, a catch on one of the uprights secures the frame in position. The threaded rod is then turned, to cause the former to move horizontally, and spread the plastic material against the side of the mould. Finally, the back end of a lever carried on the top of the frame,

and free to move backward by means of slot or otherwise, is inserted into a hole formed for the purpose, and its forward end is pressed down by hand, so that the lever bears forcibly upon the frame, and prevents all vibration or movement of the former. When the crucible is finished, the handle is turned to bring the former to the centre of the crucible, the lever is moved forward out of its hole, the catch released, and the frame raised up by a balance-weight. The operation is then repeated for the next crucible, and so on.

This invention further consists in the employment of a brake to stop the revolution of the lathe or *jigger*, when the driving belt is moved from the fast to the loose pulley of the lathe-shaft. This brake is preferably composed of a horizontal bar hinged behind the apparatus, while one end extends to the front of the apparatus, near to the attendant. The bar carries a block, and when the brake is to be applied, the attendant, by his foot or otherwise, moves the bar on its hinge, so as to cause the block to bear against a collar or other revolving portion of the lathe.

Fig. 3002 is a front elevation; Fig. 3003 a side elevation; and Fig. 3004 a section through the line A A of Fig. 3002, of an apparatus constructed according to Morgan's improvements. *a* is the former, or forming tool; it is fitted to a block *b*, which is, as before stated, free to be moved horizontally in a frame *c* by means of a horizontal threaded rod *d*, taking into a corresponding thread *e* in a nut b^1 in the block *b*; the ends of the rod *d* work in fixed nuts *f f*, on the frame *c*, and the right-hand end is provided with a handle *g*, which is turned according as the former *a* and block *b* are required to be moved. The frame *c* is free to move up and down in slots *h h*, formed in two uprights *i*, *j*, and its weight is counterbalanced by weights *k k*, on the end of chains or cords *l l*, passed over pulleys *m m*, and connected to the frame *c*. *n* is a catch on the upright *j*, to secure the frame *c* in position when the former *a* reaches its lowest position. *o* is the mould into which the plastic material is fed; this mould is carried on an ordinary lathe or jigger *p*, to which rotary motion is imparted as usual. When the frame *c* is caught by the catch *n*, and the mould is caused to rotate, the threaded rod *d* is turned by its handle *g*, so as to cause the former *a* to move horizontally, and spread the plastic material against the side of the mould *o* and when it has been moved to the required distance, which is regulated by a scale on the frame *c*, the back end of a lever *q* carried on the top of the frame *c* and free to move backward by means of a slot *r* is inserted into a hole *s* formed in an upright *t*, and its forward end is then pressed down by the attendant so that this lever bears forcibly upon the frame *c* and prevents vibration or movement of the former *a*. When the crucible is finished, the handle *g* is turned to bring the former *a* to the centre of the crucible, the lever *q* is moved forward out o its hole *s*, the catch *n* is released, the frame *c* is raised up, and the mould is removed in the ordinary manner; all being then ready for the next operation. *u* is a horizontal bar under the platform *v* and hinged at *w*, while its front end extends to the front of the apparatus. *x* is a block on the bar *u*, and *y* is a collar on the lathe-shaft. When it is required to stop the revolution of the lathe, the attendant moves the bar *u* on its hinge *w*, so as to bring the block *x* against the collar *y*. *z* is a horizontal bar or guide for the bar *u*.

3002.



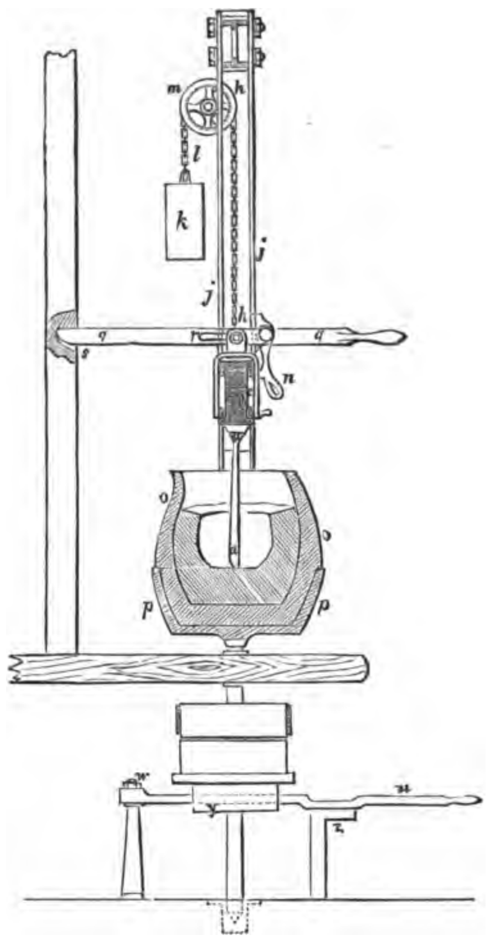
The Morgan steel crucible, Fig. 3005, so highly valued, is made of about 1 part fire-clay, and 2 of graphite or plumbago. This paste is worked to great perfection by the machine, Figs. 3002 to 3004. During the burning the Morgan crucible undergoes no change internally, as only the surface of the graphite burns.

Fig. 3006 is a section of a Morgan crucible, capacity 50 kilos., that is, a little over 100 lbs. English. The Plumbago Crucible Co., Battersea Works, London, designates this crucible No. 50.

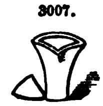
A B = 11·1 in.; B C = 1·3 in.; E F = 8·0 in.; F G = ·9 in.; and *m n* = 1·4 in.

The crucibles, Figs. 3005 to 3012, are selected from among the vast variety of crucibles manu-

3004.



3007.

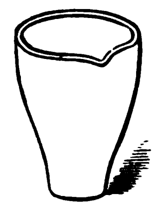


London clay crucible for refining gold.

3009.

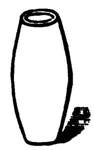
Scarifier

3010.



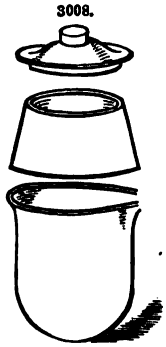
London clay crucible for refining gold.

3012.



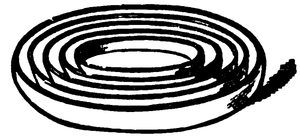
Skittle pot for refining jeweller's sweep.

3008.



Crucible, cover, and whiffle for melting silver.

3011.



Roasting dishes.

factured at the Battersea Works. The crucibles of this company have been in most successful use for many years, and are now used exclusively by the English, Australian, and Indian mints; the French, Russian, and other Continental mints; the royal arsenals of Woolwich, Brest, and Toulon; and have been adopted by most of the large engineers, brass-founders, and refiners in England. Their great superiority consists in their capability of melting on an average forty pourings of the most difficult metals, and a still greater number of those of an ordinary character, some of them having actually reached the extraordinary number of ninety-six meltings. (See Crucibles.)

These crucibles never crack; become heated much more rapidly than any other description, and require only one annealing—may be used any number of times without further trouble, change of temperature having no effect on them. Mons. C. Dierick, master of the French mint, writes:—"Each crucible runs from forty to sixty pourings, and can with safety be dipped in cold water when at a red heat, and used again immediately, as if it had not undergone any change of temperature." A large amount of time is daily saved at starting, other crucibles requiring to be annealed every morning before using, whilst these, although lasting a very considerable number of heats, *only* require to be annealed *once;* the *metal* is also fused much more rapidly, saving *time, fuel, labour,* and *waste;* the saving also of *metal* is very great, as to each worn crucible there adheres a certain amount of metal—the commoner the crucible the greater the absorption and adhesion. In this respect, comparing the Morgan plumbago with the common crucible, the saving of metal and fuel is equivalent to the cost of the plumbago crucible.

This company have introduced crucibles especially adapted for the following purposes, namely;—Malleable Iron Melting, the average working of which has proved to be about *seven days;* Steel Melting, which are found *to save nearly a ton and a half of fuel to every ton of steel fused;* and for Zinc Melting, lasting much longer than the ordinary iron pots, and saving the great loss which arises from mixture with iron.

Crucibles have been in use for melting and refining metals from that distant point of time when man exchanged his stone hatchet and bone chisel for implements of bronze. The earliest melting pots were doubtless made of the plastic and infusible substance clay, and there is no reason to suppose that they differed essentially from the earthen crucibles now commonly used in our foundries.

As an instrument of scientific research, the crucible has held an important position for at least a thousand years. It was constantly used by the first alchemists, and may, indeed, be truly styled the cradle of experimental chemistry.

At the present time, crucibles of one form or another are extensively employed by the refiner of gold and silver, the brass-founder, the melter of copper, zinc, and malleable iron, the manufacturer of cast steel, the assayer, and the practical chemist. They are made in many different shapes and sizes, and of many materials, according to the purposes for which they are intended. For certain chemical experiments, requiring high temperature, vessels of platinum, porcelain, and lime, are adopted; but for ordinary metallurgical operations clay crucibles and plumbago crucibles are exclusively employed. We, in this place, confine our remarks to these two important classes of crucibles. On examining a clay or plumbago crucible it seems to be merely a rough specimen of pottery that might be easily imitated; yet the successful makers of crucibles are so few that they might almost be counted on the fingers of two hands. When we take into consideration the qualities which are required in a crucible to enable it to pass victoriously through the ordeal by fire, the paucity of good makers becomes intelligible. The crucible should resist a high temperature without fusing or softening in a sensible degree. It should not be liable to break or crumble when grasped with the tongs, and it ought to be but little affected by the chemical action of the ashes of the fuel. Again, it may be required to withstand the corrosion and permeation of such matters as melted oxide of lead. In some cases crucibles should resist very sudden and great alternations of temperature, so that they may be plunged while cold into a furnace nearly white hot without cracking. In other cases they are merely required to resist a high temperature after having been gradually heated. Some crucibles are specially remarkable for one quality, and others for another, so that in selecting them the conditions to which they will be exposed must be kept in view.

The writer of this article, being an experienced metal-worker, speaks, for the benefit of others, without reserve; he knows from experience that the crucibles which present the finest combination of good qualities are those of the Battersea Plumbago Crucible Company. They support, even when of the largest size, the greatest and most sudden alterations of temperature without cracking; they can be used repeatedly, and their inner surface can be made so smooth that there is no fear of the particles of metal hanging about the sides. Their first cost is necessarily high, as plumbago is an expensive raw material; but the fact that they may be used for a great number of meltings makes them, in reality, cheaper than the ordinary clay pots. As fire-clay contracts considerably when exposed to a high temperature it cannot be used alone for large crucibles. The so-called clay crucibles are made of a mixture of the plaster clay with some other substance, such as highly-burnt fire-clay, silica, or coke, which counteracts in a measure the evil done to contraction, and so lessens the tendency of the vessels to crack. The large Stourbridge clay crucibles, so extensively employed by the brass-founders of Birmingham, contain both burnt clay and coke. The Cornish and Hessian crucibles are made of peculiar kinds of clay in admixture with sand. The great superiority of the plumbago crucibles over these can be easily accounted for by the fact that graphite or plumbago is the most impressible of all substances known, and at the same time a material that can be thoroughly incorporated with the clay without impairing its plasticity.

With respect to fire-clay, W. H. Stephenson, writing in the Transactions of the S. of E., observes;—Among the various deposits which have succeeded the formation of the primitive rocks upon the surface of the globe, there are certain earthy strata of very considerable extent composed chiefly of silica and alumina, partly in combination, and partly in mere mechanical mixture with other less prominent and essential ingredients. These strata are characterized by the very minute

state of division of their particles, and their want of firm connection or solidity. It is to this peculiar structure that the most valuable property of clay must be ascribed—that is, its plasticity, or the property of forming dough with water, sufficiently soft to take the most delicate impression from a mould, and so deficient in elasticity that even the slightest indentation is lasting and persistent.

By far the greater number of clays are so intermingled with substances foreign to them in their original localities, or have been primarily derived from such compound species of rock, or, lastly, have been so very far removed by the agency of water from the sources of their different constituents, that it is next to impossible to trace back the course of their formation to its very commencement; although the clays may be viewed in general as the remains of certain rocks which have been decomposed by various agents, chiefly atmospheric, which have, in a word, been weathered; yet there are few cases in which the production of clay has occurred in the immediate locality of the rock whence it is derived, and in such a simple manner as to enable its origin to be traced in all particulars, and established indubitably by chemical facts.

The most prominent physical properties of clay are its plasticity and behaviour when exposed to heat. By simple drying, at a temperature far below red heat, its particles collapse, the primary pores become contracted, and a very much more dense mass is obtained, which becomes so hard that it will no longer take impressions, although it is still sufficiently soft to be cut with a knife, and when treated with water is again converted into clay with the ordinary properties.

Exposed to the most intense heat that can be artificially produced, clay refuses to become liquid, and acquires at most a slight degree of flexibility. Its particles then cohere so strongly together that the burnt mass is hard and sonorous, although still porous enough to absorb water with avidity. Although it no longer falls to pieces, but retains its connected form, it will easily be conceived that the nature of clay must be very much modified by an admixture of foreign matters possessing other properties. These foreign matters may either be constituted of undecomposed detritus of the rocks from which the clay itself derives its origin, or of others which do not belong to the class of substances which yield clay by decomposition. The character of these foreign admixtures causes great variation in the nature of the different clays, and gives rise to the various denominations by which they are known. The ingredients which most affect the quality of the clay are sand, iron, lime, and magnesia.

The plasticity of clay diminishes with the amount of any one of these substances which it contains, as they are not plastic.

The quality is affected in the most marked manner by sand, somewhat less by lime, and very little by oxide of iron. When clay contains iron and lime, the action of heat upon it is very different: the silica, alumina, lime, and iron then form together a mixture similar to that employed in the manufacture of bottle glass, which melts in the fire with more or less ease, according as it contains much or little of the two latter ingredients. Magnesia exerts less influence upon the character of the clay; the more quartz and silica enter into the composition of the clay, the less easy will it be of fusion, and an excess of iron or lime can be corrected by a large quantity of this ingredient.

Fire-clay is commonly found in the coal-measures, at a great depth from the surface, but it not unfrequently happens that it lies on the top. Stephenson's experience was with clay at some considerable depth, and lying (at Throckley, Newcastle-upon-Tyne) immediately underneath the coal formation; its thickness varies according to circumstances, in some places 3 ft., and in others reduced to 18 in. As a rule, it is very strong and hard, and cannot be worked to advantage without the aid of gunpowder. It would be needless to recapitulate the ordinary working of a coal mine; but suffice it that the clay, on being raised to the surface, is laid out in long parallel heaps, say 20 ft. high, being 20 ft. wide at the bottom, and tapering to 5 ft. at the top. A series of ridges is thus formed, purposely, however, in order to collect as much rain and snow as possible, which, combined with the direct action of the atmosphere, soon reduces that which was at one time hard and retentive, to a soft, comparatively plastic state. Difference of opinion exists among manufacturers as to the policy of adopting this system, inasmuch as to carry it out fully a very large capital is necessary, and which for the time being lies dormant.

The sole advantage accruing in keeping so large a stock is, that it is more easily pulverized and reduced to powder, thereby causing a considerable saving in engine-power, labour, and expense. To carry out this method to its fullest extent, no clay ought to be used until it has been exposed to the action of the elements for at least two years. After the clay is brought to the works, the first process is that of grinding; the most approved plan is that of two large stones, say 10 ft. in diameter and 20 in. wide, hooped all round with iron, and revolving slowly on a cast-iron pan, or bed-plate, which in some works is also made to revolve very slowly the contrary way to the stones. The rough clay from the pit being conveniently placed for the workman, is cast under the edge stones, when it is ground to a coarse powder, which falls through an open grating in the centre of the bed-plate, whence it is lifted in the sifting cylinder by an endless chain of buckets. The clay, as it passes down the cylinder, is separated into two parcels; the coarse, or that which is too large to admit of its being passed through the meshes of the cylinder, is returned by a long wooden spout to the mill, where it a second time is ground, whilst the fine particles are received into an endless belt composed of glazed sack-cloth, and conveyed into the mixing pan, or pug-mill.

Some manufacturers prefer allowing the pugged clay to lie and *sweat* for a few days in a dark place, thereby giving greater ease and facility in working, the clay being rendered of a more plastic nature by the delay. Others remove it immediately from the pug-mill to be moulded into bricks, retorts, and so on.

Brick moulds are made of various materials, some of brass, cast in four pieces and riveted together, others of sheet iron cased with wood in the two longest sides. Iron moulds are *sanded*, but not wetted. Copper moulds are an improvement on the iron, as they require neither sanding nor wetting, and do not rust; they, however, are expensive, and do not last long, as the edges wear down very fast.

The cost of moulding bricks bears so small a proportion to the total cost, that it is questionable whether the application of machinery for this purpose in small works would effect any ultimate saving; numerous inventions have been patented, but few of them can be said to have proved successful.

The moulding operation in the ordinary brick-works is simpler than is the case with any other kind of clay ware.

The workman is supplied with a stock of clay (from the pug-mill) by his side, a table or bench before him, and two boys or helpers. The mould is larger in proportion than the finished brick, owing to the contraction of the clay in drying and burning; this, of course, varies under different circumstances, the tougher and finer the clay the greater the contraction, and *vice versâ*; in general, 1 in. to the foot is the calculation for contraction, and the moulds must be made accordingly.

The usual size of a brick is 9 in. long, 4½ in. broad, and 2½ in. thick.

The mould itself only makes the four narrow sides of the brick, the one broad surface being produced by the table which supports the mould, the other by a straight piece of wood, with which the workman removes away the excess of clay, by drawing it straight along the upper edge of the mould. To prevent the clay adhering to the mould, it is from time to time damped with water, which causes the moulded brick to separate from the mould without bending or loss of time. The operation is conducted as follows;—The workman throws a lump of clay with great force into the mould before him; the mass, which has become flattened by the shock, is forced into the corners by one or two rapid strokes with the hand, and that which projects beyond the mould is taken away with the flat board. By a sudden and peculiar twist of both hands, the workman deposits the brick from the mould on to a thin board previously placed before him for the purpose; one of the boys in attendance immediately places another similar board on the top of the newly-made brick, and thus carries it away between these two boards. Meanwhile, another brick is made as described, and thus the process continues during the hours of labour. The bricks are placed in long rows edgeways on the dry flats, a space equal to the thickness of the board, say ½ in., being left between each brick, in order to give vent to the steam generated in drying.

The drying sheds or flats consist of long floors, say 90 ft. by 30 ft., with flues running the whole extent of the building. It is desirable not to have the length of these flues more than, say, 40 ft., in order to ensure a good draught without any additional coals being used.

In most manufactories these drying flats are so constructed that there is ample room or accommodation for two days' work; in this case the moulders are never stopped, and are not required to remove their tables or benches from place to place. From thirty-six to forty-eight hours is calculated quite sufficient for drying bricks; so that while the moulder and his boys are depositing bricks on one part of the flat a gang of men and boys are engaged in clearing away the bricks from another part.

The number of bricks which a workman can mould in a day of ten hours is always considerable, but depends much upon the ability and strength of the moulder. With clay in good order a skilled workman can make 2000 to 2500 marketable bricks in a day.

It is clear that the relative merits and value of fire-bricks depend upon their fire-resisting qualities, and hence depend upon the proportion of silica they contain.

In an analysis of several kinds of Newcastle clay, Dr. Richardson found—

Nos.	1.	2.	3.	4.	5.	6.	7.
Silica	51·10	47·35	48·55	51·11	71·28	83·29	69·25
Alumina	81·35	29·50	30·25	30·40	17·75	8·10	17·90
Oxide of iron	4·63	9·13	4·06	4·91 }	2·43 }	1·88 }	2·97
Lime	1·46	1·34	1·66	1·76 }			
Magnesia	1·54	0·71	1·91	trace	2·30	2·99	1·30
Water and organic matter	10·47	12·01	10·67	12·29	6·24	3·64	7·58

whilst the amount of silica in No. 6 is to the total amount of the bases as 100 : 16, in No. 2 it is as 100 : 85. These clays are mixed in different proportions, according to the object of the manufacturer.

When, therefore, it is desirable to procure a first class article, a chemical analysis, although it cannot supersede an actual trial, may be of the greatest service, as the clays seldom or never come up to what is required of them, and only acquire the requisite properties by certain additions, and the choice of these additions must, in the first instance, be guided by the results of the chemical analysis; such additions are absolutely necessary, as fire-clay must not only be infusible in the fire, but must likewise not be subject to crack and fly. These properties are most important. The chief cause of the cracking, or the contraction of the clay, must therefore be lessened by the addition of substances which do not shrink themselves, and, on the other hand, do not impair the refractory nature of the clay.

Pure sand and previously-burnt fire-clay are the substances most commonly and appropriately used.

The Process of Fire-clay Retort Making.—Referring to the period when the fire-clay has been drawn from the mine and undergone the process of weathering, that which is intended for retorts has been kept separate for that purpose, while greater care and attention has been bestowed on it, in order to pick out any pieces of coal or iron with which it may have been associated. This, although seemingly an insignificant, is a very important part of the manufacture, inasmuch as a very small piece or particle of ironstone is sufficient to damage and spoil a whole retort, and thereby occasion considerable loss.

The clay having been thus thoroughly examined and approved, is next ground in a similar manner to ordinary fire-brick clay, excepting that the particles are not ground so fine (the average size of the meshes through which the clay passes for bricks is, say 5 × 6 to the inch, whereas for retorts it is as large as 3 × 4 to the inch), and in order to render the retorts porous, a proportion of coke or sawdust, say $\frac{1}{4}$ to $\frac{1}{5}$ the weight of the whole, is added to the fire-clay, and mixed up with it, both in the grinding and pugging process. The pug-mill, through which this retort clay passes, is generally longer and wider than the ordinary brick-clay pug-mill; or, instead of this, it is not unusual to pass the clay through two pug-mills, the one delivering into the other, so as to ensure the clay being well worked and of a proper consistency.

The manufacture of *clay retorts* was formerly carried on by machinery, but now the same objection may be said to exist against this method, as is the case with regard to machinery for brick-making. The result has, therefore, been that retort-making by hand has now become the rule, and by machinery the very rare exception.

The hand building is performed by small lumps of clay being pressed against the side of a mould or drum the required shape, and this continued till a height of 8 in. or 10 in. is obtained, the walls being gradually built up according to two wooden guides, the one of which indicates the thickness, say $2\frac{1}{2}$ in. to 3 in., the other the outward shape of the retort.

Some clays are more plastic than others, and will consequently bear a higher or longer building, but in general 9 in. are sufficient at once, in order to ensure soundness and firmness. This process of building is continued every day, or as often as necessary, till any length of retort is obtained, the top end always being kept perfectly moist, to guarantee perfect adhesion throughout the whole. The flats or sheds in which these retorts are made, are constructed in like manner to the brick flats, excepting that more height is allowed from the level of the floor to the joists, to contain the longest retorts. Fires are constantly kept burning under the floor on which the retorts are being built, and this process of drying is perhaps one of the most important of the manufacture. If not carefully and properly dried, cracks will show all over the surface, the colour of the fracture will not be uniform, and the retorts essentially bad.

It was stated that coke and sawdust were mixed with the clay in order to make the whole mass porous. To provide against the porosity of the retorts causing a loss of gas, a composition or mixture composed of about equal parts of unburnt and calcined fire-clay finely pulverized with the addition of as much water as renders it a consistency of thick *paste*, is applied day by day to the internal and external surfaces of the retorts, and well worked in (by the hand) to the body of the retort; thus an even, smooth, and unbroken surface, free from cracks and flaws, is produced, and the retort presents a uniform appearance throughout.

The burning of the retorts requires much care and attention, and generally continues for a period of ten to twelve days. The retorts being placed vertically on rows of bricks on the bottom of the *kiln*, the great desideratum is to procure a steady draught, the exclusion of atmospheric air, and a gradually progressive heat.

Opinions differ very widely as to the best shape of clay retorts, the circular, oval, or elliptical, and ⌓, being those commonly advocated and in use, while the egg-shaped, or combination of round and oval, and the round curved ▭ have each their supporters. In the leading metropolitan works the 15 in. round, and 21 in. × 15 in. oval, in settings of five and seven retorts in a bench, appear to be in favour; these retorts being from 18 ft. 6 in. to 20 ft. in length (open throughout, and charged at each end), are constructed in three or four pieces to suit convenience.

The comparative merits of clay and iron retorts is a subject which has attracted much attention from the gas engineering profession during the past few years. The results of numerous practical trials, comparing their relative durability, economy, and carbonizing power, have from time to time appeared in the various serials devoted to the gas-light interest, and many facts worthy of attention have been elicited by the controversy respecting their comparative excellence. It may seem a matter of much surprise to those unacquainted with the details of these practical essays, that a substance apparently so friable and brittle in its nature as clay should have superseded cast iron to a great extent, and received the highest encomiums from nearly every responsible source. Yet such has been the case, and this important reform, which but a few years ago met with many obstructions, in having to withstand a rigorous prejudice, has lately been gaining ground with great rapidity, and promises ere long to meet with universal approbation.

Brass Founding.—Pure copper is moulded with difficulty, because it is often filled with flaws and air-bubbles, which spoil the casting; but by alloying it with a certain quantity of zinc, a metal is obtained free from this objection, harder, and more easily worked in the lathe. Zinc renders the colour of copper more pale; and when it exists in certain proportions in the alloy, it communicates to it a yellow hue, resembling that of gold; but when present in larger quantity, the colour is a bright yellow; and lastly, when the zinc predominates, the alloy becomes of a greyish white. Various names are given to these different alloys. The one most used in the arts is *brass*, or *yellow copper*, composed of about $\frac{2}{3}$ of copper and $\frac{1}{3}$ of zinc. Other alloys are also known in commerce, by the names of *tombac*, *similor* or *Mannheim gold*, *pinchbeck* or *prince's metal* (chrysocale), &c.; they contain in addition greater or less quantities of tin.

Tombac, used for ornamental objects which are intended to be gilded, contains 10 to 14 per cent. of zinc; the composition of *Dutch gold*, which can be hammered into very thin sheets, being nearly the same. Similor, or Mannheim gold, contains 10 to 12 per cent. of zinc, and 6 to 8 of tin; and pinchbeck contains 6 to 8 per cent. of zinc, and 6 of tin. The statues in the park of Versailles are made of the following alloy;—

Copper	91	Tin	2
Zinc	6	Lead	1

The alloys of copper and zinc are altered by a high temperature, and a portion of the zinc is

crucible in a forge-fire, the zinc is nearly wholly

zinc; rosette copper being used, fused in a cru-
zinc is broken into small pieces. The fusion is
rom 30 to 40 lbs.
roportion of ⅛ of
are added. A
g-shaped furnace
supported by a
the flame of the
neath the dome.
r opening of the
by a lid having
ter beneath the
xtraction of the
les are removed
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ly added to brass
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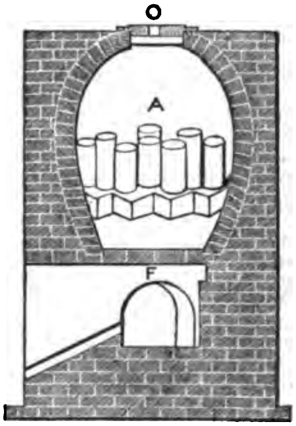


culty, and their union is never very perfect. By
fusing point, a large portion of the tin will separate
he melted alloys solidify slowly, causing circum-
e pieces.
copper and tin, according to their composition and
al, *bell-metal*, *telescope-speculum metal*, &c. All these
come hard and frequently brittle, when slowly
ble when they are plunged into cold water, after
duces, therefore, in these alloys an effect precisely

in the air, the tin oxidizes more rapidly than the
continuing the roasting for a sufficient length of

per and tin;—

Cymbal and tam-tam metal, composed of;-

Copper	80
Tin	20
	100

Telescope-speculum metal, made of;—

Copper	67
Tin	33
	100

position, and generally consists of;—

	95
	5
	some thousandths.

al objects generally contains larger quantities of
e of alloys of copper and tin; and although the
w sous coined under the Republic, from a metal
rage 86 of copper and 14 of tin.
al important conditions. It should be very tena-
normous pressure caused by the explosion of the
to be injured by the ball, which strikes the sides
tly, it should be fusible, because large guns can

possess sufficient tenacity; but as pure iron will
e for it cast iron, the tenacity of which is much
s too soft; and in rapid service would soon be so
be had to alloys of copper with other metals; and

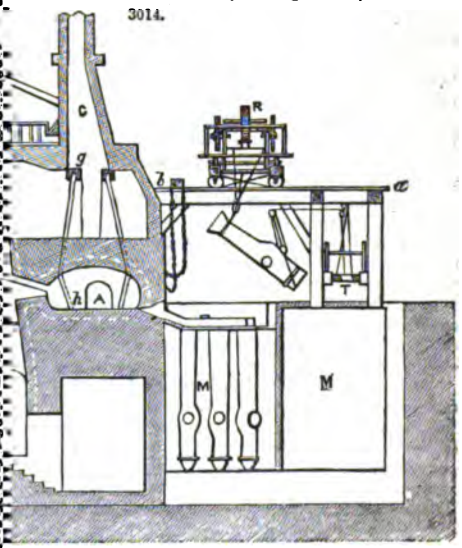
...na tin are the most suitable; but as, while tin ...hes its tenacity, it becomes necessary to stop at ...he alloy possesses both the requisite degree of ...have been determined by numerous experiments, ...have been fixed at 11 of tin for 100 of copper. ...of a calibre below 8, an alloy of 8 or 9 per cent. ...been made to ascertain if the alloy could not be ...iron, or lead; but these complicated alloys have ...on of their results; and pieces were frequently ... of obtaining such alloys homogeneous and of

...nnon is long subsequent to that of brass. As it ...uted for bronze, but it is very brittle, and pieces ...f the latter metal, thus becoming too ponderous ...nary batteries, fortifications, coast defence, and ... those of bronze, and for this reason are prefer- ...lower-deck batteries, would make a noise insup- ...e with charcoal, should alone be used for artil- ...ed for this purpose.

... contain no oxidizing gases, and the atmospheric ...stion, as far as possible, of its oxygen, because ...ould constantly separate from the alloy in the ...at the time of casting, would not be known with

...e, used in the cannon-foundry at Toulouse. It ...urbased dome, heated by the grate F, on which

3014.



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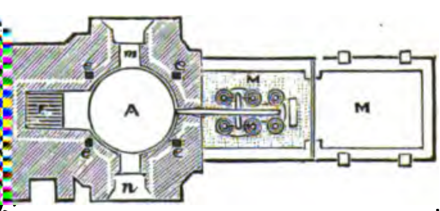


...breech being downward. Between the tap-hole ...he liquid bronze into each mould; and above is ...n, by means of which the moulds, when filled,

...ding of cast iron and other metals, has been sub- ...made, but never with success, as the walls of the ...to gases. Now, immediately after the casting of ... bubbles, which pass through the porous walls ... high column of melted metal, while in the sand

moulds, the gases not being able to escape through the sides, produce a constant bubbling in the mass, giving rise to numerous flaws, and assisting the separation, by eliquation, of the tin, or alloys rich in tin.

The charge of a furnace is composed of old brass, chiefly condemned cannons, and masselottes taken from pieces previously cast, with brass turnings taken from the lathe or the boring machine and a certain quantity of new metals, copper and tin, besides *white metals*, or alloys very rich in tin, which separate by eliquation in the moulds. The proportions of copper and tin in the several components being determined by analysis, they are mixed in the proportion of 100 copper to 13 or 14 tin, which is reduced by oxidation of tin in the furnace to the normal proportion of 100 : 11.

The condemned cannons and masselottes are laid on the hearth-sole, near the bridge, where the temperature is highest; while the copper, which should be very pure, in bars, and the turnings, are placed thereon, the white metals and tin being added at a later period. In six or seven hours the mass is almost entirely fused, and the flame escapes by every avenue. The smelter first stirs the material with sticks of very dry wood, and draws the portions which are not melted toward the bridge; after which he completes the charge by adding the white metals and tin, which he runs in the form of pigs into different parts of the bath. He stirs it a second time, in order to render it homogeneous, and, after skimming off the superabundant scoriæ, closes the door of the furnace, and blows up the fire, to bring the alloy to a proper state of liquidity, stirs and skims it a third time, and then opens the tap-hole. Other workmen direct the melted metal into each mould.

A remarkable phenomenon ensues in a few moments after the casting. A bubbling takes place in the upper part of the mould, proportioned to the size of the piece and the elevation of temperature, and a portion of the bronze rises in the form of a mushroom, being an alloy much richer in tin than the cast metal. A partial eliquation therefore takes place during the cooling, which causes the separation of an alloy more fusible, and containing more tin. The composition of the piece itself is not uniform, as the proportion of tin diminishes from the breech to the upper part of the masselotte. The intention of the masselottes is, not only to exert considerable hydrostatic pressure on the lower strata of the piece, but also to furnish metal necessary to compensate for the contraction of the metal by cooling, and its loss of substance by eliquation.

Twelve hours after the casting, the earth is cleared away in order to hasten the cooling of the moulds; and the latter are removed after forty-eight hours, broken, and the cast guns carried to the boring and turning shops.

When the surface of the piece is turned, and it has been bored to a certain point, it is examined to ascertain if it be free from such defects as would render it unserviceable. Such defects are various, and called by different names; but they are nearly all produced by eliquation of the tin or very fusible alloys.

Flaws, or *bubbles*, are cavities with smooth surfaces, produced by bubbles of gas which have been unable to escape; while *honeycombs* are cavities with rough surfaces, arising from irregular distribution of the materials or badly-proportioned alloy; and *worm-holes* are similar, but smaller, cavities. *Cendrures* are owing to impurities in the alloy, remaining in the metal, or detached from the sides of the mould; and *tin-spots* are produced by small, very hard masses of an alloy containing 20 or 25 per cent. of tin, which became separated by eliquation, and were unable to ascend as far as the masselotte. *Blasts*, or *cracks* (sifflets), which are longitudinal or transverse grooves, sometimes extending through the whole thickness of the piece, are likewise owing to a separation of the tin.

If the piece is found to be perfect, the boring and turning are completed, and it is subsequently examined and proved according to the regulations of the service.

Tinning of Copper and Brass.—The use of copper and brass for culinary purposes is dangerous, on account of the ease with which copper, on oxidizing by contact with the air and acid substances, forms very poisonous salts, unless the vessels are lined with a coat of tin, which prevents the liquids from coming in contact with the copper. The tinning of copper is effected by cleansing the pieces with chlorohydrate of ammonia, and spreading with a piece of cloth or tow, melted tin over their surface when properly heated. The tin thus adheres to the copper, and covers it completely.

Pins are made of brass wire, and whitened by being covered with a thin coat of tin by the humid way. The pins are first cleansed by heating them in a solution of cream of tartar, and then placed in a copper basin with a solution of cream of tartar and tin. The liquid is boiled for about one hour, when the tin dissolves in the cream of tartar with disengagement of hydrogen gas, and is precipitated on the brass of the pins, covering them with a very thin pellicle of metal.

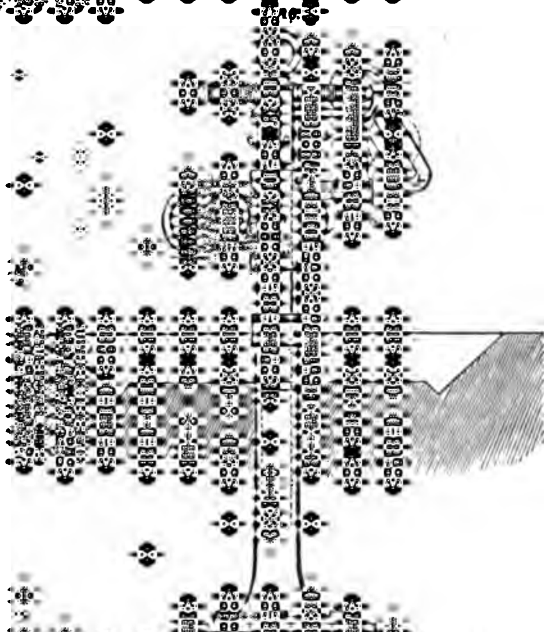
The Apparatus for Moulding Toothed Wheels, invented by G. L. Scott, Figs. 3016 to 3026, is designed to supply the means of obtaining accurate castings by machine moulding, with a portable and self-contained machine of small cost, capable of being readily and quickly applied at any part of a foundry.

The accuracy and perfection of the teeth of wheels are of great practical importance in all cases of gearing, and especially where large amounts of power are transmitted by them; and it is requisite that the transmission of power should be uniform and continuous through the teeth of the wheels, corresponding to the continued frictional contact of two circles rolling upon each other. To maintain this uniform and continuous action in toothed wheels, all the teeth throughout the circumference of the wheel are required to be precise duplicates of one another in form, size, and spacing; and all to be placed in a perfect circle round the centre of the wheel. Should these conditions be imperfectly carried out, the essential continuous contact will be destroyed, and a serious intermittent knocking between the teeth will be caused, leading to the fracture of the wheel, and risking a stoppage of the machinery. Any defective fitting of toothed wheels also involves a waste of driving power from the irregular shocks in transmitting the power; and as a consequence the wheel will not last so long in such a case, owing to the friction causing extra wear of the teeth.

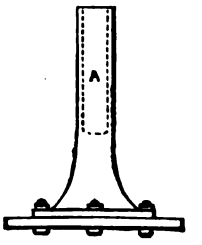
...ls, the teeth were chipped out by hand from the ...out and shaped to template. Subsequently the ...l, and moulded from this model according to ...f having a separate expensive pattern for each ...ll as in diameter. The result has been a vast ...uirements of ordinary trade demands; and this ...ruction and of the storage space occupied, that ...nge of pitch of wheels, in order to reduce the ... patterns for entire wheels involves further the ...n the general contour of the wheel and in each ... and contraction in the component parts of the ...tion in the forms and dimensions of the several ...each receives. The uncertainty, too, attending ...ld, and the distortion of the pattern that occurs ...e, are additional obstacles to the manufacture of ...th the correctness that is desirable.

...ties is by employing only a small segment as the ...ference by repetition of this small portion; em...ing it, and for spacing out the teeth round the ...ne certainty of accuracy throughout as is shown

...r moulding wheels from 12 in. to 5 ft. diameter, ...iameter. The smaller machine is shown in Figs. ... of the machine, Fig. 3019 a side elevation, and



3017.

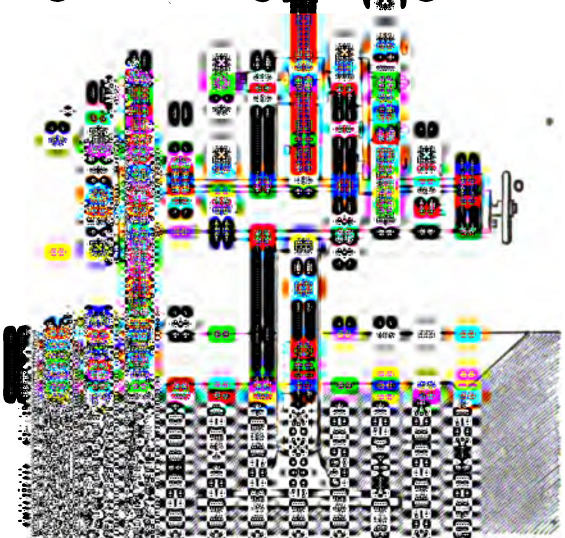


3018.

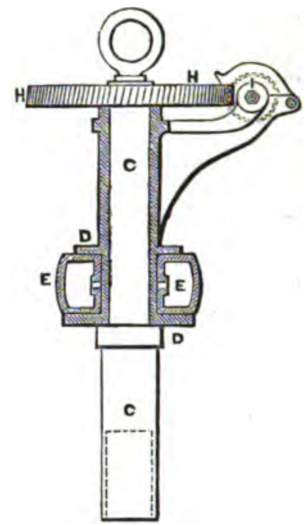
...n B, which has a collar to bear upon the pedestal, ...e a recess in the top of pedestal, whereby it is ...indle C is bored to fit on the centre pin B, and is ...tus, which it supports, as shown in section in Fig. ...sed to fix it firmly on the centre pin B, and this ...cal centre spindle is thus obtained. Loose collars ...ntre pin B, are used for the purpose of elevating ... more readily to adapt it for moulding different ...n at V in Fig. 3026, and they are of 1, 2, and 3 in.

...n in section in Figs. 3021, 3022, and in this head ... their front ends by the transverse piece F, which ... The arms E E are secured to the head D in any ...ssing through slots in the arms and through ears ... bind the arms and head firmly together. The ...rms a stationary centre pillar for the machine, on ...p of the spindle is keyed the worm-wheel H, from ...he dividing apparatus, shown in Figs. 3016, 3019,

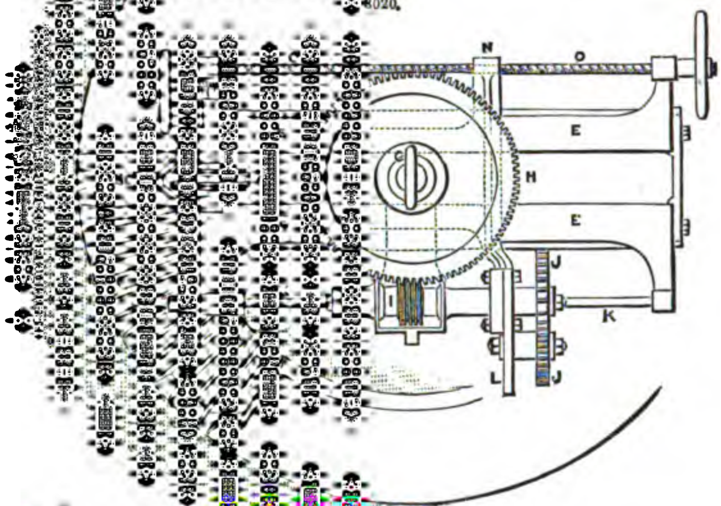
[illegible] the wheel H, and the change wheels J J J, the [illegible] lowest one keyed on the shaft K, which is carried [illegible] loose collar acting as a bearing, so that the shaft



3021.

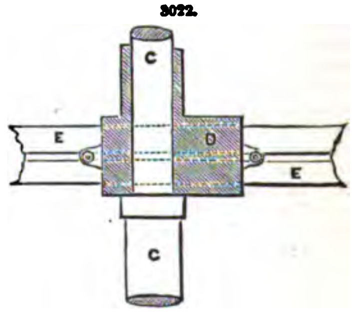


3020.



[illegible] eels when [illegible] nge wheels [illegible] required. [illegible] hich fits a [illegible] man in the [illegible] traversing [illegible] and passes [illegible] t by turn- [illegible] e arms are [illegible] wheels to [illegible]

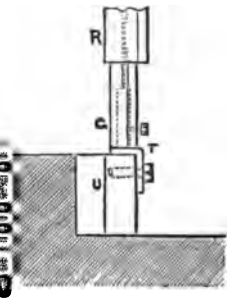
3022.



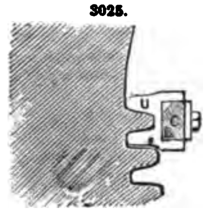
[illegible] G, which [illegible] Fig. 3023; [illegible] uired posi- [illegible] the angle- [illegible] ns; and to this is attached the segment pattern U [illegible] moved up or down by a hand-wheel Y, having a

[illegible] which is a pulley Z; from this pulley two [illegible] secured to the bottom of the ram and the other [illegible] lock-nuts. An adjustable brass collar W is fitted [illegible] ram is sufficiently lowered. An eye-bolt is fixed [illegible], for attaching the foundry crane in order to [illegible]

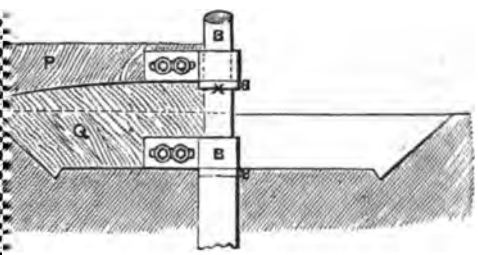
3024.



3025.



3026.



[illegible] machine is as follows. A core box for the arms of [illegible] boards for strickling the form of the top and bottom [illegible] profiles of the face and back of the wheel. The [illegible] edge the profile of the back of the wheel; and [illegible] counterpart profile of the back of the wheel, and [illegible] is also made of a segment of the toothed rim [illegible] permits of moulding one space at a time.

[illegible] ing machine is obtained by sinking in the sand [illegible] pedestal of the machine, which is bolted to a cast- [illegible] rammed solidly upon it, and the pedestal levelled [illegible] is shown in Fig 3018, which is used for fixing [illegible] pedestal is placed about 15 in. below the floor [illegible] dth of wheel that can be moulded. The centre [illegible] of the pedestal, for the purpose of forming the [illegible] the top box or other arrangement used to cover [illegible] ng laid aside for the present.

[illegible] is placed upon the centre pin B, of such thick- [illegible] the floor level as the breadth of the rim of the [illegible] is level with the floor, for convenience of fitting [illegible] a set screw, and an upper loose collar X is also [illegible] upper face at the same height above the collar V [illegible] collar thus exactly indicates the level of the bed [illegible] f the back of the wheel. The hole is then filled [illegible] d the iron trammel carrying the top board P is [illegible] n the collar X, forming a mould of the back of [illegible] sand to form the parting for the top box. An [illegible] placed on, and rammed up with sand, and the [illegible] he purpose of marking its correct position rela- [illegible] driven into the sand and fitting by the side of [illegible] is then lifted off, carrying with it the impression [illegible] ished by turning the box over, and strickling it [illegible] bottom board Q. A centre is provided in the top [illegible] in which are two bolts that pass through the [illegible] box. This loose collar fits the spindle B, and is [illegible] ctly accurate centre. By this arrangement the [illegible] ordinary top box, giving strict accuracy in the [illegible] the purpose.

[illegible] r X is then removed, and the mould being dug

out to the level of the bottom collar V, the sand is strickled with the bottom radial board Q, worked round upon the bottom collar V. This forms the mould for the lower and outer faces of the teeth, and finishes the mould ready to receive the teeth and the cores for the arms; and as both the back and the face of the wheel have been struck from the same trammel and the same centre, accuracy is ensured in the wheel.

The segmental pattern of the teeth U, Figs. 3024, 3025, is then fitted truly square and central and secured by screws upon the angle-bracket T of the vertical sliding ram G, Fig. 3019. The upper portion of the machine is then placed upon the spindle B, the trammel having been removed; and the fixing screws in the spindle are screwed up, to maintain the central axis continuous through the machine. The segmental pattern U is adjusted by the traversing screw O, Fig. 3020, to the correct radius of the wheel, measuring from the top of the tooth to the centre of the machine. The ram G is then lowered to the level of the bed of the wheel, and secured at that point by the locking screw S; and the brass collar W is adjusted on the ram and fixed by a set screw, to ensure the ram always stopping at the same level, when lowered for moulding each successive tooth. The locking screw S prevents the ram rising from the pressure of ramming the sand. One space of the wheel-teeth is then moulded by ramming sand in the space left between the pattern and the edge of the mould previously formed by the strickle-board. The locking screw S being released, the ram carrying the pattern is raised clear of the mould, and is traversed round through the exact distance of the pitch of the wheel, by means of the dividing handle and the change wheels previously arranged for the required pitch. The segmental pattern is again lowered, and a second space moulded as before.

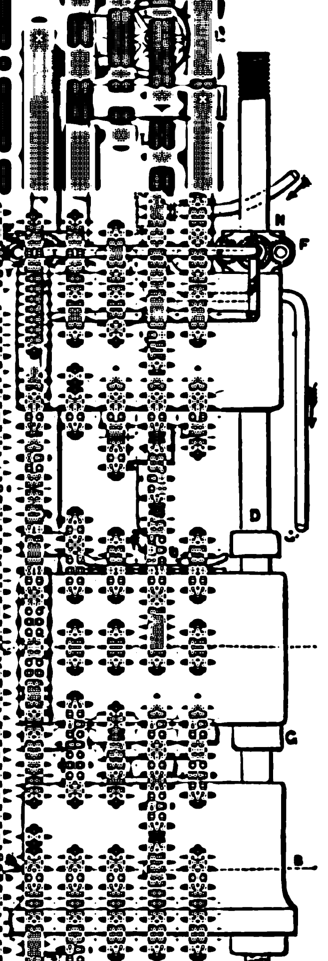
When all the teeth have been moulded, the fixing screws of the centre spindle are released, and the whole machine is then lifted away by the foundry crane laying hold of the eye-bolt on the top of the spindle, leaving the mould entirely clear to receive the cores for the arms and boss. The hole in the top of the pedestal is fitted with a cover to keep out the sand, and is then covered over with sand, which protects the pedestal against the action of the hot metal. The centre core for the wheel is adjusted as usual from the circumference, and the cores for the arms are set to their places by means of wood gauges showing the thickness of the arms and rim. The top box is then put on, to cover the mould, being placed in its correct position by the stakes; the runner is formed, the box duly weighted, and the whole is ready for casting.

Whitworth's Apparatus, Figs. 3027 to 3037, for subjecting steel to a high pressure during the process of casting. In casting some articles, such as hoops and other hollow forms, Whitworth, when using rams arranged to give a pressure to the melted metal in the mould, after applying the pressure for some time, and when the mass has become solidified, withdraws the internal resisting instrument, or core, to allow the metal to contract freely in cooling. In forming other articles, such as those of considerable length, Whitworth applies the pressure to the outer surfaces of the mould, and makes the latter in sections, between which dried loam or sand is placed, so as to allow the air to escape, and to permit of the sections being brought closer together. The object of Whitworth is to obtain sounder castings, and to do away with the necessity for great "heads" of metal.

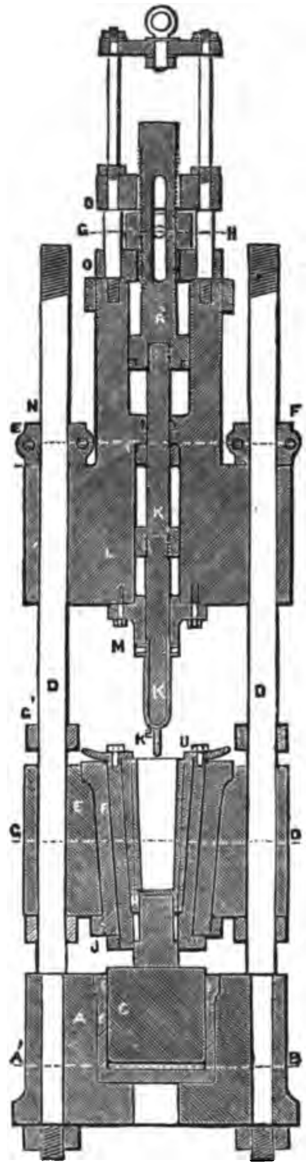
In our illustrations, Fig. 3027 shows an elevation, and Fig. 3028 a vertical section, of the apparatus. Figs. 3029 to 3032 are horizontal sections, at the lines A B, C D, E F, and F G, in Figs. 3027, 3028, respectively. In the figures just mentioned, A is a cast block, having in its centre a cylinder B of steel, within which a plunger C works; this plunger, when water or liquid is forced into the cylinder B, raising the ram Q. D D are two screw-columns, the lower ends of which are securely fixed through the casting A, whilst the upper parts above A have threads formed upon them, so that the cast block E may be supported in any desired position upon them by means of screw-nuts, G, G, G^1, G^1. The mould E F is of steel, in order that it may be of sufficient strength to sustain the great pressures to which it is subjected. This steel mould is secured in the casting E by a screw-nut F^1; within the mould is a filling piece J, which is of cast iron, and is securely retained by a nut J^1. Within the filling piece J is the lining H, of cast iron, perforated with numerous holes to facilitate the passage of air and vapour, or gases; and at the outer surface of this lining are numerous grooves, in order that the air and gases, as they pass through the perforations, may get away freely. The interior of the perforated metal lining H has a lining of sand, loam, clay, or other refractory material, which is moulded in the metal lining to the required form, and is then dried and put into position to receive the melted steel. The metal lining H is retained in its place by the turn-buckles or stops U U, and the casting E is arranged to turn on one of the screws D D as on an axis, so as to come outside the press when it is desired to remove the article from the mould, and when introducing a fresh mould into position. K is a core, which is of metal, and is coated with sand, loam, or other suitable refractory material; the coating is formed separately from the metal core, and dried, and is then placed on the metal core, and is retained in its position by pins, together with the lower nose K^2 of metal, as shown. The upper end of the stem of the metal core K is fixed into the bar K^1, which is fixed in the under-side of the piston R, the latter working in a hydraulic cylinder formed at the upper part of the iron block or casting L.

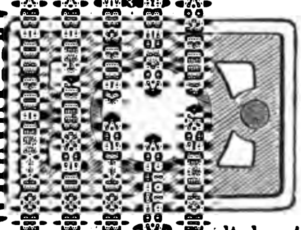
The stem of the core K is capable of sliding through the tubular plunger M, fixed, as is shown, to the casting L, and the lower end of this tubular plunger is formed of steel, and is faced with sand, loam, or other refractory material. It closes on the melted metal in the mould when the casting L is lowered, and resists the passing away of the metal when the pressure of the ram Q of the plunger C is applied to the bottom. The upper cylindrical part of the casting L is hooped with steel, and when of considerable dimensions it may be lined with a cylinder of steel. The piston-rod R^1 of the piston R has a screw-thread cut on its outer surface for the purpose of adjustment, according to the length of the core K required; and O O are two collars, secured, as is shown, to the upper or cylindrical part of the casting L. The piston-rod R^1 is capable of sliding freely up and down through the holes in the collars O O, except when locked by the screw-clip N^1. The

screw-clips N N, capable of being raised or
N N and N¹ are each in two parts, which are
t and right handed screws on the shafts P P

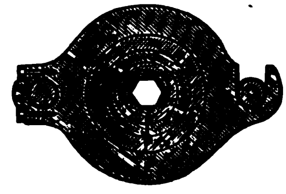


3028.





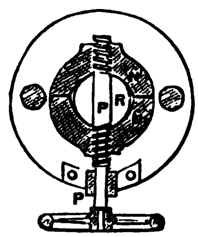
3030.



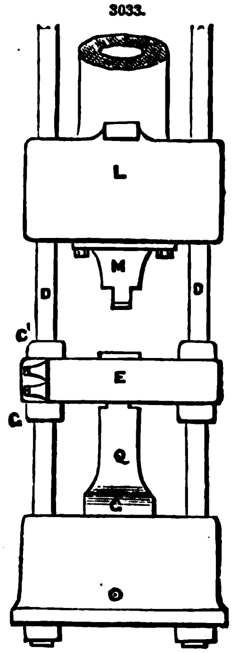
melted metal may be freely performed after the
troduced into its position before pouring.

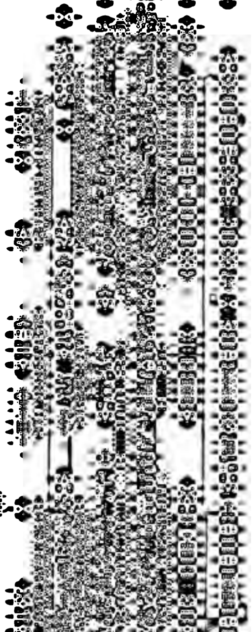
poured into the mould, and the block L is quickly
en to be locked, and the ram Q brought up by its
l to great pressure. When the metal has become
vill become free from the coating of sand, loam, or
by admitting water under pressure to the space
unlocking the screw-clip N^1. The core K is raised
tion, so that the metal remains still compressed
hing 150 lbs., Whitworth finds the pressure should
K is withdrawn. The lifting of the metal core K
tract in the latter part of the cooling. The dis-
D D, is left greater than the depth of the block E,
the pressure is being applied, and to prevent the
l the metal lining H pushed out. The mould F,
ing used with it, are suitable for casting hexagon
for casting other hollow bodies, such as hoops or
where it is desired to have the power of with-
r iron still remains under pressure in the mould,

3032.

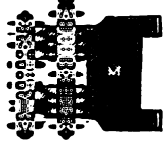


3033.

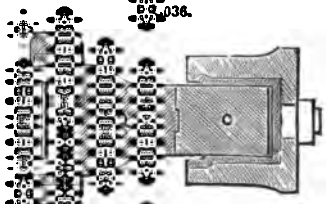




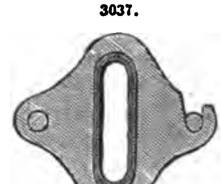
3035.



3036.



3037.



principal parts of a press similar to that already
d Figs. 3035 to 3037 show vertical sections of the
at Fig. 3034. The mould here shown is suitable
being no central core, as in the arrangement just
suitable for chill-cast ingots of the form shown.
f the mould; the part F is made with inclined
E, which is retained in any desired position on the
f the mould is carried by the ram Q, actuated by
part F^1 of the mould is not made so long as the
open, through which the melted metal is poured,
by the two parts F^2 F^3. The parts F^1, F^2, and F^3
may be the case when required with the sides of

the mould, and where necessary provision is to be made for the getting away of the air and gases, as before described; these coatings of sand prevent the sudden chilling of the metal, and enable smaller ingots to be cast and pressed than would otherwise be possible. In using this arrangement of moulds, the melted metal having been poured in, the block or casting L is lowered, and by this means the upper part or side of the mould is made complete; the screw-clips N are then locked together, and the part F¹ being pressed on by the part M, the hydraulic plunger C is then put into motion, and pressure applied to the fluid metal in the mould. When the article is set and sufficiently cooled, the clips N are unlocked, and the parts connected with the block L raised, and then, by a further motion of the ram Q, the mould and ingot may be lifted out of the block E. Either, or both, of the top and bottom surfaces, F¹ and F², may be actuated by hydraulic or other power; when both are so actuated, they should be simultaneously caused to approach each other. In this manner various forms of solid castings may be produced, the moulds being formed accordingly, such, for instance, as cranked or other axles or shafts, the connecting and other rods of steam-engines, and other similar articles; and when the length is considerable, the movable sides of the steel moulds used may be actuated by several hydraulic rams. No rule can be given as to the extent of pressure which may be most advantageously applied in all cases, as the thickness, quantity, and quality of metal acted on vary so largely, whilst the forms of the articles to be cast also differ very largely. Careful observation, however, on the part of the workman will enable him quickly to judge.

It may, however, be desirable to remark, that where the metal in the interior of a casting is found, on cutting or removing the ends or other parts, to have formed itself into irregular crystals, it has not been subjected to sufficient pressure, or the pressure has not been continued for a sufficient length of time. The character of the metal of such a casting, if of steel, may be improved and rendered more uniform throughout by heating it to a moderate red heat, and then subjecting the casting to further pressure, either on end or lengthwise, or both, according as it may be desired to contract or extend the length; this heating and subsequent pressure is also advantageous in removing or breaking off the very hard coating of sand, loam, or other refractory matter employed on the surfaces of the moulds.

By applying the pressure in the manner described, by reference to Figs. 3033 to 3037, to the whole length of the article which is being produced, the pressure may be maintained uniformly on every part until the operation is complete; whereas with plungers acting at the end or on comparatively small parts of the surface of the article, this is not the case, the pressure then ceasing to be uniform when the metal is no longer fluid.

Whitworth remarks that in subjecting fluid steel or iron to very high degrees of pressure in steel moulds, and at the same time cooling it in them, it is of importance that the amount of the pressure applied should always exceed that produced by the shrinking or cooling which is simultaneously going on; or in other words, the pressure applied should be sufficient to overcome the counteracting forces resulting from the rapid cooling of the surfaces of the article, and the slower cooling of the interior metal, so that the atoms are caused to approach each other by the pressure more rapidly than the counteracting forces can separate them.

Caslon and Fagg's Type-founding Machine, Figs. 3038 to 3046.—This invention relates, first, to breaking off the lump or piece of superfluous metal that is cast with and adheres to the body of the type when discharged from the ordinary moulds or machines, and which lump or piece of superfluous metal is usually broken off from the type by hand; and, secondly, to the arrangement of apparatus for rubbing the sides of the cast type and thereby removing the burr or rough edges at the angles of the body of the type.

Figs. 3038 to 3042 show two arrangements for effecting the severance of the lump of superfluous metal from the body of the type.

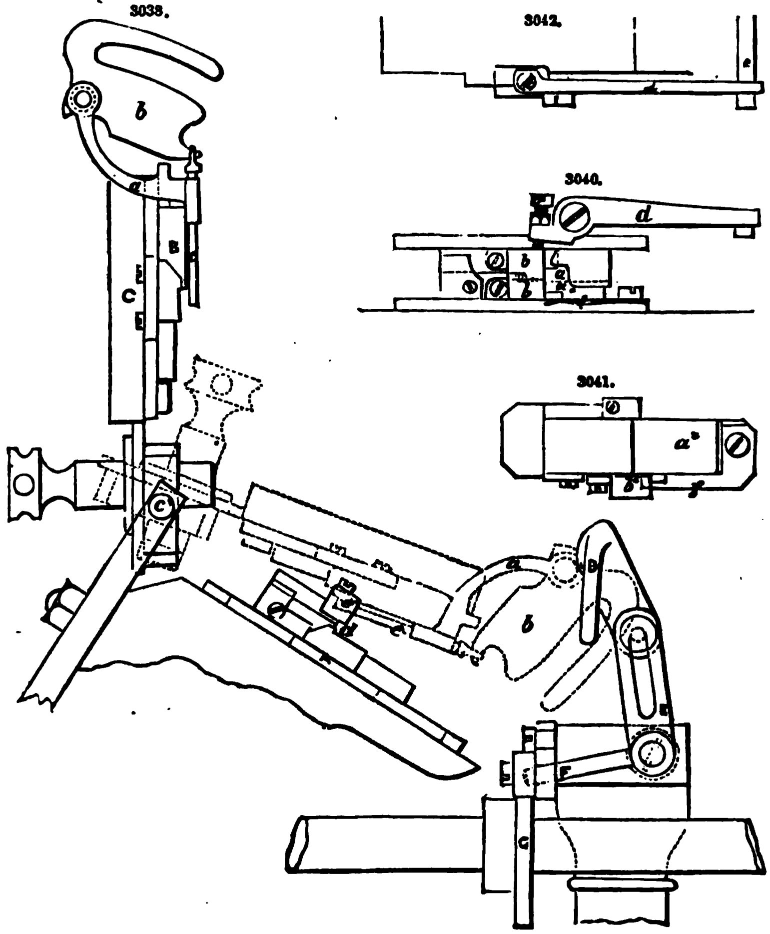
Figs. 3038, 3039, are side and front elevations of this machine; the movable half of the mould being thrown up out of action the better to explain the construction of the parts. A is the fixed die or half of the mould, and B the movable die or half thereof. The movable die is attached to a swinging arm C having its fulcrum at C¹; affixed to the arm C is a bracket *a*, on which is mounted a hook-shaped rocking plate *b*. To the forward end of this plate is connected a finger *c*, which slides in a guide formed for it in the bracket-arm *a*. The object of this finger is to advance at a proper time and move over the gate or entrance of the mould, and assist in severing the lump from the type. When the mould B is brought down into position a curved and cranked bar D will enter the recess formed in the hook-shaped rocking plate *b*. This curved bar is secured by a bolt to the slotted arm of a crank-lever E, the other arm of which enters a slot in a rock-lever F furnished with a roll that lies in contact with a cam G. The rotation of this cam will give a rocking motion through the levers F and E, and the cranked curved bar D to the plate *b*, and thereby cause it to move the finger *c* to and fro in its guide. The bar D is curved to allow of the type-moulds when closed being rocked towards the metal supply cylinder to receive the jet of metal without the position of the plate *b* being affected thereby. When the type-mould is being closed the finger *c* will be drawn to its backward position clear of the surfaces of the moulds, and in that position it will remain until the dies open. As the upper die rises it will carry with it the cast type, suitable provision having been made in that die to secure adhesion, and the finger *c* will be pushed forward over the lump as shown at Fig. 3039. So soon, however, as the upper die B rises to the position shown in Fig. 3038 the head of the type by projecting will come in contact with a shoulder *e* on the stationary die, and the lump or superfluous metal being retained in the die by the finger *c*, the type when hard metal is used in the casting will be broken off and discharged from the die into a suitable receptacle below. The finger *c* will then be withdrawn and the waste end or lump will fall from the die. From this explanation it will be understood that the severance of the waste metal from the body of the type will be effected by an automatic operation.

Another mode of effecting this object, and designed chiefly for use when soft type-metal is employed, is shown at Figs. 3040 to 3042. The action of this modification may be best described

as a shearing off the superfluous or waste metal. The divided gate of the mould instead of being formed with the dies as usual is made separate and capable of an independent motion. Fig. 3040 is a side view of a mould constructed according to this modification, and Fig. 3041 is a plan view of the lower die, and Fig. 3042 a partial top view, showing the levers for operating the gate.

a, *a**, are the top and bottom dies, and *b*, *b**, the parts that form the gate. They are jointed to the dies, and upon the part *b* bears an adjustable screw *c*, which is carried by a rock-lever *d*. The tail end of this lever is borne up by a lever *e*, which may be operated in any convenient way according to the construction of the machine to which the apparatus is to be fitted. The part *b** is formed with a projection in the under-side of which bears a spring *f*. When a type has been cast by the injection of the metal through the gate as usual the lever *d* will be rocked by the lever *e*, and the gate will thereby be forced down, taking with it the lump or waste piece contained therein; the top die will then rise, lift out the type from the bottom dies, and discharge it as before described, while the waste piece falls out of the gate by its own gravity. The part forming the lower half of the gate will be thrown up into position.

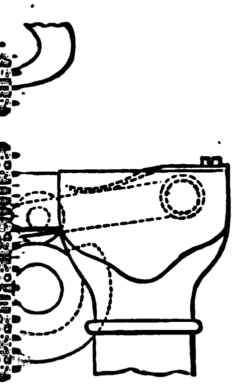
3038. 3042. 3040. 3041.

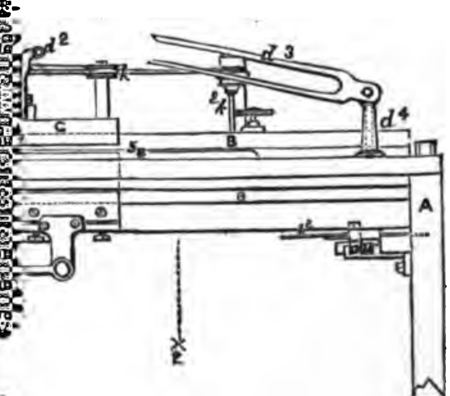


The second part of the invention, which relates to an apparatus for effecting the rubbing or finishing of the body of printing types, which operation has heretofore been effected by hand-labour, is shown in several views, Figs. 3043 to 3046. Fig. 3043 represents the apparatus in longitudinal elevation; Fig. 3044 is a plan; Fig. 3045 is a cross-section taken in the line 1, 2; and Fig. 3046, a cross-section taken in the line 3, 4, of Figs. 3043, 3044. In these views A, A, is the main framing; B, a V-shaped guide for supporting two vertical slides C and D, the uses of which will be presently explained. E is a sliding table, which has a slow endway motion imparted to it, and to which is securely attached a bracket-arm E[1]. At F a stationary file is represented secured to the main framing A, and intended to receive the types as they are fed into the machine, and finish on one side. The opposite side is in like manner finished by a file surface carried by the slide D. Mounted on guide-pulleys, on the extremities of the bracket-arm E[1], are vulcanized rubber bands *a*, *a*, which form a kind of endless apron for receiving the type to be operated upon. This type, shown in Fig. 3044, is laid across the bands by an attendant, and by the rotation of the bands the types are carried forward until they are brought under a vulcanized rubber roller *b*,

the hinder types pressing forward the forward ith the file F it is pushed on to that file, the ding off freely. This endway motion of the e d^1, which is actuated through the instru- , carried by the slide C, on which slide is g pad e that now comes down upon the type, with the file F, while by the back traverse s the type towards the rear end of the file, it on to a stationary padded table *f*. The rward again to effect, through its roll d^2, the type on to the file F. Simultaneously with t of this slide C moves the slide D, which ce for operating upon the upper side of the table *f* this slide rubs the type lying thereon, ion by the end of the file F, which presents a ot pass, and by this means the upper side of finished like the under side. Mounted in a to the rear end of the slide D is a rotary through the continued forward traverse of the finished type, and by its rotation brushes by the files, and also discharges the type on receiver H, from which it may be taken up by

riving shaft, furnished with fast-and-loose , for receiving motion through a strap from On this shaft I is keyed a band-pulley I^1, from d I^2 over guide-pulleys *i i i*, and a double guide-pulley i^1, mounted on a bracket attached to the main framing. This band I^2 passes once around a pulley *g* on the axle of the traversing brush G, then forward to a fixed guide-pulley *h*, back to the pulley i^1, and so to the driving pulley I^1; by this means, therefore, rotary motion is communicated to the brush G. From the double pulley i^1 a band *k* passes to a pulley k^1, keyed to a vertical screw-shaft k^2, which rests in a foot-step, and is supported at its upper end by a bracket from the main framing. The screw

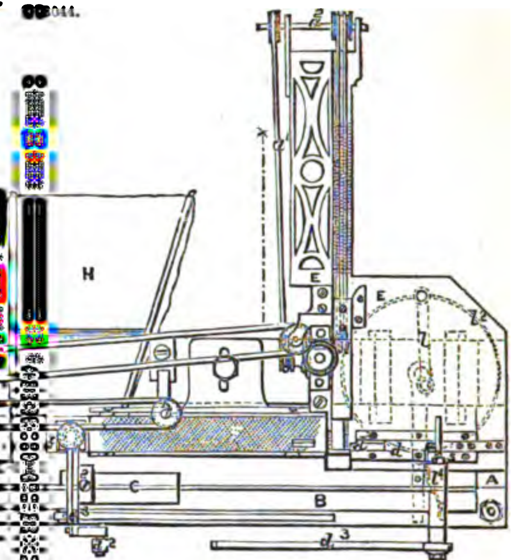


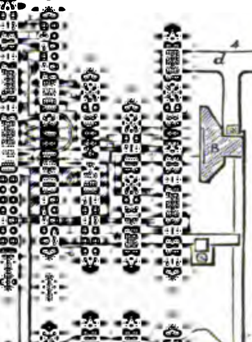


to the shaft of the vulcanized rubber roller *b*, bands *a a*, and passes them forward under the rotation, therefore, of the screw-shaft gives,

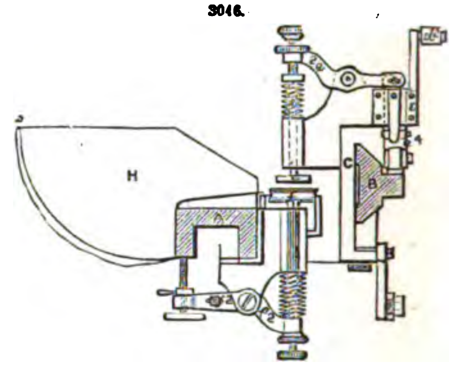
to the roller b. The rotation of the bands a is
orm-wheel b^1 by means of a band a^1, which passes
aft a^2, on the opposite end of which is keyed the
er pulley, which keeps these bands at tension, is
otion of the slides C and D is obtained from the
C[1], jointed at its forward end to the slide C, and at

3044.





3046.



of the shaft I. A metal strap connects the slides
e finger d which pushes the type on to the file F,
ter the forked arm d^2 of a crank-lever carried by
ries a crank-pin, which works in a fork projecting
roll d^2 traverses within the forked arm, that arm
the slide d^1 in its guides, and thrust forward the
type on to the file surface F. In order to bring
type on the file surface while travelling back with
n e^1, which works in a guide fixed to the slide C.
justing screw carried by a rock-lever e^2, which has
opposite end of this rock-lever is a sliding bar e^3,
jointed to the lower end of this sliding bar is a
end a small friction-roll. This roll works over a
g the forward progress of the slide C the tumbling
wever, the back traverse commences, the tumbling
hereby driving up the sliding bar e^3, the effect of
of the rock-lever e^2 on to the stem e^1 of the pad e,
upon the type last pushed forward on to the file
f the slide will enable the pad e to draw the type
harge it on to the table f. Having effected this
eans of a coiled spring surrounding its stem, the

tumbler meanwhile having arrived at the end of the raised bar e^5, and thereby removed the upward thrust from the rock-lever, and allowed the spring to act. A similar motion to that described for the pad e is imparted to the table f, for the purpose of pressing the type up against the file surface carried by the slide D, which surface, as before mentioned, operates upon and finishes the upper side of the type. This table f is carried by a vertical stem f^1, and is held down by a coiled spring. An adjustable screw, carried by a rock-lever f^2, having its fulcrum on the stem-guide which is attached to the main framing, serves to press up the table when required. This rock-lever f^2 is itself operated by a rock-lever f^3, Fig. 3043, which enters a slot in the lever f^2, and carries at its other end a roll that bears a cam f^4 on the driving shaft. The fulcrum of the lever f^3 is carried by a bracket pendent from the framing A, and as the lever is rocked by the *cam* it will raise the table f, and keep the type in contact with the rubbing surface of the passing slide D. When, however, that slide has acted, the rock-lever will allow the table to fall into the position for receiving another type.

In order that the successive types may take different lines of traverse over the file surface F, a continuous lateral motion is given to the table E. This is effected by mounting it on guides, and connecting it by a link l, with a vertical crank-shaft l^1, carried by the main framing. At the lower end of this shaft is keyed a ratchet-wheel l^2, into the teeth of which takes a click which is carried by a loose arm l^3. As the slide C moves forward, it strikes against this arm, and causes the click to drive round the ratchet-wheel a certain distance; a spring l^4, when the slide retires, throws back the arm to its quiescent position. This action being repeated, the crank-shaft will be caused slowly to rotate, and thus shift each successive type into a different position relatively to the file surface F, thereby causing each portion of the surface to act in turn upon the successive types.

See ALLOYS. ASSAYING. BLAST FURNACE. FURNACE. GEARING. MOULDING. PIN-MAKING MACHINE. REAGENTS AND FLUXES. See also articles on the various Metals.

FOUNDRY. FR., *Fonderie*; GER., *Giesserei*; ITAL., *Fonderia*; SPAN., *Taller de fundicion.*

A *foundry* is a building arranged and fitted for casting metals. See FOUNDING AND CASTING.

FRICK'S METAL. FR., *Métal de Frick*; GER., *Frick'sches Metall.*

See ALLOYS.

FRICTION. FR., *Frottement*; GER., *Reibung*; ITAL., *Attrito*; SPAN., *Friccion*; *rozamiento.*

We usually distinguish two kinds of friction. One, called *friction of sliding*, is produced when bodies slide one upon the other, whence it results that the primitive points of contact are found ceaselessly at distances respectively different from new points of contact, which is expressed in saying that they have experienced displacements, relatively unequal, and in opposite directions. The second kind of *friction*, improperly called *rolling friction*, takes place when bodies roll one upon the other, when the distances of the new points of contact from the old are the same upon both bodies, and when the relative displacements are equal. As the word friction implies, generally, the idea of sliding, and not that of rolling, it will be proper to admit only one kind of friction, that of sliding, and to designate the other by the name of *resistance to rolling.*

Review of Ancient Experiments.—The first experiments known upon the friction of sliding are due to Amontons, and are inserted in the Memoirs of the Ancient Academy of Sciences, 1699. This philosopher knew that friction was independent of the extent of surfaces, but he estimates its value at a third of the pressure for wood, iron, brass, lead, &c., coated with lard, which is far too much.

Coulomb in 1781 presented to the French Academy of Sciences, experiments made at Rochefort, and much more complete than those of Amontons. The apparatus he used consisted of a bench, formed of two horizontal timbers 6 ft. long, upon which a sledge loaded with weights slid by the action of a weight suspended to a cord, which, passing over a fixed pulley, was attached horizontally to the sled.

By means of this disposition, Coulomb at once determined the effort necessary to produce motion after the bodies had remained some time in contact. This is what he called the *resistance or friction of departure.* He saw that this friction was proportional to the pressure, and he expected to find it composed of one part proportioned to the extent of the surface of contact, which he termed adhesion—and of another part independent of this surface. He then sought the value of friction during motion, and for this effect he observed, with a stop-watch of half seconds, the time employed by the sled in running successively the first 3 ft. and the next 3 ft. of its course.

But as in these durations, sometimes equal to 1″ or 2″, he might be mistaken by a half second at the end, and also at the commencement of the experiment, there resulted very great uncertainties which did not admit of establishing his conclusions in a positive manner, and we may say he rather conjectured than observed the laws which he inferred from his experiments. Still he admitted that, generally, friction during motion is;—

1st. Proportional to the pressure.

2nd. That it is independent of the extent of the surfaces of contact.

3rd. That it is independent of the velocity of motion, with some restrictions, which subsequent experiments did not confirm.

Coulomb also first established the fact, that for compressible bodies, the friction at starting, or after a contact of some duration, was greater than it was after the first displacement.

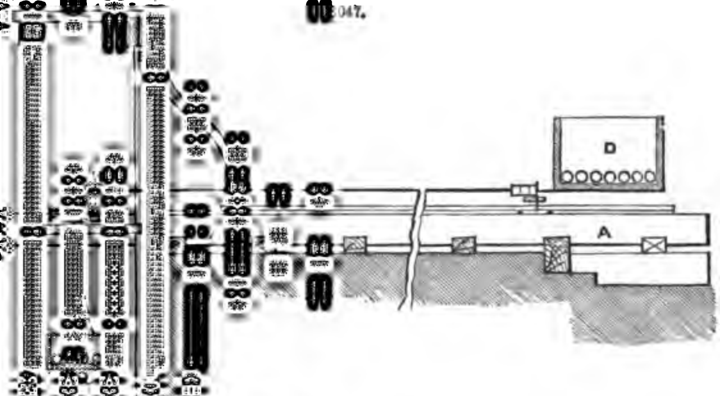
Experiments at Metz.—The uncertain observations, and the restrictions adduced by Coulomb, and above all the more general use of metals in the construction of machines, called for a new series of experiments, which Morin made at Metz, in 1831, '32, '33, and '34, by means of new processes.

Summary Description of the Apparatus used.—In the smelting yards of this ancient foundry, upon a flag-stone foundation, and at the side of a trench, Fig. 3047, was established a horizontal bed, composed of two parallel oak beams A A, 0·98 ft. square, and 26·24 ft. long, connected and supported by sleepers 3·28 ft. apart. These beams, which jutted about 4·26 ft. beyond the edge of the trench, were connected with four uprights B B, between which was placed a platform F F, which bore the pulley for passing the cord, to which was suspended the motive weight, placed in a box K,

rged with weights, under which was placed the body

to the sled, was fastened to the front plate of a
red the tension of the cord, both at its starting and

047.



perfectly smooth and covered with a sheet of paper.
uniform motion to a style, formed of a brush filled
0·459 ft. in diameter. The parallelism of the plane
ctly established by precise methods, and the contact
ll.

rs for holding weights, which, after producing the
cleats, so that the motion continues only in virtue
neans, we may at will obtain, with the box Q alone,
xes, a motion at first accelerated, then uniform or
s sufficient to overcome the friction or is inferior to

iments.—From the synchronism of the two motions,
known velocity, and the other unknown, correspond-
ibed by the sled, there must result a curve whose
he sled. We may then, by this abstract, form a table
times, and construct a curve whose abscissæ are the
e curves thus constructed are perfectly continuous,
they are parabolas, that is to say, their abscissæ are

la, we are justified in the inference that the motion
ight being constant, the motive force producing the
ight above the friction, and since this excess is con-
s constant and independent of the velocity.
sed in the construction of machines, with or without
equences, we are authorized in regarding this law as
city of observation; that is to say, of about 11·5 ft.,
ch Coulomb anticipated have no existence in reality.

of Experiments.—The apparatus which we have just
e in which the motion is variable, and enables us to
reviously pointed out. We take advantage of it to

ox, including its load and that portion of the cord
g, however, the quantity by which it is increased in
e tension of the horizontal strip; $q = 13·79$ lbs., the

instant considered.
in an element of time t.
lley and of the pieces turning with it.
periments, of the friction to the pressure, for the iron
greased; $R = 0·032\,T$ the rigidity of the twisted

n the journals.

n of variable rotation, pp. 43, 103, 303, 1349, we shall
ulley, the sum of the moments of the exterior forces
forces of inertia.

Now, the sum of the moments of the exterior forces is $P r - T r - R r - f N . r'$. The sum of the moments of the forces of inertia answering to a velocity v_1 of angular velocity is easily found; for, one of these forces, relative to a molecule of the mass m, situated at a distance r_1 being $m . \frac{v_1 r_1}{t}$, its moment in respect to the axis is $m r_1^2 \frac{v_1}{t}$, and the sum of the similar moments is $I \frac{v_1}{t}$, for all parts turning around the axis.

The moment of inertia of the weight P is $\frac{P}{g} \frac{v_1 r}{t} r$, and must be added to the preceding; we have then, at each instant of variable motion of the pulley, the relation

$$P r - T r - R r - f N r' = I \frac{v_1}{t} + \frac{P}{g} \frac{v_1 r}{t} r.$$

The pressure N upon the axle of the pulley being the resultant of two perpendicular forces, the one horizontal equal to the tension T, the other vertical and equal to the weight P of the box, increased by the weight of the pulley, and diminished by the force of inertia $\frac{P}{g} \frac{v_1 r}{t}$, which is developed in the acceleration of the vertical motion of the weight P, and is opposed to its acceleration; we have then

$$N = \sqrt{\left(P + q - \frac{P}{g} \frac{v_1 r}{t}\right)^2 + T^2}.$$

Now, according to an algebraic theorem of Poncelet, the value of a radical of the form $\sqrt{a^2 + b^2}$, in which we know beforehand that $a > b$ is given to nearly $\frac{1}{25}$ by the formula $0 \cdot 96 a + 0 \cdot 4 b$. In applying it to the case in hand, where we have always $P + q - \frac{P}{g} \frac{v_1 r}{t} > T$, since the weight P exceeds the resistance T and the friction of the sled, we have to $\frac{1}{25}$ nearly

$$N = 0 \cdot 96 \left\{P + q - \frac{P}{g} \frac{v_1 r}{t}\right\} + 0 \cdot 4 \, T.$$

The relation of the equality of moments becomes then, in making $R = 0 \cdot 032 \, T$,

$$P r - T r - 0 \cdot 032 \, T r - 0 \cdot 96 f r' \left\{P + q - \frac{P}{g} \frac{v_1 r}{t}\right\} - 0 \cdot 4 f r' T = \frac{I v_1 r}{r t} + \frac{P}{g} \cdot \frac{v_1 r}{t} r,$$

and in deriving from this equation of the first degree the value of T, the tension of the horizontal strip of the cord, we find

$$T\left\{1 + 0 \cdot 032 + 0 \cdot 4 \frac{f r'}{r}\right\} = P\left\{1 - 0 \cdot 96 \frac{f r'}{r}\right\} - 0 \cdot 96 f q \frac{r'}{r} - \frac{P}{g} \frac{v_1 r}{t}\left\{1 - 0 \cdot 96 \frac{f r'}{r}\right\} - \frac{I}{r^2} \frac{v_1 r}{t}.$$

In substituting for the known quantities their values, which are

$$f = 0 \cdot 164, \quad r' = 0 \cdot 030512 \text{ ft.}, \quad r = 0 \cdot 36417 \text{ ft.}, \quad I = \cdot 04551,$$

whence $\frac{I}{r^2} = 0 \cdot 34317$, we have for the practical formula which gives the tension T, when we know the weight P of the box, $T = 0 \cdot 95 \left\{P - \left(\cdot 34685 + \frac{P}{g}\right) \frac{v_1 r}{t}\right\} - 0 \cdot 1753$ lb.

When experiment has demonstrated that the acceleration $\frac{v_1 r}{t}$ is constant, and the abstract of the curves, in giving their equation $T^2 = 2 \, C \, E$, shall have furnished for the acceleration the value $\frac{v_1 r}{t} = \frac{1}{C}$, in calling 2 C the parameter of the parabola, we shall have all the elements required to calculate the value of the tension of the cord in the experiment. It will be

$$T = 0 \cdot 95 \left\{P - \left(\cdot 34685 + \frac{P}{g}\right) \frac{1}{C}\right\} - \cdot 1753 \text{ lb.}$$

When the motion is uniform the acceleration $\frac{v_1 r}{t} = \frac{1}{C}$ is zero, and the above formula is reduced to $T = 0 \cdot 95 \, P - 0 \cdot 1753$ lb., or simply $T = 0 \cdot 95 \, P$, on account of the small value of the second term $\cdot 1753$ lb.

In extracting directly from the curves of tension of the dynamometer, the values of T relative to more than forty experiments, in which the loads have varied from 26 to 209½ lbs., we have found that the ratio of the tension to the load, thus furnishing a direct measurement, was at 0·96, which shows that all the data introduced in the above formula leads to a result which accords with this measure, within very satisfactory limits of correctness.

Relations between the Tension of the Cord and the Friction of the Sled.—Knowing the tension of the cord T, by means of the dynamometer, or having calculated it by the preceding formula, it is quite easy to deduce the value sought, of the friction of the sled, in applying directly the principle of action equal and opposite to reaction. In fact, the tension T, and the friction sought F, are two external forces with opposite directions, whose difference T — F produces the motion of acceleration of the sled. On the other hand, the resistance which the inertia of the weight Q of the sled opposes to this *acceleration* is $\frac{Q\, v_1\, r}{g\, t}$.

We have then for the equality of action and reaction, $T - F = \frac{Q}{g} \frac{v_1\, r}{t} = \frac{Q}{g} \cdot \frac{1}{C}$, whence

$$F = T - \frac{Q}{g} \cdot \frac{1}{C}.$$

When, by direct observation, or by the formula of the preceding number, we shall have determined the tension of the cord, we must for the value of the friction subtract from it the quantity $\frac{Q}{g} \frac{1}{C}$, easily calculated when we know by the abstract the parameter 2 C of the curve of motion.

Such is the method which was adopted for the calculation of all the experiments where the motion was accelerated; as for those where the motion is uniform, we have simply F = T.

We see that the law of the motion being once known by the abstract of the curves, and being that of a uniformly accelerated motion, we may, after having proven the constancy and the generality of this law, pass to the use of the dynamometer, and rest content with the indications of the chronometric apparatus.

Results of Experiments.—The friction of oak, sliding upon oak without unguent, with the fibres parallel to the direction of the motion.

In this experiment we have Q = 295·22 lbs.; P = 203·38 lbs.

The trace of the curve gives for the parameter 2 C = 0·6339 ft., whence $\frac{1}{C} = 3{\cdot}154$, and consequently the tension $T = 0{\cdot}95 \left\{P - \left(0{\cdot}34685 + \frac{P}{g}\right)\frac{1}{C}\right\} - {\cdot}1753 = 173{\cdot}05$ lbs.

The other formula gives for the value of friction $F = T - \frac{Q}{g} \cdot \frac{1}{C} = 144{\cdot}1$ lbs.

The ratio of the friction to the pressure is here then $\frac{F}{Q} = \frac{144{\cdot}1}{295{\cdot}2} = 0{\cdot}488$.

EXPERIMENTS UPON THE FRICTION OF OAK UPON OAK, WITHOUT UNGUENTS; THE FIBRES OF THE WOOD BEING PARALLEL TO THE DIRECTION OF MOTION.

Extent of Surface of Contact.	Pressure, Q.	Motive Weight during Motion, P.	Tension of the Cord, T.	Parameter.	Value of the Acceleration, $\frac{1}{C}$.	Friction, F.	Ratio of Friction to Pressure, $\frac{F}{Q}$.	Velocity of Motion. Uniform.	Velocity of Motion. Acceleration at 9·84 ft. of its Course.
sq. ft.	lbs.	lbs.	lbs.	feet.		lbs.		feet.	feet.
2·798	295·22	148·58	141·15	..	..	141·15	0·477	2·264	..
	295·22	203·38	173·02	0·634	3·123	144·1	0·488	..	7·77
	333·52	171·03	162·48	..	..	162·48	0·487	..	..
	970·63	504·32	479·10	..	..	479·44	0·491	1·345	..
	970·63	610·01	536·64	0·850	2·352	466·41	0·480	..	6·726
	1499·13	930·23	819·18	0·862	2·320	709·33	0·472	..	6·693
	2291·56	1273·69	1164·91	1·688	1·184	1080·60	0·471	..	6·299
	2291·56	1114·91	1059·16	..	..	1059·16	0·462	3·511	..
1·062	102·09	64·95	54·17	1·914	1·044	50·86	0·498	..	4·495
	108·53	56·59	53·77	..	..	53·77	0·496	4·20	..
	120·55	62·90	59·75	..	..	59·75	0·495	4·92	..
	120·55	98·39	76·44	0·384	5·208	56·95	0·472	..	10·072
	226·81	186·83	152·57	0·472	4·237	110·38	0·486	..	8·924
	227·63	132·64	117·77	1·054	1·897	104·36	0·458	..	6·102
	332·76	162·72	154·58	..	..	154·58	0·464	4·101	..
	440·03	211·37	200·80	..	..	200·84	0·456	2·001	..
	440·24	210·45	199·93	..	..	199·93	0·454	2·789	..
0·33	215·67	108·62	103·19	..	..	103·19	0·478	3·478	..
	321·47	175·49	164·74	0·933	2·145	133·34	0·414	..	6·918
	604·06	468·80	389·44	0·506	3·952	293·51	0·484	..	8·858

When the motion is uniform, as in the sixteenth experiment of the above Table, we have simply for

$$Q = 440{\cdot}37 \text{ lbs.}, \quad P = 211{\cdot}37 \text{ lbs.},$$

$$F = 0{\cdot}95\,P = 200{\cdot}84, \quad f = \frac{F}{Q} = \frac{200{\cdot}84}{440{\cdot}37} = 0{\cdot}456.$$

EXPERIMENTS UPON THE FRICTION OF ELM UPON OAK, WITHOUT UNGUENTS; THE FIBRES OF THE WOOD BEING PARALLEL TO DIRECTION OF MOTION.

Surface of Contact.	Pressure, Q.	Motive Weight during Motion, P.	Tension of the Cord during Motion, T.	Parameter, 2 C.	Accelera-tion, $\frac{1}{C}$.	Friction.	Ratio of Friction to Pressure, f.	Velocity at 9·84 ft. of its Course.
sq. ft.	lbs.	lbs.	lbs.	feet.		lbs.		feet.
1·338	260·05	161·31	139·19	0·732	2·734	117·18	0·45	7·55
	260·05	187·42	153·06	0·469	4·261	118·88	0·45	9·45
	921·38	506·69	450·40	0·984	2·031	392·27	0·43	3·60
	921·38	480·24	440·46	1·859	1·075	408·73	0·44	4·62
	921·38	454·13	416·77	1·902	1·051	386·62	0·42	4·48
	921·38	664·42	525·72	0·377	5·291	374·53	0·41	12·46
	1980·10	1113·83	976·94	0·802	2·494	821·76	0·42	7·41
	1980·10	1007·77	927·09	1·993	1·003	865·54	0·44	4·04
	1980·10	1113·83	911·42	1·206	1·657	787·42	0·40	5·68
	1980·10	1298·70	1104·86	0·600	3·330	899·99	0·45	8·10
·063	244·81	135·42	122·36	1·414	1·414	108·93	0·45	5·25
	389·58	311·19	240·06	0·347	5·756	171·43	0·44	10·50
	917·79	479·76	439·82	1·734	1·153	408·60	0·44	4·76
						Mean ..	0·434	

EXPERIMENTS UPON THE FRICTION OF SOFT OOLITIC LIMESTONE OF JAUMONT, NEAR METZ, UPON STONE OF THE SAME KIND, WITHOUT UNGUENT

Surface of Contact.	Pressure, Q.	Motive Weight during Motion, P.	Tension of the Cord during Motion, T.	Parameter, 2 C.	Accelera-tion, $\frac{1}{C}$.	Friction, F.	Ratio of Friction to Pressure, $\frac{F}{Q}$.	Velocity of 9·84 ft. of its Path.
sq. ft.	lbs.	lbs.	lbs.	feet.		lbs.		feet.
0·861	314·04	254·18	222·40	0·829	2·412	198·89	0·633	6·890
	314·04	254·18	218·36	0·682	2·929	187·60	0·597	7·579
	1264·18	999·63	853·54	0·621	3·216	727·50	0·575	7·940
	1374·94	1034·92	859·41	0·499	4·001	700·86	0·549	8·858
	1274·94	1034·92	859·41	0·499	4·001	700·86	0·549	8·858
						Mean ..	0·580	
0·499	309·56	293·88	245·40	0·536	3·725	209·56	0·677	8·498
	331·62	293·88	244·65	0·524	3·815	207·97	0·627	8·662
	1257·50	1034·92	925·13	1·066	1·874	851·95	0·677	6·070
	1257·50	1140·78	943·99	0·488	4·101	783·89	0·623	8·990
	1257·50	1140·78	924·32	0·426	4·687	741·28	0·589	9·613
						Mean ..	0·639	
Rounded edges.	298·40	240·95	218·30	1·426	1·402	205·31	0·688	5·249
	298·40	240·95	211·02	0·841	2·377	189·01	0·633	6·824
	298·40	293·88	239·18	0·451	4·433	198·10	0·664	9·350
	597·93	465·91	421·45	1·341	1·491	393·79	0·659	5·413
	597·93	465·91	431·15	2·499	0·800	416·28	0·696	3·970
	597·93	571·77	485·40	0·597	3·347	423·25	0·709	8·104
						Mean ..	0·675	
					General	Mean ..	0·631	

When the soft limestone slides upon soft limestone, and especially when the moving body rests upon surfaces of small area, the latter are destroyed rapidly during the experiment. This circumstance, and the presence of the dust powder resulting from it, have not changed the laws observed.

Though leather is a soft and very compressible substance, its friction is proportional to the pressure, and independent of the velocity, throughout the whole range of the experiments in the next Table.

EXPERIMENTS UPON THE FRICTION OF STRONG LEATHER, TANNED, AND PLACED FLATWISE UPON CAST IRON.

Area of Surfaces in Contact.	Nature of the Unguent.	Pressure.	Motive Weight during the Motion.	Tension of the Cord.	Parameter.	Value of the Acceleration, $\frac{1}{C}$.	Friction.	Ratio of Friction to Pressure.	Velocity at 9·84 ft. of its Course.
sq. ft.		lbs.	lbs.	lbs.	feet.		lbs.		feet.
0·4155	Nothing.	471·02	320·35	291·83	1·548	1·292	272·75	0·579	5·02
		1106·42	637·94	606·04	..	..	606·04	0·547	..
							Mean ..	0·563	
0·4155	Water.	291·01	188·02	154·78	0·497	4·024	118·63	0·408	8·86
		291·01	161·55	133·85	0·524	3·816	96·44	0·342	8·66
		291·01	135·08	118·83	0·926	2·159	99·49	0·342	6·56
		1115·03	977·58	689·54	0·244	8·196	407·11	0·365	12·70
							Mean ..	0·364	
0·4155	Tallow.	1114·10	214·48	193·80	2·042	0·979	163·21	0·146	4·53
		1114·10	214·48	198·54	2·584	0·776	172·38	0·155	3·87
		1114·10	320·36	279·52	0·795	2·516	192·43	0·172	7·09
		1114·10	426·10	350·41	0·475	4·210	182·99	0·164	9·06
							Mean ..	0·159	
0·4155	Oil.	298·49	39·26	37·30	..	..	37·29	0·124	..
		299·17	92·19	76·29	0·548	3·649	42·52	0·142	8·46
		1114·10	148·32	140·91	..	..	140·91	0·126	..
		1114·10	214·48	196·22	1·804	1·108	157·93	0·141	4·59
							Mean ..	0·133	
	Oily surface.	1114·10	320·35	294·48	2·011	0·944	260·07	0·233	4·66
		478·92	135·08	123·77	1·950	1·025	108·66	0·227	4·66
							Mean ..	2·30	

EXPERIMENTS UPON THE FRICTION OF BRASS UPON OAK, WITHOUT UNGUENT; FIBRES OF WOOD PARALLEL TO THE DIRECTION OF MOTION.

Surface of Contact.	Pressure.	Motive Weight during Motion.	Tension of the Cord.	Parameter.	Acceleration, $\frac{1}{C}$.	Friction.	Ratio of Friction to Pressure.	Velocity at 9·84 ft. of Course.
sq. ft.	lbs.	lbs,	lbs.	feet.		lbs.		feet.
·433	257·13	161·46	153·36	..	..	153·39	0·60	..
	257·13	161·61	153·61	..	..	153·54	0·60	..
	1539·90	981·99	932·69	..	..	932·89	0·60	..
	1539·90	1114·32	1068·80	1·548	1·291	1007·79	0·65	..
	1539·90	1273·11	1101·97	0·707	2·828	967·05	0·62	7·48
	1989·83	1378·97	1290·72	4·346	0·460	1262·61	0·63	3·05
0·141	248·31	161·72	153·60	..	..	153·61	0·61	..
	248·49	188·36	169·56	1·283	1·558	157·58	0·63	5·21
	763·97	532·07	487·11	1·956	1·022	462·99	0·60	4·92
	1531·26	981·89	932·69	..	..	932·89	0·61	..
	1531·26	1273·11	1103·69	0·719	2·780	971·73	0·63	7·51
						Mean ..	0·616	

For the experiments where we have not indicated the value of the parameter of the law of motion, and that of the acceleration, the motion was slow and somewhat uncertain.

The results contained in this Table confirm the three laws before enumerated, but we remark that the mean value of the friction, which is here 616, is more considerable than in the case of oak rubbing against oak, or than that of elm upon oak, for which the results are consigned to the Tables of pages 1572 and 1573.

We shall see, by the following Table, that the coefficient diminishes considerablv when the friction occurs between two metallic surfaces

EXPERIMENTS UPON THE FRICTION OF CAST IRON UPON CAST IRON.

Surfaces of Contact.	Unguent.	Pressure, Q.	Motive Weight during the Motion.	Tension of the Cord during the Motion.	Parameter, 2 C.	Acceleration, $\frac{1}{C}$.	Friction.	Ratio of Friction to Pressure.	Velocity at 9·84 ft. of Path.	
sq. ft.		lbs.	lbs.	lbs.	feet.		lbs.		feet.	
0·3874	Noth.	496·10	108·62	95·78	0·993	2·012	64·49	0·130	6·37	
		496·10	135·09	113·38	0·585	3·417	60·79	0·122	8·20	
		1091·14	320·37	283·32	0·938	2·130	211·15	0·193	6·50	
		1091·14	426·08	336·38	0·378	5·291	157·18	0·144	10·17	
		1104·80	174·79	166·05	..	..	166·05	0·150	..	Slow.
		4412·70	796·73	745·58	4·267	0·468	681·74	0·154	8·25	
		4412·70	929·06	865·85	3·316	0·604	783·54	0·177	8·48	
		4412·70	1054·77	949·52	1·158	1·726	712·94	0·161	5·81	
							Mean ..	0·154		
0·3874	Water.	1104·37	399·74	361·17	1·402	1·426	812·32	0·282	8·90	
		1104·37	505·61	432·96	0·646	3·095	324·60	0·293	9·25	
		2202·70	770·26	731·36	..	..	731·36	0·332	..	Uniform.
		2202·70	876·13	806·43	2·036	0·982	739·30	0·336	4·53	
							Mean ..	0·311		
0·3874	Soap.	1091·14	201·25	191·15	..	..	191·15	0·175	..	Slow.
		1091·14	320·37	287·77	1·251	1·598	231·00	9·211	5·68	
		1091·14	373·30	321·78	0·695	2·878	224·47	0·205	7·09	
							Mean ..	0·197		
0·3874	Tallow.	496·10	52·49	49·87	..	..	49·87	0·100	..	Slow.
		496·10	78·96	65·48	1·950	1·024	50·40	0·101	4·56	
		1103·43	108·64	103·17	..	..	103·17	0·093	..	Slow.
		1103·43	201·25	179·20	1·060	1·885	114·64	0·104	6·17	
		1103·43	240·95	212·87	0·939	2·130	117·81	0·106	6·47	
		2214·98	293·88	271·14	2·286	0·875	211·30	0·095	4·20	
		2214·98	293·88	274·54	4·023	0·497	243·34	0·109	3·08	
		2214·98	426·10	379·70	0·999	2·000	243·33	0·109	6·30	
		6185·82	624·70	593·47	..	..	593·47	0·096	..	Very slow.
		1108·14	108·62	103·17	..	..	103·17	0·093	..	Slow.
							Mean ..	0·101		
0·3874	Lard.	1103·43	129·48	118·70	2·011	0·994	84·62	0·076	4·53	
			129·48	118·13	1·767	1·131	79·44	0·071	4·72	
			133·89	121·19	1·395	1·432	72·16	0·065	5·61	
			133·89	120·99	1·414	1·414	72·54	0·066	5·58	
			138·31	126·29	1·767	1·131	85·82	0·077	4·59	
			138·31	124·41	1·295	1·544	71·55	0·065	5·51	
			138·35	123·55	1·341	1·491	72·71	0·066	5·44	
			138·35	124·41	1·295	1·544	71·55	0·065	5·51	
			193·44	168·15	0·783	2·553	80·65	0·073	7·12	
			193·44	167·07	0·731	2·734	72·94	0·066	7·28	
			193·44	168·92	0·823	2·430	85·68	0·078	6·82	
							Mean ..	0·070		

This Table, besides verifying the laws of the proportionality of the friction to the pressure, and its independence of the velocity, shows that water rather increases than diminishes the friction of cast iron. We see also that tallow, somewhat hard, does not reduce the friction so much as lard.

Consequences of the Experiments.—The experiments made by Morin upon the friction proper of plane surfaces upon each other comprise 179 series, answering to different cases, according to the nature or condition of the surfaces in contact; and they all, without exception, lead to the following results;—

The friction during the motion is—

1st. Proportional to the pressure.

2nd. Independent of the area of the surfaces of contact.

3rd. Independent of the velocity of motion.

Experiments upon the Friction at Starting, or when the Surfaces have been some time in contact.—The same apparatus has served for the experiments upon friction at the start, or after a prolonged con-

tact, whose aim was to establish in what cases there is a notable difference between it and that produced during motion. This difference, which, according to the case, arises from very different causes, may in general be attributed to the reciprocal compression of the bodies upon each other, and to a kind of gearing of their elements. The time or duration of the compression probably exerts an influence upon the intensity of the resistance opposed by their surfaces to sliding. But generally this resistance reaches its maximum at the end of a very short period.

EXPERIMENTS UPON THE FRICTION OF OAK UPON OAK, WITHOUT UNGUENTS, WHEN THE SURFACES HAVE BEEN SOME TIME IN CONTACT; THE FIBRES OF THE SLIDING PIECES BEING PERPENDICULAR TO THOSE OF THE SLEEPER.

Extent of the Surface of Contact.	Pressure, Q.	Motive Effort or Friction, F.	Ratio of the Friction to the Pressure, f.
sq. ft.	lbs.	lbs.	
0·947	120·55	67·15	0·55
	282 49	150·23	0·53
	495·01	252·34	0·51
	1995·23	1171·10	0·58
	2526·65	1287·16	0·51
0·043	389·35	203·80	0·52
	402·98	212·44	0·53
	1461·08	854·77	0·58
		Mean ..	0·54

The friction seems to be proportional to the pressure, which varied from 120 lbs. to 2526 lbs., and independent of the surfaces of contact, which varied in the ratio of 1 to 22, the smallest being ·043 sq. ft., and the greatest 0·947 sq. ft.; this last value exceeds those usually employed for sliding surfaces in mechanical constructions.

The ratio of the friction to the pressure is here raised to 0·54, while it was only 0·48 during the motion, as was the result of the Table, page 1572. The friction at the start is raised then about an eighth above that which we first considered. A similar increase occurs in all similar cases.

EXPERIMENTS UPON THE FRICTION OF OAK UPON OAK, WITHOUT UNGUENTS, WHEN THE SURFACES HAVE BEEN SOME TIME IN CONTACT. THE SLIDING PIECES HAVE THEIR FIBRES VERTICAL, THOSE OF THE FIXED PIECES ARE HORIZONTAL AND PARALLEL TO THE DIRECTION OF MOTION.

Extent of the Surface of Contact.	Pressure, Q.	Motive Effort or Friction, F.	Ratio of Friction to Pressure, f.	Time of Contact.
sq. ft.	lbs.	lbs.		
·6845	432·12	184·88	0·427	5 to 6″
	432 12	184·88	0·427	10′
	432·12	157·43	0·364	1′
	696·77	354·59	0·509	6′
	696·77	304·31	0·436	30″
	696·77	342·03	0·498	8 to 10′
	882·01	405·32	0·459	8 to 10′
	1106·99	555·73	0·502	10′
	1106 99	430·03	0·388	5 to 6″
	2205·30	810 24	0·367	15′
	2205·30	882 60	0·400	10′
		Mean ..	0 434	

This Table shows that for wood the friction at the start presents for equal surfaces and pressures great differences from one experiment to another, and that the resistance attains its maximum in a short time of contact, which seems not to exceed some seconds. We, in fact, see that the figures answering to five and six seconds are not inferior to those relating to a contact of fifteen minutes, the longest of any recorded in the Table.

The mean value of the ratio f of friction to the pressure is 0·434, but it would be well in application to reckon it at 0 48 or even 0 50.

We still see by these experiments, in the following Table, that the friction at starting, as well as the friction in motion, is independent of the extent of the surface of contact, and is proportional to the pressures.

These figures, moreover, differ so little from each other, that we may place all confidence in the general mean 0·74, and employ it in all similar cases.

EXPERIMENTS UPON THE FRICTION OF OOLITIC LIMESTONE UPON OOLITIC LIMESTONE, WHEN THE SURFACES HAVE BEEN FOR SOME TIME IN CONTACT.

Surface of Contact.	Pressure, Q.	Motive Effort or Friction, F.	Ratio of Friction to the Pressure, f.	Time of Contact.
sq. ft.	lbs.	lbs.		
0·8611	314·01	228·88	0·728	15′
	330·85	239·25	0·723	15′
	1162·72	949·64	0·752	15′
	1274·93	932·87	0·731	5 to 6″
	1274·93	958·02	0·751	5 to 6″
		Mean ..	0·737	
0·4992	309·55	228·88	0·739	2′
	1257·49	983·16	0·781	10′
	1257·49	983·16	0·781	1′
		Mean ..	0·783	
Edges rounded.	298·38	228·88	0·774	2′
	602·32	442·58	0·740	5 to 6″
		Mean ..	0·757	
	General Mean ..		0·740	

EXPERIMENTS UPON THE FRICTION OF OOLITIC LIMESTONE UPON OOLITIC LIMESTONE, WHEN THE SURFACES HAVE BEEN SOME TIME IN CONTACT WITH A BED OF FRESH MORTAR.

Surface of Contact.	Pressure, Q.	Motive Effort or Friction, F.	Ratio of Friction to Pressure, f.	Time of Contact.
sq. ft.	lbs.	lbs.		
0·8611	325·66	253·98	0·780	10′
	506·08	404·87	0·800	10′
	783·98	580·87	0·740	15′
	783·98	608·22	0·773	10′
	783·98	555·73	0·709	10′
	1167·73	983·16	0·841	15′
		Mean ..	0·773	
0·4992	309·55	239·21	0·772	10′
	489·97	379·74	0·775	10′
	781·10	568·30	0·727	10′
	1164·86	807·15	0·792	15′
	1164·86	907·74	0·779	10′
	1169·27	807·15	0·690	10′
	1548·61	1159·17	0·748	15′
		Mean ..	0·745	
0·1636	319·82	254·02	0·794	10′
	500·25	304·30	0·608	10′
	791·37	480·26	0·607	10′
	1161·90	731·69	0·629	15′
		Mean ..	0·659	
	General Mean		0·735	

These experiments show that the friction at starting is for these stones very nearly the same with the interposition of mortar as without.

In recapitulating, recent trials have caused us to see that the friction at the moment of starting, and after a very short time of contact, is—

1st. Proportional to the pressure.

2nd. Independent of the area of the surfaces of contact; and that furthermore, for compressible bodies, it is notably much greater than that which takes place during motion.

Observation relative to the Expulsion of Unguents under Heavy Pressures, and by Prolonged Contact.—We have observed metallic bodies with unguents of grease or oil, under very great pressure, compared to their surfaces, and find, after a contact of some duration, that the unguents are expelled, so that the surfaces are simply in an unctuous state, and thus have double the friction of surfaces well greased. This shows us why the effort required to put certain machines in motion is, disregarding the influence of inertia, often much greater than that required for maintaining a rapid motion, and proves that, for an experimental appreciation of the friction of machines in motion, we need not, as is sometimes done, make use of the same methods as for machines starting from repose.

Influence of Vibrations upon the Friction at Starting.—Another remarkable circumstance noted in the experiments at Metz is, that when a compressible body is solicited to slide by an effort capable of overcoming the friction of motion, but inferior to the friction at starting, a simple vibration, produced by an external and apparently a slight cause, may determine the motion. Thus, for oak rubbing on oak, the friction at starting is 0·680 of the pressure, and the friction during motion is 0·480; so that, to produce the motion of a weight of 2205 lbs. it is necessary then to exert an effort of 1500 lbs., while there is only needed 1059 lbs. to maintain it. Still, under an effort equal, or a little above 1059 lbs., and by the effort of a vibration, the body may be started.

This important observation applies to constructions always more or less exposed to vibrations, and shows that, if in the calculations for machines for producing motion, we should take into account the greatest value of the friction, we should in those relating to the stability of constructions, on the other hand, introduce its smallest value, that for motion. It seems, finally, to explain how it sometimes happens that buildings, for the stability of which no uneasiness was felt, have fallen at the passing of a wagon, and how the firing of salutes from a breach battery may, at certain times, accelerate the fall of a rampart or a building.

Influence of Unguents.—Fat unguents considerably diminish friction, and the consequent wear of surfaces. But from the observations made, p. 1575, we see that though the friction is in itself independent of the extent of the surfaces, it is well to proportion them to the pressures they are appointed to sustain, so that the unguents may not be expelled. We would also remark that all the experiments in consideration were made under pressures more or less considerable, and their results should only be applied to analogous cases. In fact, we may conceive that if the pressures were so great, in respect to the surfaces, as to occasion a marked defacement, the state of the surfaces, and consequently the friction, would vary; or that, on the contrary, if the surfaces were great, and the pressures very slight, the viscosity of the unguents, usually disregarded, might then exert a sensible influence.

We would observe that, in general, and especially for metals, pure water is a bad unguent, and often increases rather than diminishes the friction.

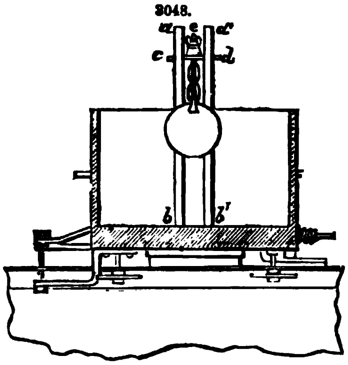
Adhesion of Mortar and Solidified Cements.—But, for mortars which have set and acquired a proper degree of dryness, there exists a different condition of things. Adhesion and cohesion take the place of friction, and the resistance to separation becomes sensibly proportional to the extent of the surface of contact, and independent of the pressure exerted, either at the moment of rest or that of separation.

For limestones bedded with mortar of hydraulic lime of Metz, the resistance is about 2112 lbs. per square foot of surface. With other limes, undoubtedly common, M. Boistard has found 1426 lbs. With plaster, the resistance seems to follow the same law; but it varies considerably with the instant of the setting of the plaster, which seems to exert a great influence upon the cohesion.

Observation upon the Introduction of Friction and Cohesion in Calculations upon the Stability of Constructions.—Finally, we would remark that friction cannot, in the case of beddings in mortar, or in plaster, show itself until the cohesion or adhesion is overcome, and that consequently these two resistances cannot coexist. In calculations upon the stability of structures, we should only reckon upon one of these, and that the weakest.

Experiments upon Friction during a Shock.—Poisson, in his Traité de Mécanique, expresses himself in these terms;—"Though no observations have been made upon the intensity of friction during a shock, we may suppose, by induction, that it follows the general laws of friction of bodies subjected to pressures, since percussion is only a pressure of very great intensity exerted during a very short time."

3048.



To verify by direct observation the correctness of this supposition, and at the express invitation of Poisson, Morin undertook many experiments, choosing for that purpose the case of strips of cast iron sliding upon bars of cast iron spread with lard, since this had been the subject of careful study in his preceding experiments, and is the case which most frequently occurs in practice.

Description of the Apparatus employed in the Experiments.—The apparatus which Morin employed differs from that described in p. 1570, only in the following disposition necessary for suspending to the sled, at a desired height, the body designed to produce the shock, and allowed to fall at will during the motion.

Upon the sides of the box of the sled, Fig. 3048, are raised two frames of firm uprights *a b* and *a′ b′*, pierced with holes at intervals of ·16 ft., through which pass two iron pins; upon these pins rests a movable cross-piece *c d* of oak. By raising and lowering the pins, the height of the cross-piece *c d* above the sled may be varied at will. A screw *e* and nut passes freely across a hole cut in the middle of the cross-piece, and

bears a plier with ring legs, upon which is suspended a shell to give the shock. The two legs of the pliers are bound with strips of wick with quick match, holding them shut. By means of the screw e the height of the shell above the surface shocked can be exactly regulated.

We may easily conceive from this description, the box and uprights being firmly fastened to the sled, that the whole system partakes of a common motion, and that if at any instant of its course the shell falls upon the sled, it falls there with a vertical velocity due to the height of the fall, and with a horizontal velocity which, as we shall see hereafter, was sensibly the same as that of the sled. By means of the ligature of the legs of the pliers we accomplish the fall of the shell, without any external concussion or disturbance. For this purpose a man sets fire to the match, and gives the signal for the starting of the sled; combustion is communicated to the upper part which keeps the pliers closed; these open suddenly and let loose the shell, without any possibility of disturbing the common motion of the system of the two bodies.

General Circumstances of the Experiments.—The experiments were made in impressing the sled, sometimes with a uniform, and sometimes with an accelerated motion. The first of these motions was obtained at will, by giving to the descending box a weight just sufficient to overcome the friction, and in suspending under this box a shell of 110 lbs. weight, which only descended 1·64 ft. when its action ceased. As for the accelerated motion, it was produced whenever the motive weight surpassed the friction. The law of these motions was moreover determined, in each case, by means of curves traced by the style of the chronometric apparatus.

General Examination of what occurred in these Experiments.—We can readily appreciate the mode of action during the experiments. We take, for example, a case where the system of the sledge and the shell suspended above it is impressed with a uniform motion. At the instant when the combustion of the wick allows the legs of the pliers to separate, the shell is free, and falls; during its fall, until the moment it reaches the sledge, the latter being freed from the weight of the shell, acquires an amount of motion precisely equal to what would be consumed by the friction due to this weight. The horizontal velocity of the sledge, at the instant of the shock, is then a little greater than that of the shell. After this the forces of compression developed by the shock produce a friction variable as themselves, at each instant, which consumes a certain quantity of motion; so that the sledge, whose progress was accelerated during the fall of the shell, is afterwards retarded during the action of the shock.

Formulæ employed in Calculating the Results of the Experiments.—As it is desirable to prove whether the friction remained proportional to the variable pressures produced during the short intervals of the phenomena, we proceed to give some formulæ relative to this hypothesis, which we will hereafter compare with the results of experiment. We consider first the case of uniform motion, and call

> Q the weight of the sledge, and the suspending apparatus of the shell;
> q the weight of the shell producing the shock;
> f the ratio of friction to the pressure for the surfaces in contact;
> h the height of fall of the shell above the sledge;
> U the velocity due to this height;
> T the time of the fall;
> V the horizontal velocity of the sledge and shell at the instant when the latter is let loose by the pliers;
> V' the velocity of the body after the shock;
> $g = 32{\cdot}1817$ ft.

At the instant when the shell is freed, the quantity of motion of the system is $\frac{Q+q}{g}V$.

The weight of the shell, when connected with the sledge, produces a friction fq which, in each element of time t, consumes a quantity of motion fqt, and which, during the time of the fall, would consume the quantity fqT.

But since, on the other hand, the shell ceases to press upon the sledge during this time, it follows that the quantity of motion gained by the system by reason of this diminution of pressure, is precisely fqT.

At the instant when the shell reaches the sledge, the quantity of motion possessed by the system is then $\frac{(Q+q)V}{g} + fqT$.

From this instant, and during the whole period of the shock, the shell loses, in each element of time, an element of velocity, and consequently a quantity of motion $\frac{q}{g}u$, whence results a force of compression $\frac{q}{g} \times \frac{u}{t}$, producing a friction $\frac{fq}{g} \times \frac{u}{t}$. This friction consumes in an element of time a quantity of motion $\frac{fq}{g} \cdot \frac{u}{t}$, and when all relative motion in a vertical direction is destroyed, the friction due to the forces of compression has finally consumed a quantity of motion equal to $\frac{fq}{g}U$.

Consequently, when the shock has terminated, we should have between the quantities the relation $\frac{(Q+q)V}{g} + fqT - \frac{fqU}{g} = \frac{Q+q}{g}V'$, or $fqgT - fqU = (Q+q)(V'-V)$.

Now, the shell falling with a uniformly accelerated motion by virtue of gravity, we have, evidently, $U = gT$, whence it follows that $V = V'$; that is to say, that in our apparatus the quantity of motion destroyed by the friction resulting from the forces of compression must be precisely equal to that which it gains during the fall of the shell.

These two effects are successive, but take place in a short interval of time, and therefore occasion in the curve of motion undulations in opposite directions, which do not affect the general law, and are scarcely appreciable, either in the draughted curve or that made from the abstract of the Table.

The Acceleration of the Motion of the Sledge during the Fall of the Shell may be neglected.—It is easy to be assured *a priori* that the acceleration of the velocity of the sledge during the fall of the shell was always very small in our experiments, though the height of the fall has reached 1·97 ft. We observe, then, from what has just been said, that calling V_1 the horizontal velocity of the sledge at the moment when the shell reaches it, we shall have $\frac{Q}{g} V + f q T = \frac{Q}{g} V_1$, whence

$$V_1 - V = \frac{f q g T}{Q} = \frac{f q U}{Q}.$$

Making, for example, $q = 110·27$ lbs., $h = 1·968$ ft., and $U = 13·80$ ft., $Q = 590·68$ lbs., $f = 0·071$, which answers to one of the most intense shocks produced during the experiments, we find $V_1 - V = 0·1829$ ft.

Now, the shock of the shell in the horizontal direction taking place only in virtue of this difference in velocity, we see that its effect upon the general motion should be quite insensible, and we may, as we have done in the preceding calculation, neglect its influence upon the general motion of the sledge.

Case where the Motion of the Sledge is Accelerated.—The preceding reasoning applies to the case where the system of the shell and of the sledge is impressed with an accelerated motion, and it follows that if, as we have admitted, the friction during the shock remains proportional to the pressure, the general law of motion in our apparatus cannot be affected; or, in other words, that if, before the fall of the shell, the motion is uniform or accelerated, according to a certain law, it will still be so after the shock, according to the same law. The only disturbance which will result will be sometimes manifested by undulations, which, in most cases, would hardly be appreciable. Moreover, the hardness or compressibility of the body in contact should not have any influence upon the result, and in causing the shell to fall upon the beechwood joists composing the sledge, or upon a mass of soft loam placed upon it, we should, for circumstances otherwise similar, find the same law of motion, which should be the same as though there had been no shock.

Results of Experiments.—It remains now for us to compare the results of the formulæ with those of experiments which have been made, some when the sledge was impressed with a uniform motion, and some when the motion was accelerated. In these experiments we have varied the weight of the shells imparting the shock from 26·43 lbs. to 110 lbs., or nearly 1 to 4; the ratio of the weight of the body imparting the shock to that of the body shocked, from $\frac{1}{18}$ to $\frac{1}{4}$, and the height of the fall from 0·328 ft. to 2·29 ft., or from 1 to 7. The shock was produced upon wood, and upon loam placed upon the sledge. If, then, the laws which we have admitted in the preceding formulæ are verified by experiments within such extended limits, we may conclude that they subsist for pressures developed during the shock, as well as for others without shocks.

Experiments upon the Friction of Cast Iron upon Cast Iron, with an Unguent of Lard during the Shock.

Weight of the Sledge.	Weight of the Sphere.	Total Pressure, Q+q.	Height of Fall of the Sphere, h.	Motive Weight during Uniform Motion.	Friction, F.	Ratio of Friction to Pressure, f.	Velocity of Uniform Motion.	Remarks.
lbs.	lbs.	lbs.	feet.	lbs.	lbs.		feet.	
			0·328				2·761	
			0·328				2·624	
492·30	26·42	518·72	0·328	41·312	39·264	0·075	2·715	No shock.
			0·984				2·643	
			0·984				2·682	
	26·42	504·68	1·968	37·708	35·832	0·071	2·460	-
	26·42	504·68	1·968	37·708	35·832	0·071	2·558	
	55·13	533·39	0·984	40·887	38·843	0·072	2·534	
	55·13	533·39	0·984	40·887	38·843	0·072	2·678	
	55·13	533·39	0·984	40·887	38·843	0·072	2·755	
	55·13	533·39	1·968	40·887	38·843	0·072	2·797	
	55·13	533·39	1·968	40·887	38·843	0·072	2·659	
478·26	55·13	533·39	1·968	40·887	38·843	0·072	2·624	No shock.
	55·13	533·39	1·968	40·887	38·843	0·072	2·656	
	55·13	533·39	2·952	40·887	38·843	0·072	2·672	
	55·13	533·39	2·952	40·887	38·843	0·072	2·814	No shock.
	110·27	588·53	0·984	44·946	42·698	0·072	3·033	
	110·27	588·53	1·968	44·946	42·698	0·072	2·961	
	110·27	588·53	1·968	44·946	42·698	0·072	3·043	

Note.—The shock is produced by the fall of a cast-iron sphere upon beech joists, while the system slides with a uniform motion.

EXPERIMENTS UPON THE FRICTION OF CAST IRON UPON CAST IRON, WITH AN UNGUENT OF LARD DURING THE SHOCK.

Weight of the Sledge.	Weight of the Sphere.	Total Pressure, Q + q.	Height of Fall of the Sphere, *h*.	Motive Weight during Uniform Motion.	Friction, F.	Ratio of Friction to Pressure, *f*.	Velocity of Uniform Motion.	Remarks.
lbs.	lbs.	lbs.	feet.	lbs.	lbs.		feet.	
			0·98				2·829	
			0·98				2·744	
			..				2·427	No shock.
			..				2·460	No shock.
			1·96				2·576	
590·68	55·12	646·83	..	48·16	45·94	0·071	2·935	No shock.
			2·95				2·547	
			2·95				2·347	
			2·95				2·702	
			0·98				2·328	
			0·98				2·853	
590·68	110·24	700·96	1·97	52·36	49·74	0·071	2·675	
							2·853	No shock.

Note.—The shock is produced by the fall of a sphere upon a mass of loam, while this mass and the sledge slide with a common uniform motion.

We see by these Tables that the velocity of uniform motion has been the same in the experiments made with the shocks as in those without them, whatever may have been the height of the fall. This velocity, in all cases, has depended solely upon the load or total pressure of the motive weight and the state of the surfaces.

An examination of the curves of motion shows from the vibrations produced by the shock throughout the apparatus—which are felt even at the style—in what place the shock was produced, and whether it occurred in the period of its course, when the motion had become uniform, or in that when it was accelerated, the draughted curve and the abstract of the Tables afford but slight undulations, and the motion remains or becomes uniform with the same velocity.

Finally, these experiments show that in the shock the frictions due to the pressures developed are still proportional to these pressures and independent of the velocity.

Friction of Journals.—Besides the experiments previously reported upon the friction of plane surfaces, Morin has made a great number upon that of journals by means of a rotating dynamometer with a plate and style.

The axle of this dynamometric apparatus was hollow and of cast iron. It could receive, by means of holders exactly adjusted, a change of journals of different materials and diameters. Its load was composed of solid cast-iron discs weighing 331 lbs. each, whose number could be increased so as to attain a load of more than 3042 lbs. A pulley, the friction of whose axle was slight, and which transmitted the motion by the intervention of a spring, received by a belt, the motion of a hydraulic wheel, and the difference of tension of the two parts of the belt was measured by the dynamometer with the style.

Journals were from ·11 to ·22 ft. in diameter. The velocities varied in the ratio of 1 to 4. The pressures reached 4145 lbs., and within these extended limits we have proved that the friction of journals is subject to the same laws as that of plane surfaces. But it is proper to observe that from the form itself of the rubbing body the pressure is exerted upon a less extent of surface, according to the smallness of the diameter of the journal, and that unguents are more easily expelled with small than with large journals.

This circumstance has a great influence upon the intensity of friction, and upon the value of its ratio to the pressure. The motion of rotation tends of itself to expel certain unguents, and to bring the surfaces to a simply unctuous state. The old mode of greasing, still used in many cases, consisted simply in turning on the oil, or spreading the lard or tallow upon the surface of the rubbing body, and in renewing the operation several times in a day.

We may thus, with care, prevent the rapid wear of journals and their boxes; but, with an imperfect renewal of the unguent, the friction may attain ·07, ·08, or even ·1 of the pressure.

If, on the other hand, we use contrivances which renew the unguents without cessation in sufficient quantities, the rubbing surfaces are maintained in a perfect and constant state of lubrication, and the friction falls as low as ·05 or ·03 of the pressure, and probably still lower. The polished surfaces operated in these favourable conditions became more and more perfect, and it is not surprising that the friction should fall far below the limits above indicated.

These reflections show how useful are oiling fixtures in diminishing the friction, which, in certain machines, as mills with complicated mechanism, consume a considerable part of the motive work. We cannot, then, too much recommend the use of appliances to distribute the unguent continuously upon the rubbing surfaces of machines, and it is not surprising that a great number of dispositions have been proposed for this purpose within a few years. We should be careful to select those which only expend the oil during the motion, excluding those which feed by the capillary action of a wick of thready substances. These constantly drain the oil even during the repose of the machine, thus consuming it at a pure loss.

EXPERIMENTS UPON THE FRICTION OF CAST-IRON JOURNALS UPON CAST-IRON BEARINGS.

Diameter of Journals.	Nature of Unguent.	Velocity of the Circumference in 1″.	Weight of the Axle and its Load.	Ratio of the Friction to the Pressure.	Remarks.
feet.		feet.	lbs.		
0·328	Oil.	0·196 0·222 0·488 0·445 0·345	2269·4	0·082 0·082 0·082 0·079 0·079 0·081	In these experiments the oil was poured only upon the surface of the journals.
0·328	Oil.	0·212 0·262 0·409 0·488	2269·4	0·054 0·052 0·052 0·052	In these experiments the oil was poured ceaselessly upon the rubbing surfaces.
			Mean	0·053	
0·177	Oil.	0·429 0·409 0·465	2241·8	0·101 0·109 0·101	In these experiments the oil was expelled by the pressure, and the surfaces were simply very unctuous.
			Mean	0·104	
0·177	Lard.	0·190 0·268 0·328 0·393 0·445 0·465	2240·7	0·070 0·069 0·075 0·084 0·070 0·060	In these experiments the surfaces themselves supplied the lard.
0·328	Lard.	0·222 0·331 0·380 0·409 0·429	4157·	0·049 0·050 0·052 0·040 0·042	In these experiments the unguent was renewed.
0·328	Lard.	very slow. 0·150 0·238 0·321 0·321 0·380 0·492	2276·	0·037 0·039 0·025 0·026 0·035 0·026 0·832	In these experiments the unguent was continually renewed.

The examples contained in this Table suffice to show that the friction of journals is in itself subject to the same laws as that of plane surfaces; but they also show the great influence which the constant renewal of the unguent possesses in diminishing the value of the ratio of the friction to the pressure, which sometimes falls as low as ·025.

We see also that the diameter of the journals seems to have some influence upon the more or less complete expulsion of the unguent, and consequently upon the friction, so that the dimensions to be given them should not be determined from a consideration solely of their resistance to rupture.

Recapitulating the summary of the experiments which Morin has made upon the friction of journals shows that it is nearly the same for woods and metals rubbing upon each other, and that its ratio to the pressure may, according to the case, take the values given in the following Tables.

STATE OF SURFACES.

With rotten-stone and perfectly greased. f.	Continually supplied with unguent. f.	Greased from time to time. f.	Unctuous. f.
0·025 to 0·030	0·050	0·07 to 0·08	0·150

Advantage of Granulated Metals.—It is not true, as is generally supposed, that the friction is always less between substances of different kinds than between those of the same kind. But it is well generally to select for the rubbing parts granulated rather than fibrous bodies, and especially not to expose the latter to friction in the direction of the fibres, because the fibres are sometimes raised and torn away throughout their length. In this respect fine cast iron, which crystallizes in round grains, as well as cast steel, are very suitable bodies for parts subjected to great friction.

Thus, for several years past, a cast-iron packing has come into very general use for the pistons of steam-engines. If for the boxes of iron or cast-iron axles, brass continues in use, it is chiefly because it is less hard, and wears out before the axles, and because it is easier to replace a box than an axle.

Remarks upon very light Mechanisms.—In very light mechanisms, and especially with very rapid motion, the viscosity of the unguent may offer a resistance similar to that produced by friction proper; in such cases the results of experiments made under considerable pressures in relation to the surfaces of contact, should only be applied with extreme caution.

Use of the Results of Experiments.—The results obtained from the experiments of Morin are resumed in the three following Tables, which give the ratio of the friction to the pressure, for all the substances employed in construction. The first of these Tables relates to plane surfaces which have been some time in contact. The values which it gives for the ratio f of friction to the pressure, should be employed whenever we are to determine the effort necessary to produce the sliding of two bodies which have been some time in contact. Such is the case with the working of gates and their fixtures, which are used only at intervals more or less distant.

I.—Friction of Plane Surfaces which have been some time in contact.

Kind of Surfaces in Contact.	Disposition of the Fibres.	Condition of the Surfaces.	Ratio of Friction to Pressure, f.
Oak on oak	Parallel	Without unguent	0·62
	,,	Rubbed with dry soap	0·44
	Perpendicular	Without unguent	0·54
	,,	Moistened with water	0·71
	Wood upright on wood flatwise	Without unguent	0·43
Oak on elm	Parallel	,,	0·38
Elm on oak	,,	,,	0·69
	,,	Rubbed with dry soap	0·41
	Perpendicular	Without unguent	0·57
Ash, pine, beech, on oak	Parallel	,,	0·53
Tanned leather on oak	The leather flatwise	,,	0·61
	The leather on edge	,,	0·43
		Moistened with water	0·79
Black curried leather or belt on plane oak surface	Parallel	Without unguent	0·74
Black curried leather or belt on oak drum	Perpendicular	,,	0·47
Hemp matting on oak	Parallel	,,	0·50
		Moistened with water	0·87
Hemp cord on oak	,,	Without unguent	0·80
Iron on oak	,,	,,	0·62
		Moistened with water	0·65
Cast iron on oak	,,	,,	0·65
Brass on oak	,,	Without unguent	0·62
Ox-hide for piston packing on cast iron	Flatwise	Moistened with water	0·62
	On edge	With oil, lard, tallow	0·12
Black curried leather, or belt upon cast-iron pulley	Flatwise	Without unguent	0·28
		Moistened with water	0·38
Cast iron upon cast iron	,,	Without unguent	0·16*
Iron upon cast iron	,,	,,	0·19
Oak, elm, yoke elm, iron, cast iron, and brass, sliding two and two one upon the other	,,	Spread with tallow	0·10†
		With oil, or lard	0·15‡
Calcareous oolite upon oolite limestone	,,	Without unguent	0·74
Hard calcareous stone called muschelkalk upon oolite limestone	,,	,,	0·75
Brick on calcareous oolite	,,	,,	0·67
Oak on ,, ,,	Wood upright	,,	0·63
Iron on ,, ,,	,,	,,	0·49
Hard muschelkalk on muschelkalk	,,	,,	0·70
Calcareous oolite upon ,,	,,	,,	0·75
Brick on muschelkalk	,,	,,	0·67
Iron upon ,,	,,	,,	0·42
Oak on ,,	,,	,,	0·64
Calcareous oolite on calcareous oolite	,,	With mortar, three parts fine sand, and one part of hydraulic lime	0·74§

* The surfaces being somewhat unctuous.
† When the contact had not been long enough to press out the unguent.
‡ When the contact had been long enough to press out the unguent and bring the surfaces to an unctuous state.
§ After a contact of from ten to fifteen minutes.

II.—Friction of Plane Surfaces in Motion upon each other.

Surfaces in Contact.	Position of Fibres.	State of Surfaces.	Ratio of Friction to Pressure, *f*.
Oak on oak	Parallel	Without unguent	0·48
	"	Rubbed with dry soap	0·16
	Perpendicular	Without unguent	0·34
	"	Wet with water	0·25
	Upright on flatwise	Without unguent	0·19
Elm on oak	Parallel	"	0·43
	Perpendicular		0·45
	Parallel		0·25
Ash, pine, beech, wild pear, on oak	"	"	0·36 to 0·40
Iron on oak	"	"	0·62
		Wet with water	0·26
		Rubbed with dry soap	0·21
Cast iron on oak	"	Without unguent	0·49
		Wet with water	0·22
		Rubbed with dry soap	0·19
Copper on oak	"	Without unguent	0·62
Iron on elm	"	"	0·25
Cast iron on elm	"	"	0·20
Black curried leather on oak	"	"	0·27
Tanned leather on oak	Flatwise on edge	"	0·30 to 0·35
		Wet with water	0·29
Tanned leather upon cast iron and brass	Flatwise and on edge	Without unguent	0·56
		Wet with water	0·36
		Unctuous and wet with water	0·23
		Spread with oil	0·15
Hemp strips or cords upon oak	Parallel	Without unguent	0·52
	Perpendicular	Wet with water	0·33
Oak and elm on cast iron	Parallel	Without unguent	0·38
Wild pear on cast iron	"	"	0·44
Iron upon iron	"	"	*
Iron upon cast iron and brass	"	"	0·18†
Cast iron on cast iron and brass	"	"	0·15†
Cast iron on cast iron	"	Wet with water	0·31
Brass on brass	"	Without unguent	0·20
Brass on cast iron			0·22
Brass on iron			0·16‡
Oak, elm, yoke elm, wild pear, cast iron, iron, steel, steel and brass, sliding upon each other or themselves	"	Lubricated in the usual way with tallow, lard, soft coom, &c.	0·7 to 0·8§
Calcareous oolite on calcareous oolite	"	Slightly unctuous to the touch	0·15
		Without unguent	0·64
Muschelkalk upon " "	"	"	0·67
Common brick upon " "	"	"	0·65
Oak on oolitic limestone	Wood upright	"	0·38
Forged iron upon oolitic limestone	Parallel	"	0·69
Muschelkalk upon muschelkalk	.. ..	"	0·38
Oolitic limestone upon "	.. ..	"	0·65
Common brick on "	.. ..	"	0·60
Oak on "	Wood upright	"	0·38
Iron on "	Parallel	"	0·24
		Wet with water	0·30

* Surfaces worn when there was no unguent.
† The surfaces still being slightly unctuous.
‡ The surfaces slightly unctuous.
§ When the unguent is constantly supplied, and uniformly laid on, this ratio may be lowered to 0·05.

Table II. relates to plane surfaces in motion upon each other; Table III. applies to journals in motion upon their bearings. The values given by these Tables ought not to be used except to calculate the friction of two surfaces in motion upon each other, after the period in which the coefficient of friction at the starting has been introduced.

III.—FRICTION OF JOURNALS IN MOTION UPON THEIR PILLOWS.

Surfaces in Contact.	State of Surfaces.	Ratio of Friction to the Pressure when the Unguent is renewed.	
		In the Common Way.	Continuously.
Cast-iron journals on cast-iron bearings	Unguents of olive oil, of lard, of tallow, or of soft coom	0·07 to 0·08	0·030 to 0·054
	With the same unguents and moistened with water	0·08	..
	Asphalte	0·054	..
	Unctuous	0·14	..
	Unctuous and wet with water ..	0·14	..
Cast-iron cushions on brass cushions	Unguents of olive oil, of lard, of tallow, and of soft coom	0·07 to 0·08	0·03 to 0·054
	Unctuous	0·16	..
	Unctuous and wet with water ..	0·16	..
	Slightly unctuous	0·19	*
Cast-iron journals on lignum-vitæ bearings ..	Without unguent	0·18	†
	Unguents of oil or lard	..	0·090
	Unctuous with oil or lard	0·10	..
	Unctuous, with a mixture of lard and black-lead	0·14	..
Wrought-iron journals on cast-iron bearings	Unguents of olive oil, tallow, lard, or soft coom	0·07 to 0·08	0·030 to 0·054
Wrought-iron journals on brass bearings	Unguents of olive oil, tallow, lard	0·07 to 0·08	0·030 to 0·054
	Unguents of soft coom	0·09	..
	Unctuous and wet with water ..	0·19	..
	Slightly unctuous	0·25	‡
Iron journals on lignum-vitæ bearings	Unguents of oil or lard	0·11	..
	Unctuous	0·19	..
Brass journals on brass bearings	Unguents of oil	0·10	..
	Unguents of lard	0·09	..
Brass journals on cast-iron cushions	Unguents of oil or tallow	..	0·030 to 0·052
Lignum-vitæ journals on cast-iron cushions	Unguents of lard	0·12	..
	Unctuous	0·15	..
Lignum-vitæ journals on lignum-vitæ cushions ..	Unguent of lard	..	0·07

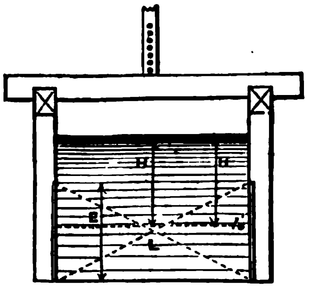
* The surfaces began to wear. † The wood being slightly unctuous. ‡ The surfaces began to wear away.

Application to Gates.—Let L be the horizontal width of a gate under a certain head of water, and H' the head or height of level above a horizontal section of this gate, of a thickness h' infinitely small. The pressed surface of this element will be L h', and the pressure which it will experience will be 63·32 L H' h'. The total pressure upon the entire surface of the gate being equal to the sum of all the similar pressures upon each of the elements, will have for its value

$$62 \cdot 32\, L\,(H'\,h' + H''\,h'' + H'''\,h''' + \&c.).$$

Now, the products L H h', L H'' h'', &c., are the moments of the elementary surfaces L h', L h'', &c., in relation to the plane of the level, and their sum is equal to the moment of the whole surface equal to L E H. Calling E the height of the gate pressed, and H the distance of the centre of gravity from the surface of the level, or the head upon the centre of the figure. Then the total pressure is 62·32 L E H, and the friction which results against the slides of this gate is 62·32 f. L E H, f being the ratio of the friction to the pressure for the surfaces in contact, a ratio whose value should be taken from the first Table, if we are to calculate the effort required to put the gate in motion.

3049.



Example.—If L = 6·56 ft., Fig. 3049, E = 1·148 ft., H = 4·92 ft., the first Table gives for a wood gate of oak sliding with crossed fibres upon oak wet with water f = 0·71; we have then for the friction 62·32 × 0·71 × 6·56 × 1·148 × 4·92 ft. = 1639·4 lbs.

The effort should be transmitted in the direction of the racks fixed upon the gate; and as it is considerable, it will be proper to arrange a kind of screw-jack, suitably proportioned, for the establishment of which we may take as the effort to be exerted by a man upon the winch, at any instant, from 55 to 66 lbs. at most, and during the motion from 22 to 26·5 lbs.

When the gate is in motion, the effort to be transmitted to the racks is much less, because

the ratio of the friction to the pressure diminishes, and is reduced for a gate with moistened slides to $0 \cdot 25$, which gives for the friction during motion,

$$62 \cdot 32 \times 0 \cdot 25 \, \text{L E H} = 62 \cdot 32 \times 0 \cdot 25 \times 6 \cdot 56 \times 1 \cdot 148 \times 4 \cdot 92 = 577 \cdot 2 \text{ lbs.},$$

at the first instant, and a value decreasing with the raising of the gate, or as the head H upon its centre is lessened.

We hardly need to say that, in working the gate, we must calculate for the maximum effort.

Application to Saw-frames.—If we have, for example, the frame of a saw for veneering, subjected to a pressure of $110 \cdot 274$ lbs., and provided with iron strips sliding in brass grooves, greased with lard, we have, if the surfaces are well lubricated, for the friction, $0 \cdot 07 \times 110 \cdot 274 = 7 \cdot 719$ lbs., and if they are unctuous, $0 \cdot 15 \times 110 \cdot 274 = 16 \cdot 54$ lbs.

If the stroke of the frame is $3 \cdot 936$ ft., and the number of strokes 180 in 1′, the space described in 1″ will be $11 \cdot 81$ ft., and the work consumed by the friction of the frame in 1″ will be in the first case, $2 \times 11 \cdot 81 \times 7 \ 719 = 182 \cdot 32$ lbs. ft. $= \frac{1}{3}$ horse-power nearly; in the second case,

$$2 \times 11 \cdot 81 \times 16 \cdot 54 = 390 \cdot 66 \text{ lbs. ft.} = \frac{2}{3} \text{ horse-power nearly.}$$

Application to Journals.—To calculate the work consumed by the friction of the journals of a revolving axle, we begin by seeking the resultant of the forces acting around this axle, and decompose this into two, the one horizontal and the other vertical, and we take separately the resultant of each of these groups. Calling X the sum of the horizontal components, Y the sum of the vertical components, the general resultant will be $\sqrt{X^2 + Y^2}$, and the friction produced by it will be $f \, . \sqrt{X^2 + Y^2}$.

The theorem of Poncelet, already cited, informs us that when we do not know the order of magnitude of X and Y, we may calculate to nearly $\frac{1}{6}$ of the value of the radical by the formula $0 \cdot 83 \, (X + Y)$, and that if we know beforehand that one of the terms, X for example, is greater than the other, which is most usually the case, we shall have the value of the radical to $\frac{1}{25}$ nearly, by the expression $0 \cdot 96 \, X + 0 \cdot 4 \, Y$.

Suppose, for example, that we have a hydraulic bucket-wheel weighing 88,219 lbs., transmitting a useful effect of 50 horse-power to the exterior circumference, and imparting motion to a pinion, so that the useful resistance may be horizontal and represented by Q. Suppose the radius of the wheel $R = 9 \cdot 84$ ft., the velocity at its circumference to be $5 \cdot 249$ ft., and the radius of the gearing wheel $R' = 6 \cdot 56$ ft. The effort P transmitted to the circumference of the wheel will be

$$P = \frac{50 \times 550}{5 \cdot 249} = 5239 \text{ lbs.}$$

The pressure upon the journals of the hydraulic wheel will be $\sqrt{(M + P)^2 + Q^2}$, or, since $M = 88219$ lbs., and consequently $M + P$ is greater than Q, we may take for an approximate value of the radical to $\frac{1}{25}$ nearly, $0 \cdot 96 \, (M + P) + 0 \cdot 4 \, Q$.

For uniform motion, the moment of the power P must be equal to the sum of the moments of resistances. We have then, in calling r the radius of the journal $= 0 \cdot 393$ ft., $f = 0 \cdot 07$,

$$5239 \text{ lbs.} \times 9 \cdot 84 = Q \times 6 \cdot 56 + 0 \cdot 96 (0 \cdot 07)(88219 + 5239)(0 \cdot 393) + 0 \cdot 4 (0 \cdot 07)(Q \times 0 \cdot 393);$$

$$\text{whence } Q = \frac{5239 \times 9 \cdot 84 - 0 \cdot 96 \times 0 \cdot 07 \times 93458 \times 0 \cdot 393}{6 \cdot 56 \times 0 \cdot 4 \times 0 \cdot 07 \times 0393} = 7469 \text{ lbs.},$$

while if we had neglected the friction of the journals, we should have found

$$Q = \frac{5239 \times 9 \cdot 84}{6 \cdot 56} = 7858 \cdot 6 \text{ lbs.}$$

The velocity of the gearing wheel being $5 \cdot 249 \times \frac{2}{3} = 3 \cdot 499$ ft. The work transmitted to this circumference in 1″ is $7469 \cdot 8$ lbs. $\times 3 \cdot 499 = 26137$ lbs. ft. $= 47 \cdot 5$ horse-power. The loss by the friction of the journals is then $50 \cdot 00$ horse-power $- 47 \cdot 5 = 2 \cdot 5$ H. P.

If the surfaces of the journals had not been unctuous the loss would have been double.

The space described by the rubbing points, being one of the factors of work consumed by the passive resistance, it is important to diminish it as much as possible, and consequently to give the journals only such dimensions as will ensure a proper strength.

To calculate their diameter in the establishment of the wheel, we disregard the friction, which will give us a first value of $Q = 7858$ lbs., a little too much, and consequently for the resultant of the efforts to which the journal is subjected, $\sqrt{(93458)^2 + (7858 \cdot 6)^2} = 93787$ lbs.

Each journal supports then nearly 46893 lbs. of pressure, and its diameter, calculated by the formula for journals of hydraulic wheels, will be $d = \cdot 00364 \sqrt{46893}$; whence $d = 0 \cdot 788$ ft.

This is the value which we have adopted in the preceding calculation.

C. Schiele's Anti-friction Curve.—This invention consists in the application of a curved form (instead of a rectilinear form usually employed) to the construction of cocks and valves, and also to the construction of axles, journals, bearings, or other rubbing surfaces in machinery in general, in order to reduce their friction and consequent wear and tear.

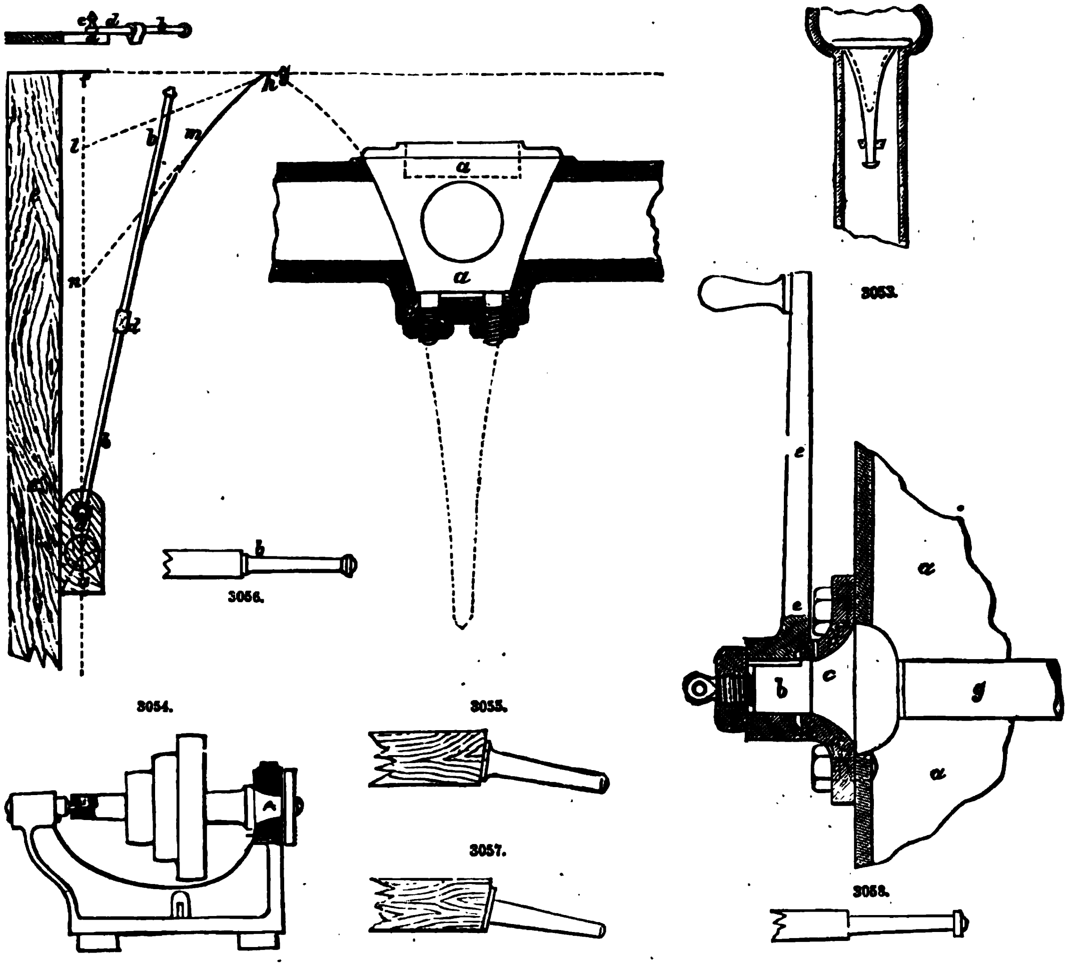
Fig. 3050 represents a plan and end view of a small apparatus for describing such a curve.

a a is a small wooden slide, to which the rod *b b* is jointed by means of a pin *c*. *d* is a slide or bush, to which a drawing pen is affixed, and *e e* is a ruler, along the edge of which the slide *a* is to be guided. If the slide *a* and rod *b b* be so placed that the pin *c* shall be at *f* and the pen *d* at the point *g*, the centre line at the rod *b b* will then be over the dotted line *g f*, at a right angle with the dotted line *l n*; and if the slide *a* be then guided along the edge of the ruler *e e* the pin *c* will move along the dotted line *l n*, dragging the pen *d* after it, which in travelling over a horizontal plane will describe the curved line *h m*. The pen *d* can be moved upon the rod *b* to the proper distance for the curve required, and is kept in that and in a vertical position by a spring which fits in a groove. *l n* is the axis of the curve, and *g f*, *h l*, *m n*, represent some of the tangents above mentioned.

Fig. 3051 represents a vertical section of the shell of a stop-cock, showing the application of this invention to the seats or surfaces of contact. The dotted lines near the top of the plug *a* represent a groove in the plug for the reception of a key.

Fig. 3052 represents the application of this curve to the seats or surfaces of contact of lift-valves for pumps.

Fig. 3053 shows its application to the journal and bearing of a regulator for a locomotive engine, to be used instead of a stuffing box. *a* is part of the boiler of a locomotive engine; *b* is the spindle of the regulator, and *c* is the journal; *d* is the bearing, and *e* is the lever or handle by which the regulator is turned. The spindle *b* is furnished on that end which is inside the boiler with a square hole for the reception of the squared end of a rod *g*, which has to transmit the motion to the valve.

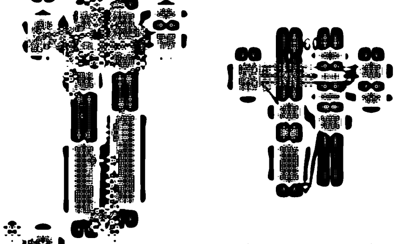


3050. 3051. 3052. 3053. 3054. 3055. 3056. 3057. 3058.

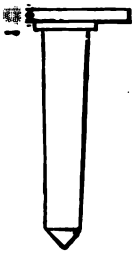
Fig. 3054 shows the application of Christian Schiele's curve to the journals and centres of turning lathes.

Figs. 3055, 3056, show the application to axles on the parts *a*, *b*, *c*. Here the pressure acts only at intervals in the direction of the axis, and must therefore be borne separately. The difference of their construction from that commonly in use will be seen on comparing Figs. 3055, 3056, with Figs. 3057, 3058.

curve to pivots or axes for astronomical, or survey-
ee Fig. 3061, which shows a mode of construction
own in Fig. 3060, is also applicable to footsteps of

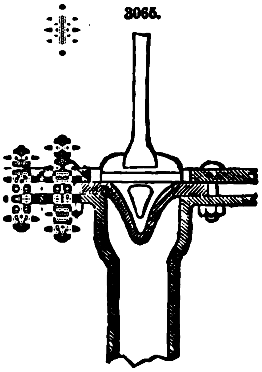


3061.



3062.

ntion to the construction of the threads of screws.
structed in the mode described acts in the direction
ences the construction of the curve with that part
see Fig. 3051, at *b*, *c*, and *d*; Figs. 3052, 3053, at
and Fig. 3062. When part of the pressure acts
part of the curve which in its inclination to the
dle pressure of the combined forces; for examples,
at *a* and *b*, and Fig. 3059.
curve to a safety-valve. Fig. 3064 to a lock-up
ump. Fig. 3066 a universal cock adjustable at any
rnal of a screw propeller. Fig. 3068, turning joint
for astronomical instruments; also applicable to
Fig. 3071, pivot for shafting. Fig. 3072, pivot
for lathe-spindle. Fig. 3074, castor. Fig. 3075,
Artesian wells. Fig. 3077, screw-collar. Fig. 3078,
and wheel. Fig. 3080, screw-jack. Fig. 3081, pivot



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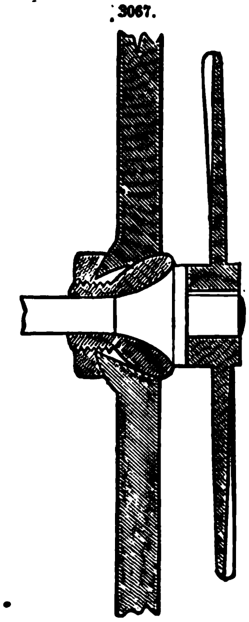

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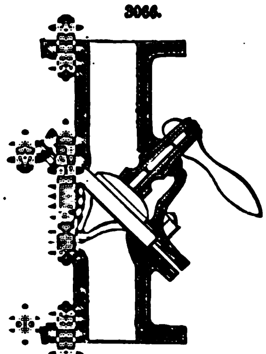

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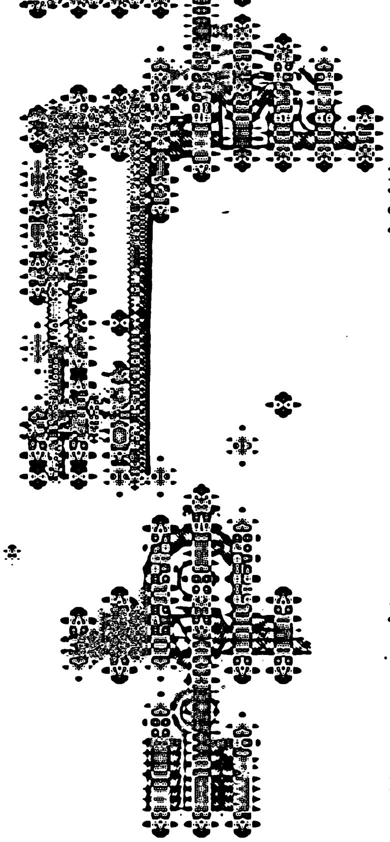

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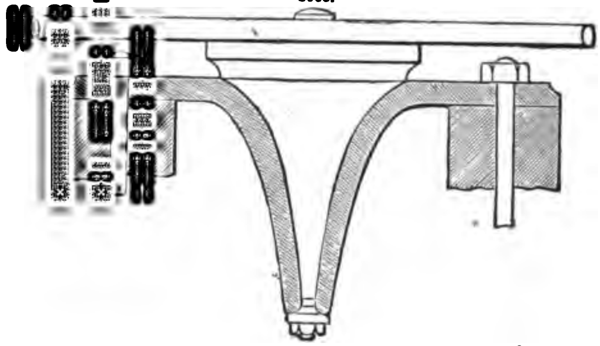

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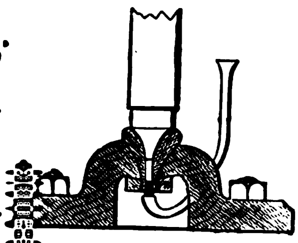

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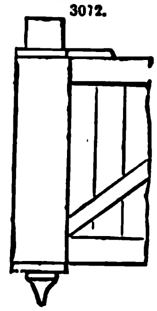
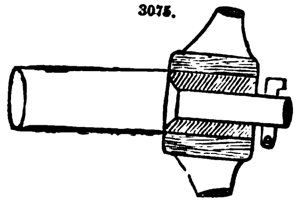


3074.

3075.



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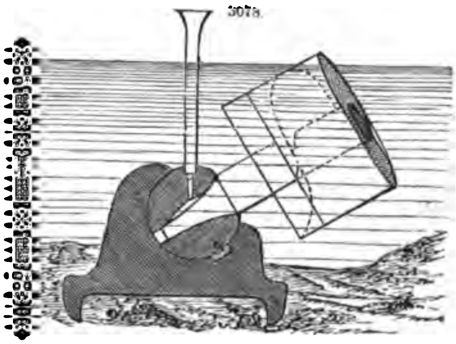

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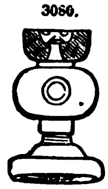

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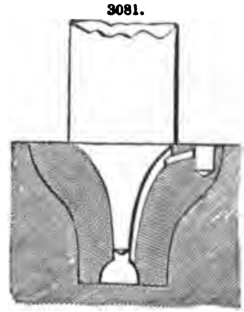
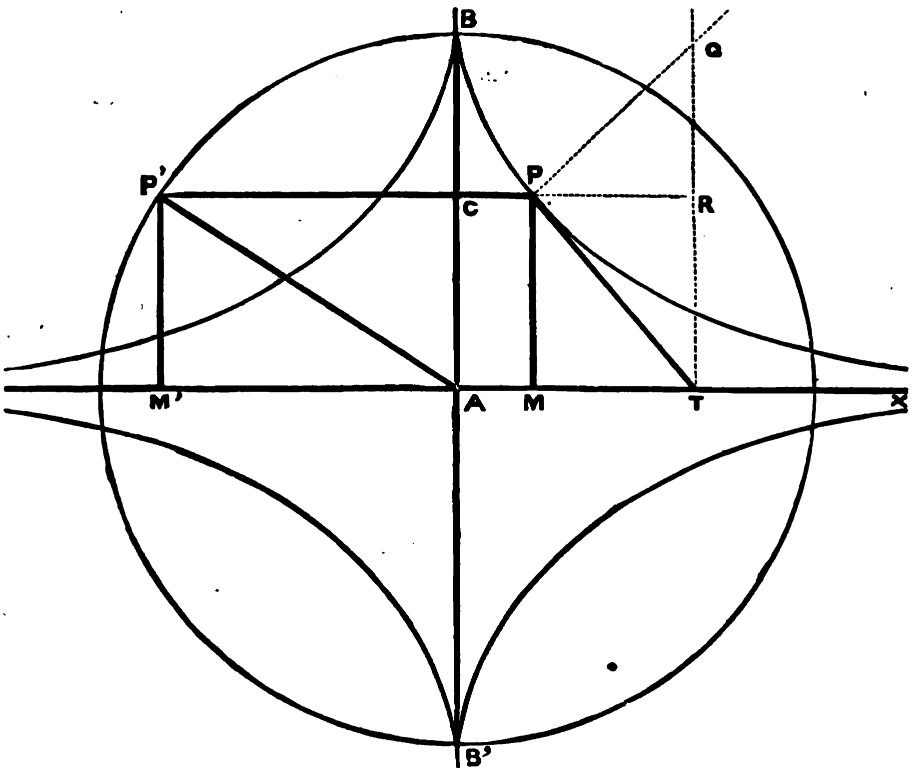


This curve, termed the anti-friction curve by Christian Schiele, is known to mathematicians as the *tractrix;* it has been erroneously identified with the *catenary*. This curve was invented by Christian Huygens and received its name from a supposition that it is the curve which would be described by a weight drawn on a plane by a string of a given length, the extremity of which is carried along the directrix A B, Fig. 3082. Euler has shown that this conclusion is wrong, unless

3082.



the momentum of the weight which is generated by its motion be every instant destroyed. See Euler, *Nova*. Comm. Petrop. 1784. However, to Schiele is due the credit of applying this curve to effect an important mechanical requirement.

The characteristic property of the anti-friction curve, or tractrix, is that the locus of a point T, on the tangent P T, at a given distance from the point of contact, is a straight line A X, which is called the directrix of the curve.

To find the equation to the Anti-friction Curve.—Let the intercept (P T) of the tangent between the directrix A X and point of contact P be put $= a$. Then by the general formula for the subtangent

$$-\frac{y\,dx}{dy} = (a^2 - y^2)^{\frac{1}{2}} \,. \qquad [1]$$

From integrating [1] we obtain

$$x = a \downarrow, \left(\frac{a + (a^2 - y^2)^{\frac{1}{2}}}{y}\right) - (a^2 - y^2)^{\frac{1}{2}} \,; \qquad [2]$$

In general terms $\downarrow, (y)$ is put to represent the dual logarithm of y to the base $B = 1 \cdot 00000001$; but $\frac{\downarrow, (y)}{10^8}$ corresponds with the hyperbolic log. of y to eight places of decimals. $\downarrow, (y) = \frac{\downarrow, (y)}{10^m}$, to any of the dual bases B_n or b_n, is termed the logarithm of y. Equation [1] may be found by differentiating [2], and thus verify the integration.

[2] may be put under the form

$$x = \downarrow, [a + (a^2 - y^2)^{\frac{1}{2}}] - a \downarrow, (y) - (a^2 - y^2)^{\frac{1}{2}} \,;$$

$$\therefore \frac{dx}{dy} = -\frac{a\,y}{(a^2 - y^2)^{\frac{1}{2}}\,[a + (a^2 - y^2)^{\frac{1}{2}}]} - \frac{a}{y} + \frac{y}{(a^2 - y^2)^{\frac{1}{2}}} \,;$$

$$\therefore -\frac{y\,dx}{dy} = \frac{a\,y^2}{a\,(a^2 - y^2)^{\frac{1}{2}} + (a^2 - y^2)} + a - \frac{y^2}{(a^2 - y^2)^{\frac{1}{2}}} \,;$$

$$\text{and} - \frac{y\,d\,x}{d\,y} - a = \left[\frac{a\,y^2}{a + (a^2 - y^2)^{\frac{1}{2}}} - y^2\right]\frac{1}{(a^2 - y^2)^{\frac{1}{2}}};$$

$$= \frac{a\,y^2 - y^2[a + (a^2 - y^2)^{\frac{1}{2}}]}{a + (a^2 - y^2)^{\frac{1}{2}}}\left(\frac{1}{(a^2 - y^2)^{\frac{1}{2}}}\right);$$

$$= -\frac{y^2}{a + (a^2 - y^2)^{\frac{1}{2}}};\ \text{consequently,}$$

$$-\frac{y\,d\,x}{d\,y} = a - \frac{y^2}{a + (a^2 - y^2)^{\frac{1}{2}}} = \frac{a^2 + a\,(a^2 - y^2)^{\frac{1}{2}} - y^2}{a + (a^2 - y^2)^{\frac{1}{2}}} = (a^2 - y^2)^{\frac{1}{2}}.$$

To find the equation of a tangent through any given point P on the Curve.—Let the co-ordinates of the given point be $x_1 = $ A M and $y_1 = $ M P.

$$\text{From [1]}\ \frac{d\,y}{d\,x} = -\frac{y}{(a^2 - y^2)^{\frac{1}{2}}}; \qquad [3]$$

Hence the equation to the straight line passing through P, and touching the curve at P, is

$$y - y_1 = -\frac{y_1}{(a^2 - y_1^2)^{\frac{1}{2}}}(x - x_1); \qquad [4]$$

The geometrical construction for applying a tangent to the anti-friction curve is obviously pointed out by [4]. With the centre A and the radius $a = $ A B let a circle be described; through any point P of the curve let the ordinate P M be drawn, and P P′ parallel to A X, meeting the circle in P′, and let P′ A be drawn; a line P T parallel to P′ A is a tangent to the curve at P. For tangent of P′ A M′ $= \frac{y_1}{(a^2 - y^2)^{\frac{1}{2}}} = $ tangent of the angle P T M. It is evident that when $y = \pm a\ x = 0$, therefore if A B $= + a$ and A B′, Fig. 3082, $= - a$, the curve meets the axis of y at the points B, B′; and in [4], if $y_1 = \pm a$, and $x_1 = 0$, the equation becomes $x = 0$, which shows that the axis of y touches the curve at the points B, B′.

If [3] be differentiated, we have $d^2\,y = \frac{a^2}{(a^2 - y^2)^2}\,y\,d\,x^2$, hence, $d^2\,y$ and y have always the same sign, and the curve must be everywhere convex towards the directrix.

By [2] it appears that for each value of y there are two equal and opposite values of x, and for each value of x there are two equal and opposite values of y. Therefore the four branches of the curve, included in the four right angles round the origin A are perfectly equal and similar, and such as if placed upon each other would coincide. It also appears by equation [2] that, as x increases without limit, y diminishes without limit, and consequently the directrix A X is an asymptote.

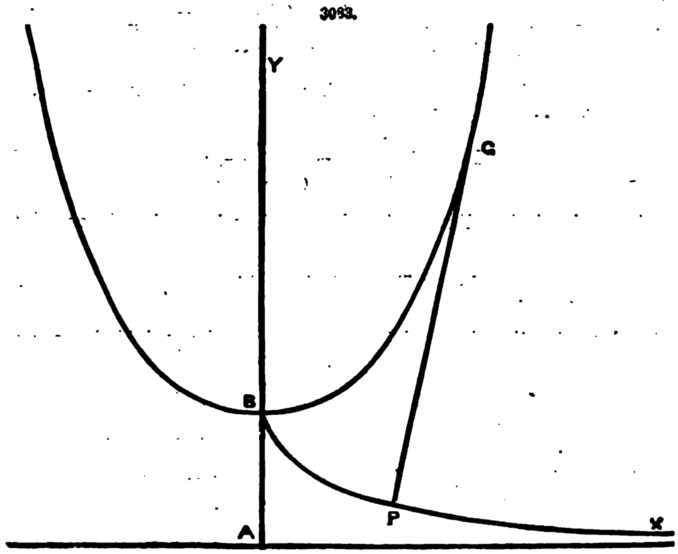
To quadrate the Anti-friction Curve, from [1] we have $y\,d\,x = (a^2 - y^2)^{\frac{1}{2}}\,d\,y$. On one side of this equation, $y\,d\,x$ is the differential of the area A B P M; and since $-(a^2 - y^2)^{\frac{1}{2}} = $ A M′, = P′ C, the other side is the differential of the area B P′ C, and therefore taking the integrals of both sides we have P B A M = B P′ C. Also, since the triangle P′ A M′ = P T M, the area B P T A is equal to the sector B A P′. It also follows that the whole area included by the four branches is equal to the area of the circle B P′ B′. To rectify this curve, we find from [1], $-\frac{a\,d\,y}{y} = (d\,y^2 + d\,x^2)^{\frac{1}{2}}$ the general expression for the length of any plane curve referred to rectangular co-ordinates. The negative sign is appended to $\frac{a\,d\,y}{y}$, because the length of the arc increases as y diminishes. By integration we have $-a \downarrow, (y) + C = \int (d\,x^2 + d\,y^2)^{\frac{1}{2}}$. To find the constant C, let the arc K be supposed to begin at B, so that when K $= 0, y = a$; hence $-a \downarrow, (a) + C = 0, \therefore C = a \downarrow, (a)$, and hence we find the length of the arc K $= a \downarrow, \left(\frac{a}{y}\right)$. If $a = 3$ in. and $y = \cdot 3$ in.; then the length of the arc K = 6·90775527 in. For $\downarrow \left(\frac{3\cdot}{\cdot 3}\right) = \downarrow, (10) = 2\cdot 30258509$; and $2\cdot 30258509 \times 3 = $ 6·90775527 in. When K and y are given a can be found by dual arithmetic, but by no other known means.

Putting r for the radius of curvature at any point P, Fig. 3082, and substituting in the general formula for the radius of curvature the values of the first and second differential coefficients, we find $r = -\frac{a\,(a^2 - y^2)^{\frac{1}{2}}}{y}$. Hence by geometrical construction the radius and centre of the osculating circle may be found thus;—let P Q be perpendicular to the tangent at P, and produced to meet a

perpendicular to the directrix at T, the intercept P Q is the radius, and Q the centre of the osculating circle; for P M : P T :: T M : P Q, by the similar triangles M P T, R P Q.

To find the evolute of the Anti-friction Curve.—Let the co-ordinates of the centre of the osculating circle be x_1, y_1. By substituting in the general formulæ for the values of these the particular values of the differential coefficients, the results are $y_1 = \frac{a^2}{y}$ and $x_1 = x + \frac{a\,(y_1^2 - a^2)^{\frac{1}{2}}}{y_1}$. Eliminating x and y by means of these equations, and that of the curve, the result is $x_1 = a\,\mathrm{l.}\left[\frac{y_1 + (y_1^2 - a^2)^{\frac{1}{2}}}{a}\right]$, which is the equation of the evolute. The evolute of the anti-friction curve of Schiele is therefore a catenary, whose parameter is a = A B, Fig. 3083, whose vertex is at B, and whose axis is A Y.

3083.



If a string P G applied to a catenary G B, have its extremity P at the vertex B, and be wound off, its extremity P will describe the anti-friction curve, in the examination of which mathematicians have blundered much. See ACCELERATION. BELTS. BRAKE. DYNAMOMETER. GEARING.

FRICTIONAL GEARING. FR., *Engrenage de frottement;* GER., *Frictionscheiben;* ITAL., *Ruote ad attrito;* SPAN., *Organo de trasmision de movimiento sin friccion.*

See GEARING.

FUEL. FR., *Combustible;* GER., *Brennmaterial;* ITAL., *Combustibile;* SPAN., *Combustible.*

Great quantities of combustible substances, of immense importance in metallurgy and the various arts, are found in the bosom of the earth. They are evidently produced by the decomposition of vegetables which grew in the vicinity, or the *débris* of vegetables carried down by rivers. Peat mosses exhibit, though on a smaller scale, an example of this formation; as they consist of innumerable herbaceous vegetables, spontaneously decomposed by the action of water and atmospheric air; and their various stages of alteration may be followed, from the perfectly herbaceous turf to the earthy turf presenting but few or no recognizable remains.

The vegetable structure is frequently perfectly preserved in the mineral combustibles of the tertiary formation, where pieces of wood, called *lignite*, are found still retaining their original form, but having become friable, and yielding a brown powder by pulverization.

In the mineral fuel of older formations, the vegetable structure has generally disappeared, and it forms black, brilliant, compact masses, of a schistose texture, yielding a black, or more or less brown powder; it is called *pit-coal*, or *sea-coal*, and is rare in the secondary, but very abundant in the transition formation, in the upper stratum of which they are so frequent as to characterise them by the name of *coal formation.*

In the upper strata of the transition rocks the mineral fuel, which is sometimes called *anthracite*, is generally very compact, rich in carbon, difficult to ignite, and yielding but little volatile matter by calcination. Anthracite is sometimes, though rarely, found in the superior strata, and even in the secondary rocks.

Pit-coal of the coal formation yields on calcination a great quantity of volatile substances and inflammable gases, and experiences, prior to decomposition, an incipient fusion, while the coal remaining, or the *coke*, presents the appearance of a swollen or bloated mass. Although the structure of plants can no longer be recognized in certain combustible minerals, their vegetable origin is undoubted, for in the layers of schist or sandstone which bound the layers of coal, impressions of plants are frequently found, which are so distinct and clear as to enable the botanists to detect the family to which they belong, and thus, partly, to restore the flora of antediluvial epochs.

In the tertiary rocks a mineral fuel is also found, which is soft, or easily fusible, forming irregular masses, or a kind of strata, and presenting a bearing analogous to that of the lignites,

while at other times they permeate layers of schist or sandstone belonging to various geological formations, and then seem to arise from the decomposition, by heat, of other combustible minerals contained in the earth. Some of these substances, which are called *bitumen*, contain a large amount of nitrogen, and are fetid, yielding, on distillation, considerable quantities of carbonate of ammonia. They appear to have been generated by the putrefaction of animal matter, chiefly by that of fishes, the impressions of which are frequently found in the neighbouring rocks.

Coals may be divided into five classes;—

1. The anthracites.
2. *Fat and strong*, or *hard* pit coal.
3. *Fat blacksmiths'* or *bituminous* coal.
4. *Fat* coal *burning with a long flame.*
5. *Dry* coal *burning with a long flame.*

1. Calcination scarcely changes the appearance of anthracites, as their fragments still retain their sharp edges, and do not adhere to each other. They have a vitreous lustre, and their surface is sometimes iridescent, while their powder is black or greyish black. They burn with difficulty, but generate a large amount of heat when their combustion is properly effected. In *blast furnaces* anthracites require a great blast, and those only can be used which do not soon fall to powder, as otherwise the furnace would be speedily choked. We find that anthracite is used in Wales for heating reverberatory furnaces; and it is now proper to remark, that the flame produced by the combustible under these circumstances is not owing to the combustion of the volatile substances given off by the anthracite, but rather to the combustion of the carbonic oxide formed by the passage of air through a thick layer of fuel.

2. Fat and strong, or hard pit-coals, yield a coke with metallic lustre, but less bloated and more dense than that of blacksmiths' coals. They are more esteemed in metallurgic operations requiring a lively and steady fire, and yield the best coke for blast furnaces. Their powder is brownish black.

3. Fat bituminous, or blacksmiths' coals, yield a very bloated or swollen coke, with metallic lustre, and are more highly valued for forging purposes, because they produce a very strong heat, and allow the formation of small cavities, in which the pieces to be forged can be heated. Blacksmiths' coal is of a beautiful black colour, and exhibits a characteristic fatty lustre: its powder is brown. It is generally brittle, and breaks into cubical fragments, which adhere to each other in the fire.

4. Fat coals burning with a long flame generally yield a swollen, metalloid coke, less bloated, however, than that of blacksmiths' coal. These coals are much esteemed in a reverberatory furnace, particularly when a sudden blast is required, as in puddling, and are also well adapted to domestic purposes, and are preferred for the manufacture of illuminating gas. They yield a good coke, but in small quantity, and their powder is brown.

5. Dry pit-coal burning with a long flame yields a solid, metalloidal coke, the various fragments of which scarcely adhere to each other by carbonization. This coal is also applicable to steam-boilers, and burns with a long flame, which however soon fails, and does not produce the same amount of heat as the coals of the preceding class.

The elementary analysis of combustible minerals, which easily explains their various properties, and indicates the uses to which each is most applicable, is effected like that of organic substances; but as coal is generally difficult to burn, it is necessary at the close of the experiment by which the quantity of water and carbonic acid it contains is determined, to pass a current of oxygen gas through a tube, which burns the last particles of carbon. The organic analysis of coal yields the hydrogen, carbon, and nitrogen which they contain; but it is also necessary to determine the proportion of earthy matter which exists in very various degrees in them, and which remains in the ashes after combustion.

For this purpose two grammes of the coal are ignited in a thin platinum capsule, heated by an alcoholic-lamp, and the ashes remaining are weighed. This method of incineration is difficult, and requires considerable time, only in those anthracites which do not burn readily, and it is then more easily effected if the coarsely-powdered anthracite be placed in a small platinum vessel, heated in a current of oxygen in a porcelain tube.

It is essential carefully to examine the nature of the ashes. Sea-coal of the coal formation frequently leaves argillaceous ashes, in which case there is a trifling error in the supposed composition of the fuel, because the small quantity of water always contained in clay, and which it loses at a red heat, is regarded as existing in the state of hydrogen; and this error, which is of no importance if the quantity of ashes is small, may be considerable in the opposite case. The ashes often contain, likewise, peroxide of iron, which metal generally exists in coal in the state of pyrites, and the analysis is thus inaccurate for two reasons: the proportion of ashes is valued at too low a rate, because, instead of the iron pyrites, sesquioxide of iron is weighed, the weight of which for the same quantity of iron is less; and again, in combustion by oxide of copper, the substance may yield sulphurous acid, which interferes with the determination of hydrogen and carbon. The latter cause of error is avoided by placing in the combustion-tube, in front of the oxide of copper, a column of one or two decimètres of oxide of lead, which completely retains the sulphurous acid. The quantity of pyrites in the coal may be ascertained by determining, on the one hand, the quantity of sesquioxide of iron which exists in the ashes, and, on the other, the quantity of sulphuric acid yielded by a known weight of coal, powdered very finely, and acted on by fuming nitric acid, or ordinary nitric acid, to which small quantities of chlorate of potassa are gradually added. It is evident that these determinations are necessary only when the combustible produces a large quantity of ashes, and when the latter are very ochreous.

Coal belonging to the secondary and tertiary formations often yields calcareous ashes, in which case it becomes necessary, before weighing them, to sprinkle them with a solution of carbonate of ammonia, which is subsequently evaporated at a gentle temperature. But the determination of

the carbon is generally inaccurate, because the carbonate of lime of the ashes gives off, by contact with the oxide of copper in the combustion-tube, a portion of its carbonic acid; and the oxide of copper must then be replaced by chromate of lead, intimately and largely mixed, with the coal reduced to impalpable powder, after which the carbonic acid produced by the carbonates of the ashes, which has been determined by direct weighing of these carbonates, is subtracted from the carbonic acid formed by combustion.

Coal also retains one or two per cent. of hygrometric water, which must be previously driven off by drying it in a stove at 270° or 280°.

It is necessary, in order to form a correct judgment of the nature of a combustible, to determine the weight of coke it yields by burning; and it is indispensable that this operation should always be conducted under the same circumstances, as the quantity and nature of the coke depend on the manner of calcination. The best method consists in placing 3 grammes of the coal in a thin platinum crucible, accurately covered by its lid, and rapidly heating to a red heat. The crucible is kept at a red heat for eight minutes, and after cooling without being uncovered, the coke is weighed and carefully examined.

The calorific power of fuel is calculated from its chemical composition; admitting that this power is equal to the sum of that of the carbon it contains, and that of the hydrogen obtained by subtracting from the total quantity of hydrogen that which would form water with the oxygen contained in the fuel. This hypothesis is not strictly true, but it may be admitted when the quantities of heat afforded by various kinds of fuel are only to be compared by approximation.

This comparison is generally made in another way, based on the supposition that the calorific powers of combustibles are in proportion to their reducing powers; that is, to the weight of the same oxide which they can reduce to the metallic state. An intimate mixture of 1 gramme of finely-powdered combustible and 40 grammes of litharge being introduced into an earthen crucible, 20 grammes of litharge are added, and the crucible is covered with its lid and rapidly heated to a red heat. It is allowed to cool, and, after being broken, the lump of lead is weighed, which rapidly separates from the scoria of the litharge; and it is assumed that the calorific powers of combustibles are in proportion to the weight of lead yielded by this experiment. This supposition is not absolutely exact, because combustibles yield, before attaining the temperature at which they act on the litharge, a small quantity of volatile substances possessing a reducing power—which substances are more abundant in combustibles of recent formation than in those containing a larger proportion of oxygen.

The following Table exhibits the composition of a large number of kinds of mineral fuel, taken from various geological formations, and from the kinds best marked and most extensively applied in the arts. The fragments containing least ashes have also been chosen, in order to cast no uncertainty on the composition of the combustible itself.

The Table contains, 1st, the actual composition of the coal, as afforded by direct analysis; and, 2ndly, the composition calculated by abstracting the ashes contained.

In order to see how the composition of mineral combustibles varies with their qualities in the arts and geological age, the numbers contained in the last three columns of the Table must be compared; that is, those which exhibit the composition of these combustibles after the ashes are removed. On assuming as a standard of comparison the coals of the third class, and ascending from this to those of the second, it will be found that the quantity of hydrogen is nearly the same, but that the oxygen has remarkably decreased and been replaced by carbon. On passing from the second class to the first, it will be observed that both the hydrogen and oxygen decrease, while the carbon increases in the same ratio.

Starting always from the blacksmiths' coal, we descend toward the fourth class, and remark that generally the hydrogen exists in greater quantity, and that the carbon decreases remarkably and is replaced by oxygen. Lastly, in the fifth class, the oxygen has still increased, and taken the place of a corresponding quantity of carbon.

Fat pit-coal may become dry in two ways: either by passing into anthracite, the hydrogen and oxygen both decreasing, and the carbon increasing in the same ratio, or by approaching the more modern combustibles, the *lignites*, the carbon decreasing and being replaced by oxygen; in which latter case the ratio between the oxygen and hydrogen increases.

By now comparing the combustibles of the secondary with those of the coal formation, it will be seen that, in the inferior stratum of the latter formation, the same variety can be distinguished. Thus, the anthracites of Lamure and Macot, which are found in the lower part of the Jurassic rocks, present the same composition as those in the transition rocks; while the coal from Obernkirchen, which also exists in the Jurassic formation, has the same properties and composition as those of the carboniferous formation. Lastly, the coal from Céral, which also occurs in the Jurassic formation, belongs, on account of its composition and applications in the arts, to the class of fat coal burning with a long flame.

The coal found in the upper stratum of secondary rocks resembles, on the contrary, the combustibles of the tertiary rocks or the lignites, which differ from the coal of the older rocks by containing less carbon and more oxygen; and as their formation approaches a modern period, their composition resembles more closely that of wood. The charcoal they yield by calcination becomes more and more dry: thus, the jet of chalk still yields a fritted metalloid coke, while the lignites of the tertiary rocks produce a non-metalloid charcoal, the fragments of which do not adhere to each other, and resemble in appearance wood-charcoal.

The bitumens, which are evidently products of distillation of older combustibles, or produced by the spontaneous decomposition of animal substances, differ essentially from coal properly so called, by containing much larger quantities of hydrogen.

See ASSAYING. BALLAST, p. 218. BLAST FURNACE. BOILER. BRICK-MAKING MACHINE. CHIMNEY. COAL MINING. COPPER. DISTILLING APPARATUS, p. 1218. ENGINES, VARIETIES OF, p. 1418. HEAT. VENTILATING AND WARMING.

[illegible]

Transition Rocks.	Newcastle ..	[illegible] schistose and prismatic; coke swollen	1·280	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
IV. Fat pit-coal burning with a long flame ..	Flênu of Mons, 1	Coal formation; rhomboidal fragments; coke swollen	1·276	69·8	83·51	5·29	9·10	2·10	85·30	5·40	9·30
	Idem, 2	Coal formation; less marked rhomboidal cleavage; coke swollen	1·292	..	82·72	5·42	8·18	3·68	85·88	5·63	8·49
	Rive-de-Gier (cemetery), 1	Coal formation; lustre feeble, texture schistose; coke swollen, but less brilliant	1·288	70·9	80·92	5·27	10·24	3·57	83·91	5·46	10·63
	Idem, 2	The same as No. 1	1·294	69·1	83·67	5·61	7·73	2·99	86·25	5·77	7·98
	Rive-de-Gier, Couzon, 1 ..	Coal formation; lustre more marked, texture very schistose; coke swollen, but less brilliant	1·298	64·6	81·45	5·59	10·24	2·72	83·73	5·75	10·52
	Idem, 2	Coal formation; lustre very feeble; fracture unequal, and not schistose; coke less swollen ..	1·311	65·6	80·59	4·99	9·10	5·32	85·12	5·27	9·61
	Lavaysse	Coal formation; lustre brilliant; fracture conchoidal; coke swollen and light	1·284	57·9	81·00	5·27	8·60	5·13	85·38	5·56	9·06
	Lancashire ..	Coal formation; English *cannel coal*; without lustre; fracture conchoidal; coke fritted and brilliant	1·317	57·9	82·60	5·66	9·19	2·55	84·63	5·85	9·52
	Epinac	Coal formation; lustre brilliant, texture schistose; adherent metalloid coke, but slightly swollen	1·353	62·5	80·01	5·10	12·36	2·53	82·08	5·23	12·69
	Commentry ..	Coal formation; resembling cannel coal; fracture conchoidal; metalloid fritted coke ..	1·319	63·4	81·59	5·29	12·88	0·24	81·79	5·30	12·91
V. Dry pit-coal burning with a long flame	Blanzy	Coal formation; fracture laminated; lustre brilliant; coke slightly adherent, but not swollen	1·362	57·0	75·43	5·23	17·06	2·28	77·19	5·35	17·46

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Tertiary Rocks.		Mt. Meissner	Brilliant; fracture conchoidal; coke feebly adherent	1·351	48·5	70·73	4·85	22·65	1·77	72·00	4·93	23·07
		Lower Alps	Black; lustre greasy; coke slightly swollen ..	1·276	49·5	69·05	5·20	22·74	3·01	71·20	5·36	23·44
	II. Imperfect lignites ..	Greece	Laminated; of a dull black; indices of vegetable organization; coke not adherent ..	1·185	38·9	60·36	5·00	25·62	9·02	66·36	5·49	28·15
		Cologne ..	Umber coloured; friable; streak reddish brown; texture ligneous; coke not adherent	1·100	36·1	63·42	4·98	27·11	5·49	66·04	5·27	28·69
		Usnach ..	Fossil wood; woody texture; very hard	1·167	..	55·27	5·70	36·84	2·19	56·50	5·83	37·67
	III. Lignites passing into bitumen ..	Ellebogen ..	Compact, homogeneous; fracture conchoidal; very light metalloid coke	1·157	27·4	72·78	7·46	14·80	4·96	76·58	7·85	15·57
		Cuba	Velvet black colour; lustre greasy; coke swollen and very light	1·197	39·0	74·82	7·25	13·99	3·94	77·88	7·55	14·57
	IV. Asphaltum	Mexico ..	Black; very brilliant; strong smell; melts below 212°; coke exceedingly swollen	1·063	9·0	78·10	9·30	9·80	2·80	80·34	9·57	10·09
Alluvial Formation.	Turfs or Peats	Vulcaire ..	In a very advanced stage of alteration, though still exhibiting some remains of vegetables ..	..	..	56·25	5·63	32·54	5·58	59·67	5·96	34·47
		Long	Similar to the foregoing	..	..	57·29	5·93	32·17	4·61	60·06	6·21	33·73
		Champ-du-Feu	In a less advanced stage of alteration, though still containing some vegetables	..	..	57·00	6·11	31·56	5·33	60·21	6·45	33·34
	Wood	.. ..	Average composition	..	..	49·60	5·80	42·56	2·04	50·62	5·94	43·44

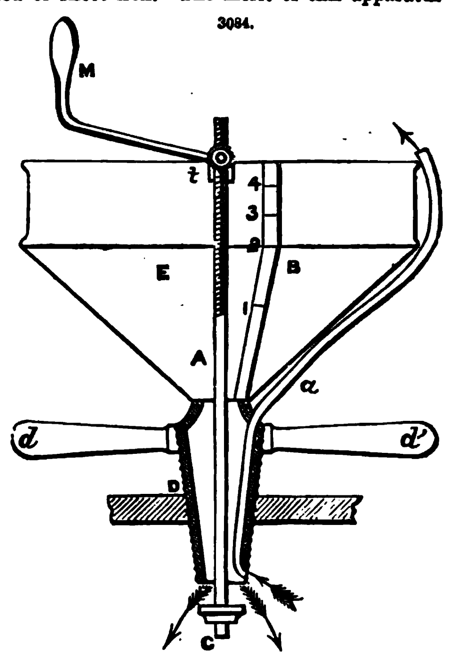
FULLER'S EARTH. Fr., *Argile smectique, Terre à foulon;* Ger., *Walkererde;* Ital., *Creta da sodare i panni.*

Fuller's Earth.—The fuller's-earth pits of Nutfield, near Reigate, are extensively worked, and supply large quantities of this substance to the clothing districts. There are two kinds, one greener than the other, owing to the presence of silicate of iron; but both exist under the same geological conditions, occurring in the lower cretaceous series, and differing little in chemical condition. Fuller's earth consists of about 45 silica, 20 alumina, and 25 water. When placed in water it almost dissolves, and when exposed to great heat it melts. It combines readily with grease, forming a kind of earthy soap, and for this reason is valuable in the manufacture of cloth made of animal fibre. The following is the mode of purifying and preparing the raw material for use;—The fuller's earth, after it comes from the pit, is baked or dried by exposure to the sun, and then thrown into cold water where it falls into a powder, and the finer parts are separated from the coarser by a method of washing in several tubs, through which the water is conducted, and where it deposits the different kinds in succession. These are used for different kinds of cloth, the coarser part for the inferior, and the fine for the better kinds of cloth. The soapy combinations formed by fuller's earth with the greasy portions of cloth during the fulling of cloth, are supposed in some measure to serve the purpose of mordants.

FUNNEL. Fr., *Entonnoir, Évent, Hatte de cheminée;* Ger., *Trichter, Luftschacht, Rauchfang;* Ital., *Camino;* Span., *Chimenea.*

A funnel, possessing many advantages over those in ordinary use, invented by M. Bignon, is shown in Fig. 3084. Bignon's funnel is constructed of sheet iron. The merit of this apparatus consists in its being furnished at its lower extremity with a conically-shaped projection, which is, in fact, a screw, since it has a thread cut upon the whole of its length. The result of this arrangement is, that it will adapt itself indifferently to any sized bung-hole. At the bottom of this screw is placed a small clack-valve, which can be opened and shut as required. Fig. 3084 is a vertical section passing through the axis of the apparatus. The body E is of sheet iron, tinned inside and outside, and soldered to the upper part of the copper tube D, which has two little handles, $d\ d^1$, cast in one piece with itself, and which serve to screw and unscrew the funnel. As the tube hermetically closes the orifice of the cask, the air, which endeavours to escape from the interior, passes out through the little tube *a*, which has one extremity inside the screw-tube D, and the other outside the funnel. The lower extremity of the tube forms a seat for the valve *c*, which is attached to the central rod A, whose upper extremity passes through a socket *t*, which retains it in its proper place. The upper half of this rod has a thread cut upon it to receive the boss of the handle M, which acts upon it like a nut. The *modus operandi* is very simple and efficacious. On turning the handle M to the right or to the left, the opening or shutting of the valve *c* ensues, and the communication with the interior of the cask is thus opened or cut off, as required. There is a scale B attached to the interior of the funnel, which can be graduated to the measures of capacity in ordinary use, and which therefore registers the exact quantity of liquid poured into the cask. This is a very great advantage when it is necessary to complete the filling of a cask already partially filled, and saves the trouble of first ascertaining the quantity in the cask and then of calculating that necessary to fill the cask.

3084.



FURNACE. Fr., *Fourneau;* Ger., *Ofen;* Ital., *Forno;* Span., *Horno.*

Furnaces, or *roast-ovens*, are used for roasting ores; they differ greatly in their construction, according to the method of their use. Iron ores are roasted in ovens similar to a common *lime-kiln* of large size, and one that may serve for either roasting or burning *lime*. No fine or small ore can be roasted advantageously in an oven of this kind. For other ores than carbonates of iron, argillaceous ores, these kilns are not well adapted. Pyrites cannot be roasted in them, neither most other ores, because it is impossible to regulate the heat so as to prevent the melting of the ore; and if this happens, of course that ore is either lost or is with difficulty recovered. In roasting poor iron ores it is extremely difficult to regulate the fire so that no parts of the ore are burned dead or melted. For these reasons, kilns for roasting are not so much in use as would naturally be expected; they save fuel, but are more expensive in labour than the open heap.

Reverberatory Furnace.—This apparatus forms one of the best furnaces for roasting; but as its application is by no means general, and as the form of a roasting furnace is modified according to the kind, form, and uses of ore, we shall allude to this method when treating of those substances to which it is applied.

There is a variety of forms in the apparatuses for roasting, but we cannot perceive any advantage in the use of them; neither in the form of lime-kilns, for wood; nor cupola furnaces, con-

structed like porcelain kilns; nor large chambers, into which the flame from a grate is conducted through the ore; nor other forms of apparatus. These contrivances are not calculated for our smelt-works; they cause more labour, and absorb more capital, than a smelting business can afford.

Principles of Roasting.—Roasting means to heat a substance, a metal, or a metallic ore, or matt, to at least a red heat, or such a heat that the mineral does not melt, but only the volatile or combustible substances are expelled, and at the same time as much oxygen becomes combined with the ore as it possibly can absorb. It is therefore a principal condition, that with the heat a liberal quantity of atmospheric air, or oxygen, is admitted. In some cases chlorine, carbonic acid, carbon, or steam, is required along with the air, or in their pure conditions. In most instances, the object is merely to oxidize the ore to a higher degree, or to drive off volatile matter and in the meantime oxidize the ore, or to combine chlorine with a certain metal, as silver; or to reduce ore to metal, and evaporate the latter, which is the case with *arsenic, zinc*, and *antimony*.

The operation proceeds faster when the ore is fine than when it is coarse, because more surface is offered to the oxidizing agent; but this method includes the motion of the particles, so as to expose their various sides to the heat. It is not always necessary that the ore should be a fine powder; but it is of great advantage to have it in pieces of uniform size, because the action of heat and air is more regular, and the surfaces acted upon are larger. In roasting more or less fine powder, it should be stirred and moved while hot. The melting of the substance must by all means be prevented, for in that case neither evaporization nor oxidation can be accomplished. In the large operation, and in the reverberatory furnace, the melting of any kind of substance which is to be roasted is easily prevented. Roasting is always applied to oxidize iron ores, in order to obtain the highest degree of oxidation. A simple oxidation is performed when magnetic ores are exposed to heat and air, and transformed into peroxide. Chlorides are produced when, for instance, hot silver ore is brought in contact with chlorine, or a salt of chlorine, such as common salt; the roasting operation is here performed to reduce the oxide. When arsenic is to be evaporated, we put carbon in the mixture, and produce metal, which is more easily evaporated than its oxide. An evaporating, roasting process, is that which is performed on hydrated oxides, when only water is evaporated; a compound operation is performed when evaporation and oxidation are produced at the same time. In roasting pyrites, blende, and arseniurets, the volatile substances are driven off by heat, and the remaining metal is at the same time oxidized, which is brought, in most instances, to the highest degree of oxidation.

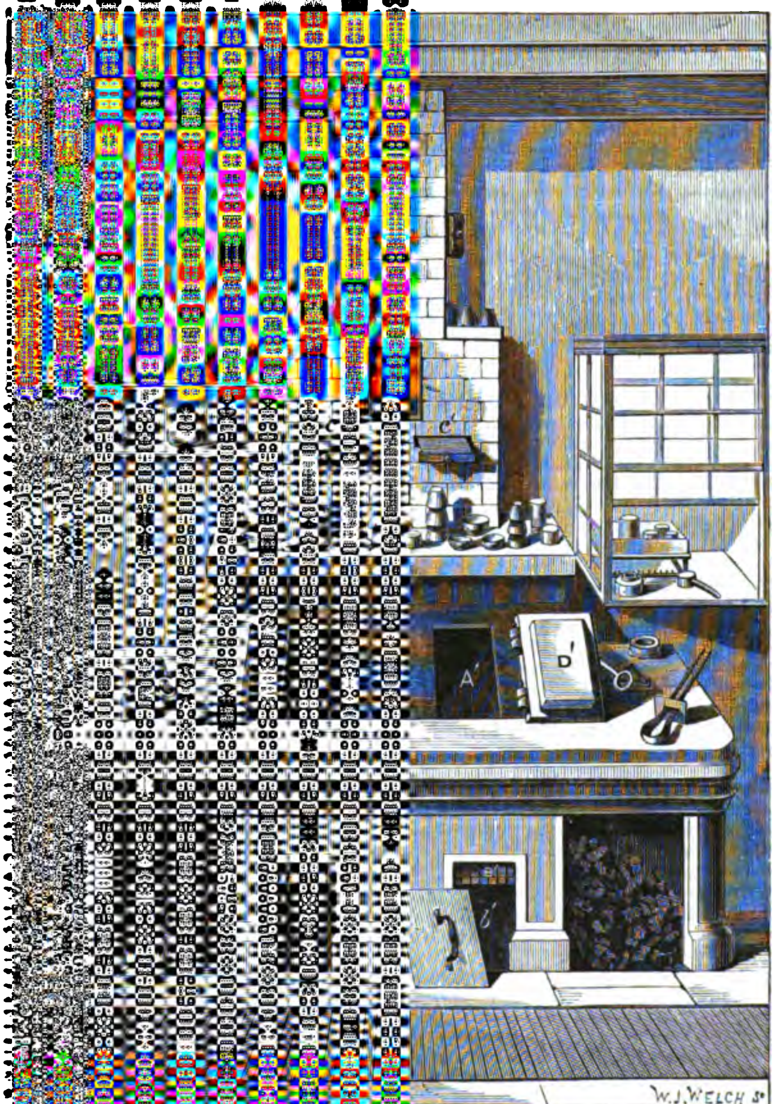
The affinity of the metals for other substances than oxygen, and the form in which these combinations appear, modify the process of roasting considerably. We shall allude to those particulars in the proper places; but it may be right to state here some general circumstances which have a bearing upon the subsequent operations. Iron cannot by any means be entirely freed from sulphur, phosphorus, or arsenic, by roasting; the presence of the vapours of water facilitates the expulsion of these substances, but the roasted ore never can be made entirely free from them. Blende, or sulphuret of zinc, is extremely slow to oxidize, and never can be purified from all the sulphur. Sulphuret of bismuth is equally slow of oxidation, not for want of affinity for oxygen, but because it is so highly fusible that its melting cannot be prevented. Sulphuret of copper is easily purified from all its sulphur. Galena is of very difficult oxidation, almost as much so as bismuth. Sulphuret of silver is easily liberated from its sulphur, and forms metal; the same is true with gold. Mercury acts in a similar manner, but it requires some caution to avoid evaporating the sulphuret of mercury with the sulphur. Sulphuret of antimony is of difficult oxidation, because it is extremely fusible. Sulphuret of arsenic is easily decomposed, but the result of the oxidation evaporates; the arsenious as well as the sulphurous acid both evaporate. The sulphurets of nickel and of cobalt are easily oxidized, and form pure oxides. Phosphorus and arsenic act in a similar manner as sulphur, and what applies to the latter applies to the former, with slight modifications. Phosphoric acid is more permanent than sulphurous acid, and silver cannot be entirely freed from arsenic if once combined with that substance.

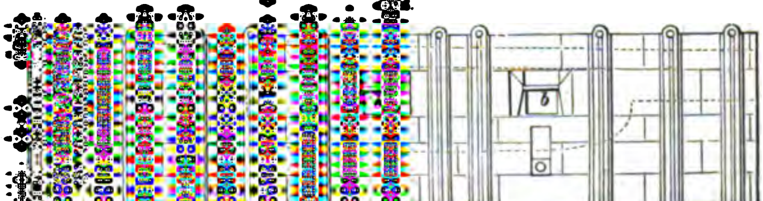
In quartz-crushing establishments generally, the assays of ores and tailings may be conveniently conducted in the furnace employed for retorting and melting, which is generally of sufficient size to admit of three or four fusions being carried on at the same time. In regular metallurgical laboratories, for the sake of durability, and to prevent the cracking of the brickwork, the outside of the melting furnace is usually secured by iron plates, as shown, Fig. 3085, which represents that employed by F. Claudet, and in which A A′ are the fire-places, B B′ the fire-bars, and *b b′* the ash-pits. The dampers C C′ permit of regulating the draught, and the mouths of the furnaces are closed by the hinged doors D D′, lined with baffle-plates. Instead of hinged doors, sliding plates are sometimes employed, and are, for general purposes, probably preferable. The dimensions of this furnace are, 10 in. square and 16 in. in depth above the fire-bars, which can, when necessary, be drawn out from the front for the purpose of allowing the coke to fall into the ash-pits *b b′*, or for unclinkering the grate. When used with charcoal, the crucibles must be supported on the bars on pieces of fire-brick; but when coke is employed, it has in itself sufficient resistance to allow of their being imbedded in the fuel without any other support.

The whole of the work lead produced at Pontgibaud is purified before it can be treated by Pattinson's process, and this is done by exposing it at a low red heat to partial oxidation in a *reverberatory furnace* specially adapted for that purpose. The chief impurity contained in the lead is antimony; the others are sulphur, iron, arsenic, and copper. All are in relatively small proportion, but are still in sufficient quantity to render the lead *hard*. The accompanying drawings, Figs. 3086 to 3088, show the arrangement and dimensions of the furnace employed.

Fig. 3086 is an elevation; Fig. 3087 a horizontal section at the level of the top of the pan; and Fig. 3088 a vertical section through the tap-hole. The fire-place A is separated from the pan B by a bridge 3 ft. 3 in. wide, and the furnace is provided with two doors *b*, through which the dross may be removed. In principle this resembles the ordinary softening furnace with its cast-iron pan,

…er it much more economical than the furnaces
…that the pan is not only much larger than those



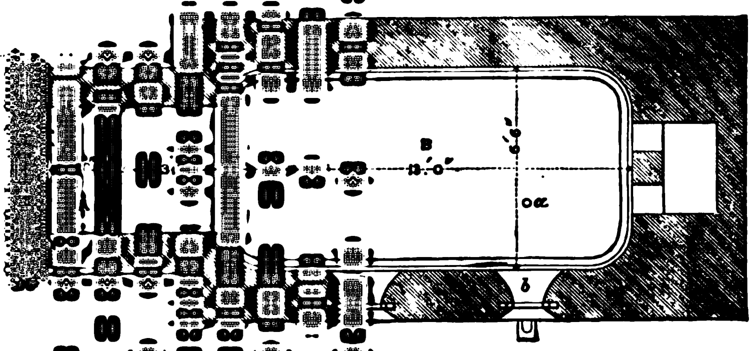


…Furnace.

…n; the object in giving it this shape being to …re-sided pans are so liable. Another essential …make them perfectly lead-tight, in case the iron

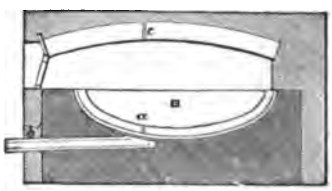
setting the pan on a bottom of well-beaten brasque … of masonry. The sides of the furnace must be … In either case the space between them and the … with hard-beaten brasque.

3087.



… small hole … tom of the … tube b', … into about … rging, the … inted bar … e roof, cor- … vertically … hole quite … working a … illed with … been pre- … e channel … the hole in the pan. This horizontal bar is even … of the latter being to take off the pressure of the … al in tapping. The horizontal bar is put in and … e same way as, though more easily than, a tapping

3088.



… with bricks, as shown in Figs. 3087, 3088, to protect … the surface of the metallic bath. This precaution … required being to avoid overheating the furnace, and … found that a dull red heat is the best temperature … int the oxides neither melt nor exert any corrosive … from time to time added. The usual charge of a … lining diminishes its capacity by about one-fifth. … res sixty hours to become sufficiently soft for treat- … necessary for the operation, including charging and

… ying the pan, and are paid 2 francs each per charge. … the working doors by means of a long charging bar, … nd rapidity. Except for charging and discharging … is attended to by one of the men working at the

… as follows ;—

…	…	… Lead …	Dross. Tons.	Coals consumed. Tons.	Lime used. Kilos.
[illegible]	[illegible]	…7	8·675	11·5C0	554

… llic iron is chiefly employed for smelting galenas … gangue. The reduction of galenas rich in lead, of … ch generally contain but a small amount of silver, … of a *reverberatory* furnace, and a subsequent fusion

… double decomposition which takes place between the … tions of the ore which have, by roasting, been con- … way the decomposition of one equivalent of sulphide … tal, give rise to the production of an equivalent o … uivalents of metallic lead,

$$… O = SO^2 + 3\,Pb.$$

...f sulphide of lead and one of sulphate, results ...l, and the liberation of two of metallic lead,

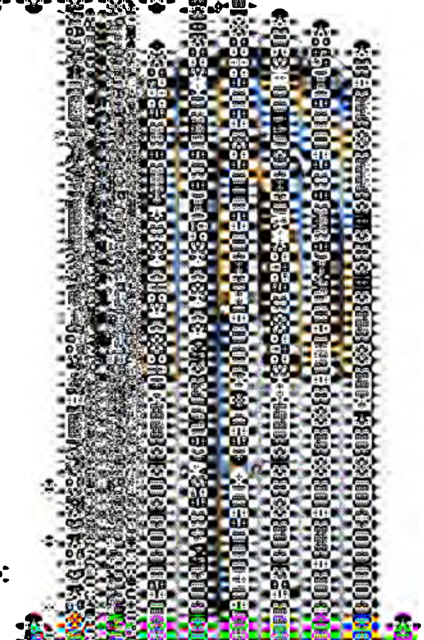
...2 SO² + 2 Pb.

...of silicious or argillaceous gangue renders the ... when contained in the ores to the extent of ...obtained in the reverberatory furnace by these ...rge amount of silver cannot be advantageously ... moderate percentage; and it is consequently ... in the raw state, or, after a partial roasting, ...s case, by combining with the sulphur of the ...duced. When ores only partially roasted are ... of a somewhat more complicated character; ... lead, are the same.

...in about 30 oz. of silver a ton, and the furnace ... which are added certain secondary products,

...r cent. of lead, 34 parts; old cupel bottoms, ...ge from cupel, 1 part; slags from a previous ...; granulated cast iron, 4½ parts.

...ing from 18 to 20 ft. in height, and having a ...3091: Fig. 3089 being a vertical section at right ...ne of the nozzles; and Fig. 3091 a horizontal



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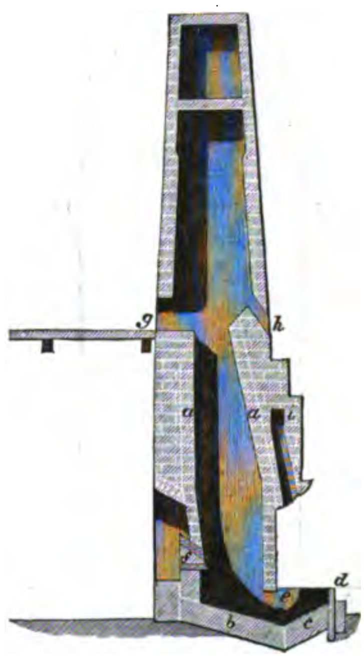


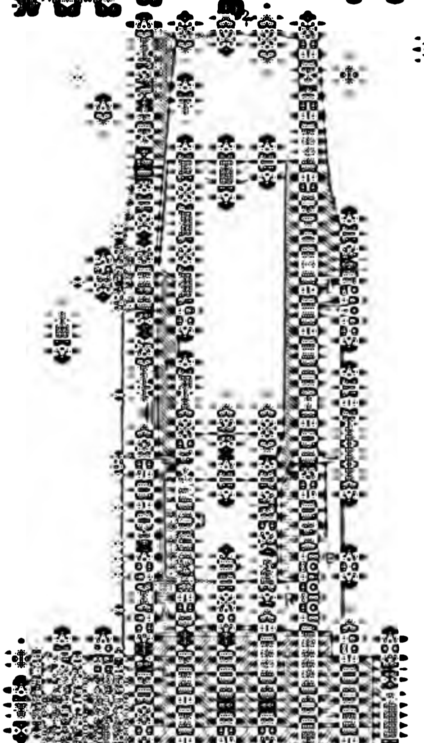
...usthal Furnace. Vertical Section through Tuyere.

3091.

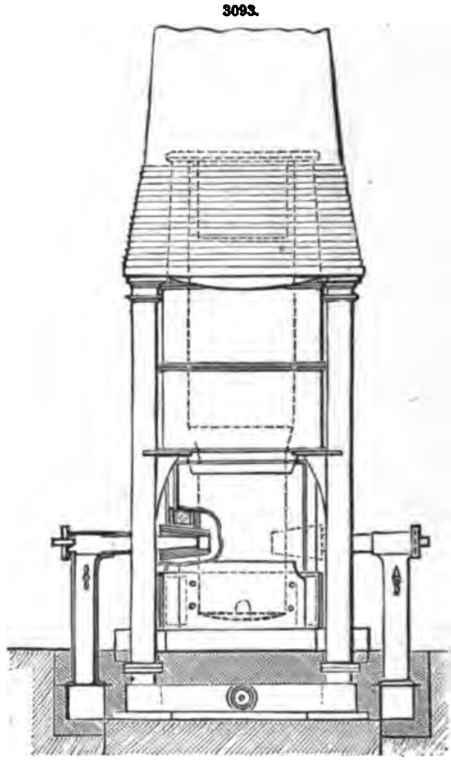
Clausthal Furnace.
Horizontal Section through Tuyeres.

...the interior ...e c are of ...on plates d. ... front wall ...ion is thus ...n flow into ...and have a ...other. At ...this furnace ...allelogram, ... The dis...yeres, 4 ft., ... hole is 2 ft. ...ening g, and the small aperture h enables the ... flame which may escape at the top; it being

lead, to keep the charging hole constantly dead.
chambers is seen, Figs. 3089, 3090. The side flue *i*
, and is for the purpose of carrying off any arsenical
-pan *e*. In the back wall of the furnace are small
the various deposit chambers and flues. On each
the establishment, is a float or tapping pan *l*, into
run out; whilst the slags flow off in a continuous
the brasque which forms the breast.
in which are placed the tuyeres, whilst the fuel is
is opposite to them. The blast, passing through
with the liquid slags in the body of the furnace, so
e. This is so managed by the smelter as to extend
e basin and hearth have become full of matt and
ir contents are run off into one of the floats *l*, where
s to solidify, when it is removed in discs, as before
ulds ready for desilverization.
The chief object of the inventor of this furnace
the better withstand the action of heat and scouring
metals, or in the smelting of their ores. The main
neral arrangement of the parts, are the construction
on the water-boshes. Fig. 3092 shows a sectional
gles; and Fig. 3094 a sectional plan of one of these



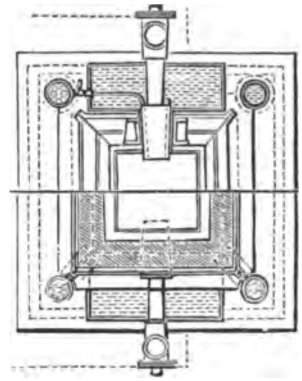
3093.



s first makes a foundation A, and fixes therein a
he furnace, except at the front. There is a branch C
g provided with a valve and a valve-well D. There
on which is fixed the hollow columns E E. These
ture is fixed on them, on which the chimney is built.
from a high level, so as to rise and fill the columns
er and carry the chimney. Brackets *a a* are cast on
the upper brickwork of the furnace. Upon the foun-
bottom of the furnace, as shown at F. He then
on, all round the furnace, except at the front, where
, or a dam, as in blast furnaces. These boshes are
ide, to allow of a thickness of lining G of fire-brick.
above those just described; these upper boshes go
hey have spikes *b b* cast all over their inner sides,

may be of any suitable length, but 1½ in. is
boshes against the spikes are lined with stiff
anister and fire-clay, or road grit and fire-clay.
iron plates O O, which enclose the four sides.
of the furnace. On the bottom edge of these
of the furnace, independently of the boshes N,
introduced

3094.



the top of
rial. The
e columns
nns; each
A hole is
c c; there
, as shown
n form to a
ottom, and
rm of fur-
treatment,
d fall like
sirable, the
face of the
the upper
the spiked
fluxed away
back of the
furnaces for
f the furnace is built in the usual way up to
ater-boshes are then set around the sides and
ainst, as in the cupola or blast furnace. For a
ht iron, the spiked water-boshes are set round
-smelting furnace.

This process has been successfully introduced
percentage of all the silver produced in that
acted by its use. The process has lately been
over the old plan of roasting and reverberatory
reby very considerably reduced, and the roast-
time than in the reverberatory. One man can
by which the process is performed.

ler iron, lined in a durable way with strongly-
re-box and a flue; from the fire-box the flame
rom there, together with the gases produced in
the latter escapes through a smoke-stack.

n angle of about 15° to the axis of revolution,
ngth of the cylinder, for the purpose of moving
posing it to the action of heat and atmospheric
anically the work of transferring the ore from
earest to the fire-bridge, and *vice versâ*, which
hand once every hour.

le length of the cylinder, the latter is provided
about 45°, for the purpose of conveying the ore

any person in a few days, and is as follows:—
s. of salt, is introduced through a man-hole into
n heated to red heat, the opening closed, some
made to revolve at one-half to one revolution a
after one hour's time the sulphur contained in
gulated that the ore is kept all the time at a
Very little fuel, and in ores containing much
e sulphur has been oxidized; but then some
the temperature of the ore pulp to an intense
lly consistency, in consequence of the mutual
and salt, chloride of sodium, and of the chlorine
as soon as a sample taken from the furnace
be perceived any more, the chloridizing roasting
l, and while the cylinder is kept revolving the
veyor, which conveys it through an iron trough
hanical contrivance, the formerly so expensive,
a large cooling floor is done away with, and is
labour, within fifteen minutes' time, since the
into the screen and the hopper ready for further

tor has also made some additional improvements
f the cylinders, and to prevent at the same time
olatile chlorides from being carried off by the

5 K 2

draught. It must be borne in mind that the greater the quantity of atmospheric air which comes into contact with the heated ore the quicker the roasting process is performed.

The arrangement consists in the use of a steam suction-pipe set, in the direction of the draught, into the flue between the cylinders and the condensing chambers; said pipe being arranged in such a manner that the draught through the furnace is considerably increased, and all volatile matter condensed and collected at the bottom of the chambers provided for this purpose.

This furnace can be used for roasting any kind of refractory silver ore; also for desulphurizing auriferous pyrites previous to chlorination or smelting; for roasting ores of zinc, lead, copper, &c.; also for burning cement, and for the manufacture of soda from cryolite.

The Stetefeldt Furnace.—This invention is one of the most important steps of progress yet achieved in silver metallurgy; and its direct effects in stimulating the production of bullion, by reducing its cost, will be felt immediately. Already the mines of many a half-abandoned district are augmented in value and importance by the mere announcement of its success.

The following description of the Stetefeldt furnace is from the notes of the inventor himself, and from the records of actual experience at Twin River, Reno, &c.

Since the discovery and exploration of the numberless mineral deposits in the Western States and Territories, no branch in metallurgy has received so much attention as the process of roasting ores of all descriptions. One can hardly look over a file of mining journals, or newspapers from some mining district, without finding descriptions of new devices for roasting ores, all of which claim to surpass everything else in this line which was known before. The devices are as strange as they are many, and much time and money have been wasted to test impracticable inventions. Indeed, the high expense which the roasting in the old reverberatory furnace entails was a strong inducement to invent some cheaper, and at the same time more effective, method. This is especially of importance where silver ores are found which require a chloridizing roasting preparatory to their amalgamation. In such cases the expense of roasting is frequently more than one-half of the total expense of reduction, and consequently low-grade ores cannot be worked with a profit. But in spite of the necessity to adopt some improved and more economical process of roasting, it has been extremely difficult to introduce two inventions, which are based upon the most simple and rational principles—so simple, indeed, that it seems impossible to simplify them any more. We refer to the Gerstenhöfer, or Terrace, furnace, first introduced about six years ago at Freiberg, and the Stetefeldt furnace, invented three years ago at Austin, Nevada, but first introduced for regular working at the mill of the Nevada Silver Mining Company, near Reno, Nevada. The nature of these inventions can be expressed as follows;—

Gerstenhöfer discovered that sulphurets are completely roasted or oxidized if they fall against a current of hot air rising in a shaft which is filled with shelves, so as to check and retard the fall of the ore particles at certain intervals.

Stetefeldt discovered that silver ores, no matter in what combination the silver occurs, mixed with salt are completely chloridized if they fall against a current of hot air rising in a shaft with no obstructions whatever to check or retard the fall of the ore particles.

It is a matter of course that in both cases a certain degree of fineness is required for the ore to be treated, and that a much coarser material can be successfully roasted in the Gerstenhöfer furnace than in Stetefeldt's.

In the Gerstenhöfer furnace only such ores can be successfully treated which, at a red heat during roasting, have no tendency to sinter or stick together. But the small particles of a charge of ore mixed with salt are exactly in such a condition while roasting as to have the greatest possible inclination to sinter and adhere to the shelves. They would thus soon obstruct the whole shaft, and prevent any further work. This has been demonstrated by actual experiments on a working scale. It is apparent, therefore, that the application of the Gerstenhöfer furnace, even for desulphurizing purposes, is very limited, and that certain classes of ore must be entirely excluded from it. This is especially the case with galena ores, which are the most expensive to roast in reverberatory furnaces.

In Stetefeldt's opinion, the shelves in the Gerstenhöfer furnace are perfectly superfluous, and all ores, even galena, can be desulphurized by dropping them through a plain shaft heated by fire-places below, if they are reduced to a sufficient degree of fineness. The escape of unroasted dust from the shaft is of no consequence, as a separate fire-place is constructed for the roasting of these suspended particles in the Stetefeldt furnace. Furthermore, the feeding machinery of the Stetefeldt furnace is based upon a principle entirely differing from that used with the Gerstenhöfer furnace.

That a furnace without shelves is cheaper and easier to construct, more durable, less liable to get out of order, and that it requires less labour and skill to run it, will be readily conceded.

Much difficulty was experienced to provide suitable feeding machinery for the Stetefeldt furnace. Gerstenhöfer's apparatus, consisting of fluted rollers, which force the ore through slits in the top of the furnace, would not answer at all. The ore fell down in lumps, and arrived at the bottom of the shaft almost raw. The reason for this behaviour is simply the tendency of the particles of all finely-pulverized mineral substances to adhere to each other if a slightly-compressed mass of them falls through the air. It is, therefore, necessary to introduce the ore pulp so finely divided, that all the particles can be penetrated by the heat within the short time of their fall through the shaft. To feed the pulp with a blower, as in Keith's desulphurizing furnace, was not considered desirable for the following reasons;—

1. The fall of the ore would be accelerated.

2. The draught of the fire-places would be impeded by the downward current of the air from the blower.

3. The formation of dust would be considerably increased.

The feeding machinery in its present shape can be briefly described as follows;—

A hollow cast-iron frame, kept cool by a small stream of water, rests on top of the furnace. In this frame is inserted a cast-iron grate, which is covered by a punched screen of Russia iron, No. 0,

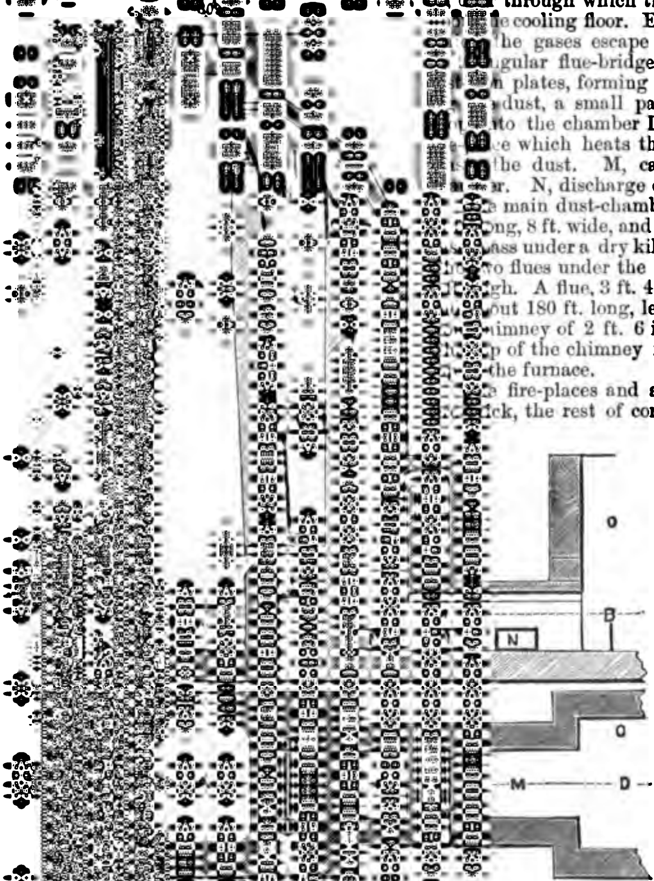
screen moves, inside of the hopper, a coarse frame. The frame has flanges resting upon receives its motion from a crank, with 1¾-in. lp with the coarse screen, a number of thin ir lower edges reach close down to the coarse is set in motion, the meshes of the coarse wire openings of the punched screen. In this way furnace. The motion of the crank-shaft was tions a minute.

ying drawing, Fig. 3095, will give a correct

shaft upon which the feeding machine is eens of the feeding machinery are exchanged. through which the roasted ore is discharged e cooling floor. E, fire-place. F, flue through he gases escape near the top of the shaft. gular flue-bridges of cast iron. H, grate of n plates, forming the bottom of flue F, allow- dust, a small part of which settles here, to to the chamber I. K, discharge door. L, e which heats the lower part of flue F, and the dust. M, canal connecting with dust- r. N, discharge door. O, dust-chamber.

main dust-chamber of the furnace at Reno is ng, 8 ft. wide, and 10 ft. high. From there the ass under a dry kiln, 39 ft. long and 7 ft. wide. o flues under the dry kiln are 3 ft. wide and gh. A flue, 3 ft. 4 in. wide and 4 ft. 6 in. high, out 180 ft. long, leads from the dry kiln to an imney of 2 ft. 6 in. diameter on a hill-side. p of the chimney rises about 40 ft. above the the furnace.

fire-places and arches are built of the best ck, the rest of common brick. All the walls are built double, with a space between. The furnace is well anchored with iron rails and ¼-in. rods.



The following changes are contemplated in the construction of the furnace;—

1. The use of oxide of carbon as fuel; the gas to be made out of charcoal in generators. The construction adopted for the latter will be similar to that of the copper-refining furnace at Mansfeld, Prussia. In this way a much more uniform heat can be obtained than by using wood, and labour will be saved, as the generators have to be charged only every three or four hours. charcoal will be even a cheaper fuel than wood. e F brought down directly on the side B B (see

ll be connected with the furnace.

ry amount of salt on the dry kiln, and crushed een. A conveyor takes the pulp to a revolving used by the breaking of a battery screen. The of the furnace and discharged into a bin, which

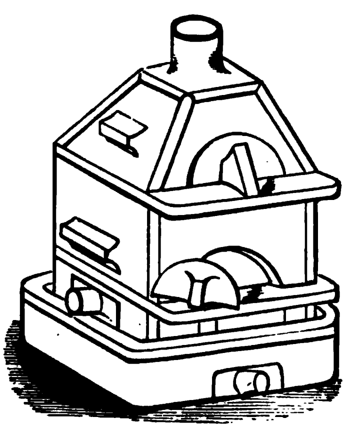
s possible, and such a degree of heat is main- shaft is red hot, but does not sinter or stick 1000 lbs. to a ton has accumulated, and cooled e is discharged through the door N, where most L.

he first glance it would seem that, considering ore is exposed to the flame, a perfect chlorin- with the known facts apparent in the common

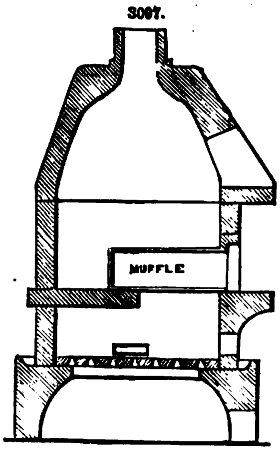
roasting furnace—that is, that sulphurous acid is formed under influence of a dark red heat, by aid of the oxygen of the air, while the metal, deprived of its sulphur, becomes an oxide. The oxygen of the air and of the oxide act on the sulphurous acid, converting it into sulphuric acid, which again combines with the metal oxide to a sulphate. The sulphate reacts now on the salt, setting the chlorine free, and the formation of chlorides begins.

This reaction and transformation requires time, which is not offered in Stetefeldt's furnace, but the chlorination is effected nevertheless, and very perfectly, with less salt and in a few seconds. The chemical action in Stetefeldt's furnace is as follows;—As soon as the ore enters the furnace each sulphuret particle ignites, being surrounded by a glowing atmosphere, evolving at the same time sulphur, which, in presence of atmospheric air entering undecomposed through the grates, is converted into sulphurous acid, and the metal into an oxide. In contact with ore particles and oxygen the sulphurous acid becomes sulphuric acid. This acid does not combine with the metal oxide to a sulphate, as is the case in a common furnace; or if so, only to an insignificant degree, on account of the temperature, which, nearly from the start, is too high. The sulphuric acid, therefore, turns its force directly against the glowing salt particles, setting free the chlorine. All these reactions are, so to say, *in statu nascenti.* From the burning fuel steam is present among the gases, giving rise to the formation of hydrochloric acid. This hydrochloric acid not only originates directly by decomposition of the salt, but also from the chlorides of the base metals, which are formed in the upper part of the furnace, and again decomposed to oxides and hydrochloric acid in passing through the hot flame. The whole space of the furnace is then filled with glowing gases of chlorine, hydrochloric acid, sulphuric acid, sulphurous acid, oxygen, steam, and volatile base metal chlorides; all of them acting on the sulphurets and oxides with great energy. The chlorine decomposes the sulphurets directly, forming chloride of metal and chloride of sulphur. It decomposes and combines also with oxides and sulphates. The hydrochloric acid does the same. The sulphuric acid decomposes the salt and oxidizes the sulphurets, while the oxygen creates sulphurous and sulphuric acid and oxides. The red-hot ore falls down, and accumulating, continues evolving gases of chlorine, &c.

3096.

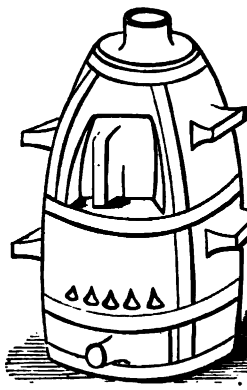


3097.

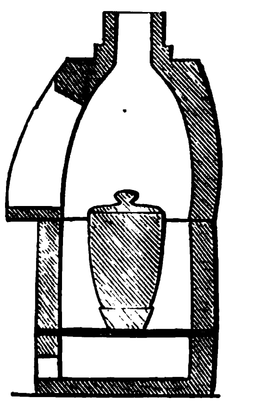


Considering now a minute particle of ore (for only as such, not as a mass, can the ore be considered in falling) in a red-hot state being attacked contemporaneously by all those gases which have free access from all sides; the principle of the Stetefeldt furnace is, that the chloridizing result must be effected before the particle reaches the floor The dust which passes the flame of the small fireplace is even in a better condition for chlorination, being surrounded and acted upon longer by all the chloridizing gases which are formed in the main shaft.

3098.



3099.



Practical Results of the Stetefeldt Furnace in Chlorination.—A great number of tests were made during the first weeks of running the furnace at Reno. Between 88 and 92½ per cent. of the silver contained in the ore was found to be chloridized, all of which is easily extracted in amalgamation. The roasted dust discharged through the door N is generally 1 per cent. better chloridized than the ore discharged from the main shaft. With an improved system of firing, the chlorination should never be less than 90, and we have no doubt that much higher figures will be obtained. Only very skilled roasters achieve such results in the reverberatory furnace. With ordinary care, a charge cannot be burned in the Stetefeldt furnace, and the roasted pulp is in a splendid condition for barrel amalgamation, as it contains no lumps or sintered matter. Ores

of the most various characters have been roasted with equal success. Even ore containing nothing but silver-bearing galena was treated without any difficulty. In this respect the furnace is admirably adapted to roast ores with large amounts of antimonial and lead-bearing minerals.

Amount of Salt.—In reverberatory furnaces 10 per cent. of salt is generally used. This amount may be safely reduced to 6 per cent. for very rich ores, and to 3 and 4 per cent. for low-grade ores, in the Stetefeldt furnace. No experiments have as yet been made to determine if this percentage can be reduced still more. The difference in the percentage necessary is explained by the fact that in the Stetefeldt furnace all the salt is decomposed and utilized, while in the reverberatory furnace a large percentage remains in lumps and entirely unchanged.

Fuel.—The amount of fuel necessary to heat the shaft depends very much upon the character of the ore. The more sulphurets an ore contains the less fuel is required to roast it. The furnace in Reno uses on an average about two cords in twenty-four hours. With this amount between 12 and 15 tons of ore are roasted daily, which is as much as the battery crushes. But the same fuel would just as well roast 20 tons of mainly sulphuret ores, which increase the heat in the shaft when introduced in larger quantities. How many bushels of charcoal a furnace with gas generators would require we are not able to estimate reliably at present; but for most localities in Nevada charcoal will be as cheap if not cheaper than wood.

Fig. 3098 is of a portable melting furnace; Fig. 3099 is section of 3098, which shows the interior arrangement and the position of the *crucible.*

This furnace, lately introduced by the Plumbago Crucible Company, Battersea Works, can be employed in a very confined space; no blower is necessary; its heat may be increased to melt gold by merely lengthening the funnel. This furnace is of great use to the gold-beater, as it renders him independent of the gold-refiner.

Figs. 3096 and 3097 are of a muffle furnace for assayers, dentists, and enamellers.

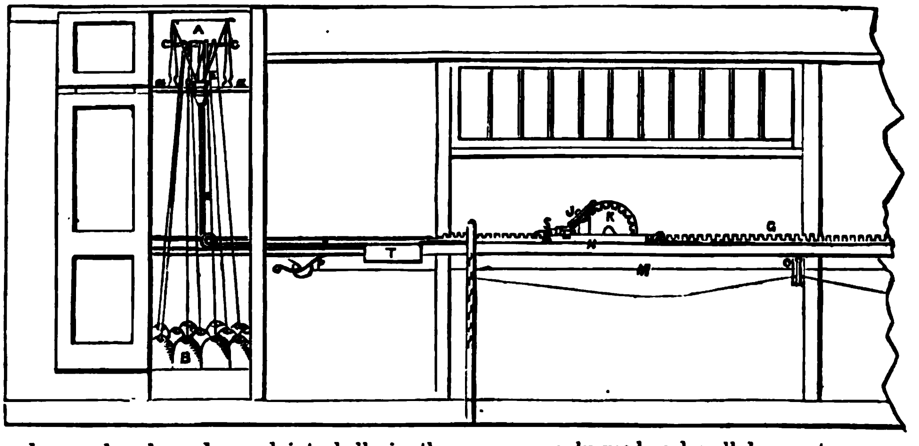
See ANEMOMETER. ANIMAL-CHARCOAL MACHINE. ASSAYING. BLAST FURNACE. BOILER, p. 466. CHIMNEY. ELECTRO-METALLURGY, p. 1374. FOUNDING AND CASTING. KILN. OVENS. See also articles on the various Metals.

FUZE. FR., *Fusée;* GER., *Zünder;* ITAL., *Fuso;* SPAN., *Espoleta.*

A *fuze* or *fuse* is a tube filled with combustible matter, and used for discharging shells, in blasting, and so on. See BORING AND BLASTING.

Fuze-making Machine, Bickford's, Figs. 3100 to 3105.—The fuze invented by Bickford for igniting gunpowder when used in the operation of blasting of rocks and in mining, which he called the miner's safety fuze, is manufactured by the aid of machinery, and of flax, hemp, or cotton, or any other suitable materials spun, twisted, and countered, and otherwise treated in the manner of twine-spinning and cord-making. Bickford, in describing his machinery, observes;—"I embrace in the centre of my fuze, in a continuous line throughout its whole length, a small portion or compressed cylinder, or rod of gunpowder, or other proper combustible matter prepared in the usual pyrotechnical manner of firework for the discharging of ordnance, and which fuze so prepared I afterwards more effectually secure and defend by a covering of strong twine made of similar material, and wound thereon, at nearly right angles to the former twist, by the operation which I call countering, hereinafter described, and I then immerse them in a bath of heated varnish, and add to them afterwards a coat of whiting, bran, or other suitable powdery substance, to prevent them from sticking together, or to the fingers of those who handle them; and I thereby also defend them from wet or moisture, or other deterioration, and I cut off the same fuze in such lengths as occasion may require for use; each of these lengths constituting, when so cut off, a fuze for blasting of rocks and mining, and I use them either under water or on land, in quarries of stone and mines for detaching portions of rocks, or stone, or mine, as occasions require, in the manner long practised by and well known to miners and blasters of rock. In Fig. 3100 I represent that part of the manufacture of my fuze called twisting, the yarn or other material being assumed to be already prepared

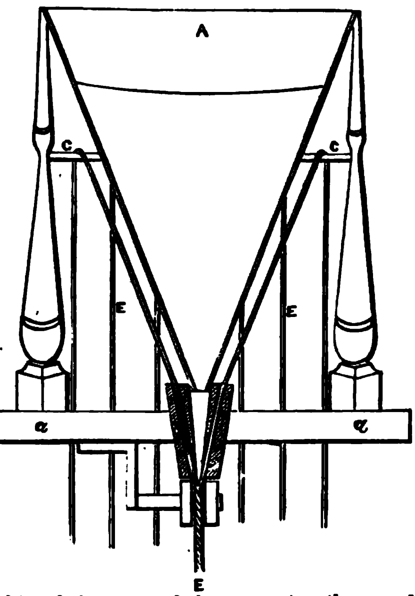
3100.



and spun loosely, and wound into balls in the way commonly used and well known to manufacturers of twine and cord; and I also therein represent the mode of charging the fuze with the combustible.

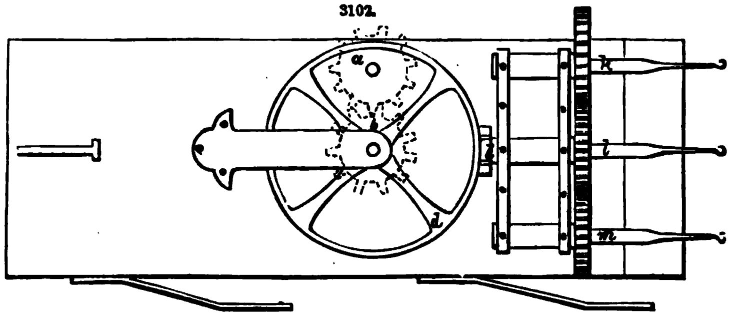
"At the left-hand end of an apartment, which is 65 ft. long, is made an enclosed recess or closet of about 2 ft. square and 6 ft. high, with a door or doors in front; in which closet, at the height of about 4 ft. 10 in., is placed a wooden shelf, marked A, about 1 in. thick at least, extending the whole length and breadth of such closet. In the centre of this shelf is made a hole, into which hole is inserted a collar, marked D. This collar is of metal, in form the frustum of a cone inverted. It is 3 in. long, 2 in. diameter at the upper end, and $1\frac{1}{4}$ in. at the bottom; through the centre of this is a hole, $\frac{3}{4}$ of an inch diameter at the top, and $\frac{3}{16}$ of an inch diameter at the bottom. Around this, in a circle, are twelve holes, of about $\frac{1}{8}$ of an inch diameter, which converge toward the centre hole at the bottom, so as to be separated from it only by a fine edge of the metal, Fig. 3101. D is a vertical section of the collar, and *r* the top and S the bottom of it. This collar, when so placed in the hole of the shelf, projects both above and below the wooden shelf. In the upper projecting part of the cone or collar is then to be placed a common funnel, 12 in. high and 10 in. in diameter at the top, as represented by the letter A; around this funnel, at about 10 in. high from the before-mentioned shelf, is placed a ring, made of cane, marked *c c*, and supported by a small frame of two or more pillars, resting on the before-mentioned shelf. At about 2 ft. 6 in. high from the floor of this room, and passing through the said closet, and extending the entire length of the said room 65 ft. long, is a stage or shelf or bench, marked F F; the outside of this stage or shelf or bench has a ledge or raised edge rising 1 in. above its surface, and on a similar raised edge on the inside, rising $1\frac{1}{2}$ in., is a line-rack marked G G, with teeth or cogs, twenty teeth to a foot. This stage or shelf or bench is intended to support thereon a machine, being part of the apparatus thus used by me in my invention, called the monkey, marked H, which monkey consists, first, of a plane piece of board, 20 in. long and 6 in. wide, supported by and running upon three wheels of $1\frac{1}{2}$ in. diameter; on this board, supported by, and turning in two centres, is a transverse axle, marked 2, placed quite across the plane bed of the monkey, supported by brackets, marked 1, 1, and turning round in holes made in the brackets, on the inside end of which is fixed a wheel marked K, of 10 in. diameter. Close to this wheel K, and directly over the line-rack, is placed on the said axle a pinion marked Z, which works on the cogs of the line-rack G G, on the side of which wheel K at its outer edge are twenty-four teeth or cogs; these teeth or cogs work into corresponding teeth on the inner circle of the wheel J, the wheel J having two circles of teeth or cogs, the inner and smaller circle working, as already described, in those of the wheel K, and the outer circle of cogs working into the pinion L; connected with this pinion L is the crook S, and to this crook the threads of twine or other material are attached for twisting. A string or cord M is fastened to the board of the monkey, and stretching along the stage or shelf or bench F F, passes over the pulley N, and returning through the holes O, O, O, made in the supports of the stage or shelf or bench, is attached to the winding roller P; in this situation of the apparatus twelve balls of twine marked B or other material intended to form the fuze are placed in the recess or closet on a floor raised there for that purpose 6 in. higher than the floor of the room, and the running threads from these balls are each led perpendicularly up through holes of 1 in. diameter, made for that purpose in a circle of 12 in. diameter in the shelf in which the before-mentioned collar is placed, and also perpendicular to and passed from the outside to the inside over the said cane ring next to the funnel hereinbefore mentioned, and from thence the said threads are again led down by the side of the funnel to and through the holes in the upper and under side of the aforesaid collar, which threads will then converge towards the lower point of the inner cone or collar, and there hang parallel and near to each other, and from thence they are to be led together to the pulley Q. Thence passing under the said pulley at a right angle they go to the monkey on the stage or shelf or bench, and are there made fast to the crook S. The winding roller P being now put in motion communicates that motion to the monkey H which travels on the stage or shelf or bench marked F F, towards N, and at the same time by the pinion marked Z working on the line-rack G G; the cog-wheels before mentioned marked J and K work the pinion marked L, and turning thereby the crook S, completely twist the twelve threads marked E so made fast thereto, and continue that twist up to the very point of the cone projecting downwards from the collar under the funnel; at the very same time of putting the monkey in motion the funnel is charged with the gunpowder or other combustible matter for making the fuze, and then it is important to carefully watch the progress of the threads, and prevent or rectify any entangling thereof; and also regulate the exit of the powder and prevent the dispersion of any surplus that falls to waste through the point of the cone or collar under the funnel during the operation of twisting. In this operation of twisting the powder passing the funnel lodges in the centre of the twelve threads, and is simultaneously embraced by all the threads, and the twisted part thus containing the powder is by the monkey drawn down and passes under the pulley Q, and continues its course with the monkey twisting the fuze along the stage or shelf or bench to the end of the room at N. Here the monkey stops, and this part of the fuz e so charge

3101.

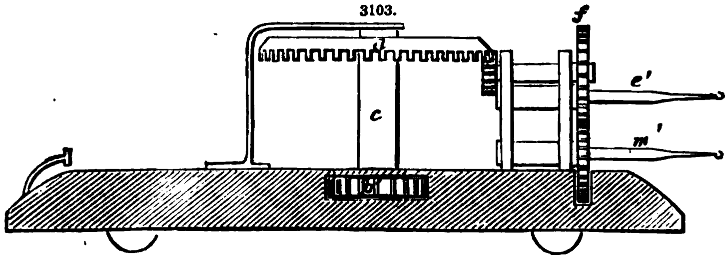


and twisted is then cut off, and over a box marked T placed on the said stage or shelf or bench to receive any gunpowder or other combustible matter used therein that may fall from the ends when so cut asunder; the two ends thus separated are secured by a knot or tie made on each; the part so twisted and separated is put aside for the subsequent operation of countering." By the monkey just described only one fuze could be spun at the same time, whereas it is desirable that several fuzes should be spun at once. By Bickford's improved machine, Figs. 3102 to 3105, three or more fuzes may be spun at the same time by one apparatus. The nature of this improvement, and the construction of the improved monkey for this purpose, will be seen by reference to the annexed figures. Fig. 3102 is a plane; Fig. 3103, a vertical longitudinal section; and Fig. 3104, a vertical transverse section of a monkey for spinning three fuzes at the same time. The wheel *a* works another of equal size and number of cogs *b*, shown in dotted lines in Fig. 3102, and seen in section in Fig. 3103. This wheel *b* has a vertical spill or centre *c*, Fig. 3103, on the upper part of which is a crown-wheel *d*, Figs. 3102, 3103, with teeth on its under edge working the pinion E, Figs. 3102, 3103, having on the outer part of its centre the cog-wheel *f*, Figs. 3103, 3104, with eighteen teeth working into the wheels *g*, *h*, *i*, Fig. 3104, of the same size and number of teeth; on the centres of the wheels *g*, *h*, *i*, are the wires and crooks *k*, *l*, *m*, Fig. 3102. To these crooks are attached the several

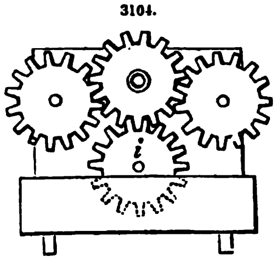
3102.



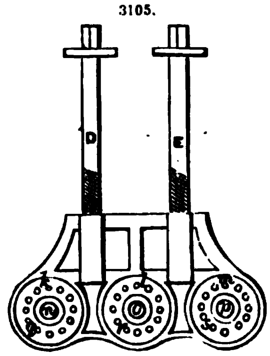
3103.



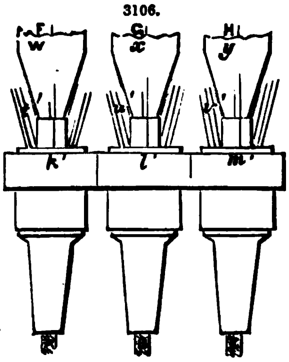
3104.



3105.



3106.



yarns, which as the monkey travels along the bench are spun into fuzes. We prefer to have fifty-two teeth in the crown-wheel *d*, eight teeth in the pinion *e*, twelve teeth in each of the wheels *a* and *b*, and twenty-four teeth to a foot in the side rack. In order to use this improved monkey it is necessary to increase the number of funnels for the gunpowder and collars, which may be arranged as shown in Figs. 3105, 3106. Fig. 3105 is a plan of these collars cast in one united piece of brass-work, *k*, *l*, *m*, being the three several collars, the whole being fixed to an upright frame by the screws D, E; the centre holes *n*, *o*, *p*, being intended for the reception of the gunpowder funnels *w*, *x*, *y*, Fig. 3106, and the several circles of holes *q*, *r*, *s*, being intended to receive the yarns, which meet below and embrace the gunpowder as the fuze is spun. Fig. 3106 is a vertical view of this same part of the machine. *k*, *l*, *m*, are the three collars; *t*, *u*, *v*, are the yarns passing into the several

circles of holes around the mouth of the funnel; *w*, *x*, *y*, are the three gunpowder funnels, and A, B, and C, the three fuzes issuing from the interior tube beneath. By placing the wheel *f*, Fig. 3103, in such a position that a large number of wheels may work into its teeth, and making such other necessary additions and arrangements as will be obvious to any competent workman, four, five, or more fuzes may be spun at one single operation.

Our second improvement in manufacturing fuzes relates to the mode of introducing the gunpowder according to the method described in the said specification; the gunpowder is not supplied from the funnel with the rapidity, regularity, and certainty which may be attained by our improvement. We introduce into the centre of the fuze a small strong thread or yarn, smaller and less fibrous than the yarns employed for the fuze. The thread or yarn which we employ is that known as No. 135 white-brown thread; this we supply from a reel or other source conveniently placed above the funnel containing the gunpowder; this thread or yarn is passed down through the gunpowder and spun into the centre of the fuze by being attached to the monkey and drawn on with it as the fuze is spun. The position of these threads is shown by F, G, H, Fig. 3106. By means of this thread so drawn on as the fuze is spun the gunpowder in the lower part of the funnel is constantly kept in motion and travels on with the thread, so as to flow regularly down into the fuze. By this means the continuity and regularity of the cylinder of gunpowder is ensured.

Our third improvement relates to the coating or varnishing applied to those fuzes which are to be used for blasting in dry ground, and in close or confined situations, or which are subject to considerable variations in temperature. The coating or varnishing heretofore applied, consisting of tar or resin, burns with a great deal of smoke and heat, and is affected by changes of temperature. Instead of a coating or varnish of either of those materials, we take 4 lbs. of best glue and 2 lbs. of yellow soap, and having dissolved them in 12 gallons of water by a gentle heat, we add 56 lbs. of whiting to give it a body; the varnish so made is applied to the fuzes by any suitable arrangements. The new varnish, not being waterproof, must not be employed for fuzes which are to be immersed in water, but being less affected by temperature than the old varnish, and being non-inflammable or burning with little smoke, is much preferable for general purposes.

Our fourth and last improvement relates to fuzes which are to be employed under water. In the manufacture of fuzes to be employed under water, or waterproof fuzes as they are called, it has been usual to add a second countering, after which the fuze was coated or varnished a second time in the usual manner. But fuzes prepared in this manner occasionally failed, in consequence of the varnish becoming more hard and brittle on immersion in water and cracking, and thus admitting the water to the gunpowder. Our improvement for obviating this defect is as follows;—After the fuze has been coated or varnished with tar or resin varnish, and before the coating is hard or quite set, the fuze is fastened to crooks and made to revolve as if for countering, and a strip of brown paper is wound around the fuze in a spiral form, so as completely to envelop and cover the whole surface of the fuze. A thread is then wound round over the paper, which fixes the paper and prevents its shifting; another coat of tar or resin varnish is then applied to the paper, and by this means the fuze may be completely waterproofed and protected against the action of the water.

FUZEE. Fr., *Fusée;* Ger., *Schnecke;* Ital., *Piramide;* Span., *Rueda espiral.*

See Escapement.

GABION. Fr., *Gabion;* Ger., *Schanzkorb;* Ital., *Gabbione;* Span., *Gavion, ceston.*

See Fortification.

GAD. Fr., *Pointerolle, Aiguille du mineur;* Ger., *Setzeisen, Stufeisen;* Ital., *Zeppa;* Span., *Cuña de acero.*

A *gad* is a wedge of steel for driving into crevices or openings made by the pick or chisel.

GALVANISM. Fr., *Galvanisme;* Ger., *Galvanismus;* Ital., *Galvanismo;* Span., *Galvanismo.*

See Battery. Telegraphy.

GALVANIZED IRON. Fr., *Fer zingué;* Ger., *Galvanisirtes Eisen;* Ital., *Ferro zincato;* Span., *Hierro galvanizado.*

See Zinc.

GAS. Fr., *Gaz;* Ger., *Gas;* Ital., *Gas;* Span., *Gas.*

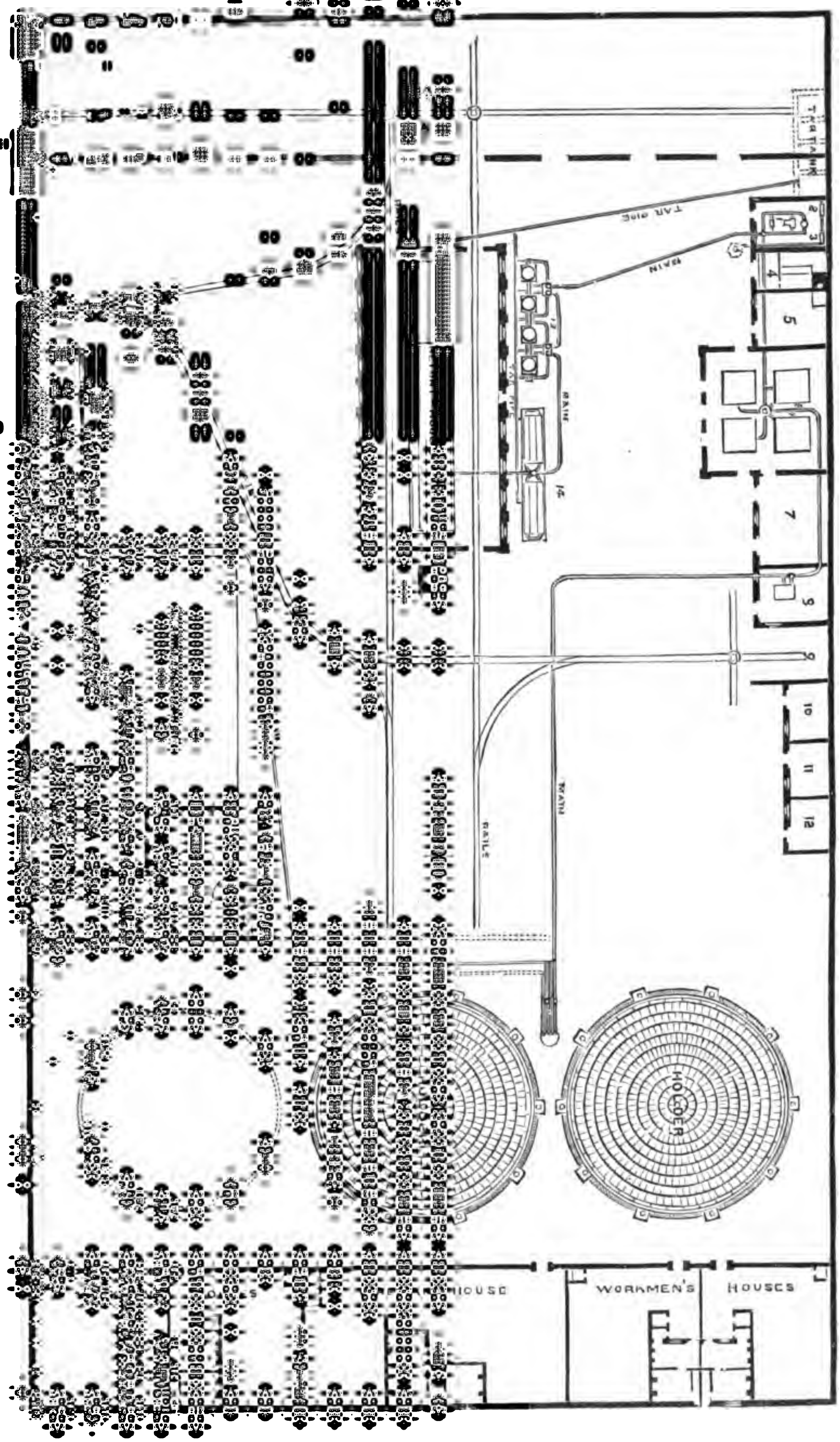
The common gas used for illuminating purposes is a mixture of carburetted hydrogen and olefiant gas, or bi-carburetted hydrogen, which gives a brilliant light when burned.

GAS, Manufacture of. Fr., *Fabrication du gaz;* Ger., *Gasbereitung;* Ital., *Fabbricazione del gas;* Span., *Fabricacion de gas.*

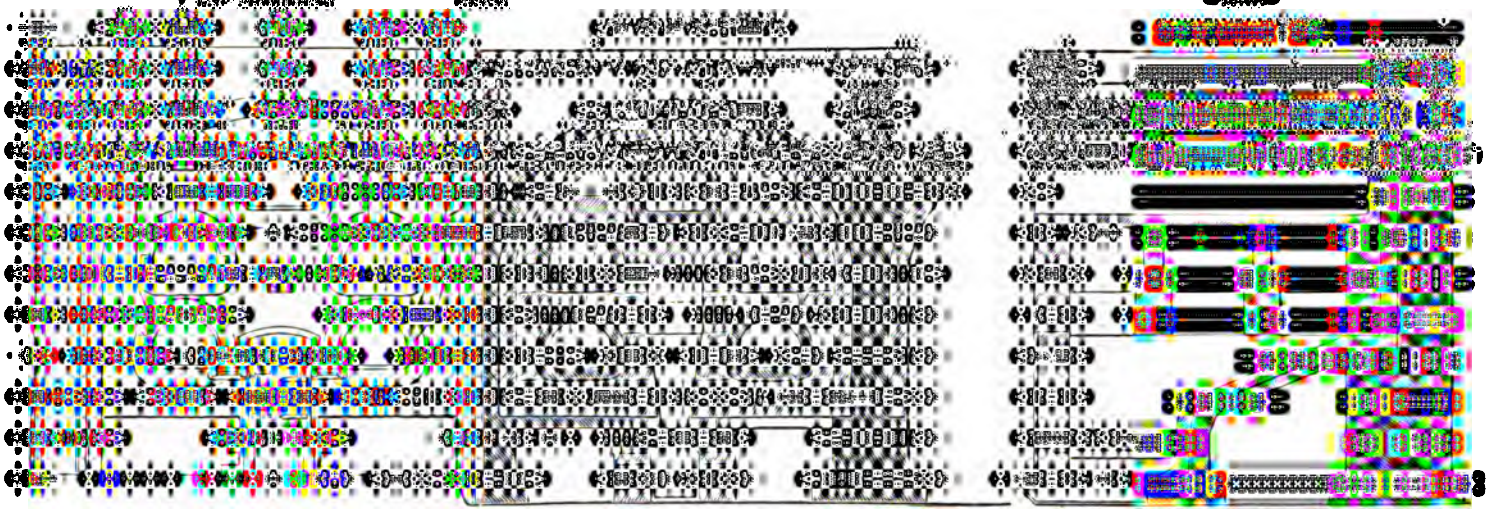
Fig. 3107 is a general plan of a gas-works by Geo. Bower, of St. Neots, Hunts. 1, is the exhauster; 2, the tar-pump; 3, water-pump; 4, boiler; 5, fitters' shop; 6, purifier house; 7, lime-stores; 8, meter; 9, spent lime; 10, 11, and 12, stores; 13, scrubbers; 14, condenser; 15, weigh-bridge; 16, water-well; 17, governor. It is very complete, and with the aid of the details shown on a larger scale in the following figures will illustrate the most modern and approved practice.

Figs. 3107 to 3113 represent sections and part elevations of the retort setting. The retorts are of *fire-clay*, provided with cast-iron mouth-pieces, from which ascension pipes conduct the gas produced in the retorts to the hydraulic main shown in the general plan, Fig. 3107. The retorts are heated in beds of five each, by means of a small furnace, under the fire-bars of which is placed a cast-iron evaporating cistern or pan, into which the ashes from the furnace are allowed to fall, and which evaporate either waste or ammoniacal liquor. The charge for each retort is about 1¼ cwt. of coal, drawn at the end of each six hours, producing at the heat of 27°, Wedgwood's *pyrometer*, about 700 cub. ft. of gas, or 3500 cub. ft. in the six hours, for each single bench of five retorts. The retorts are set in double benches end to end, and are charged from both sides of the house. A tramway is provided in front of the benches, connected with the coal-shed and the general system of rails on the works, so that the coal may be taken to, and the coke from the retorts with great facility.

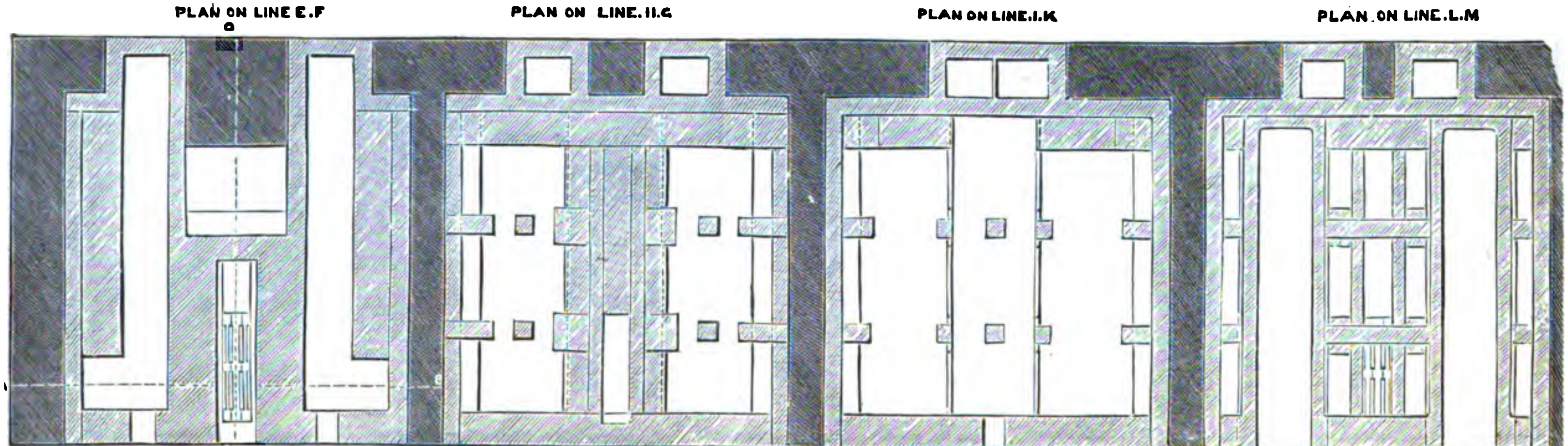
The hydraulic main consists of a horizontal pipe laid above the retort benches supported on small pillars. Into the main the whole of the pipes from the retorts are made to dip. The main


TAR TANK
TAR PIPE
MAIN
RAILS
HOLDER
WORKMEN'S
HOUSES
HOUSE
2
3
4
5
7
8
9
10
11
12
14

3107.



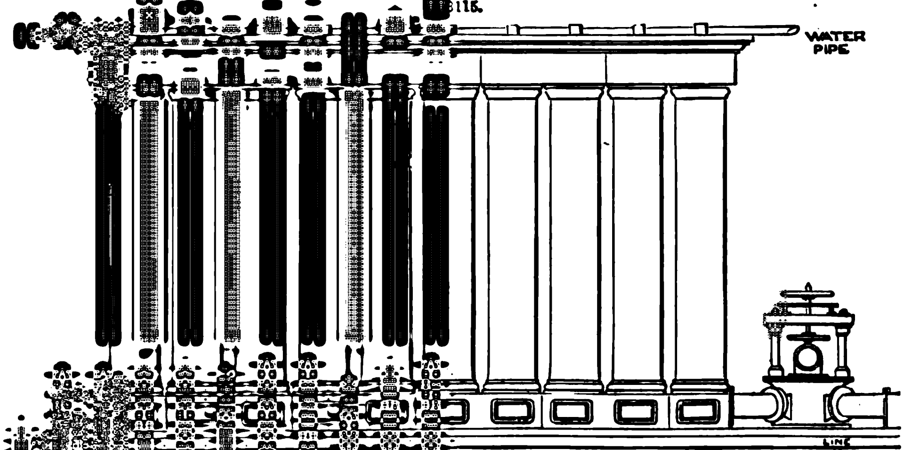
PLAN ON LINE E.F — PLAN ON LINE. H.G — PLAN ON LINE. I.K — PLAN. ON LINE. L.M



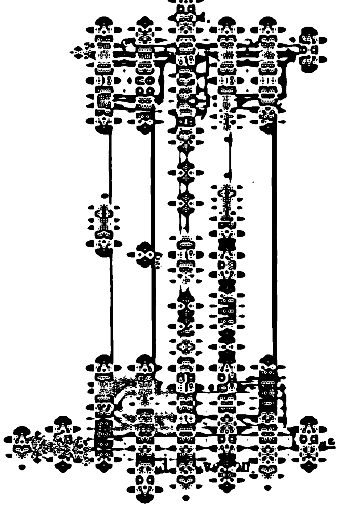
3110. 3111. 3112. 3113.

ipes are made to terminate about 1½ in. below its
prevents the return of gas to the retorts when the
ar is formed in the hydraulic main from a partial
orts on its way to the condenser shown at 14 on the
t of tar in the hydraulic main an overflow pipe is
ed at one universal level, and the surplus is con-

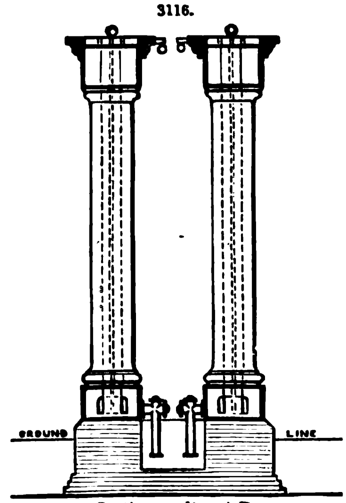
3115.



Elevation.

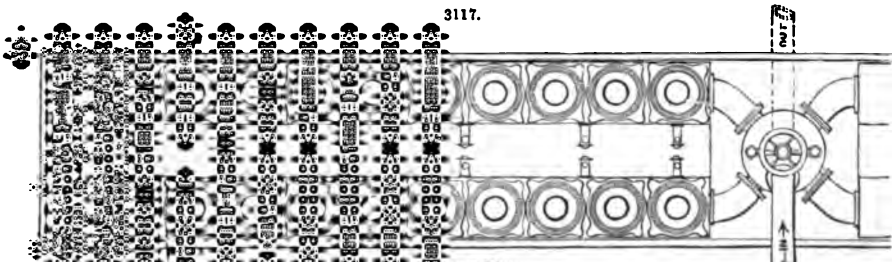


3116.



Section on line A B.

3117.

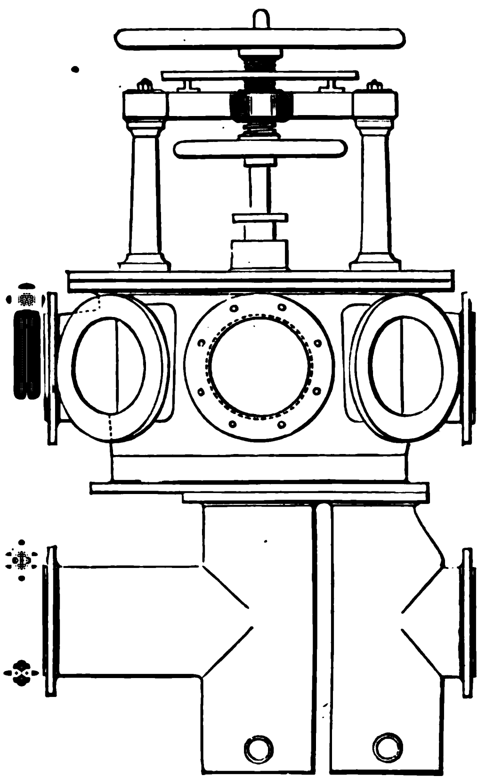


nd Plan.

eads to the tar-cistern. The condensers are formed
ich water circulates for the more effectual cooling of
ith water. and circulation is kept up by making a
n of the column inside. The construction is clearly
ensing column; *b*, the water column; *c*, the water-

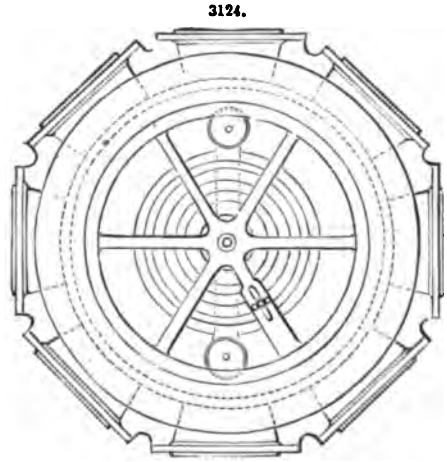
...rough which the tar is conveyed from the con-
...m overflowing by means of a tar-pipe shown on
... condensers and scrubbers, with branches from

3123.



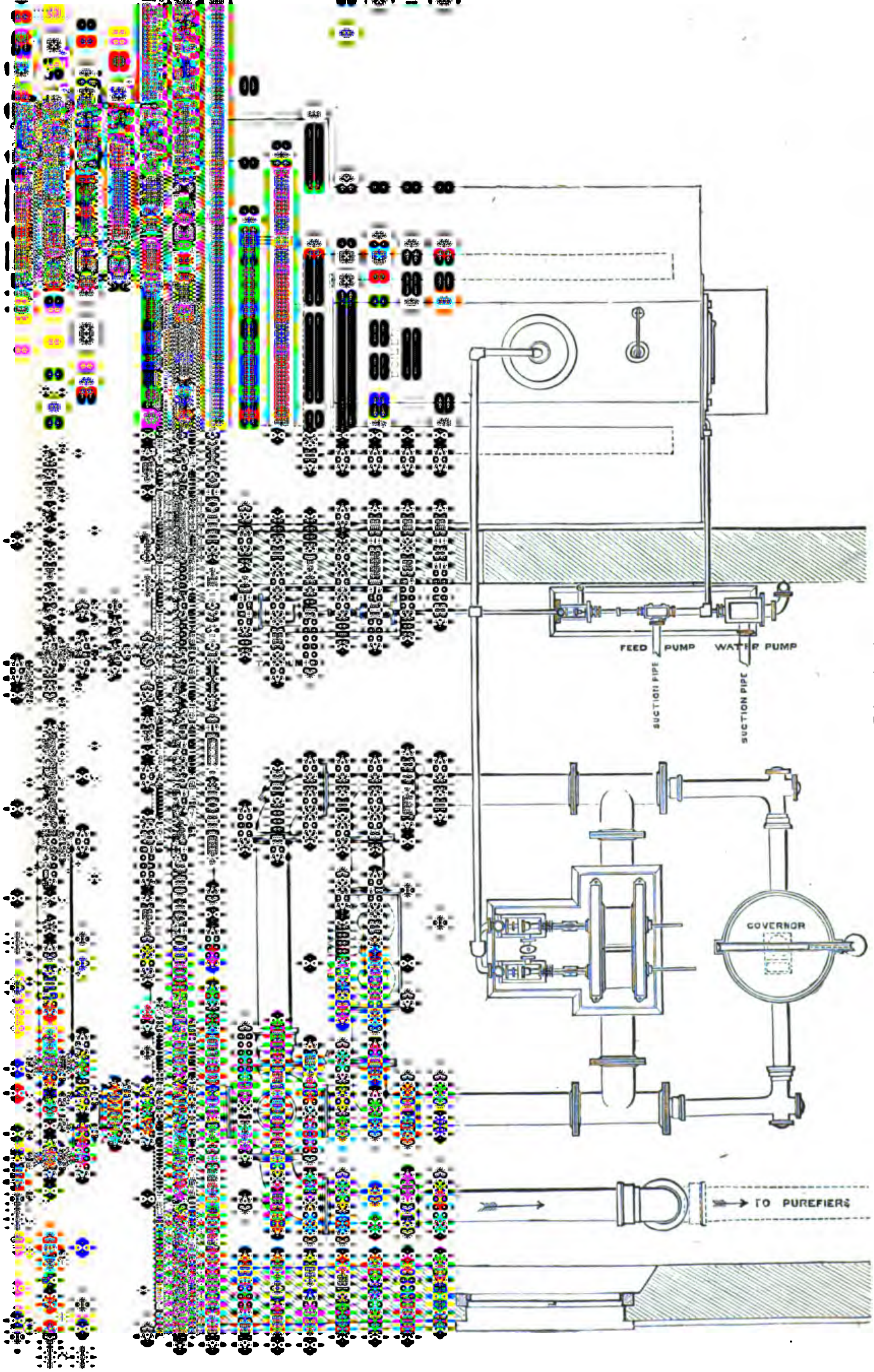
Elevation.

3124.



Plan.

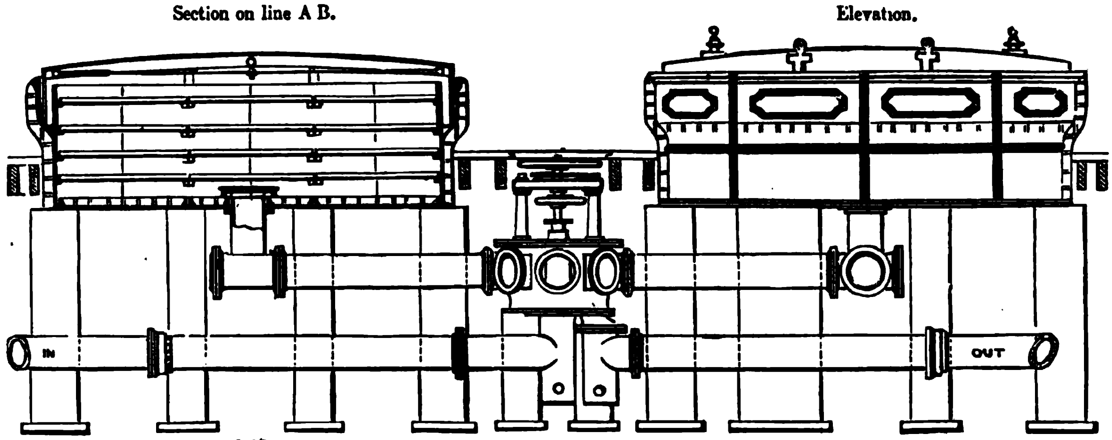
...he tar-pipe from the hydraulic main, so that all
..., the condensers, and the scrubbers, is conveyed
...densers arranged so that by means of the centre



Exhausting Apparatus.

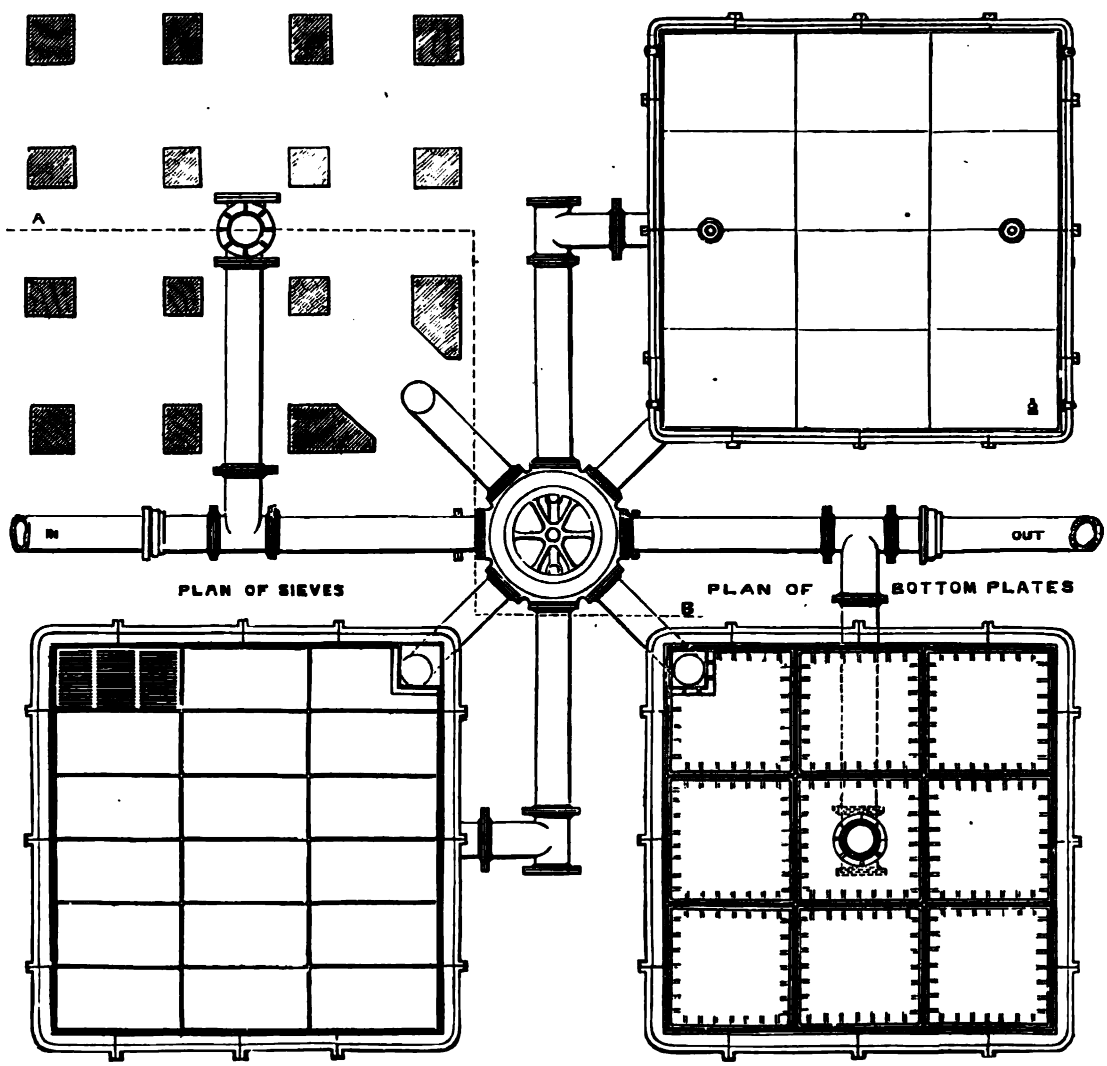
3125.
Section on line A B.

3126.
Elevation.



3127.
PLAN OF FOUNDATIONS

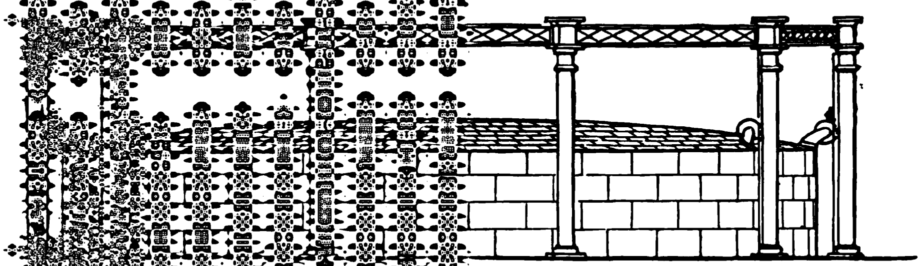
3128.
PLAN OF LID



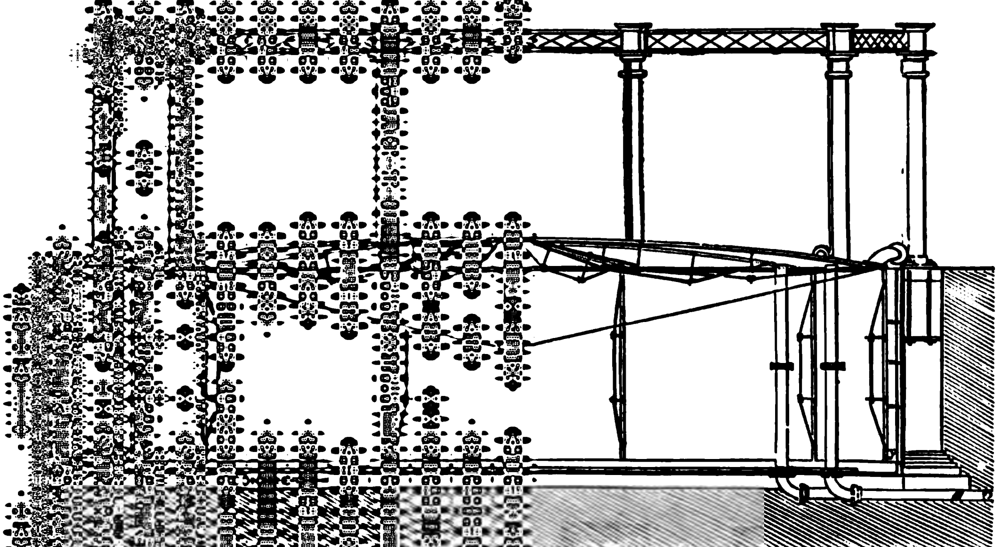
3129.

3130.

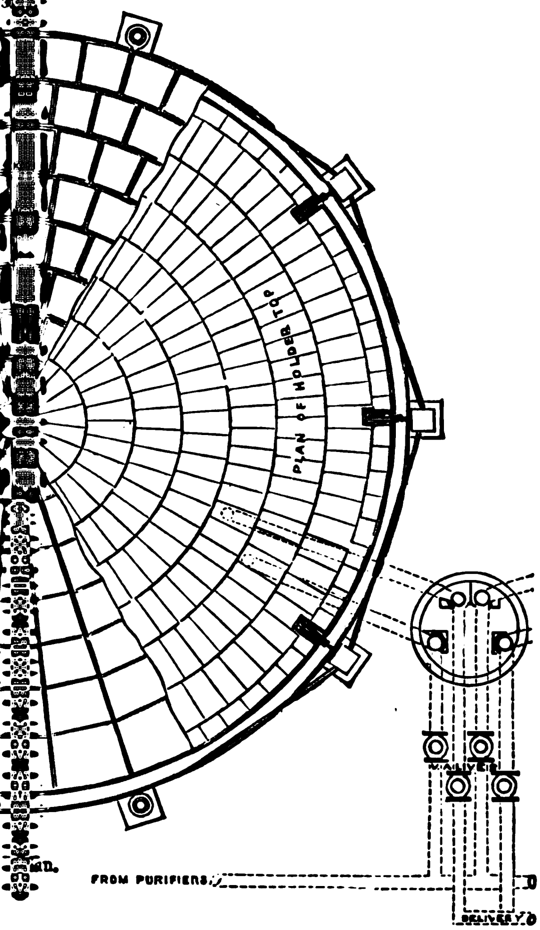
…ut the other, or both may be worked together.
…the scrubbers. The scrubber is for the purpose
… than can be effected in the condenser. The
…spection of Figs. 3118 to 3121. The gas enters
… three tiers of sieves, which are filled to a con-
…nting large scrubbing surface, which is kept
…ns of a rose or water spreader, consisting of
…reat amount of surface exposed to gas in passing
… oils, absorbs ammonia, and cleanses the gas
… are four scrubbers (shown at 13 on the general
…ans of which each or either of them may be
…e scrubbing material. From the scrubbers the
…e of the exhauster is to relieve the retorts from
… which the yield of gas is increased from 6 to
…were it not for the exhauster, would be equal
…me all other resistances on its passage to that
…de, and thus maintains the pressure of the gas-
…ruction of the exhauster is shown in Fig. 3122.
…nes, with horizontal pumps placed on a strong
…ceedingly simple; there is no fly-wheel or crank,
… directly to the pump-piston, and by means of a
… are produced; the two pumps can be worked
… the exhauster house is placed a four-way valve,
…to the purifiers and so throw the exhauster out
…placed a by-pass valve, which allows the gas to
…m accident or other cause; at the other end is
…pass back again should the exhauster be pump-
…arrangement renders the whole of the apparatus
…ed-pumps for supplying the boiler and the tar
…ner to that of the exhauster, and being complete



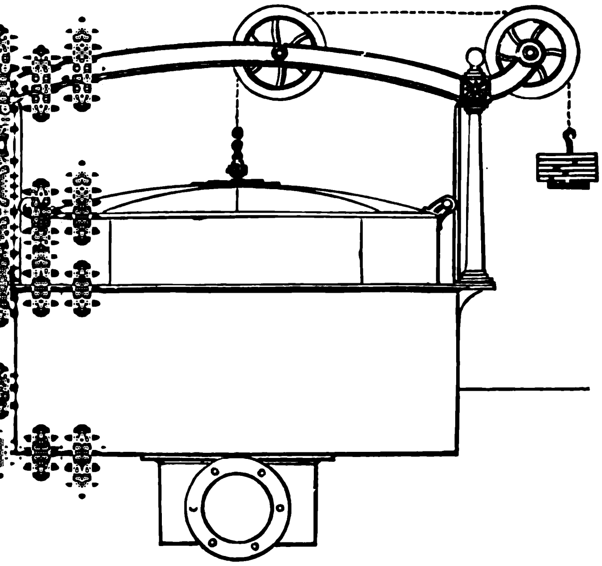
…ion.



…n.



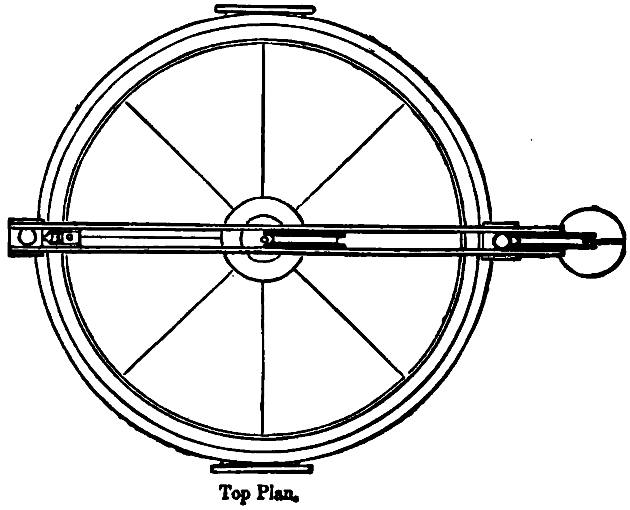
3134.



Elevation.

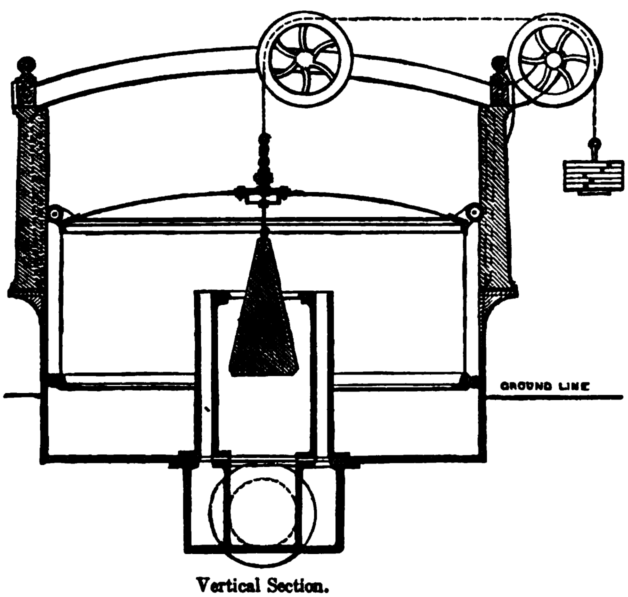
passing through the lime spread over the surface of the sieves, makes its exit through the corner pocket *d*. After passing the purifiers the gas has only to pass through the station meter before being conducted to the gas-holder. The station meter is made to register the gas produced, and by working in connection with a clock, shows the relative quantities produced at each hour of the day. The gas-holders, Figs. 3131 to 3133, are each 80 ft. diameter; a dry well is made by the side of each, through which the inlet and outlet pipes are laid; each pipe is made to dip in a box at the bottom of the well, from which the water formed in the pipes is pumped out by means of a hand-pump. The well *c*, Fig. 3106, answers for the two holders, and the other well *f* is a reserve for the third holder when erected. A stop-valve is

3135.



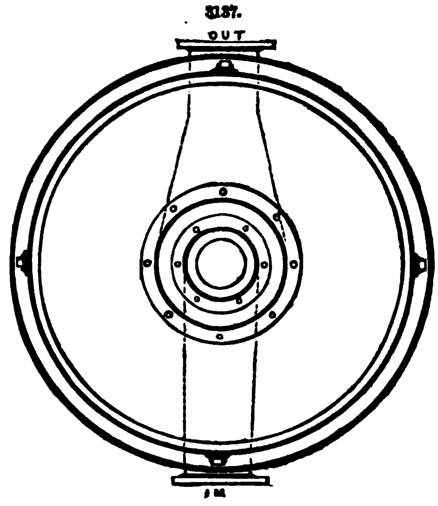
Top Plan.

3136.



Vertical Section.

3137.



Section through Tank, showing Valve, Seat, and Outlet.

placed to each of the inlet and outlet pipes. On the outlet pipe (17 on plan) is fixed the *governor*, Figs. 3134 to 3137, which governs the pressure at which the gas is supplied to the town. It is of great importance to the engineer that the pressure of gas should be preserved constant at various points on the works; and that he may be enabled to ascertain the pressure at any given point, pressure gauges are placed in various situations: the most important gauges are those placed one before and one after the governor. The former shows the pressure given by the gas-holder, and the latter the pressure at which the gas is delivered into the town. There is also a gauge placed at each of the condensers and scrubbers, one on the suction and one on the delivery side of the exhauster, and one near the purifiers. In the block of the office is provided a room for photometric observations. The general plan shows the arrangement of a works capable of supplying a town requiring about 10,000 lights burning for five hours out of each twenty-four; or it would supply 2000 public lamps burning twelve hours, and 6000 private lights burning four hours per twenty-four; or equal to the wants of an English town of 50,000 or a Continental town of 100,000 inhabitants.

Works relating to Gas:—T. S. Peckston, 'A Treatise on Gas Lighting,' 8vo, 1841. S. Hughes, 'A Treatise on Gas Works,' 12mo, cloth, 1866. W. R. Bowditch, 'The Analysis, Technical

Valuation, Purification, and Use of Coal-Gas,' 8vo, 1867. N. N. Schilling, 'Traité d'Éclairage par le Gaz,' traduit de l'Allemand par E. Servier, 4to, Paris, 1868. S. Clegg, jun., 'A Practical Treatise on the Manufacture and Distribution of Coal-Gas,' 4to, 1868. T. Newbigging, 'The Gas Manager's Handbook,' 8vo, 1870. 'The Journal of Gas Lighting,' small folio, 1849 to 1870. 'Annual Reports of the Trustees of the Philadelphia Gas Works,' 8vo, various years. 'Reports of the Proceedings of the British Association of Gas Engineers,' 8vo, various years.

GAS-HOLDER. Fr., *Gazomètre;* Ger., *Gasometer;* Ital., *Gasometro;* Span., *Gasómetro.*
See Gas.

GAS-METER. Fr., *Compteur au gaz;* Ger., *Gasmesser;* Ital., *Misuratore del gas;* Span., *Contador de gas.*
See Meter.

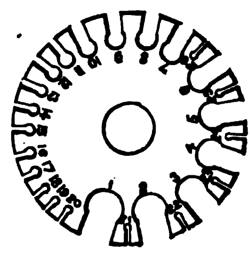
GASOMETER. Fr., *Gazomètre;* Ger., *Gasometer;* Ital., *Gasometro;* Span., *Gasómetro.*
See Gas.

GATES. Fr., *Porte;* Ger., *Thor;* Ital., *Porte;* Span., *Puertas.*
See Docks. Locks and Lock-gates.

GAUGE. Fr., *Tauge, Échantillon;* Ger., *Urmass, Aichmass.*

A *gauge* is a measure; a standard of measure; an instrument to determine dimensions or capacity; a standard of any kind. In mechanics and manufactures;—any instrument for ascertaining or regulating the dimensions or forms of particular things, as a *button-maker's gauge*, a *gun-smith's gauge*, or a *template.* The distance between the rails of a railway. When the railway-gauge is 4 ft. 8½ in., it is called narrow gauge; wide or broad gauge in England is 7 ft., in the United States 6 ft. There are also other intermediate gauges. In plastering, the greater or less of plaster of Paris used with common plaster to accelerate its setting; or the composition made of plaster of Paris and other materials used in finishing plastered ceilings for mouldings, and the like. The *gauge of a carriage* is the distance between the opposite wheels when on the track. *Joiner's gauge*, an instrument used to strike a line parallel to the edge of a board, &c. *Printer's gauge*, an instrument to regulate the margin of the page. *Rain-gauge*, an instrument for measuring the quantity of rain at any given place. *Salt-gauge*, or *salinometer*, a contrivance for indicating the degree of saltness of water from its specific gravity, as in the boilers of ocean steamers. *Standard gauges*, templates, and patterns of certain parts, and tools common to all machine work. *Steam-gauge*, an instrument for measuring the pressure of steam in a boiler. *Tide-gauge*, an instrument for determining the height of the tides. *Vacuum-gauge*, a species of barometer for determining the relative elasticities of the vapour in the condenser of a steam-engine and the air, or for indicating the difference between the vacuum of a condenser and a perfect vacuum. *Water-gauge*, a contrivance for indicating the depth of water as in a steam-boiler, as by a gauge-glass or cock. The term *water-gauge* is also applied to the height of water in a boiler, as three *gauges of water*, that is water up to the third cock. See Anemometer. Details of Engines. Dynamometer. Fan. Meters. Vacuum-gauge.

3138.



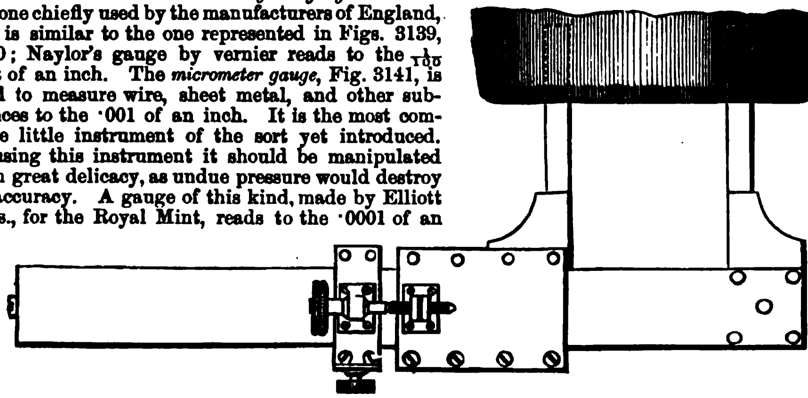
Birmingham Wire Gauge, Fig. 3138.

Table of the Birmingham Wire Gauge, by Sabine and Clark.

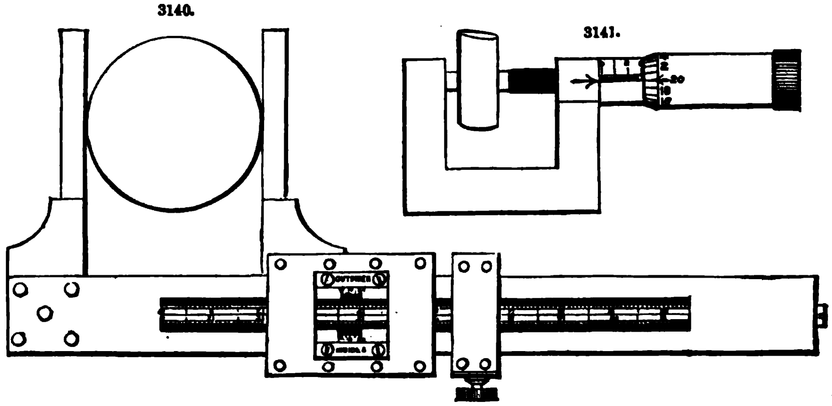
No. B. W. G.	d = diam. in inches.	d^2.	Sect. area in square inches.	No. B. W. G.	d = diam. in inches.	d^2.	Sect. area in square inches.
1 circ. in.	1·000	1·0000	·7854	13½	·089	·0079	·00622
0000	·454	·2061	·16188	14	·083	·0069	·00541
000	·425	·1806	·14186	14½	·077	·0059	·00466
00	·380	·1444	·11341	15	·072	·0052	·00407
0	·340	·1156	·09079	15½	·068	·0046	·00363
1	·300	·0900	·07068	16	·065	·0042	·00332
2	·284	·0807	·06335	17	·058	·00336	·00264
3	·259	·0671	·05268	18	·049	·00240	·00188
4	·238	·0566	·04449	19	·042	·00176	·00138
5	·220	·0484	·03801	20	·035	·00123	·00096
5½	·211	·0445	·03497	21	·032	·00102	·00080
6	·203	·0412	·03236	22	·028	·00078	·00061
6½	·191	·0365	·02865	23	·025	·00063	·00049
7	·180	·0324	·02545	24	·022	·00048	·00038
7½	·172	·0269	·02324	25	·020	·00040	·00031
8	·165	·0272	·02138	26	·018	·00032	·00025
8½	·156	·0243	·01911	27	·016	·000256	·00020
9	·148	·0219	·01720	28	·014	·000196	·00015
9½	·141	·0199	·01561	29	·013	·000169	·00013
10	·134	·0180	·01410	30	·012	·000144	·00011
10½	·127	·0161	·01267	31	·010	·000100	·000078
11	·120	·0144	·01131	32	·009	·000081	·000063
11½	·114	·0130	·01021	33	·008	·000064	·000050
12	·109	·0119	·00933	34	·007	·000049	·000038
12½	·102	·0104	·00817	35	·005	·000025	·000019
13	·095	·0090	·00708	36	·004	·000016	·000012

The gauge, Fig. 3139, manufactured by Elliott Bros., London, is one of the standard Government gauges employed in the Royal Gun Factories. These gauges measure from 36 to 40 in. outside, and 64 in. inside; they read by *vernier* to ·001 of an inch. Some of these gauges are from 4 to 5 ft. in length. The one shown in Figs. 3139, 3140 will measure from 0 to 12 in. *Naylor's gauge* for iron is the one chiefly used by the manufacturers of England, and is similar to the one represented in Figs. 3139, 3140; Naylor's gauge by vernier reads to the $\frac{1}{100}$ part of an inch. The *micrometer gauge*, Fig. 3141, is used to measure wire, sheet metal, and other substances to the ·001 of an inch. It is the most complete little instrument of the sort yet introduced. In using this instrument it should be manipulated with great delicacy, as undue pressure would destroy its accuracy. A gauge of this kind, made by Elliott Bros., for the Royal Mint, reads to the ·0001 of an inch. There are other peculiar contrivances termed gauges, such as the standard for measuring the height of recruits; the instrument employed to measure buckles and indents in targets; standard yards and rods; and the stadiometer used in the army, which require no particular description.

3139.



3140.



3141.

GAUGE, Steam. Fr., *Manomètre;* Ger., *Monometer;* Ital., *Manometro;* Span., *Manómetro.*
See Details of Engines. Gauge. Vacuum-gauge.

GEARING. Fr., *Engrenage, Communication de mouvement;* Ger., *Verzahnung, Verbindungstheil, Geschier;* Ital., *Ruote dentate;* Span., *Aparato de trasmision de movimiento.*

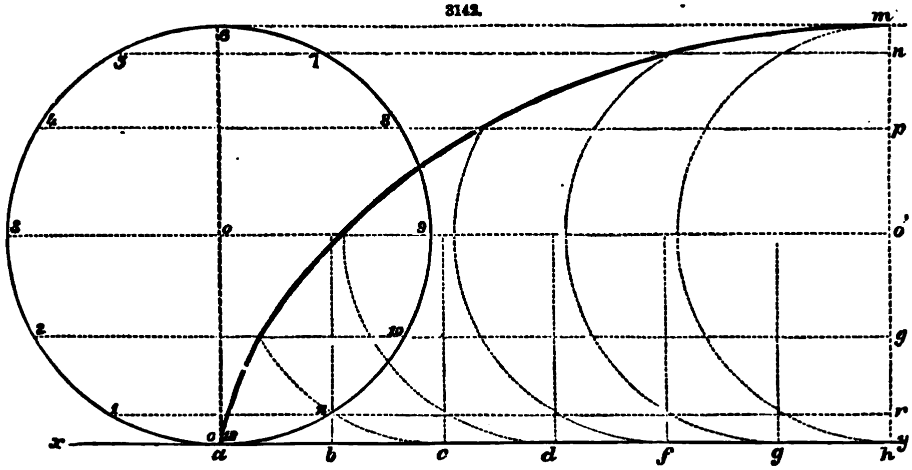
Nomenclature of the Curves used for the Form of the Teeth of Wheels.—The form to be given to the teeth of wheels is an essential point in their construction, for on it depends, in a great measure, the regularity of the motion and the wear of the wheels. Originally the cylindrical form was considered the most advantageous, and was exclusively employed. Numerous experiments have shown, however, that the best forms are those of certain curves, which give a result rigorously exact from a theoretical point of view, and very advantageous in practice. These curves, employed according to the nature of the transmission of the motion, are the following;—The common cycloid, the elongated cycloid, the common epicycloid, the elongated epicycloid, the contracted epicycloid, or common hypocycloid, the elongated hypocycloid, the involute of a circle, and the elongated involute of a circle.

The Common Cycloid.—This curve is generated by the point a in the plane of the circle o which, while turning about its own axis, rolls upon the straight line $x\,y$. This point a and the circle o are called the generators of the curve, the line $x\,y$ being usually called the directrix. The curve is complete when, after a complete revolution of the generating circle, the point a again touches the directrix, Fig. 3142.

Construction.—The generating circle o being given, and the line $x\,y$ upon which it is to roll, divide, beginning at the point a, the circumference of the circle into a sufficient number of equal parts $a\,1$, $a\,2$, $a\,3$ . . . to allow of each of them being considered as a straight line; and mark on the line $x\,y$ the same number of parts $a\,b$, $b\,c$, $c\,d$ . . . equal to the divisions on the circumference. With the radius $o\,a$ describe circles to the directrix in the points b, c, d . . . ; their intersection with the parallel lines drawn through the points 1, 2, 3 . . . gives points in the cycloid. It only remains to join them by a continuous curve. It will be noticed that the centres of all the circum-

ferences must be on a line parallel with the directrix, and drawn through the point o, and upon the perpendiculars raised from the points b, c, d . . .

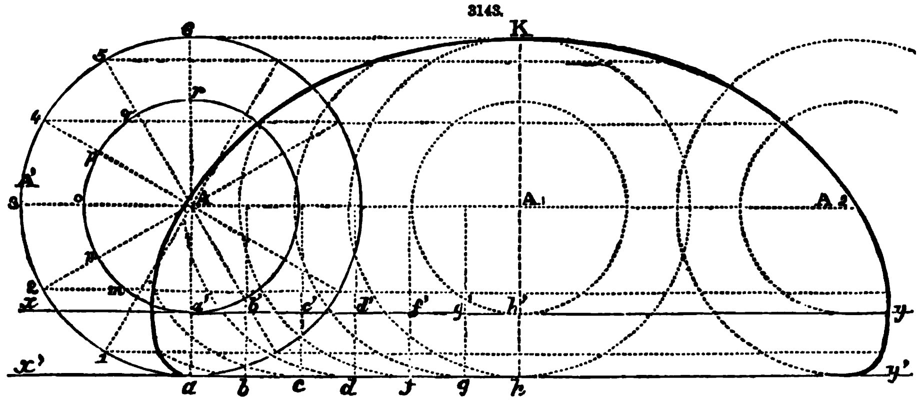
3142.



When the generating circle is tangent in h the half of its circumference will be developed upon $x y$, and consequently we have obtained the half of the curve sought. The other half may be easily determined by taking $n\,5''$, $p\,4''$ . . . equal to $n\,5'$, $p\,4'$ . . . As only a very small portion of the cycloid is ever used, it will be sufficient for purposes of gearing to determine a single element.

The Elongated Cycloid.—This curve is generated by a point a in the circle A' concentric with a circle A, which rolls upon a straight line $x y$, called the directrix, Fig. 3143. The point a and the circle A' are called the generators of the curve. The generating circle being invariably connected with the circle A, and dragged along with it in the motion of rotation, the latter might be called the conducting circle.

3143.



Construction.—The generating point being at starting on the perpendicular line drawn on the directrix through its point of contact with the circle A, divide the circumference of this circle into a sufficient number of equal parts to allow of each of them being considered as a straight line. Mark upon the line $x y$, beginning at the point a', the lengths $a' b'$, $b' c'$, $c' d'$ . . . equal to one of the divisions on the circumference. This done, with the radius of the generating circle, describe circumferences tangent to $x' y'$ in the points b, c, d . . . By producing the radii A m, A n, A o, A p . . . we get upon the circumference A' the points of division 1, 2, 3 . . . ; through these points draw lines parallel with the directrix, their intersection with the corresponding circumferences already described gives points in the curve.

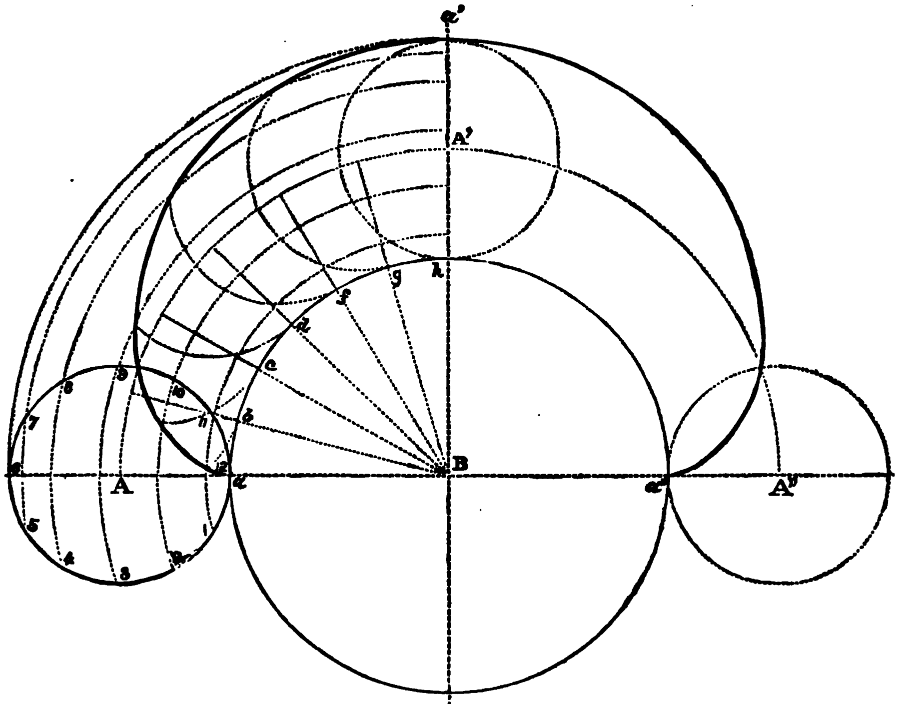
When the half of the circumference A is developed upon the line $x y$, the generating point has described the half of the elongated cycloid; the other may be found by analogous construction, or, since it is symmetric with the first with respect to the perpendicular K h, a certain number of points may be determined by the method already explained for the common cycloid.

The Epicycloid.—This curve is generated by the point a in the circumference of a circle A, Fig. 3144, which revolves along the circumference of a circle B, called the directing circle. The point a and the circle A are called the generators of the curve.

Construction.—If we suppose the diameter of the generating circle equal to the half of that of the directing circle, it is evident that, in this case only, the point a will have described the complete epicycloid when the circle A has arrived at that position upon the diameter $a a''$ which is opposite

that from which it started, that is, when it is tangent in a''. The construction given for this particular case is the same in all the others, and gives consequently an analogous curve.

3144.



Divide now, beginning at the point a, the circumference A into a considerable number of equal parts, each of them having to be considered as a straight line. In the same way, beginning at the point a, mark upon the circumference B divisions equal to the preceding, these having to represent the development. With the radius of the generating circle describe a series of arcs tangent to the directing circle in the points a, b, c, d . . . ; then from the centre B with the radii B 1, B 2, B 3 . . . , describe circumferences. Their intersection with the corresponding arcs, tangent to the circle B, gives points in the curve. One of these points, m for example, is a position of the point a, for the latter must belong to the circumference tangent in c. Also when the point a has arrived at c, the generating point will evidently be upon the circumference passing through the point 3; it must, therefore, be at m, the point of intersection of the two circumferences.

The half of the curve will be described when the generating circle is tangent to the circle B at the end of the radius perpendicular to the diameter $a\,a''$. The other half of the epicycloid may be obtained by placing, by means of parallels with $a\,a''$, a sufficient number of the points of the preceding half, in a symmetrical position with respect to B a'.

The Elongated Epicycloid.—This curve is described by a point a in the plane of a circle A', concentric and invariably connected with a second circle A, which revolves along the circumference of a third circle B, called the directing circle. The point a and the circle A are the generators of the elongated epicycloid, Fig. 3145.

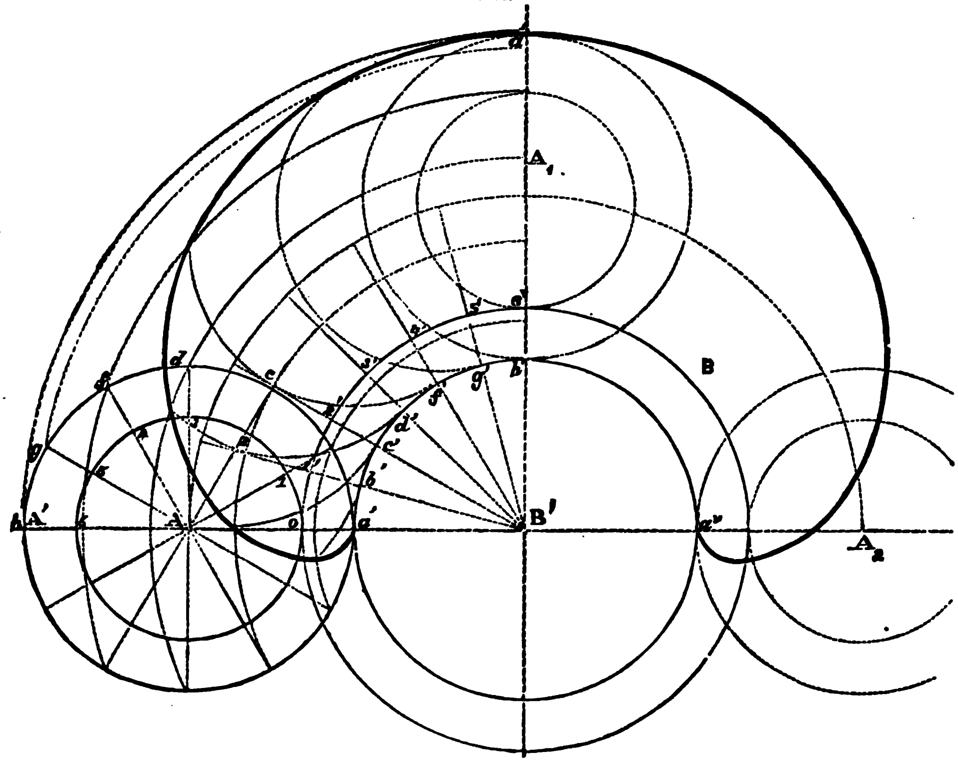
Construction.—Having taken the diameter of the circle A equal to the radius of the directing circle B, the circumference of the former will be completely developed upon that of the latter when it occupies the position opposite the extremity of the diameter $a\,a''$, and the point a will have described the whole curve when it is again in contact with the circumference B'. Now, to obtain points in the curve, divide the circumference A into a sufficient number of equal parts 0, 1. 1, 2. 2, 3 . . . to allow of each of them being considered as a straight line. Mark, beginning at the point a, these same divisions upon the directing circumference; then, through the points 1', 2', 3' . . . draw radii which give upon the circumference B' the points b', c', d' . . . With the radius of the generating circle describe circumferences tangent in these points.

Having produced the radii at the points of division 1, 2, 3 . . . , and obtained upon the generating circle the corresponding points b, c, d . . . , from the centre B with the radii B b, B c, B d . . . , describe circumferences the intersection of which with the arcs already drawn gives points in the elongated epicycloid.

The point a will have described the half of the elongated epicycloid, when the diameter $a'\,h'$ of the generating circle coincides with the radius B h' produced, perpendicular to $a\,a''$. The other half of the curve may be obtained by an analogous construction, or by placing the points obtained in a position symmetrical with respect to a' B'. To facilitate the construction, we mark equal parts

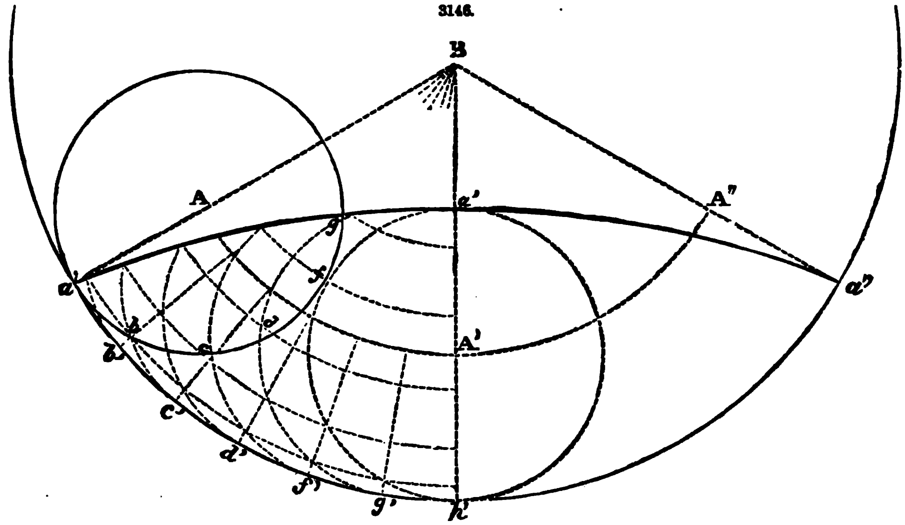
upon the circumference A, but it may be remarked that these parts may be unequal, provided they be transferred accurately to the directing circumference.

3145.



Common Hypocycloid.—This curve is described by the point a in the plane of a circle A which revolves about its own axis along the concave side of a circumference B. The circle B is called the directing circle, the circle A and the point a are the generators of the curve, Fig. 3146.

3146.

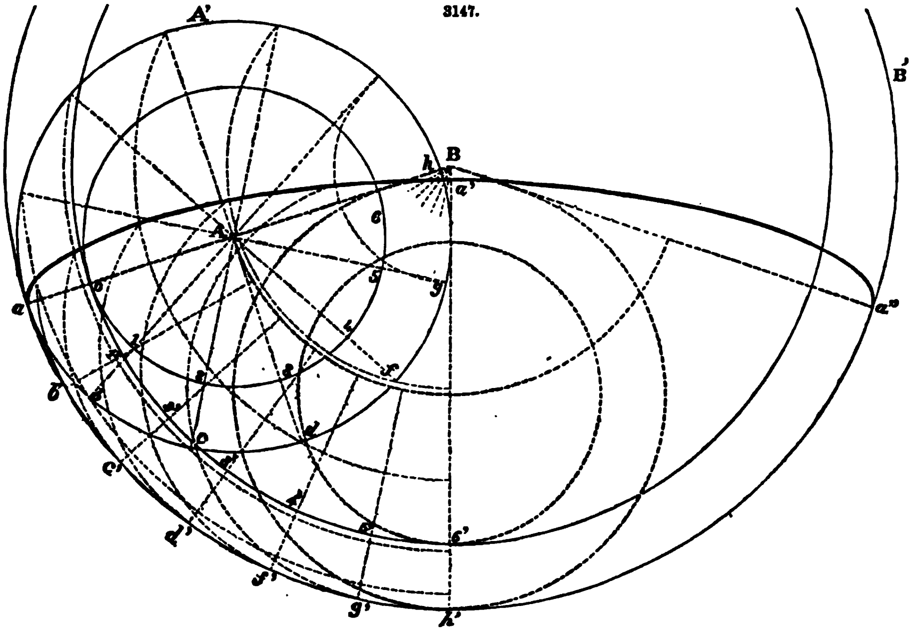


Construction.—The line of the centres B A determines by its intersection with the circumference B, the generating point a. Beginning at this point, divide the circumference A into a great number of small parts $a\,b$, $b\,c$, $c\,d$ . . . , which may be considered as straight lines. Beginning

at the same point a, mark an equal number of these parts upon the circumference B. With the radius of the generating circle describe a series of arcs tangent in the points b', c', d' . . ., and from the centre B, with the radii B b, B c, B d . . ., describe arcs the intersection of which with the former gives points in the hypocycloid. The centre of the circle A describes during its revolution a portion of a circle A A′ A″, concentric with the circle B; this is the common locus of the centres of all the arcs tangent in the points b', c', d' . . . If we wish for greater exactness than that which we obtain by considering the elements of the generating circle as straight, we may determine the development of one of them, and use this length for $a\,b'$, $b'\,c'$, $c'\,d'$ . . . This precaution is useful when the radius of the generating circle is very small with respect to the radius of the directing circle, that is, when the elements, however small they may be, form a decided curve.

Elongated Hypocycloid.—This curve is generated by the point a in the plane of a circle A′ invariably connected with a circle A, which revolves about its own centre along the inner side of a third circle B, called the directing circle, Fig. 3147.

3147.



Construction.—The line of the centres A B determines by its intersection with the circumference B′ the first position of the generating point a. Beginning at the intersection o of this line of the centres with the circumference B, divide the circumference A into a sufficient number of equal parts 0. 1, 1. 2, 2. 3 . . ., each of them having to be considered as a straight line. Mark, in the same way, beginning at 0, upon the circumference B, an equal number of divisions 0. 1′, 1′. 2′, 2′. 3′ . . . representing the development of the preceding. The radii being now produced from the points of division 1′, 2′, 3′ . . ., give upon the circumference B′ the corresponding points b', c', d' . . .; with the radius of the generating circle, describe arcs tangent in these points.

Having produced the radii A 1, A 2, A 3 . . ., and obtained upon the circumference A′ the points of division b, c, d . . ., from the centre B with the radii B b, B c, B d . . ., describe a series of arcs the intersection of which with the preceding gives points in the elongated hypocycloid.

When the point c arrives at the position c', the generating point a will be at a', and will have described the half of the curve sought. The other half may be obtained by an analogous construction, or by transferring a sufficient number of the points of the former to a symmetrical position with respect to B h'.

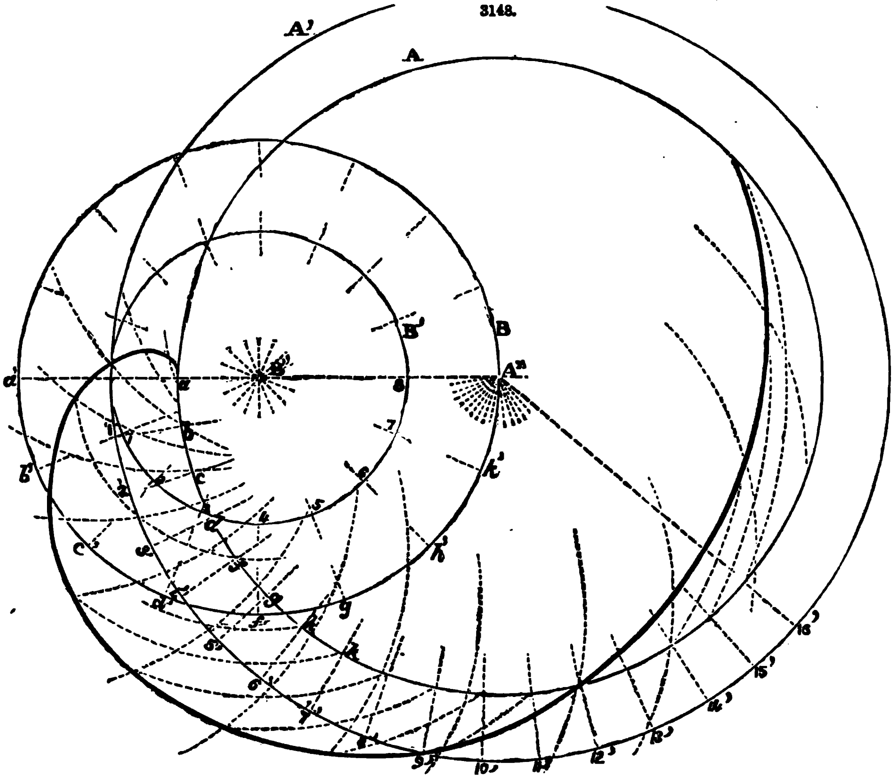
Another Elongated Hypocycloid.—Let there be two circles, A and A′, invariably connected together and forced to turn about their common centre A″. If in this revolution the circle A is made to follow the contour of a third circle B′, called the directing circle, the point a of the circle A will describe an elongated hypocycloid, Fig. 3148.

Construction.—Beginning at the point of intersection o of the circumference A with the line of the centres A″ B″ produced, mark upon the circumference B′ a sufficiently great number of divisions to allow each of them to be considered, without an appreciable error, as a straight line. Mark an equal number of these divisions upon the circumference A, and draw the radii from the points 1′, 2′, 3′, 4 . . . It will be seen that the centre A″ of the generating circle describes in its revolution a circle B concentric with B′. Producing the radii from the points of division 1, 2, 3, 4 . . to the circumference of this circle, we obtain the corresponding points a', b', c', d' . . . From each of these points with the radius of the circle A′ describe arcs, and from the point B″ with the radii B″ b, B″ c, B″ d . . ., describe other arcs the intersection of which with the former gives points in the curve sought.

One of these points, m for example, must occupy one of the positions of the point a, for the latter,

remaining upon the generating circle A′, must belong to the arc described from the point c' with a A″ as a radius; it must also be upon the circumference described from the point B′ with B″ p as a radius; it can, therefore, be only at their point of intersection.

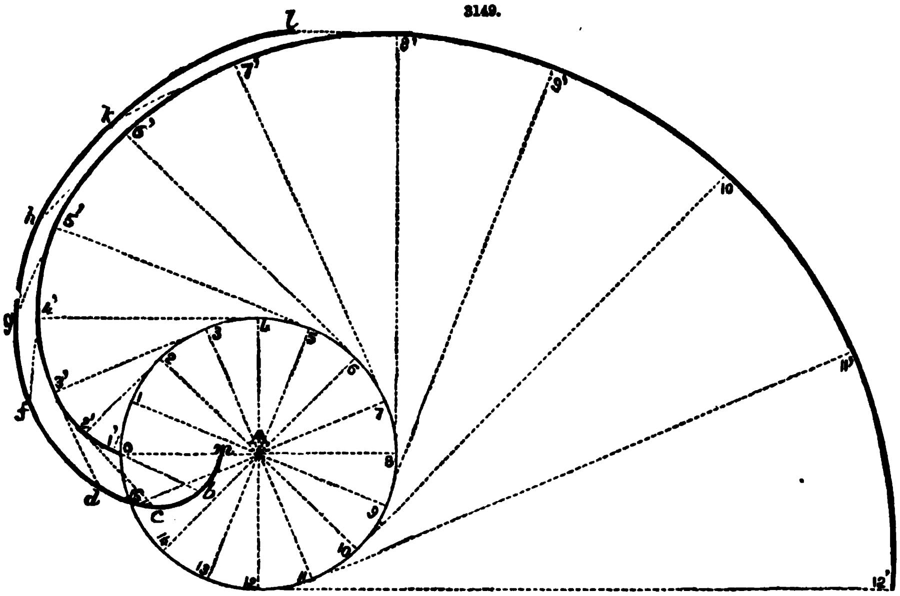
3148.



Involute of a Circle.—This curve is generated by the end of a tangent to the circle made to touch the circumference in all its points successively.

Let o A, Fig. 3149, be the radius of the given circle, and o the first point of contact of the

3149.



tangents. Beginning at this point, mark on the circumference a sufficient number of equal parts 0.1, 1.2, 2.3, 3.4 . . . , to allow of each being considered as a straight line. Draw the tangents from the points of division, and, upon the first, mark one of the elements or divisions of the circumference, upon the second two of these elements, upon the third, three, &c. Join by a regular curve the points 1′, 2′, 3′, 4′ . . . , thus obtained.

Elongated Involute of a Circle.—If at each of the points 1′, 2′, 3′, 4′ . . . , Fig. 3149, we draw perpendiculars to the tangents, and mark off on each of them a constant length, equal to *o m*, for example, the points obtained, *b, c, d, f* . . . belong to an elongated involute of a circle.

Remark.—The common involute may be wholly traced by the compasses, for any element *q′*, 10′ has its centre at the point of intersection *p* of the tangents *q, q′*, and 10, 10′. The other curves, the means of drawing which we have already given, may be obtained by means of a little piece of wood, the arrangement of which for each case may be easily imagined.

Dimensions of the parts of Gearing considered both in Detail and as a Whole.—Object of Gearing, its Advantages.—A toothed wheel is a circle or disc furnished along its contour with projecting pieces called teeth, designed to transmit to another toothed wheel a determinate force, the direction and velocity of which are known.

Gearing is used;—

1. To transmit the continuous circular motion of a shaft to another shaft fixed at a short distance from it. When the shafts are parallel the gearing is called straight or spur-gearing. If the shafts make an angle with each other, the motion is transmitted from one to the other by means of conical or bevel-gear.

2. To transform the continuous circular motion of a shaft into a rectilineal motion of a plane-toothed surface.

3. To transform the continuous circular motion of a shaft into another circular motion of an endless screw, and *vice versâ*. In all cases the axis of the toothed wheel and that of the endless screw are placed in two perpendicular planes, and not in the same plane as in the case of conical gear.

4. The ascending motion of mill-hammers and other similar contrivances is obtained by the transformation of the continuous circular motion of a shaft by means of cams. The use and construction of these cams rest upon principles similar to those which have led to the application and to the improvement of gear.

The first attempts to transmit the motion of one shaft to another was by means of friction. This friction was produced upon cylinders or drums fixed upon the shafts. The rapid wear of the drums required them to be frequently changed, or the shafts to be brought nearer together. These grave defects, and the difficulty of removing them, led to the adoption of teeth, which have lately undergone great improvement. If these teeth are carefully constructed, we may obtain very exactly relations of speed determined beforehand, a gentle and uniform motion, and a sufficient duration of material. The use of gearing enables us to make a large number of transmissions in a small space; its application is especially remarkable in watch and clock work, where it is carried to a high degree of perfection.

Relation of Velocity to be obtained.—Primitive Circles, or Pitch-Lines.—When a rotary motion is transmitted from one shaft to another, the object always is to obtain a certain determinate velocity. This velocity results from two conditions, which are mutually dependent; 1, the primitive diameter of the wheels; 2, the respective number of the teeth of each of them.

By primitive or pitch-circles we mean those which are tangent to each other at the point *a*, Fig. 3150, where the contact of the two teeth which act normally, one as the power, the other as the resistance, takes place, that is, where the principal force is exerted. The relation between the radii *o a*, *o′ a* of the primitive circles must always be the same as that of the velocities; thus if it be required to make one of the wheels revolve three times while the other revolves only once, the latter must have a radius equal to three times the radius of the former.

To calculate the primitive diameters we must know;—

1. The distance of the shafts measured from axis to axis;

2. The relation between the velocity of the wheel and the velocity which the pinion is to have. The radii of the wheels must be in inverse proportion with the velocities. Indeed the circumferences are to each other as the radii; and as the numbers of teeth depend on the circumference, these numbers are also in the same proportion as the velocities. Again, as each tooth of the wheel propels forward one of the pinions, it is evident that the number of revolutions will be inversely as the radii.

Let n be the number of revolutions of the wheel and r its radius, r' the radius of the pinion and n' the number of revolutions it is to make a minute.

According to what we have said above, we have

$$n : n' :: r' : r. \qquad [1]$$

Let us suppose that $n = 14$, $n' = 42$, and the distance of the centres $= 1^{m} \cdot 30 = r' + r$. Proportion [1] may be put under the form

$$n + n' : n' :: r' + r : r,$$

or

$$56 : 42 :: 1^{m}\ 30 : r,$$

whence

$$r = \frac{1^{m} \cdot 30 \times 42}{56} = 0^{m} \cdot 975,$$

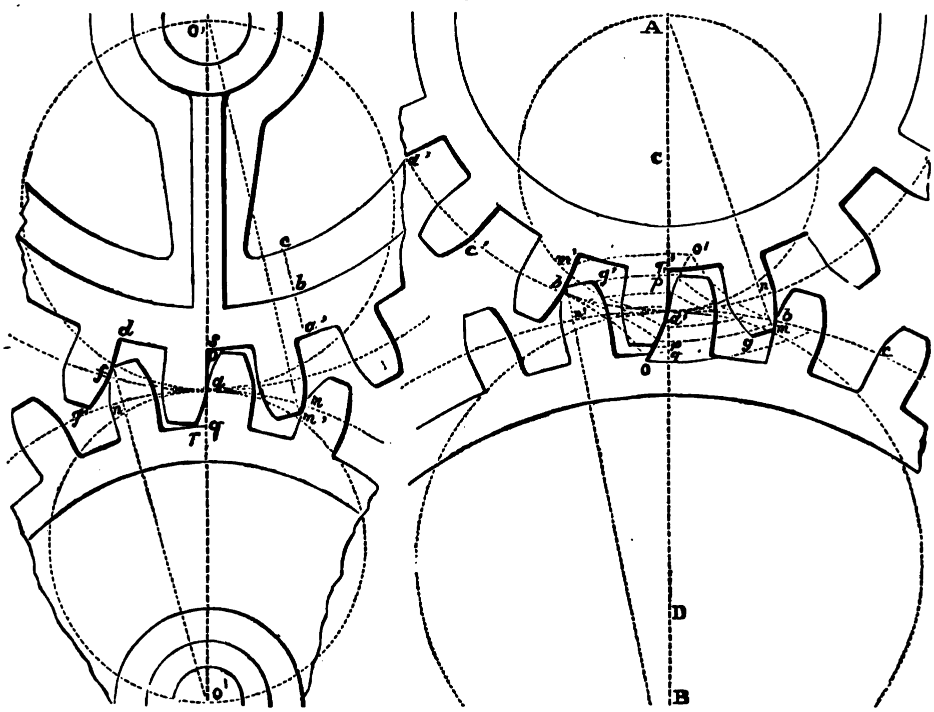
and

$$r = 1^{m} \cdot 30 - 0 \cdot 975 = 0 \cdot 325.$$

In some cases, the velocity to be obtained differs greatly from that which we have at our disposal: if we employed only two wheels, we should have to reduce the dimensions of the pinion

too much, and sometimes to increase beyond measure those of the wheel. In such cases, we interpose between the two shafts a series of wheels and pinions, the combination of which gives the result desired. But this means should be employed only in case of absolute necessity, for we must not forget that these mediums absorb a considerable quantity of the motive force.

3150.



Let us suppose that the shaft B is to make sixty-four revolutions while the shaft A makes one. If, for the sake of simplicity in execution, we make all the wheels of the same diameter, and the pinions also equal to each other, we extract a root of the number of revolutions required; the index of this root will represent the number of couples to be employed, and the root itself will denote the constant proportion between the primitive diameter of the wheels and those of the pinions. Thus if the square root of 64, which indicates the employment of two couples and gives as the proportion of the radii 8 : 1 does not appear applicable to the case required, we may take the numbers given by the cube root, that is, three couples in the proportion of 4 : 1.

When particular arrangements oblige us to employ different proportions for the couples of gearing, we resolve the number of revolutions to be obtained into several factors, choosing those which are the most convenient. Thus for the case we have alluded to above, three couples in the proportions of 2, 4, and 8 would give the velocity required.

Dimensions and Form of the Teeth.—In constructing the teeth we must have in view;—1, the thickness of the tooth; 2, the space between the teeth; 3, the distance of the teeth; 4, the number of the teeth; 5, the depth of the face; 6, the depth of the shoulder; 7, the form of the teeth; 8, the total depth.

1. *Thickness of the Teeth.*—The thickness of the teeth depends entirely upon the force they have to bear and the nature of the material of which they are made. The force should always be taken for the case in which the wheels would have to transmit the maximum power of the motive engine. In calculating the thickness of the teeth the results of experience must be referred to, and we must suppose the case of only one tooth in contact, that is, supporting alone the whole force.

Example.—A pinion makes six revolutions while the wheel which drives it makes one; the distance of the centres is $2^{m} \cdot 30$; the work effected is 940 kilogrammètres, and the wheel has a velocity of five revolutions a minute. Determining the radius of the wheel in the way we have described, we find it to be equal to $1^{m} \cdot 917$.

The velocity at the primitive circumference or pitch-line will therefore be

$$\frac{1 \cdot 917 \times 2 \times 3 \cdot 14 \times 5}{60} = 1^{m} \cdot 003.$$

The force is found by dividing the work to be transmitted by the velocity

$$\frac{940}{1 \cdot 003} = 937 \text{ kilogrammètres.}$$

Now, to find the thickness of the tooth, we multiply the square root of 937 by a coefficient determined by experience.

For cast iron	0·105
For bronze	0·131
For hard wood	0·145

Thus the teeth being of cast iron, their thickness will be $0\cdot105 \times \sqrt{937} = 0^{m}\cdot032$.

2. *Space between the Teeth.*—If the teeth were made with mathematical precision, the space between two of them measured upon the pitch-line might be made equal to the thickness. When the teeth are filed the space is taken equal to the thickness plus $\frac{1}{15}$. This fifteenth is intended to correct the defects resulting from the imperfection of the work. When the teeth are unsmoothed or of different materials, the space is made equal to the thickness plus $\frac{1}{10}$.

3. *Distance of the Teeth.*—The arc of the primitive or pitch-circle comprising a space and a tooth, that is, the distance from the outside edge of one tooth to the outside edge of the next, constitutes what is called the distance of the teeth.

It is evident that this distance must be the same upon the wheel and upon the pinions, and that it must be taken an exact number of times upon each of the two circumferences.

4. *To Calculate the Number of the Teeth.*—Let m' be the number of the teeth on the wheel; m the number on the pinion; a the distance of the teeth, and r the radius of the wheel. We will continue to operate with the values given in the preceding example. The number of teeth is evidently equal to the pitch-line divided by the distance of the teeth. Therefore

$$m = \frac{2\pi r}{a};$$

but

$$a = 0^{m}\cdot032 + (0\cdot032 + 0\cdot003) = 0^{m}\cdot067,$$

and

$$r = 1^{m}\cdot917;$$

therefore

$$m = \frac{6\cdot28 \times 1\cdot917}{0\cdot067} = 180.$$

This number is exactly divisible by the ratio 6 of the radius of the wheel and that of the pinion, which is a necessary condition, since the numbers of the teeth must be to each other as the primitive or pitch-circles, and consequently as their radii. Besides this, symmetry and readiness in putting together require, if the wheels are in several pieces, that the number of teeth be exactly divisible by the number of the arms of the wheel. In the present case, supposing six arms, the number 180 would fulfil the second condition also. In the contrary case, we take the number next smaller than that given by calculation, which is at once divisible by the number of arms and the ratio between the radius of the wheel and that of the pinion. This modification can never be attended with any objectionable result, since the teeth are taken a little stouter than the preceding operation indicates.

5, 6, and 7. *Shoulder and Face of the Teeth; Form of the Profile.*—The part df of the profile included between the primitive circumference and the base of the tooth, is called the shoulder; it is always formed by the radius from the point f, Fig. 3150. The part fg of the same profile standing beyond the primitive circle is called the face of the tooth.

The depth of the shoulder depends on that of the face, and this latter is determined by considerations which we shall come to presently.

In the motion of two wheels, the shoulder alone of a tooth comes into contact with the faces of the teeth of the other wheel. If the gear turned always in the same direction, in other words, if the same wheel always drove, this one alone would require for the faces of its teeth the particular forms adopted in their construction. But in almost all cases the wheels drive and are driven alternately; thence arises the necessity of providing each of them with a face.

8. *Limit to the Teeth.*—The limit to the depth of the teeth results from important considerations, which we will now proceed to consider merely from a practical point of view.

The teeth must be long enough to allow two couples at least to be in contact at the same time, for if the force be exerted upon only one tooth of each wheel, the whole work of a mill or of a machine will be subordinated to the rupture of this tooth. If, on the other hand, too large a number of teeth be in contact at one time, we shall find that their contact is one in appearance rather than in reality, for it is almost impossible to execute the work with such precision that all the teeth shall act simultaneously. Thus care must be had to have two teeth always ready to come into contact when two others are on the point of quitting each other. This result is obtained by limiting the teeth, the length of which depends upon what is sometimes called the driving arc. By driving arc is meant that arc in which a tooth moves while it is engaged in driving the one against which it presses. Thus in Fig. 3150 this arc would be the arc $m\,a\,n$ described upon the pitch-line by the point n from the moment when one tooth begins to press upon its neighbour till the moment when it ceases to act. In practice, and for gearing of ordinary dimensions, this arc may be taken equal to the distance of the teeth, on each side of the line of the centres.

Thus to limit the teeth of the wheel, take upon the pitch-line of the pinion the arc $a\,m$ equal to the distance of the teeth, and draw the radius $o'\,m$, cutting in m' the circumference described upon $o'\,a$ as a diameter. The circumference described from the centre o with $o'\,m$ as a radius will limit all the teeth of the wheel. This circumference and that which determines the extremity of the teeth of the pinion meet the line of the centres in the points $g\,p$; beginning at these points, mark upon the line of the centres towards o' and o, the lengths $q\,r$ and $p\,s$, equal to about $\frac{1}{4}$ of the depth of the face for large gear, and $\frac{1}{3}$ for small gear. Then from the points o and o' with $o\,s$ and $o'\,r$ as radii, describe circumferences forming the bottom of the spaces between the teeth, and determining

in profile the base of the teeth. To avoid a re-entering angle, that which is formed by the bottom of the space and the side of the tooth is slightly rounded.

This method of limiting the teeth is quite practical, and may be modified to suit the cases of the epicycloid and the involute.

Remark.—It may happen that we are obliged to employ a pinion of a very small diameter to transmit a great force, or a wheel of a very large diameter for work of inconsiderable importance. In the former case, if the driving arc were equal to the distance of the teeth on each side of the line of the centres, the teeth would be very long, and would, consequently, become too thin at their extremities. We must, therefore, take the arcs described during contact equal to $\frac{4}{5}$ or $\frac{2}{3}$ or even $\frac{1}{2}$ if necessary. In the case in which the wheels are large to transmit a small force, we should obtain by the foregoing method, teeth much too short; we must therefore take the arc of contact equal $1\frac{1}{2}$ the distance of the teeth. It is advisable, however, if the size of the teeth be increased, not to give it more than $1^{m}\cdot 5$ of the breadth taken upon the pitch-circle.

If the dimensions of a pinion are such that the number of teeth would be less than fifteen, it is preferable to employ several couples. The endless screw may be substituted for the pinion when the wheel is to be driven by it, especially if the force to be transmitted is great and the velocity to be obtained rather low.

It frequently happens that a wheel with wooden teeth works into a pinion wholly of metal. In this case the wooden teeth are evidently stouter than the iron teeth, since their thickness b is given by the formula $b = 0\cdot 143\sqrt{P}$, whilst that of the teeth of the pinion is expressed by $b = 0\cdot 105\sqrt{P}$, P representing the force to be transmitted. It follows from this that the spaces on the pinion are greater than those on the wheel to give passage to the cogs. Nevertheless, the distance of the teeth must remain the same in both wheels.

Breadth of the Teeth; particular arrangements.—The length of the teeth in the direction of the axis is commonly equal to four, five, or six times the thickness upon the pitch-line, according as the velocity is to be small, greater than $1^{m}\cdot 75$ a second, or the gear constantly wet. It does not interfere in any way with the wear of the gear or the regularity of the motion, to increase the breadth of the teeth, or at least to increase it in small degree above that indicated. Generally when this breadth exceeds certain limits, in large wheels with wooden teeth, the teeth are separated into two equal parts in the direction of the breadth of the circumference of the wheel, and these parts are so arranged that one is opposite the space between it and two others. In this way we have upon the same wheel two sets of gear quite distinct though perfectly similar and acting together. By this means we get between the mortises into which the teeth are fixed, a solid space in the middle of the rim of the wheel, which adds to its strength. When a wheel so constructed works into an iron pinion, the model of the latter is made so as to have its teeth placed inversely as those of the wheel.

Gear intended to transmit to four pumps the work of an engine of 60 horse-power is arranged in the manner described above, and each part has a breadth equal to four times its thickness upon the pitch-line; this gives eight times the thickness for the total breadth, supposing the teeth not crossed, but made of a single piece or placed end to end.

The parts of the teeth that enter the mortise are a little smaller than the teeth themselves measured at the base, so that the shoulder resulting from this difference rests upon the periphery of the wheel. The ends of the teeth project on the inner side by a quantity equal to the depth at the crown measured in the direction of the radius. There result from this, spaces having the form of equal trapeziums, into which are placed, in the manner of dove-tailing, pieces of wood that are afterwards fixed to the teeth by means of screws.

Dimensions of the Periphery, and Number of Arms of Toothed Wheels.—Instead of being placed upon the periphery of a solid disc, the gear forms part of an iron rim $a'\,b$, Fig. 3150, connected with the axle by a certain number of arms. The breadth of this rim is equal to the breadth of the teeth when the whole is of metal. When the teeth are of wood the rim is made broader by a quantity equal to twice their thickness, in order that it may not be too much weakened by the mortises; its thickness should be at least equal to that of the tooth. In gearing which is exposed to violent vibration, these dimensions must be increased; experience and the quality of the metal must in such cases determine the degree.

Whatever be the section of the rim, it is provided on the inside and in the middle of its breadth with a moulding or rib $b\,c$, the two dimensions of which, projection and thickness, are equal to the thickness of the rim itself. By proceeding in the above manner for wheels of a large diameter intended to transmit a small force, we should obtain too thin a crown, and liable to twist out of shape while cooling after casting. To avoid this defect it must be made a little stouter, and if necessary the number of the arms increased.

The number of arms to be given to wheels depends on their diameter. Those of $1^{m}\cdot 50$ and less have four arms, those of from $1^{m}\cdot 50$ to $2^{m}\cdot 50$ have six, and those from $2^{m}\cdot 50$ to 5 mètres have eight. If it is necessary to employ wheels of from 5 to 7 mètres, a case that seldom occurs, we may increase the number of arms to ten.

It is customary to give to the arms of wheels a section exactly equal to that of the rim, and they are strengthened on each side by a rib of the same thickness as that of the crown, and running into the latter. If any space remains between the bases of the arms, the rib runs round the centre or nave.

Methods of Tracing the various Forms of Gear.—Epicycloidal Gear.—We have shown the different principles upon which the construction of gear and the calculation of its dimensions rest; we will now point out the rules generally followed in tracing it.

It is known that when a circle is made to roll upon another circle, any point in its circumference describes an epicycloid. The motion which produces the curve is subject to the same laws as that of two tangent circumferences revolving about their centres when the latter are fixed. It follows from this that if we give the epicycloidal form to the profile of the teeth, the wheels will be driven

by each other in the same manner as they would be by simple contact. This form is sometimes adopted. Suppose now we have to construct a spur epicycloidal gear with the following data;—

Distance of the centres	$0^m \cdot 267$
Ratio of the velocities	$\frac{3}{5}$
Distance of the teeth	$0^m \cdot 035$

Take A B, Fig. 3150, equal to $0^m \cdot 267$, and divide this line into two parts a A and a B, so that they may be to each other as 3 is to 5. The circumferences described with a A and a B as radii are the pitch-lines of the gearing. The length or development of the circumference of the wheel is expressed by $2 \pi a \mathrm{B}$, or $6 \cdot 28 \times 0 \cdot 167 = 1^m \cdot 049$; and the number of teeth is given by $\frac{1 \cdot 049}{0 \cdot 035} = 30$.

The pitch-line of the pinion being $2 \pi a \mathrm{A}$, or $6 \cdot 28 \times 0 \cdot 100 = 0^m \cdot 628$.

The number of teeth is $\frac{0 \cdot 628}{0 \cdot 035} = 18$.

The division of $1 \cdot 049$ and $0 \cdot 628$ by the distance of the teeth cannot be exactly performed, but the approximation is such that the error of calculation when spread over all the divisions may be neglected. Also by taking the radius a B equal to $0 \cdot 167$ instead of $0 \cdot 66875$, and the radius a A equal to $0 \cdot 100$ instead of $0 \cdot 100035$, we make the error $\frac{1}{8}$ millimètre at the most.

Mark now, beginning at the point of contact a upon the circumference a B, the lengths $a\,b$, $b\,c$, $c\,d$ . . . , equal in development to $0^m \cdot 035$; mark off also these divisions upon the circumference a A, beginning at the same point a, then, upon the radii a B and a A as diameters, describe circumferences, and determine a fraction $a\,o$ of the epicycloid generated by the point a, supposing the circumference a D to roll upon the primitive circumference or pitch-line of the pinion. The face of the tooth of the pinion will be taken upon $a\,o$, and the shoulder upon the radius a A. If we now draw the radii through the points of division, and to each of these points bring the curve $a\,o$ with its concave side turned towards the axis of the tooth, we shall get all the profiles. The faces of the teeth of the wheel are determined by constructing the portion $a\,a'$ of the epicycloid described by the point a while the circumference a C is rolling upon the pitch-line of the wheel, and by bringing this curve to the end of the radii of the points b, c, d . . .

The depth of the teeth depends, as we have seen, upon the duration of contact or driving arc. If it be required to have always three teeth engaged, this arc must be made equal to the distance of the teeth of each side of the line of the centres. Thus a K being the distance of the teeth, the epicycloid which begins at K must be produced until it meets the circumference D a in m. From the point A as a centre, with A m as a radius, describe a series of arcs such as $m\,g$, which will limit all the teeth of the pinion. This operation is performed in the same manner for the wheel. The circumferences which limit the teeth of both wheels meet the line of the centres in the points p and q; mark off, beginning at these points and in the direction of the centres, the lengths $p\,q$ and $p'\,q'$ equal to $\frac{1}{4}$ or $\frac{1}{5}$ of the projection of the tooth upon the pitch-line. The circumferences described from the points B and A with B q and A q' as radii, will determine the bottom of the spaces between the teeth on the wheel and on the pinion. It only remains then to round slightly the angles formed by the periphery of the wheel and the sides of the teeth.

Remark.—Epicycloidal gear has the grave defect of being able to drive only one wheel, or if it drive several, they must be all of the same diameter. The faces of the teeth of the pinion are in fact obtained by the rolling upon its primitive circumference of a circle having as its diameter the radius of the primitive circle of the wheel. Thus the curve generated for different pinions will vary with their diameter. Again, the faces of the teeth of the wheel are determined by the rolling upon its primitive circumference of different circles having as diameters the radii of the pinions. The curves generated all comply separately with the requirements of the question, but none can be employed to drive the system. Consequently, it is better in this case to renounce the form of the epicycloid, and to employ the involute, which does not possess the same defect, as we shall have occasion to show presently. The epicycloidal form may be made use of, however, in the case of which we have been speaking, by taking a generating circle having as a diameter the radius of the smallest pinion, and by causing it to roll successively upon the primitive circumference of the wheel, and upon that of each of the pinions which it is to drive.

Cylindrical Gear.—This kind of gear which the French call from its form, *lantern gear*, is vicious from several points of view, and its use has been almost wholly abandoned. We will, however, point out the principle of its construction, as cases now and then occur in which it is necessary to employ it.

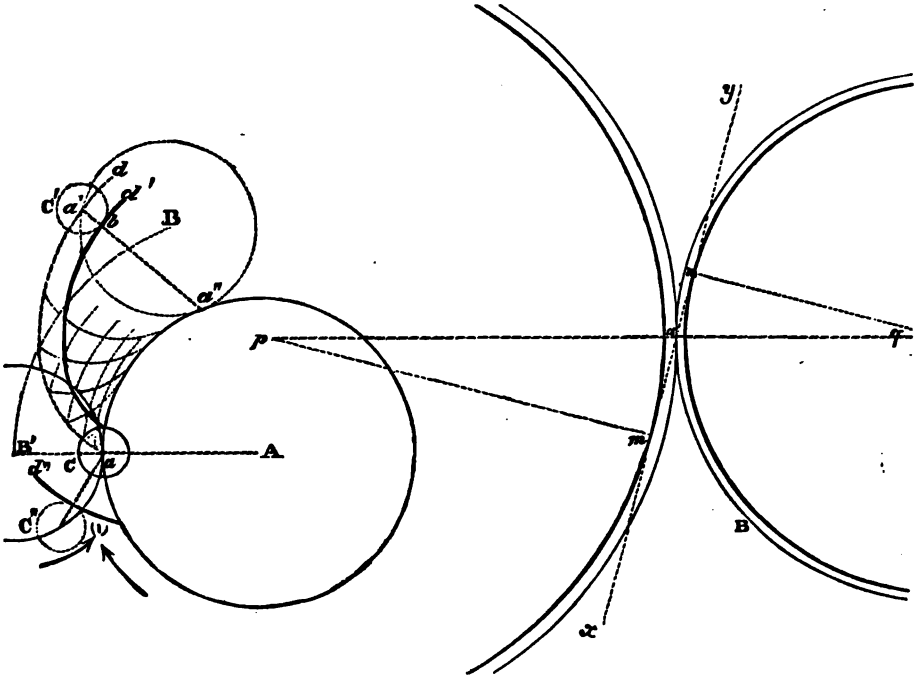
In this kind of gearing the pinion is provided, in the place of teeth, with cylindrical spindles, such as C, Fig. 3151, all of which have their centres upon the primitive circumference B. The driving wheel is provided with teeth or cogs, the profile of which we are about to determine.

Let A be the primitive circumference of the wheel and B one of the positions of the lantern gear. Suppose the wheel A fixed, and make the circle B roll round it until it comes into the position B′. During this motion the centre of the spindle will evidently describe an epicycloid d, and its new position a' will belong to this curve, and to the circumference B′. The tracing of gear rests theoretically upon this principle, namely, that the normal common to the point of contact of two conjugate teeth, passes through the point of contact of the primitive circles. If, therefore, from the point a'' we draw a normal $a''\,a'$ to the circumference C′, we shall obtain the corresponding point b of the tooth of the wheel, that is, the point in which the contact of the cog and the spindle will take place. If now we bring back the circle B′ to its former position B, the point b will describe during the motion a curve d' equidistant from d, which curve gives the true form to be adopted for the profile of the cogs of the wheel A.

It is now easy to show that this kind of gear is defective. Let C″ be a position of the spindle, and d'' the corresponding position of the tooth in contact. If the motion is in the direction of the

arrows, it is clear that the curve d' will not drive the spindle, but that it must be driven by it. Thus, *before the line of the centres, the cog cannot drive the spindle.* If the motion of the wheels is in the contrary direction, it will be seen in like manner that *beyond the line of the centres, the spindle cannot drive the cog.* This kind of gear is, therefore, very incomplete, and it must remain so, for if both branches of the curve were employed, the motion could not pass the line of the centres.

3151.



Practical Method for Epicycloidal Gear.—When the wheels are large and the teeth consequently long, in order to preserve to the curve which forms a part of the profile its particular character, it is usual to execute a portion of the drawing in full size, and to make a model from it, by means of which that form which has been graphically obtained, may be transferred exactly to the wheel.

Generally the teeth are short enough to allow of their curve being considered, without an appreciable error, as an arc of a circle. This radius of this arc must not be taken arbitrarily; and besides this, the centre of curvature must be properly placed. The common radius of the arcs for the teeth of the same wheel, and the geometrical locus of all their centres, are determined by the following construction, which gives a most satisfactory result.

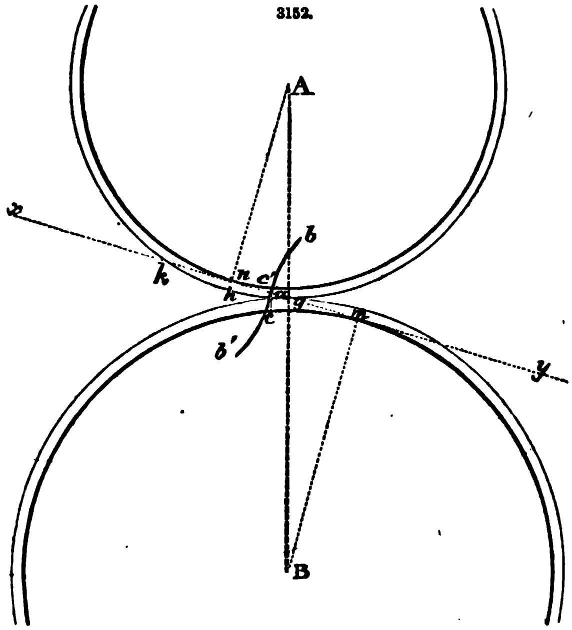
The pitch-lines A and B, Fig. 3151, being given, draw through their point of contact a straight line $x\,y$, making with the line of the centres $p\,q$ an angle of 75°; draw also from the centres p and q to $x\,y$ the perpendiculars $p\,m$, $q\,n$. The magnitudes $a\,m$ and $a\,n$ are the radii, and the points m and n the centres of the arcs which may be substituted for the epicycloid in the profile of the teeth. It is very evident that the circumferences described with $p\,m$ and $q\,n$ as radii will contain all the centres, and if care be taken to mark them beforehand when the wheels are on the axle, the operation of tracing the teeth will be quickly performed.

Involute Gear.—Let B g and A g, Fig. 3152, be the radii of the pitch-lines of the wheel and pinion, determined according to the data, and in the manner we have already described. Let us take as the profile of the teeth of the pinion, the involute $c'\,b'$ of a circumference, A p interior to A g, and determine the conjugate curve. If we draw through the point g a normal $x\,y$ to $b'\,c'$ the point of intersection a is the point in the conjugate tooth where its contact with that of the pinion is to take place. Again, as it is an established geometrical fact that every normal to the involute is tangent to the evolute, it follows that the line $x\,y$ is tangent to the circumference A p at the end of the radius A n. Having drawn the perpendicular B m, and taking this line as a radius, we describe a circumference, the involute $c\,a\,b$ of which is the curve conjugate with $c'\,a\,b'$. It may be easily demonstrated that the evolute of $c\,a\,b$ must be the circumference B m.

The triangles $a\,n\,g$, B $m\,g$ being similar, their homologous sides are proportional; but as A g, B g, and A n are constant, B m remains constant, and must be the radius of a circle. The two radii A n and A m are to each other as those of the pitch-lines A g and B g. The locus of the points of contact of the teeth is the line $x\,y$; but in practice, contact can take place only on the portion $m\,m$ of this line. Theoretically, the angle $a\,g\,x$ may be arbitrary, but it is usually made equal to 75°. Some millwrights take from the point g upon the pitch-line A g, an arc $g\,h$ K equal to twice the distance of the teeth; the line K q produced is then the common tangent of both circumferences. We have now only to divide the pitch-lines in the manner described when treating of epicycloidal gear, and to shape the teeth by the involutes passing through the points of division.

In this kind of gear the driving force is exerted throughout the whole length of the line $m\,n$, and, consequently, before and beyond the line of the centres. It may be remarked that a wheel of this kind plays no part in the construction of the conjugate wheel; it follows, therefore, that it possesses the valuable property of being able to drive at once several wheels of the same kind, of different diameters. This system works well after the shafts have been forced a little beyond their original position, and is, therefore, suited to rolling mills and similar machines.

3152.



Inner Gearing.—When the motion of two parallel shafts is to take place in the same direction, we cannot employ two wheels both toothed on the outer circumference; one of them, necessarily the larger, is toothed on the inner side. In this system the profile of the teeth differs from that which we employ in the other cases, and that on account of the following considerations.

In epicycloidal gear the shoulder of the tooth is generated by the rolling, on the inner side of the primitive circumference, of a circle having only half its diameter. The force is generated by a point of a circle rolling externally upon the same circumference. The forms of the shoulder and face may be made to run into each other, and it is easy to cut the material to the profile given by the drawing. If we adopt the same method for the inner gear, impossibilities arise in practice which oblige us to abandon the system of reciprocal shoulders. The only admissible solution of the difficulty consists in giving shoulders to the teeth of the pinion and curves to the wheel.

Suppose now we have to construct an inner gearing with the following data

Distance of the centres	$0^{m}\cdot100$
Distance of the teeth	$0^{m}\cdot035$
Ratio of the velocities	$\frac{1}{2}$

If r and n represent the radius and the number of revolutions of the wheel, r' and n' the radius and number of revolutions of the pinion, we have the proportion $r : r' :: n' : n$.

According to the data, $n' = 2\,n$; therefore $r = 2\,r'$.

But the distance of the centres $d = r - r' = 2\,r' - r' = r'$; therefore the radius of the pinion is equal to d or $0^{m}\cdot100$; and that of the wheel $r = 2\,r' = 0^{m}\cdot100 \times 2 = 0^{m}\cdot200$.

Having acquired these data, continue the operation in the following way;—

Describe the pitch-lines A a and B a tangent in a. From this point a divide the pitch-line of the wheel into parts $a\,b$, $b\,c$, $c\,d$ . . . , equal to the distance of the teeth or to $0^{m}\cdot35$. Mark in the same way, beginning at the point a, these same divisions upon the circumference of the pinion in $a\,b'$, $b'\,c'$, $c'\,d'$ . . . , then, having determined the thickness of the teeth with respect to the space between them, transfer it from the points a, b, c . . . , and a, b', c' . . . , to $a\,1$, $b\,2$, $c\,3$ . . . , and $a\,1'$, $b'\,2'$, $c'\,3'$ . . . Through these points of division in the pitch-line B a, draw radii such as B 1, which determine the profile of the teeth of the pinion, since they consist wholly of shoulder.

The teeth of the wheel may be obtained in the following manner;—

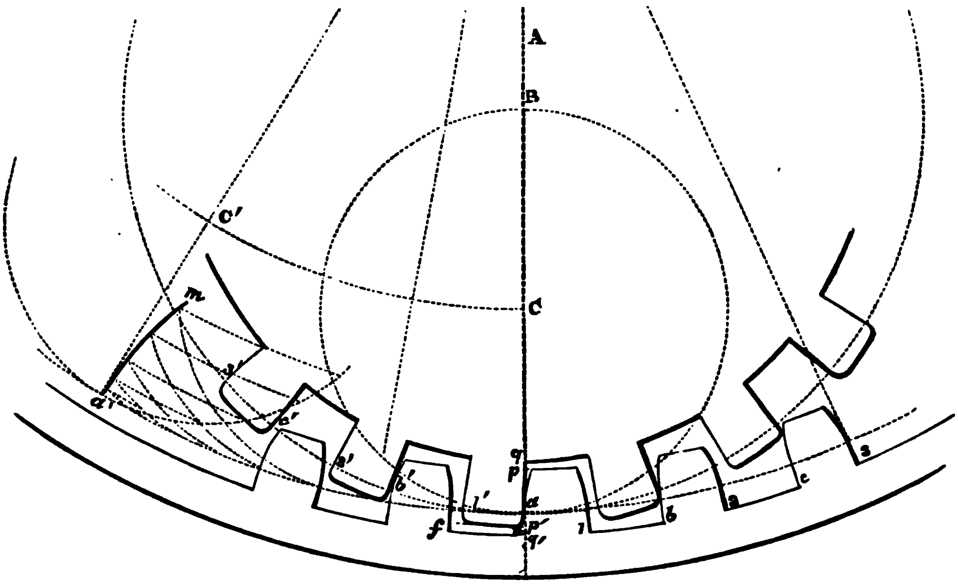
Upon the radius B a as a diameter describe a circle a C, and, in order not to crowd the diagram, take a second position a' C′ of this circle; determine a portion $a'\,m$ of the contracted epicycloid described by the point a', whilst the circle C′ a' is rolling upon the inner side of the pitch-line of the wheel. Transfer this curve to all the points of division a, b, c, d . . . , 1, 2, 3 . . . , placing the concave side towards the middle of the tooth. Having completed this operation, it only remains to limit the teeth. Those of the wheel are taken long enough to have at least three couples always in contact, and they are limited by the circumference described upon A p as a radius.

The teeth of the pinion do not terminate at the primitive circumference; they would terminate at the intersection of this circumference with the radii in sharp angles, which would have an injurious effect upon the teeth of the wheel. The profile of the shoulder is continued by arcs of circles, such as $a\,n$, which run into other arcs described with B n as a radius. We have now only the bottom of the spaces between the teeth, or in other words, that portion of the periphery of the wheel that is comprised between two teeth, to determine. The play $p\,g$ and $p'\,g'$ depends on the dimensions of the gear and on the care bestowed on its execution: the amount of play being decided upon, describe with the radii A g' and B g a series of arcs such as $g'\,f$ and $g\,f$. It is well to make the tooth join the periphery by a curve rather than to enter it at right angles.

Rack and Pinion.—If we suppose a toothed wheel to increase in diameter indefinitely, the disc which bears the teeth will become a straight surface, and its circular motion will evidently be

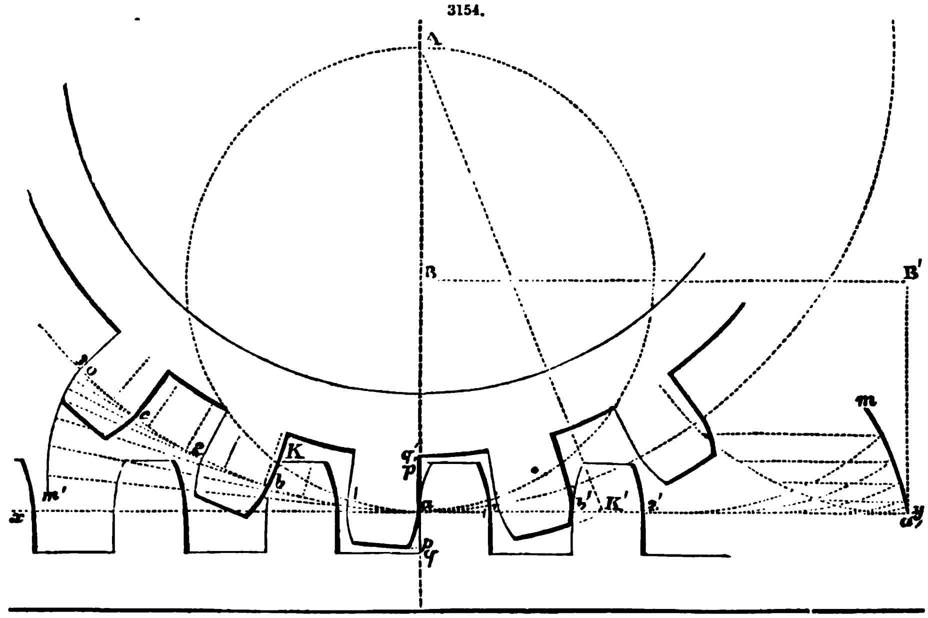
converted into a rectilinear motion, since in the proportion $r : r' :: n' : n$, r being equal to infinity, n will be equal to zero. The straight surface which receives a rectilinear motion is called a rack.

3153.



Let A a, Fig. 3154, be the pitch-circle of the pinion, and $x\,y$ the pitch-line of the rack, tangent in the point a. Divide, from the point a, the circumference A a into parts $a\,b$, $b\,c$, $c\,d$ . . . , equal to the pitch or distance of the teeth, and mark these same divisions upon $x\,y$. From the points a, b, c . . . , and a', b', c' . . . , mark the thickness of the teeth at a 1, b 2, c 3 . . . , and a 1', b 2', c 3' . . . The radii from the points a, b, c . . . , 1, 2, 3 . . . , determine the shoulders, or as they are often called, the *flanks*, of the teeth of the pinion, and those of the rack may be obtained by drawing perpendiculars to $x\,y$ through its points of division.

3154.



If now we suppose the line $x\,y$ to move about the pitch-circle of the pinion, its first point of contact will describe an involute of this circle, which should be taken as the profile of the teeth. Having determined a portion $o\,m'$ of the curve, we have only to bring it to each of the points of division in the circumference A a.

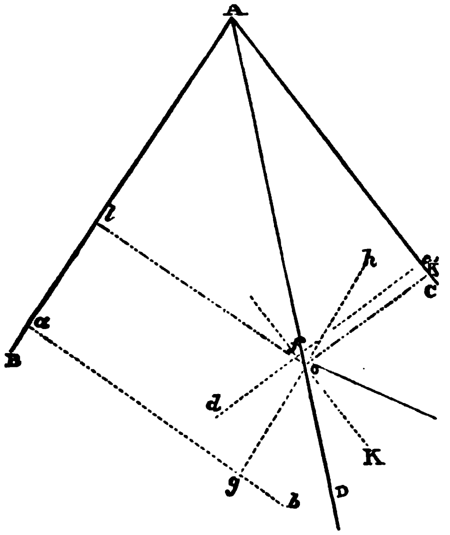
To shape the teeth of the rack we take a portion $a' m$ of the cycloid generated by the point a of a circle B a, described upon the pinion as a diameter, and rolling upon $x y$. To limit the teeth of the rack we usually take a length $a b'$ equal to the pitch, describe the cycloid at the point b', and produce it till it meets in K the generating circle B a. The line parallel to $x y$ drawn through the point K gives the extremities of the teeth. The teeth of the pinion are limited by the circumference of a circle described with the radius b K′, and determined by the position of the point K upon $x y$. The length $a k$ is usually taken equal to the distance of the teeth, but it may be altered if it is seen that there are too many teeth in contact or that the force is exerted upon too small a number of teeth.

To complete the drawing we take upon the line A a, beyond the extremities of the teeth, the short distances $p q$ and $p' q'$ to represent the play to be allowed at the bottom of the spaces. The circumference described with A q' as a radius, and the parallel to $x y$ drawn through the point q, determine the roots of the teeth.

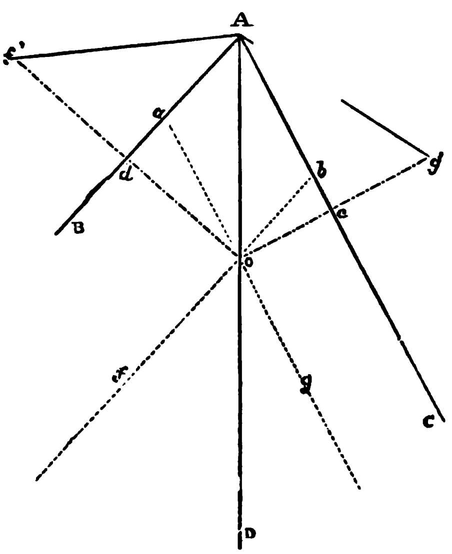
Bevelled Gear.—When it is required to transmit the circular motion of a shaft A B, Fig. 3155, to another A C, making with it any angle B A C, conical or bevelled gear is employed. This kind of gear possesses the same properties as the spur-gear. Suppose, for example, the two axes A B and A C given, as well as the ratio $\frac{m}{n}$ of the number of revolutions, at the points a and c taken arbitrarily upon each of the axes, raise the perpendiculars $a b$, $c d$; take upon $a b$ a length $a g$, and upon $c d$ a length $c f$, proportional to m and n. Through the points g and f draw $g h$ and f K parallel to the axes; the point of intersection o determines, with the summit A, the generatrix of contact A D. The perpendiculars $o k$ and $o l$, drawn from the point o to the axes, represent the radii of the bases of the cones or teeth.

Denoting the velocity of the motion by V, the angular velocities of the pinions by V'_1 and V_1, and the radii by R and R′, we have $V = V_1 R$, and $V = V'_1 R'$. But as the velocity V is dependent upon each of the pinions, we get the equation $V_1 R = V'_1 R'$, from which we deduce the proportion $V_1 : V'_1 :: R' : R$. Thus in bevelled as in spur-gear, the angular velocities are inversely proportionate to the radii.

3155.



3156.

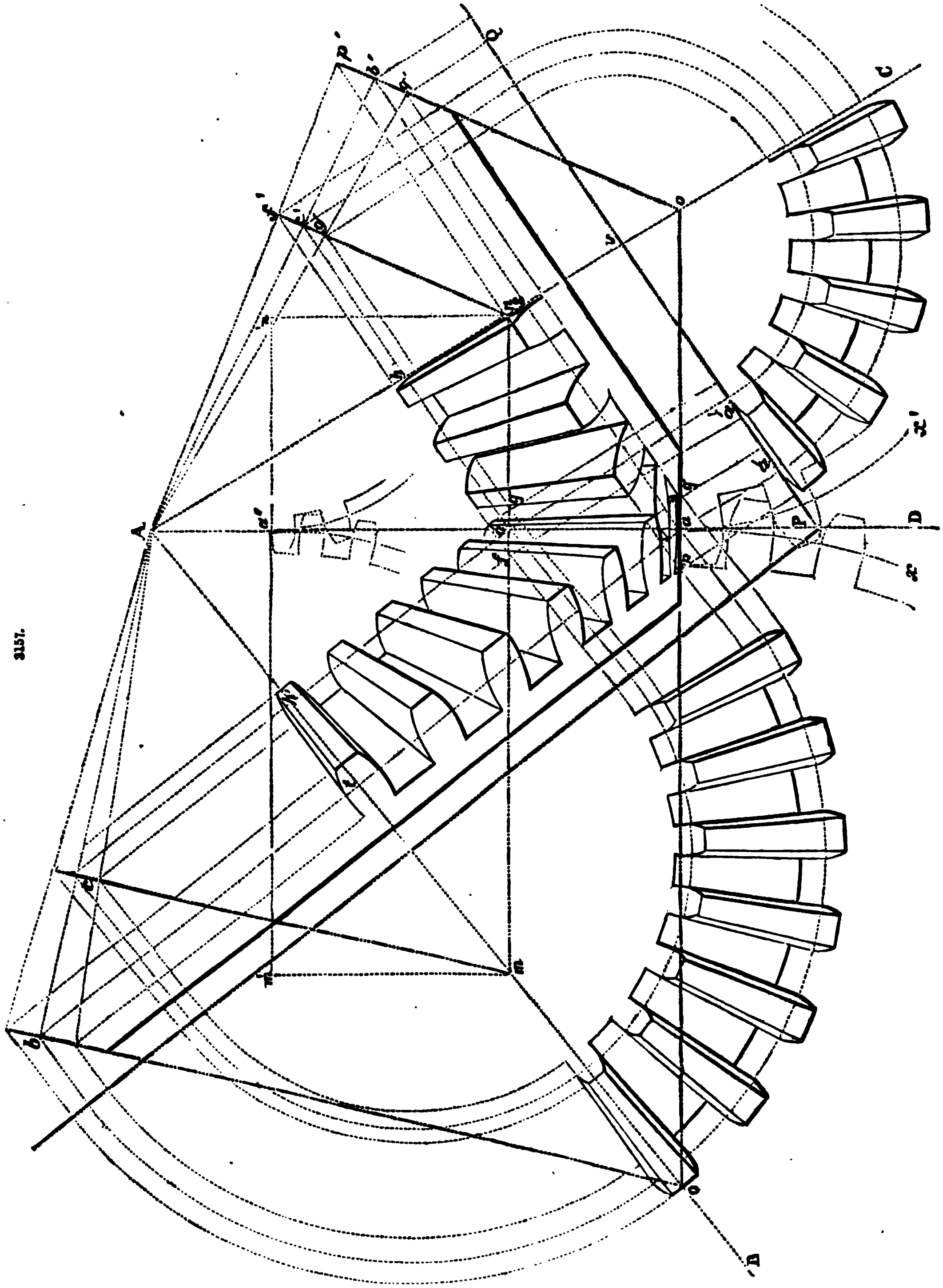


But, the two axes and the ratio of the velocities being given, we may determine the generatrix of contact in the following manner;—Upon the axis A B, Fig. 3156, take A a equal to m, and upon the axis A C, $a b = n$; through the points a and b draw $a g$, $b f$, parallel to the axes; the point o in which they meet belongs to the generatrix of contact. If we let fall upon the axes through the point o, the perpendiculars $o c$, $o d$, we get two similar triangles, which give the proportion $o d : o c :: o a : o b$; but $o a = n$ and $o b = m$; substituting these values, we have $o d : o c :: n : m$. Therefore the radii $o d$, $o c$, or the diameters $o f'$, $o g'$, fulfil perfectly the condition required.

Let us now execute a drawing of two bevelled wheels by means of the following data;—The angle B A C, Fig. 3157, is equal to 82°. The ratio of the velocities is $\frac{2}{3}$, and let E be the force to be transmitted.

By operating in the way described above, we easily obtain the generatrix of contact A D. Upon this line we determine a point a, such that the perpendiculars $a s$, $a t$, let fall upon the axes, may be to each other as 2 is to 3. By taking $s b' = a s$ and $t b = a t$, we have the primitive diameters of the bases of the cones. Through the point a draw $o o'$ perpendicular to A D; in this way we get two cones $o a b$ and $o' a b'$ which will limit the base of the gear. Having calculated the thickness of the teeth, and consequently the pitch, according to the force F in the manner described for spur-gear, take a length $a a'$ varying between three and four times the thickness of a tooth, according to the nature of the metal employed and the conditions in which the pinions work.

Draw through the point a', $m\,n$ perpendicular to A D, and $a'c$, $a'c'$ parallel to $a\,b$ and $a'b'$, so that $a'k = k\,c$, and $a'h = h\,c'$; draw $m\,c$ and $n\,c'$. We get in this way two cones $m\,a\,c$ and $n\,a\,c'$, which



3157.

limit the upper portions of the pinions. The two lines which join A to the points b and b' evidently pass through the points c and c', and are the primitive generatrices or the several positions of the generatrix of contact.

Now, from the points o and o', with $o\,a$ and $o'\,a$ as radii, describe two arcs $a\,x$ and $a\,x'$, which may be considered as the pitch-circles of two spur-wheels, and upon which the gear is constructed, with the pitch previously determined. The teeth thus obtained represent the bases of the teeth of the pinions. If we consider the teeth obtained for the wheel A C, the circumferences which limit them at their extremities and at their roots, meet the line $o\,o'$ in the points p and q; through these points draw lines parallel to $a\,b'$ till they meet $o'\,b'$ in p and q'.

Now, with $m\,a$ and $n\,a'$ as radii transferred to $m'\,a''$ and $n'\,a''$, describe two arcs which will serve as a basis to a drawing analogous to the one given above. The teeth obtained form the upper extremities of those of the pinion. By bringing, through lines parallel to the generatrix of contact, the extremities and roots of the teeth upon $m\,n$, we get the points f and g, through which we draw lines parallel to $a'\,c$ and $a'\,c'$. We might determine these parallels equally well by giving the points p', q', to the summit A, which would give the points f', g'. The outline of the pinion is thus complete. What we have said is sufficient for practice, the rest belongs to descriptive geometry; we will, however, point out in summary manner what remains to be done to complete the plan.

Take a plane P Q perpendicular to the axis A C; mark upon this plane the different radii which have one of their extremities upon the axis and the other in the points p, a, q, and f, a', g; from the point v, with these radii, describe circles which will represent the plane of the pinion, projected at first upon the plane P Q. Upon the circumferences $v\,a_1$ and $v\,a'$, considered as pitch-circles, trace the teeth in the manner shown for spur-gear, that is, recommence the operations indicated by the dotted lines; then join the extremities of the teeth and project each of them upon the vertical plane. All the operations indicated for the wheel A C must be repeated for the other.

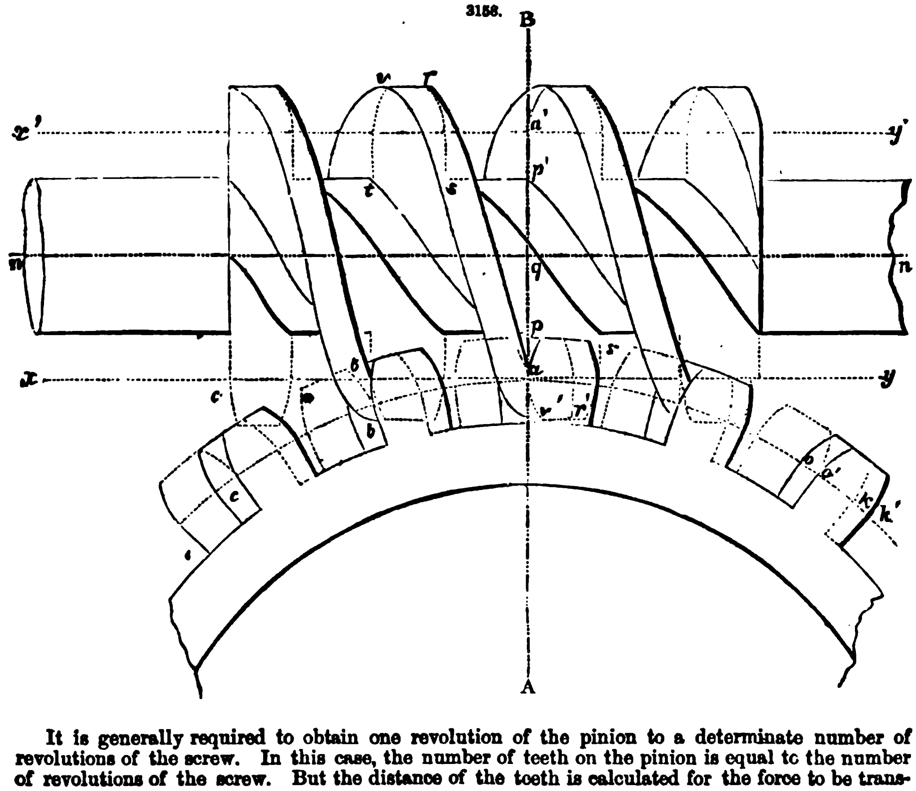
After having determined the generatrix of contact and the primitive radii of the wheels, it almost always happens that the pitch, determined in accordance with the force F, does not exactly divide the circumferences described with these radii. In this case the number of the teeth must be lessened, and consequently the pitch increased, until the division can be exactly made. No inconvenience can result from modifying in this direction the dimensions of the teeth.

We have selected as our example the case of two shafts forming an acute angle: the method of drawing is the same in all cases; but the most general case is that of two shafts placed at right angles to each other.

Endless Screw and Pinion.—When it is required to communicate the continuous circular motion of a shaft to another not situate in the same plane, the endless screw and pinion may be employed, if the given axes are at right angles and near each other.

The discussion of the principles upon which the construction of this gear rests would lead us into a somewhat complicated theory, which we will pass by for the sake of considering the drawing from a practical point of view. The data are reduced to the work and, consequently, to the force F to be transmitted.

3158.



It is generally required to obtain one revolution of the pinion to a determinate number of revolutions of the screw. In this case, the number of teeth on the pinion is equal to the number of revolutions of the screw. But the distance of the teeth is calculated for the force to be trans-

mitted and remains invariable; therefore it is necessary to make the radius of the pinion depend upon the number of teeth: this cannot be, however, unless the distance of the axes be unlimited. It must be remarked that in almost all cases, the screw drives the pinion, the object being an increase of force at the cost of speed.

Knowing the maximum work to be transmitted, we may determine the pitch of the pinion, which is also that of the endless screw. The radius of the solid portion of the screw may be taken as a function of the pitch, since it is also dependent on the force. It is usually obtained by multiplying the pitch by 5, and dividing the result by 2. To execute the trace we must take A a, Fig. 3158, equal to the primitive radius of the pinion; draw through the point a, a perpendicular $x\,y$, which will be the pitch-line of the screw. From the point a, take upon the radius of the pinion produced, a length $a\,q$ equal to $\frac{13}{10}$ the radius of the solid or central portion of the screw, so that $a\,p$ may be equal to $\frac{3}{10}$ of this radius. By taking $q\,a' = q\,a$, and drawing $x'\,y'$ parallel to the axis $m\,n$ and to $x\,y$, we obtain the other pitch-line of the screw.

Now mark upon the pitch-circle of the pinion the lengths $a\,b$, $b\,c$ . . . , equal to the pitch; mark these same divisions upon $x\,y$ in $a\,b'$, $b'\,c'$ . . . , and construct the teeth of a rack, by taking as their faces the cycloid generated by a circle of a diameter A a, rolling upon $x\,y$. Trace upon $x'\,y'$ teeth symmetrical to the preceding with respect to $m\,n$. The helicoidal portions which unite the profiles $r\,s$ and $v\,t$, for example, to $r'\,s'$ and $p\,v'$, determine the thread and its inclination with respect to the axis of the screw.

The teeth of the pinion must have upon the disc which carries them the same inclination as the thread of the screw; the profile of them will therefore need to be traced upon each face of the wheel. Thus $o\,k$ represents the profile of a tooth upon one face, and $o'\,k'$ the same profile upon the other; $o\,o'$ is therefore the inclination upon the breadth of the wheel. As to the form of this profile it has not been exactly determined, but if the teeth are large, they must be cut to receive as nearly as possible the impress of the thread of the screw. Besides this, the extremities of the teeth and the bottom of the spaces, instead of being straight, that is form of generatrices situate in planes parallel to the axis, will present circular gorges, the radii of which are a little greater than $p\,q$ and $a\,q$.

It often happens that the endless screw has several threads; the distances which separate them are, in this case, equal fractions of their common pitch. The length of the screw may be limited to the three or four threads which act simultaneously. If we trace upon a cylinder of a considerable diameter, a series of threads of a very wide pitch, and then reduce each of them to a small portion of a spiral by means of two planes perpendicular to the axis and near together, we obtain what is called a helicoidal gear.

Construction of Cams for Stamps and Hammers.—The conversion of the continuous rotatory motion of a shaft into a reciprocating rotatory or rectilineal motion is frequently made in mills, and especially in iron-works, to obtain the ascensional motion of the stamps and hammers.

Suppose a rod A, Fig. 3159, furnished with a stamp, to be guided in such a way that it can move only in a vertical direction; such a rod may be raised by teeth or cams turning about an axis o and acting against a projecting part C. The stamp falls by its own weight, to be raised again by another cam.

When a system of this kind is established it is necessary to consider—1. The number m of cams to be employed. 2. The angular velocity of the shaft, or the number n of revolutions it is to make a minute. 3. The height to which the stamp is to be raised. 4. The radius of the pitch-circle to be developed to obtain the profile of the cams.

The course or stroke of the stamp is generally determined beforehand, and upon it alone depends the velocity of impact. The number of cams and the number of revolutions are mutually dependent, and their product $m\,n$ must be equal to the number of strokes a minute. We must in all cases take one of the factors and deduce the value of the other.

If we denote by T the time of one revolution, the time which elapses between the beginning of two consecutive strokes will be expressed by $\frac{\mathrm{T}}{m}$.

The number of strokes a minute being equal to $m\,n$, we have $\frac{\mathrm{T}}{m} = \frac{60''}{m\,n}$.

As there are always some passive resistances opposed to the descent of the stamp, $\frac{\mathrm{T}}{m}$ must be increased in value by $\frac{1}{6}$ or $\frac{1}{7}$ to prevent the stamp from falling upon the cam which is to raise it. This result may be arrived at by increasing the radius of the pitch-circle upon which we operate for the construction of cams.

Thus the time of one ascent is given by $\frac{6}{7}\,\frac{\mathrm{T}}{m}$. The formula of the time occupied in the descent is $t^2 = \frac{2\,c}{g}$, or $t^2 = \frac{2\,h}{g}$, from which we deduce $t = \sqrt{\frac{2\,h}{9 \cdot 81}}$.

The time t' during which the cam will act upon the stamp is therefore expressed by

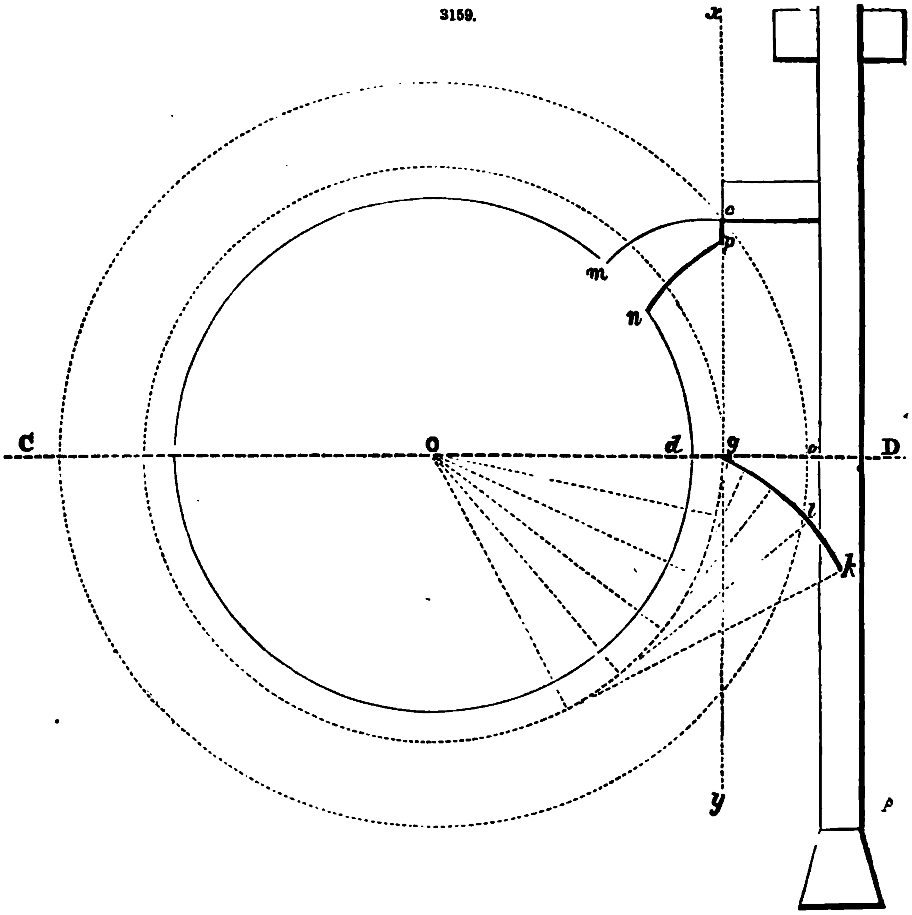
$$t' = \frac{6}{7}\,\frac{\mathrm{T}}{m} - \sqrt{\frac{2\,h}{9 \cdot 81}}.$$

The radius of the circumference to be developed will now be calculated by the formula

$$= \frac{60\,h}{t' \times 6 \cdot 28\,n}.$$

This radius may never be less than the value given by the formula, but no inconvenience can result from making it greater; this indeed, is nearly always done.

3159.



Trace of Cams.—Having described a circle with a radius r_1 draw the vertical tangent $x\,y$ upon which the point of application of the force transmitted will constantly be. Take $g\,c = h$, and draw the highest position of the projecting piece c. Determine a portion $g\,k$ of the involute of the circle $o\,g$, and from the point o, with the radius $o\,c$, describe a circle, which will limit at the point l the length of the cam. The other face of the cam is generally a nearly straight line; it is traced so that the form obtained may be nearly that of a solid of equal resistance, the greatest thickness $m\,n$ of which is calculated according to the maximum force to be developed, and the strength of the materials.

Theoretically, the cam should possess no thickness at its extremity, but in practice it has a thickness, which, instead of being marked by the circumference passing by the point c, is marked by a portion $c\,p$ of the line $x\,y$. This allows the instantaneous escape of the stud c.

Epicycloidal Cams for converting a Continuous into a Reciprocating Rotatory Motion.—The construction of cams for raising mill-hammers differs from the preceding in using the epicycloid.

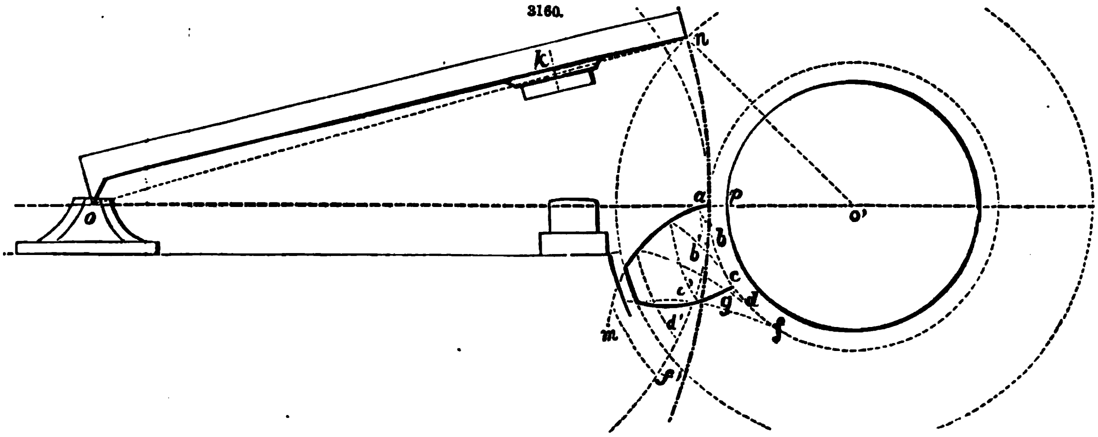
Let $o\,n$, Fig. 3160, be the whole length of the hammer; from the point o, about which the hammer revolves, describe a circle with the radius $o\,n$. Take as the radius of the pitch-circle of the cams a sufficient length to prevent the hammer from falling upon the cam which is to raise it. What we have said above in reference to this subject is equally applicable here.

To find the form to be given to the profile of the cams, describe a circle with the diameter $o\,a$, and determine a portion $a\,m$ of the epicycloid generated by the point a while this circle is rolling upon the circle $o'\,a$.

To find the limit of the cams, take from the point a, upon the circle with a radius $o\,a$, the arc $a\,n$, representing the space traversed by the end of the hammer in its ascent, and describe the circle with a radius $o'\,n$. The two profiles of the cam, as in the preceding case, need not be symmetrical, but generally they are made similar. The thickness $p\,q$ is determined in the manner described above for stamps. It may be remarked, however, that the resistance to be overcome by the cams is not in this case merely the whole weight of the hammer, and that the length of the arms of the lever turning about the point o must be taken into account. Neglecting the weight of

the beam $o\,n$, and denoting the weight of the hammer by W, and the force borne by the cam by F, we have $F = \dfrac{O\,K \times W}{o\,n}$.

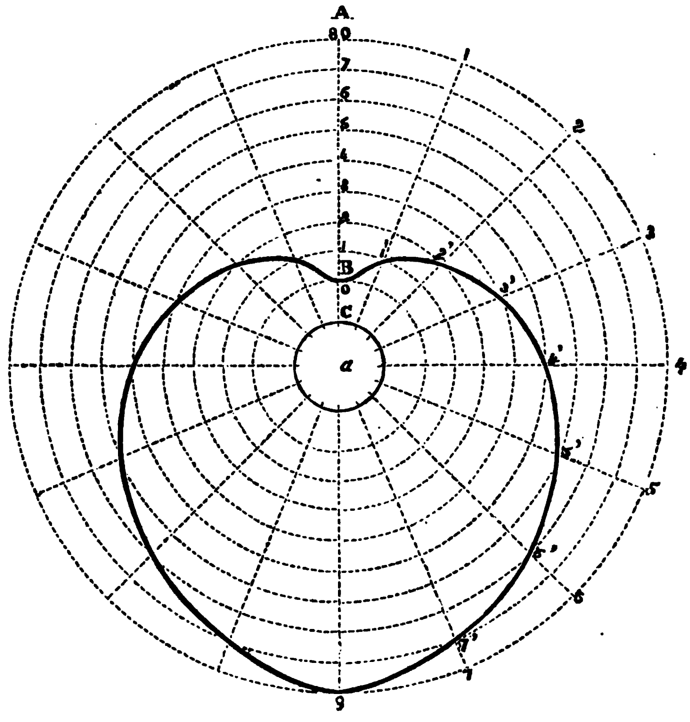
3160.



Suppose, as it is always well to suppose, that the load is constantly borne by the end of the cam, and denote by a, b, c, the three dimensions of the latter, that is, thickness in the direction of the force, breadth taken perpendicularly to the direction of this force, and the length of the cam. Neglecting the weight of the tooth, the section at the base will be given by the formula $a\,b^2 = \dfrac{F\,c}{1250000}$, if the cams are of cast iron. If they are of wrought iron, the divisor of the product $F\,c$ is 1000000; and if they are of wood, this divisor will be reduced to 100000.

Construction of the Heart-shaped Cam.—This cam, whose form is sufficiently indicated by its name, is used to convert the continuous rotatory motion of a shaft into a reciprocating rectilinear motion of a rod, and so on. The only datum necessary to the construction is the length of the stroke required.

3161



To construct the heart-shaped eccentric, take a length A B, Fig. 3161, equal to the stroke; produce it first to C, B C being equal to the least thickness of metal to be retained about the shaft; then to a, C a being equal to the radius of the shaft. Describe the circles a C, a B, a A; divide each half of the circle A a into eight equal parts, for example; the greater the number, the more exact will be the construction. Draw the radii a 1, a 2, a 3 . . .; divide in like manner A B into eight equal parts, and from the centre a describe the circles passing through the points of division; the point in which each of these circles meets the radius of the same number is a point in the curve.

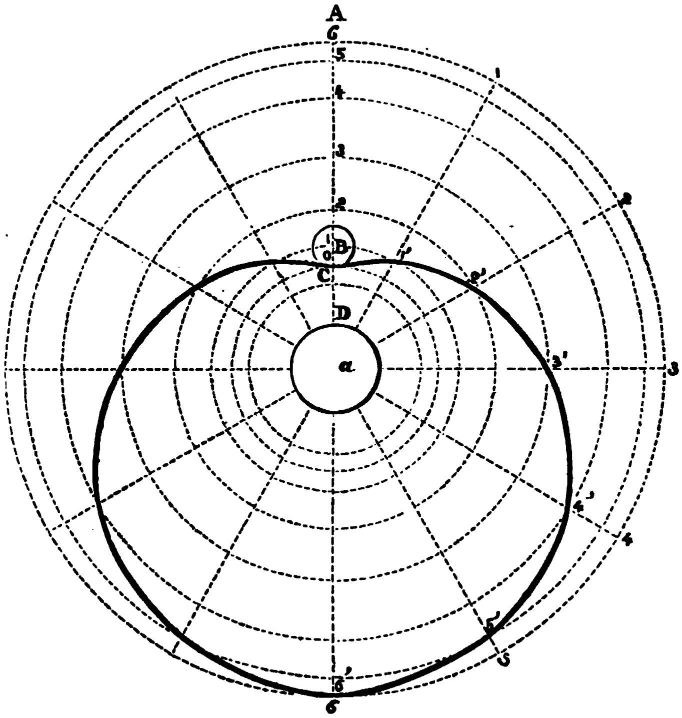
If the cam is to work against a roller, which is generally the case, the angle B must be rounded a little to give it the form of the roller; and a length equal to the radius of this roller added to the radius A a, which length is placed between the stroke A B and the thickness B C.

The heart-shaped cam possesses several valuable properties.

1. It intercepts upon all the lines passing through the centre a equal length; this enables it to turn in a frame.

2. The spaces traversed in a straight line by the frame or rod are exactly proportional to the angular velocity of the cam; whence it follows that if the rotatory motion of the shaft is uniform, the rectilinear motion produced is uniform also.

3162.



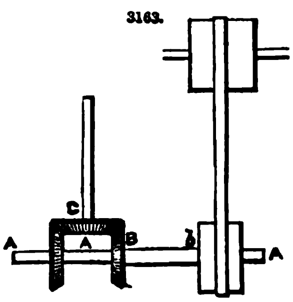
Trace of a Cam converting a Continuous Rotatory Motion into a Uniformly Periodical Motion.—Suppose the rectilinear motion to be produced to be slow at the beginning, accelerated during a certain time, and retarded towards the end of the stroke, so as to end with the velocity with which it began.

Having taken, as before, A B equal to the stroke, Fig. 3162, we add B C equal to the radius of the roller; the point C is here the beginning of the curve. Take C D as the least thickness of the metal and D a equal to the radius of the shaft. From the point C divide C A into parts proportional to the acceleration or the retardment of the velocity at the given points of the stroke, and describe from the centre a the circles passing through the points 1, 2, 3 . . . Divide each half of the circle a A into six equal parts, and draw the radii to the points of division; their intersection with the circumferences passing through the corresponding points of the stroke A C gives the points of the curve sought.

This eccentric cannot, like the last, work in a frame.

Fig. 3163. A method of engaging, disengaging, and reversing the upright shaft at the left. The belt is shown on the middle one of the three pulleys on the lower shafts a, b, which pulley is loose, and consequently no movement is communicated to the said shafts. When the belt is traversed on the left-hand pulley, which is fast on the hollow shaft b, carrying the bevel-gear B, motion is communicated in one direction to the upright shaft; and on its being traversed on to the right-hand pulley, motion is transmitted through the gear A, fast on the shaft a, which runs inside of b, and the direction of the upright shaft is reversed.

3163.



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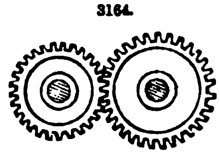

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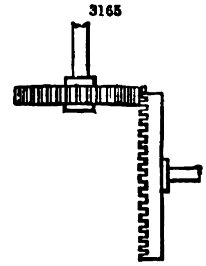
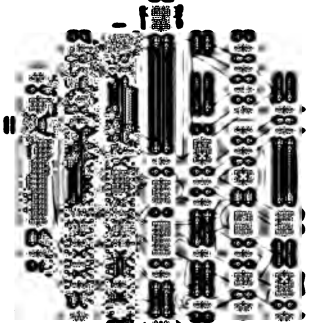


Fig. 3164. Spur-gears.

Fig. 3165. The wheel to the right is termed a *crown-wheel;* that gearing with it is a spur-gear. These wheels are not much used, and are only available for light work, as the teeth of the crown-wheel must necessarily be thin.



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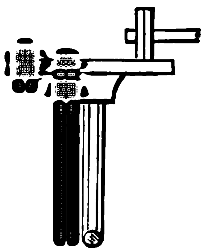

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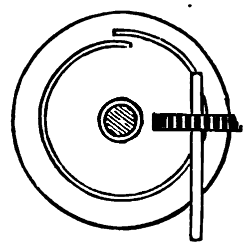

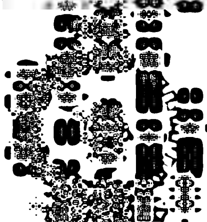

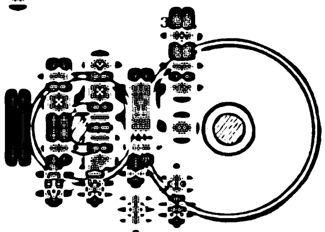

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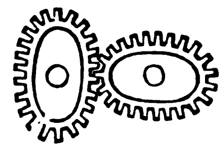

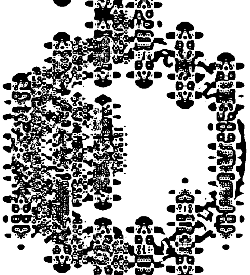

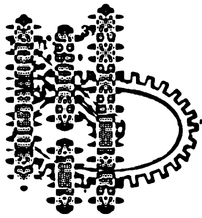

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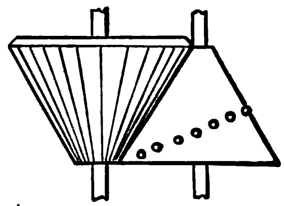

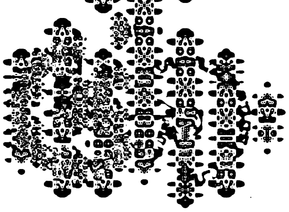

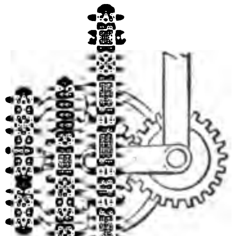

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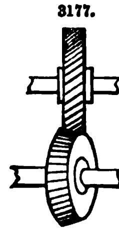

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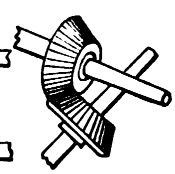

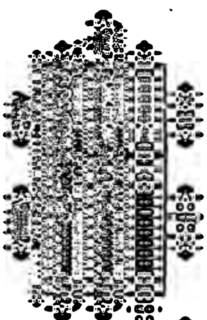

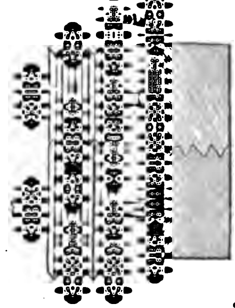

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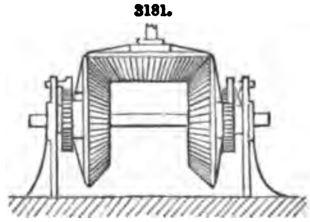

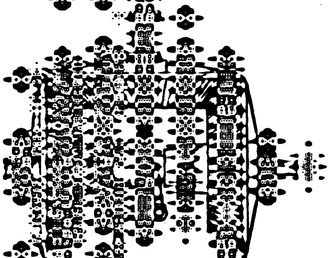

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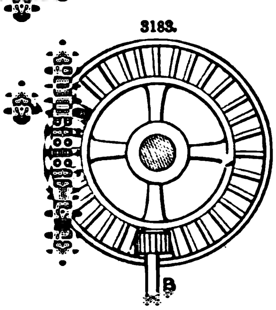

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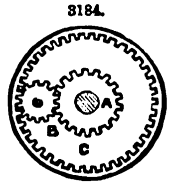


Fig. 3166. *Multiple-gearing*—a recent invention. The smaller triangular wheel drives the larger one by the movement of its attached friction-rollers in the radial grooves.

Fig. 3167. These are sometimes called brush-wheels. The relative speeds can be varied by changing the distance of the upper wheel from the centre of the lower one. The one drives the other by the friction or adhesion, and this may be increased by facing the lower one with india-rubber.

Fig. 3168. Transmission of rotary motion from one shaft at right angles to another The spiral thread of the disc-wheel drives the spur-gear, moving it the distance of one tooth at every revolution.

Fig. 3169. Worm or endless screw and a worm-wheel. This effects the same result as Fig. 3168: and as it is more easily constructed, it is oftener used.

Fig. 3170. Friction-wheels. The surfaces of these wheels are made rough, so as to *bite* as much as possible; one is sometimes faced with leather, or, better, with vulcanized india-rubber.

Fig. 3171. Elliptical spur-gears. These are used where a rotary motion of varying speed is required, and the variation of speed is determined by the relation between the lengths of the major and minor axes of the ellipses.

Fig. 3172. An internally-toothed spur-gear and pinion. With ordinary spur-gears (such as represented in Fig. 3164) the direction of rotation is opposite; but with the internally-toothed gear, the two rotate in the same direction; and with the same strength of tooth the gears are capable of transmitting greater force, because more teeth are engaged.

Fig. 3173. Variable rotary motion produced by uniform rotary motion The small spur-pinion works in a slot cut in the bar, which turns loosely upon the shaft of the elliptical gear. The bearing of the pinion-shaft has applied to it a spring, which keeps it engaged; the slot in the bar is to allow for the variation of length of radius of the elliptical gear.

Fig. 3174. Uniform into variable rotary motion. The bevel-wheel or pinion to the left has teeth cut through the whole width of its face. Its teeth work with a spirally-arranged series of studs on a conical wheel.

Fig. 3175. A means of converting rotary motion, by which the speed is made uniform during a part, and varied during another part, of the revolution.

Fig. 3176. Sun-and-planet motion. The spur-gear to the right, called the planet-gear, is tied to the centre of the other, or sun-gear, by an arm which preserves a constant distance between their centres. This was used as a substitute for the crank in a steam-engine by James Watt, after the use of the crank had been patented by another party. Each revolution of the planet-gear, which is rigidly attached to the connecting rod, gives two to the sun-gear, which is keyed to the fly-wheel shaft.

Figs. 3177, 3178. Different kinds of gears for transmitting rotary motion from one shaft to another arranged obliquely thereto.

Fig. 3179. A kind of gearing used to transmit great force and give a continuous bearing to the teeth. Each wheel is composed of two, three, or more distinct spur-gears. The teeth, instead of being in line, are arranged in steps to give a continuous bearing. This system is sometimes used for driving screw-propellers, and sometimes, with a rack of similar character, to drive the beds of large iron-planing machines.

Fig. 3180. Frictional grooved gearing—a comparatively recent invention. The diagram to the right is an enlarged section, which can be more easily understood.

Fig. 3181. Alternate circular motion of the horizontal shaft produces a continuous rotary motion of the vertical shaft, by means of the ratchet-wheels secured to the bevel-gears, the ratchet-teeth of the two wheels being set opposite ways, and the pawls acting in opposite directions. The bevel-gears and ratchet-wheels are loose on the shaft, and the pawls attached to arms firmly secured on the shaft.

Fig. 3182. The vertical shaft is made to drive the horizontal one in either direction, as may be desired, by means of the double-clutch and bevel-gears. The gears on the horizontal shaft are loose, and are driven in opposite directions by the third gear; the double-clutch slides upon a key or feather fixed on the horizontal shaft, which is made to rotate either to the right or left, according to the side on which it is engaged.

Fig. 3183. Mangle or star-wheel, for producing an alternating rotary motion.

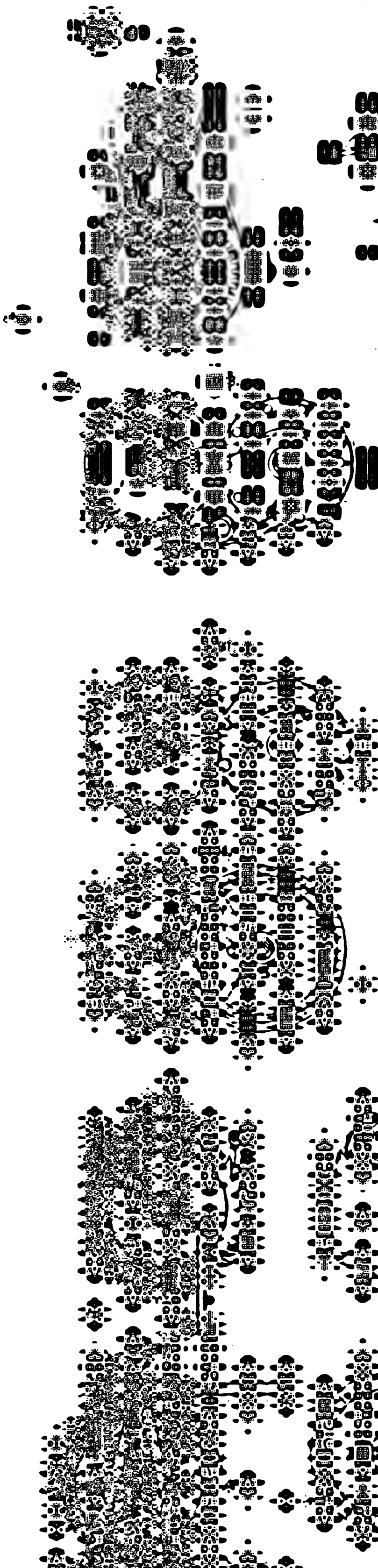
Fig. 3184. Different velocity given to two gears, A and C, on the same shaft, by the pinion D.

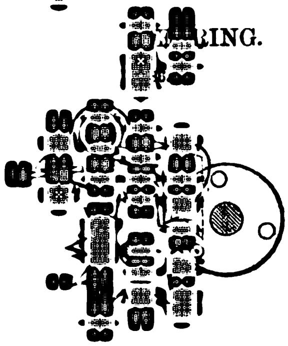
Fig. 3185. The small pulley at the top being the driver, the large, internally-toothed gear and the concentric gear within will be driven in opposite directions by the bands, and at the same time will impart motion to the intermediate pinion at the bottom, both around its own centre and also around the common centre of the two concentric gears.

Fig. 3186. Jumping or intermittent rotary motion, used for meters and revolution-counters. The drop and attached pawl, carried by a spring at the left, are lifted by pins in the disc at the right. Pins escape first from pawl, which drops into next space of the star-wheel. When pin escapes from drop, spring throws down suddenly the drop, the pin on which strikes the pawl, which, by its action on star-wheel, rapidly gives it a portion of a revolution. This is repeated as each pin passes.

Fig. 3187. Another arrangement of jumping motion. Motion is communicated to worm-gear B by worm or endless screw at the bottom, which is fixed upon the driving shaft. Upon the shaft carrying the worm-gear works another hollow shaft, on which is fixed cam A. A short piece of this hollow shaft is half cut away. A pin fixed in worm-gear shaft turns hollow shaft and cam, the spring which presses on cam holding hollow shaft back against the pin until it arrives a little farther than shown in the figure, when, the direction of the pressure being changed by the peculiar shape of cam, the latter falls down suddenly, independently of worm-wheel, and remains at rest till the pin overtakes it, when the same action is repeated.

Fig. 3188. The left-hand disc or wheel C is the driving wheel, upon which is fixed the tappet A.





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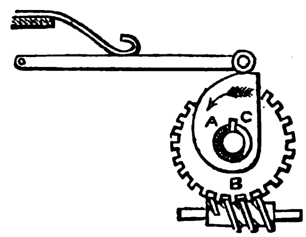

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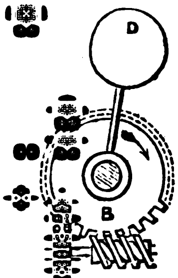

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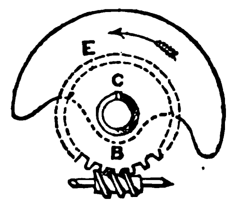

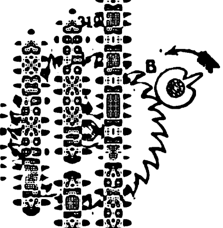

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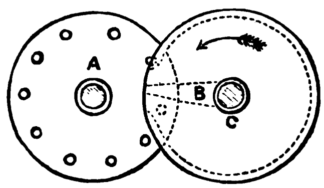

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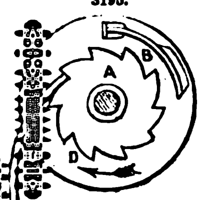

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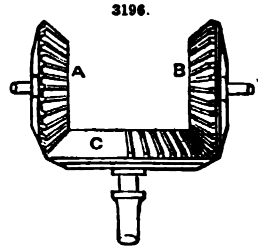

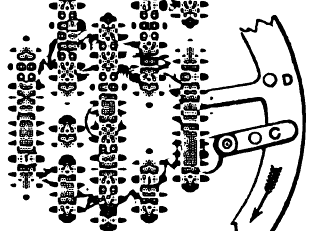

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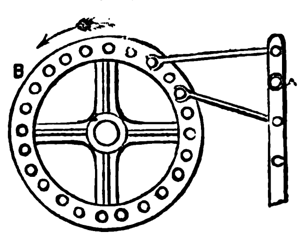

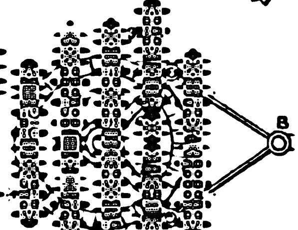

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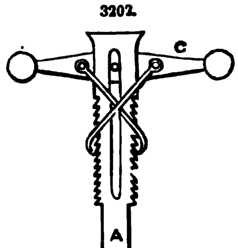

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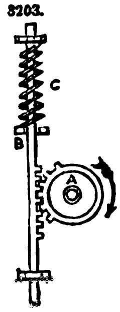


The other disc or wheel D has a series of equidistant studs projecting from its face. Every rotation of the tappet acting upon one of the studs in the wheel D causes the latter wheel to move the distance of one stud. In order that this may not be exceeded, a lever-like stop is arranged on a fixed centre. This stop operates in a notch cut in wheel C, and at the same instant tappet A strikes a stud, said notch faces the lever. As wheel D rotates the end between studs is thrust out, and the other extremity enters the notch; but immediately on the tappet leaving stud, the lever is again forced up in front of next stud, and is there held by periphery of C pressing on its other end.

Fig. 3189. A modification of Fig. 3187; a weight D, attached to an arm secured in the shaft of the worm-gear, being used instead of spring and cam.

Fig. 3190. Another modification of Fig. 3187; a weight or tumbler E, secured on the hollow shaft, being used instead of spring and cam, and operating in combination with pin C, in the shaft of worm-gear.

Fig. 3191. The single tooth A of the driving wheel B acts in the notches of the wheel C, and turns the latter the distance of one notch in every revolution of C. No stop is necessary in this movement, as the driving wheel B serves as a lock by fitting into the hollows cut in the circumference of the wheel C between its notches.

Fig. 3192. B, a small wheel with one tooth, is the driver, and the circumference entering between the teeth of the wheel A, serves as a lock or stop while the tooth of the small wheel is out of operation.

Fig. 3193. The driving wheel C has a rim, shown in dotted outline, the exterior of which serves as a bearing and stop for the studs on the other wheel A, when the tappet B is out of contact with the studs. An opening in this rim serves to allow one stud to pass in and another to pass out. The tappet is opposite the middle of this opening.

Fig. 3194. The inner circumference (shown by dotted lines) of the rim of the driving wheel B serves as a lock against which two of the studs in the wheel C rest until the tappet A, striking one of the studs, the next one below passes out from the guard-rim through the lower notch, and another stud enters the rim through the upper notch.

Fig. 3195. To the driving wheel D is secured a bent spring B; another spring C is attached to a fixed support. As the wheel D revolves, the spring B passes under the strong spring C, which presses it into a tooth of the ratchet-wheel A, which is thus made to rotate. The catch-spring B, being released on its escape from the strong spring C, allows the wheel A to remain at rest till D has made another revolution. The spring C serves as a stop.

Fig. 3196. A uniform intermittent rotary motion in opposite directions is given to the bevel-gears A and B by means of the mutilated bevel-gear C.

Fig. 3197. Reciprocating rectilinear motion of the rod C transmits an intermittent circular motion to the wheel A, by means of the pawl B at the end of the vibrating bar D.

Fig. 3198 is another contrivance for registering or counting revolutions. A tappet B, supported on the fixed pivot C, is struck at every revolution of the large wheel (partly represented) by a stud D attached to the said wheel. This causes the end of the tappet next the ratchet-wheel A to be lifted, and to turn the wheel the distance of one tooth. The tappet returns by its own weight to its original position after the stud D has passed, the end being jointed to permit it to pass the teeth of the ratchet-wheel.

Fig. 3199. The vibration of the lever C on the centre or fulcrum A produces a rotary movement of the wheel B, by means of the two pawls, which act alternately. This is almost a continuous movement.

Fig. 3200. A modification of Fig. 3199.

Fig. 3201. Reciprocating rectilinear motion of the rod B produces a nearly continuous rotary movement of the ratchet-faced wheel A, by the pawls attached to the extremities of the vibrating radial arms C C.

Fig. 3202. Rectilinear motion is imparted to the slotted bar A by the vibration of the lever C through the agency of the two hooked pawls, which drop alternately into the teeth of the slotted rack-bar A.

Fig. 3203. Alternate rectilinear motion is given to the rack-rod B by the continuous revolution of the mutilated spur-gear A, the spiral spring C forcing the rod back to its original position on the teeth of the gear A quitting the rack.

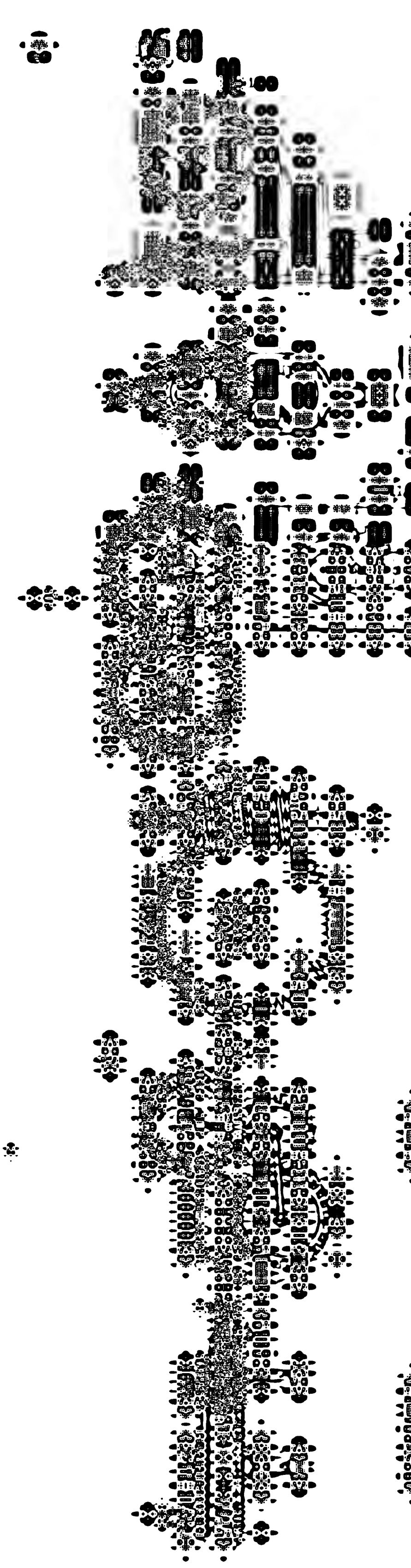
Fig. 3204. On motion being given to the two treadles D a nearly continuous motion is imparted, through the vibrating arms B and their attached pawls, to the ratchet-wheel A. A chain or strap attached to each treadle passes over the pulley C, and as one treadle is depressed the other is raised.

Fig. 3205. A nearly continuous rotary motion is given to the wheel D by two ratchet-toothed arcs C, one operating on each side of the ratchet-wheel D. These arcs (only one of which is shown) are fast on the same rock-shaft B, and have their teeth set opposite ways. The rock-shaft is worked by giving a reciprocating rectilinear motion to the rod A. The arcs should have springs applied to them, so that each may be capable of rising to allow its teeth to slide over those of the wheel in moving one way.

Fig. 3206. The double rack-frame B is suspended from the rod A. Continuous rotary motion is given to the cam D. When the shaft of the cam is midway between the two racks, the cam acts upon neither of them; but by raising or lowering the rod A either the lower or upper rack is brought within range of the cam, and the rack-frame moved to the left or right. This movement has been used in connection with the governor of an engine, the rod A being connected with the governor, and the rack-frame with the throttle or regulating valve.

Fig. 3207. Uniform circular motion into reciprocating rectilinear motion, by means of mutilated pinion, which drives alternately the top and bottom rack.

Fig. 3208. Circular motion into alternate rectilinear motion. Motion is transmitted through pulley at the left upon the worm-shaft. Worm slides upon shaft, but is made to turn with it by



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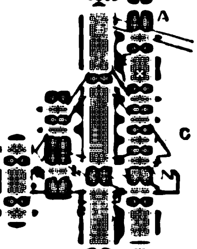

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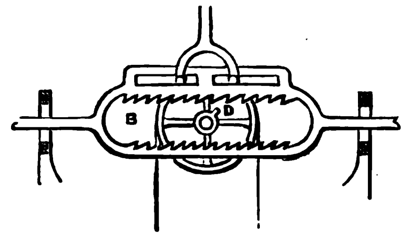

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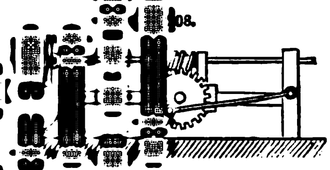

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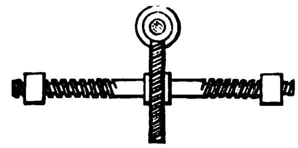

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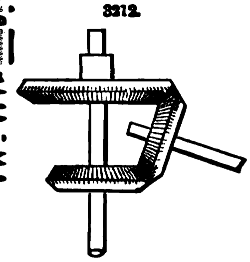

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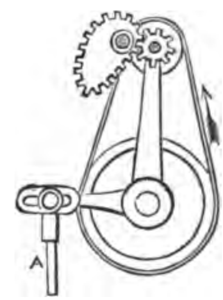

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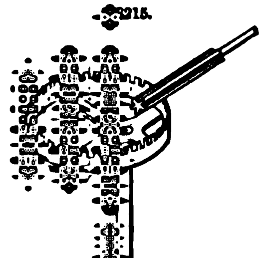

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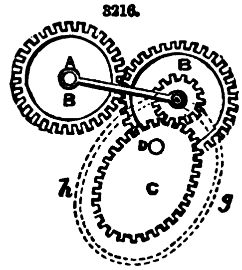

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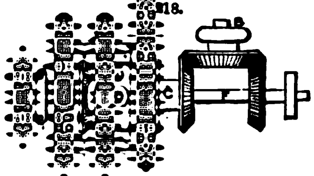

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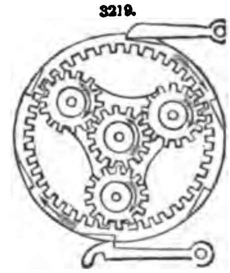

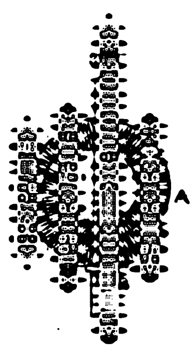

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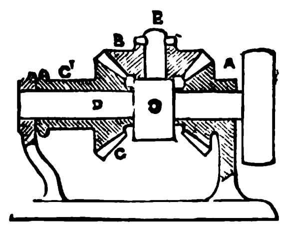


means of a groove cut in shaft, and a key in hub of worm. Worm is carried by a small traversing frame, which slides upon a horizontal bar of the fixed frame, and the traversing frame also carries the toothed wheel into which the worm gears. One end of a connecting rod is attached to fixed frame at the right and the other end to a wrist secured in toothed wheel. On turning worm-shaft rotary motion is transmitted by worm to wheel, which, as it revolves, is forced by connecting rod to make an alternating traverse motion.

Fig. 3209. Continuous circular into continuous but much slower rectilinear motion. The worm on the upper shaft, acting on the toothed wheel on the screw-shaft, causes the right and left hand screw-threads to move the nuts upon them toward or from each other according to the direction of rotation.

Fig. 3210. Scroll-gears for obtaining a gradually-increasing speed.

Fig. 3211. What is called a *mangle-rack*. A continuous rotation of the pinion will give a reciprocating motion to the square frame. The pinion-shaft must be free to rise and fall, to pass round the guides at the ends of the rack. This motion may be modified as follows;—If the square frame be fixed, and the pinion be fixed upon a shaft made with a universal joint, the end of the shaft will describe a line, similar to that shown in the drawing, around the rack.

Fig. 3212. A mode of obtaining two different speeds on the same shaft from one driving wheel.

Fig. 3213. A continual rotation of the pinion (obtained through the irregular-shaped gear at the left) gives a variable vibrating movement to the horizontal arm, and a variable reciprocating movement to the rod A.

Fig. 3214. Worm or endless screw and worm-wheel. Used when steadiness or great power is required.

Fig. 3215. Variable circular motion by crown-wheel and pinion. The crown-wheel is placed eccentrically to the shaft, therefore the relative radius changes.

Fig. 3216. Irregular circular motion imparted to wheel A. C is an elliptical spur-gear rotating round centre D, and is the driver. B is a small pinion with teeth of the same pitch, gearing with C. The centre of this pinion is not fixed, but is carried by an arm or frame which vibrates on a centre A, so that as C revolves the frame rises and falls to enable pinion to remain in gear with it, notwithstanding the variation in its radius of contact. To keep the teeth of C and B in gear to a proper depth, and prevent them from riding over each other, wheel C has attached to it a plate which extends beyond it and is furnished with a groove *g h* of similar elliptical form, for the reception of a pin or small roller attached to the vibrating arm concentric with pinion B.

Fig. 3217. If for the eccentric wheel described in the last figure an ordinary spur-gear moving on an eccentric centre of motion be substituted, a simple link connecting the centre of the wheel with that of the pinion with which it gears will maintain proper pitching of teeth in a more simple manner than the groove.

Fig. 3218. This movement is designed to double the speed by gears of equal diameters and numbers of teeth—a result once generally supposed to be impossible. Six bevel-gears are employed. The gear on the shaft B is in gear with two others—one on the shaft F, and the other on the same hollow shaft with C, which turns loosely on F. The gear D is carried by the frame A, which, being fast on the shaft F, is made to rotate, and therefore takes round D with it. E is loose on the shaft F, and gears with D. Now, suppose the two gears on the hollow shaft C were removed and D prevented from turning on its axis, one revolution given to the gear on B would cause the frame A also to receive one revolution, and as this frame carries with it the gear D, gearing with E, one revolution would be imparted to E; but if the gears on the hollow shaft C were replaced D would receive also a revolution on its axis during the one revolution of B, and thus would produce two revolutions of E.

Fig. 3219. Wheel-work in the base of capstan. Thus provided, the capstan can be used as a simple or compound machine, single or triple purchase. The drumhead and barrel rotate independently; the former, being fixed on spindle, turns it round, and when locked to barrel turns it also, forming single purchase; but when unlocked wheel-work acts, and drumhead and barrel rotate in opposite directions, with velocities as three to one.

Fig. 3220. J. W. Howlett's adjustable frictional gearing. This is an improvement on that shown in Fig. 3180. The upper wheel A shown in section, is composed of a rubber disc with V-edge, clamped between two metal plates. By screwing up the nut B, which holds the parts together, the rubber disc is made to expand radially, and greater tractive power may be produced between the two wheels.

Fig. 3221. Scroll-gear and sliding pinion, to produce an increasing velocity of scroll-plate A, in one direction, and a decreasing velocity when the motion is reversed. Pinion B moves on a feather on the shaft.

Fig. 3222. Entwistle's gearing. Bevel-gear A is fixed. B, gearing with A, is fitted to rotate on stud E, secured to shaft D, and it also gears with bevel-gear C loose, on the shaft D. On rotary motion being given to shaft D, the gear E revolves around A, and also rotates upon its own axis, and so acts upon C in two ways, namely, by its rotation on its own axis and by its revolution around A. With three gears of equal size, the gear C makes two revolutions for every one of the shaft D. This velocity of revolution may, however, be varied by changing the relative sizes of the gears. C is represented with an attached drum C′. This gearing may be used for steering apparatus, driving screw-propellers, &c. By applying power to C action may be reversed, and a slow motion of D obtained.

GEODESY. Fr., *Géodésie;* Ger., *Geodäsie;* Ital., *Geodesia;* Span., *Geodesia.*

An extensive survey made over large portions of the surface of the earth, either for the purpose of ascertaining the exact position of the principal places of a country, or of determining the dimensions and figure of the earth, is usually designated a Trigonometrical survey. This branch of surveying is termed *Geodesy.*

For this purpose a country is first divided into a number of large triangles, whose sides are usually from 10 to 20 miles in length; but sometimes they extend to 50 or 60 miles, and even occasionally, as in Spain and the west of Scotland, to 100 miles in length. All the angles of the triangles are then carefully observed, and a line situated in a level tract of country, called a *base line*, is measured with extreme care. These triangles may be said to form a species of polyhedron, circumscribing a portion of the earth, and they are reduced to others on its surface at the level of the sea, by supposing perpendiculars to be drawn from each station to the surface. The latitudes and longitudes of the different stations are then determined; and also their heights, and the angles which the sides of the triangle make with the meridional line.

The great triangles, into which the country is divided in the first instance, are denominated *principal* triangles. They are afterwards, by a second series of operations, subdivided into smaller ones, called *secondary* triangles, and these again are broken up into others of still smaller dimensions, until at length a survey of the whole country is made of any degree of minuteness which may be thought necessary. The calculations are finally *verified* by measuring other base lines, and comparing them with their lengths determined by calculation.

In the choice of stations, in a trigonometrical survey, there are two points which ought principally to be attended to;—1st. The angles should have such a magnitude, that any small inevitable errors in the observations shall produce the least effect on the sides to be calculated. 2nd. The stations should all be distinctly visible from each other.

To determine the most advantageous conditions of a Triangle.—Let a, b, c, be the sides of a triangle, and A, B, C, the angles respectively opposite to them. The angles are all known from observation, and the side a is either measured or determined from previous calculation. The side b is then found from the equation

$$b \sin. A = a \sin. B. \qquad [a]$$

Suppose now that the side a is accurately known, but that the angles A and B have not been correctly measured. Let α, β, be the respective errors in A and B, and let x be the corresponding error in the side b. We have then $(b + x) \sin. (A + \alpha) = a \sin. (B + \beta)$.

Expanding this expression, and putting $\cos. \alpha = 1$, $\sin. \alpha = \alpha$, $\cos. \beta = 1$, $\sin. \beta = \beta$, since the errors α, β, are necessarily very small, we get

$$(b + x)(\sin. A + \alpha \cos. A) = a (\sin. B + \beta \cos. B).$$

Subtracting [a] from this equation, and omitting the term $x \alpha \cos. A$, which is the second order, and extremely small compared with the other terms, we get

$$x \sin. A + b \alpha \cos. A = a \beta \cos. B = \frac{b \sin. A}{\sin. B} \beta \cos. B;$$

$$\therefore \quad x = b (\beta \cot. B - \alpha \cot. A). \qquad [1]$$

Hence, if we suppose the errors α and β to be equal, and to have the same sign, the error x will be 0 when $A = B$; that is, there will be no error in calculating the side b, although the angles A and B are not correctly observed. If the errors α and β are equal, but have different signs, this equation becomes $x = b\,\alpha (\cot. A + \cot. B)$.

Now, $$\cot. A + \cot. B = \frac{\cos. A}{\sin. A} + \frac{\cos. B}{\sin. B} = \frac{(\sin. A + B)}{\sin. A \sin. B}.$$

Also,

$2 \sin. A \sin. B = \cos. (A - B) - \cos. (A + B) = \cos. (A - B) + \cos. C$, and $\sin. (A + B) = \sin. C$,

therefore $$x = b\,\alpha \frac{2 \sin. C}{\cos. (A - B) + \cos. C}, \qquad [2]$$

an expression which is evidently the least possible when $A = B$.

Since the same reasoning is applicable to the third side c, it follows that the most advantageous conditions of a triangle are *that its sides should be as nearly equal as possible.* But, as it is frequently impossible to fulfil these conditions, surveyors are in general satisfied with rejecting all triangles which have an angle less than 30°.

If the angles are accurately known, but there is an error in the side a, it is evident that the errors in the sides b and c will be proportional to their lengths; for the angles being constant the triangles will be similar. Hence it is of the utmost importance to measure the base line correctly, for any error in this line, which is necessarily very short compared with the extent of the country to be surveyed, will be continued through the whole chain of triangles, and magnified in proportion to the length of the sides.

Description of Signals.—All the stations should be situated in the most elevated part of the country, so as to be seen from each other without difficulty. In many cases the theodolite has to be elevated to the top of some tower, church-steeple, or other building, and flagstaffs placed over the instruments. These can be more easily distinguished when their figures are seen in the sky, than when they are projected on the earth or on trees. For more distant stations, *Bengal* or *white lights* were at first used by General Roy. Afterwards the reflection of the sun from a plane mirror, as recommended by Gauss, was employed by Colonel Colby and Captain Kater, in verifying that part of General Roy's triangulation which connected the meridians of Greenwich and Paris. *Drummond lights* were used as night signals at some of the stations in Ireland and the west of Scotland; but the practice of observing by night has lately been abandoned, in consequence of the unsteadiness of the light and the quantity of vapour in the atmosphere. Signals in the English survey were sometimes formed by building a temporary shed in the form of the frustum of a cone, over the point which marks the centre of the station. When the distances are not very great a

plate of metal is sometimes used, with a narrow vertical slit cut in it; in which case the line of light passing through it may be seen very distinctly.

In elevating a signal for the purposes of observation, it is necessary that it should be sufficiently high to be easily distinguished from the surrounding objects. From the experience of the French surveyors, they state that the angle of elevation should be at least 31″; and as tan. 31″ = 0·00015, it follows that the height of the signal must be equal to 0·00015 × distance. In practice, therefore, the French usually made the height of the signal equal to a seven thousandth part of the distance from whence it was to be observed, and the base of the signal equal to half its height. Hence if the distance be 20 miles, a distance not unusual in the trigonometrical survey, the signal should be at least 15 ft. in height.

Of all the operations in which the surveyor is engaged, that which appears the most simple, but which is by far the most difficult, is the measurement of a base line. Since this line is seldom more than six or seven miles in length, and any error in its measurement is multiplied in the other parts of the survey in proportion to their linear dimensions, it is obvious that, in a tract of country 300 miles long, the error in this length would be fifty times the error in the base line. Every precaution, therefore, has been taken which art or ingenuity could devise to ensure the greatest accuracy in this most important operation.

The first thing to be done is to select a level piece of ground, from five to seven or eight miles in length, which shall be free from local obstructions, and commodiously situated with respect to surrounding objects. It is also desirable that it should not be far distant from an observatory, so that the whole chain of triangles may be connected with a fixed station, where astronomical observations are made with the utmost care and precision.

After the ground has been selected, a line is drawn in the same vertical plane, by means of a transit telescope, and marked out by pickets, the tops of which are brought exactly into the same level. The tract which is to be measured is then cleared of all obstructions, and made tolerably smooth; and the extremities of the base are permanently fixed by dots marked on cannon, or on massive blocks of stone.

Deal Rods.—In the commencement of the English survey, General Roy made use of deal rods, 20 ft. 3 in. long, about 2 in. deep, and 1¼ in. broad, on which lengths of 20 ft. were laid off by Ramsden. They were constructed in such a manner that they might be used either by butting the end of one rod against the end of another, or by bringing fine transverse lines, inlaid into the upper surface at the distance of 1½ in. from each extremity, into exact coincidence; but the method of coincidences was attended with so much inconvenience and loss of time, that General Roy was compelled to abandon it, and to proceed solely by the method of contacts. Notwithstanding all the care, however, that was taken to select rods of the best materials, they were found liable to such irregular and sudden variations of length, from the moisture of the atmosphere, that they were entirely abandoned, after the first base on Hounslow Heath had been completed. The error in this measurement was found to be about 21 in.

Glass Rods.—When it was discovered that the deal rods would not prove satisfactory, it was proposed that glass rods should be substituted in their place. Tubes were used rather than solid rods, as it was found that a sufficient quantity of melted glass could not be taken on the irons which were used at the glass-house for drawing the rods. Three hollow tubes were, therefore, selected, and converted by Ramsden into measuring rods. They were then placed in cases, to which they were made fast in the middle, and also braced at two other points; the whole together serving as stays to keep the tubes in their true places from shaking, but not binding them too closely. The ends were ground perfectly smooth, and at right angles to the axis of the bore; one end having a fixed apparatus, or metal button, attached to it, for making the contacts, and the other end a movable apparatus or slider, which was pressed outwards by a slender spring. The fixed extremity of the succeeding rod was pushed against this spring until a fine line on the slider was brought into exact coincidence with another fine line on the glass rod, in which state the distance between the extremities was exactly 20 ft.

Steel Chains.—The third method of measuring a base line, by the English surveyors, was with a steel chain made by Ramsden. This chain was 100 ft. in length, and contained 40 links of 2½ ft. each. A transverse section of these links was a square, each of whose sides was ¼ an inch. In using the chain five coffers were arranged in a straight line, and supported either by trestles or courses of bricks; the chain was placed on the coffers and stretched with a constant weight of 56 lbs. The ends were brought over the same point in this manner;—At the extremity of the chain, but unconnected with it, and on a separate post, was placed a scale. When the chain was in any position, the scale at the preceding end was moved by means of screws, until one of its divisions coincided exactly with the mark on the handle of the chain. This scale remaining in its place, the chain was carried forward into its next position, and adjusted, by means of its screw apparatus, until the mark in its following end coincided exactly with that division of the scale which had been in coincidence with the mark on the preceding end.

	Feet.
The measurement of the base on Hounslow Heath, made with deal rods, reduced to the lowest extremity, was found to be	27406·26
„ with glass tubes	27404·0843
„ with steel chain	27404·3155

The mean of the two last results, or 27404·2 ft., was assumed as the true length of the base in the future calculations.

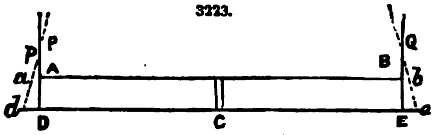
Notwithstanding the near agreement of the two last methods of measuring a base line, it has been objected to the glass rods;—1st. That some error might arise from the ends of the two consecutive rods being made to rest on the same trestle, because when the first rod was taken off, the face of the trestle being pressed by one rod only, would have a tendency to incline a little forward, the effects of which would be to shorten the apparent length of the base. 2nd. That

some error might arise from the casual deviation of the rods from a straight line in the direction of the base. 3rd. That from the manner of supporting the rods on two trestles only, they would be liable to bend in the middle. To the steel chain it has also been objected, by Legendre and others, that, as the chain is not uniformly supported at every point, some doubt must remain whether it is perfectly straight when placed in the coffers, and also that its length is liable to vary from the rubbing and wear of the joints.

Rods of Platinum and Brass.—In the French surveys, rods of platinum were used. These were two toises, or 12 French feet, in length; their breadth was about six lines, or half a French inch, and their thickness one line. On the surface of the platinum was placed another rod of brass, firmly fixed at one end to the rod of platinum, by means of three screws, but entirely free at the other end, and throughout its whole length. It was about 6 in. shorter than the rod of platinum; and, from the different expansive powers of the two metals, the two rods united might be considered as a kind of metallic thermometer. Four rods were used in the measurement, three of which were always on the ground at the same time; and, in order to prevent any derangement from bringing the ends into contact, a small interval of about $\frac{1}{4}$ of an inch was left between them, which was measured by means of a slider attached to the preceding end of each rod. The slider was then pushed gently out, until it came into contact with the following end of the next rod.

Colonel Colby's Method.—The last method adopted, in the survey of Ireland, is an ingenious apparatus made by Troughton, Fig. 3223, which supersedes all other instruments. A B is a bar of iron, 10 ft. long, $1\frac{1}{2}$ in. deep, and $\frac{1}{2}$ of an inch broad, united to a bar of brass D E, of the same dimensions, at the distance of 2 in. These bars are firmly riveted together at their centres, but are free to move at the extremities, according to their respective expansions. The base D E is covered with a non-conducting substance, to make the two bars equally susceptible of heat. P D, Q E, are two tongues of steel, attached to the rods by double conical joints, around which they are capable of turning and forming a small angle with the lines perpendicular to the bars. P and Q are two dots of platinum, so exceedingly minute as to be almost invisible to the naked eye. At the temperature of 60° the bars are exactly of the same length, and the tongues P D, Q E, are then perpendicular to the bars; but if the temperature be increased, the bars will expand in different proportions; thus, if P a d, Q b c, represent the position of the tongues at the temperature of 70°, and the expansion of iron be to that of brass as 53 to 83, then A a : D d :: P A : P D :: 53 : 83.

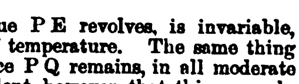
3223.



Hence the situation of the point P, about which the tongue P E revolves, is invariable, or at least is sensibly so in practice, for all moderate variations of temperature. The same thing is true with respect to the point Q, and consequently the distance P Q remains, in all moderate changes of temperature, of the exact length of 10 ft. It is evident, however, that this can only be true within certain limits; for, as P d is no longer equal to P D, the point P will have moved to p, nearer to d, making $p\,d =$ P D; and the distance of p from P D is evidently equal to P D × (tan. D P d − sin. D P d). But as the angle D P d is, in practice, always extremely small, the difference between its tangent and its sine is altogether insensible.

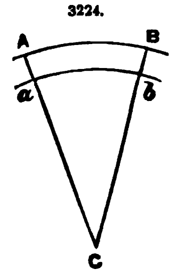
In the Irish survey, five or six sets of bars, constructed in this manner, and placed in strong deal boxes, supported on trestles, were laid along the line to be measured, and accurately levelled. They were placed at a short distance from each other, and the distance between the dots on the adjacent steel tongues of two succeeding bars was accurately measured, by means of powerful micrometers, constructed so as to form a compensating instrument of the same nature as the measuring bars. It is stated that the greatest possible error of the base, measured on the eastern shore of Lough Foyle, cannot exceed 2 in., though the length is very nearly eight miles.

The Reduction of the Hypothenuses.—As the ground on which the base is measured is seldom perfectly level, the whole distance is divided into a number of inclined lines in the same vertical plane. Let A B be one of those lines, whose length is l, B C = h, the height of this plane, and the inclination of the plane B A C = θ. In the first English surveys, B C, the height of B above A, was found from levelling, and therefore the base A C = $\sqrt{(l^2 - h^2)}$. But in the latter surveys, as well as in those on the Continent, the angle θ was measured, and therefore the correction A B − A C is equal to $l\,(1 - \cos.\,\theta)$.



Correction of Temperature.—In the English survey, the temperature of the rods and chain was found from the mean of a number of thermometers; and the rate of expansion was previously determined by Ramsden. In the French survey, the measuring rod itself is the thermometer, and the difference of the rates of expansion between the platinum and the brass is carefully ascertained before the survey commences. In either case the correction is easily found by a single proportion, or by means of tables constructed for the purpose.

3224.



Reduction to the Level of the Sea.—Let A B, Fig. 3224, be the arc which has been measured, as described above, and corrected on account of temperature and the inclinations of the hypothenuses. This arc may be supposed to be taken at a mean between the heights of the two extreme points. Let $a\,b$ be a concentric arc at the level of the sea, and C a the radius of the earth. Put C $a = r$, A $a = h$, A B = L, $a\,b = l$, we have then C A : C a :: A B : $a\,b$;

$$\therefore\ l = \mathrm{L}\,\frac{r}{r+h} = \left(1 - \frac{h}{r} + \frac{h^2}{r^2} - \&c.\right) = \mathrm{L} - \frac{\mathrm{L}\,h}{r}, \qquad [3]$$

nearly. In order, therefore, to reduce the base to the level of the sea, we must subtract the correction $\frac{L\,h}{r}$ from the length.

We will now give, as an example, the final result of the measurement of the first base, with glass rods, on Hounslow Heath. (Trig. Survey, vol. i., p. 87.)

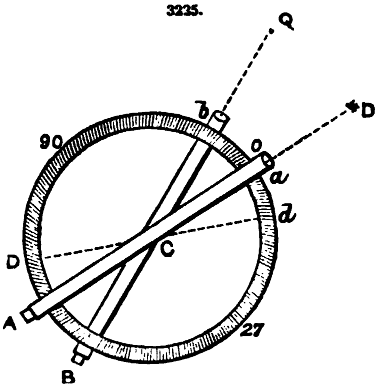
	Feet.
Hypothenusal length of the base as measured by 1369·925521 glass rods, of 20 ft. each + 4·31 ft.	27402·8204
Reduction of the hypothenuses, to be subtracted	− 0·0714
Add the difference between the expansion of the glass above, and the contraction of it below, 62°	+ 0·3489
Add also for 6° difference of temperature of the standard brass scale and the glass rods	+ 0·9864
Length of the base, in temperature 62°	27404·0843
Reduction from the height of the lower end of the base above the mean level of the sea, supposed to be 54 ft.	− 0·0706
True length of the base, reduced to the mean level of the sea ..	27404·0137

The Measurement and Reduction of the Angles.—In all the surveys made in the British dominions, the instrument for measuring angles has been a large theodolite, rendered as perfect as the ingenuity of English artists could make it. This instrument may be defined to be an altitude and azimuth instrument, or an instrument for measuring vertical and horizontal angles. The horizontal circle was 3 ft. in diameter, and angles could be measured with it to the fractional part of a second.

The instrument used by the French and Swedish surveyors was the repeating circle of Borda. The principle of the circle of repetition is to take the angle several times successively in continuation on the circle, and then divide the whole arc by the number of observations.

Let A B D, Fig. 3225, be a circle graduated entirely round the circumference from right to left on the upper side only of the instrument. A a, B b, are two telescopes, the one on the upper and the other on the under side of the instrument; these telescopes can either be moved independently or they may be clamped and moved altogether with the circle. Let P and Q be two objects whose angular distance is to be measured; and let the instrument by means of a stand be brought into the plane P C Q. Place the upper telescope A a at zero, and direct it to the object P; also direct the under telescope B b to the object Q. The two telescopes are then clamped, and the entire instrument is turned in its plane until B b be pointed to P. A a will now be in the position D d, making the angle a C d equal to a C b; unclamp it and turn it back to Q, while the circle itself remains fixed; it is evident that A a has moved through an angle d C b, equal to twice the required angle P C Q. The whole circle must now be turned again until A a points to P, then will B b be in the position D d; turn B b again through the angle d C b to Q and clamp it. As the under-side of the circle is not graduated, the angular distance of b from zero cannot be measured. Now move the whole circle until B b points to P, and turn A a again until it points to Q; the telescope A a will have been turned through four times the arc $a\,b$; and by repeating the process the arc can be multiplied any even number of times. It will readily be seen that the circle must always be turned to the *right* through the arc $b\,a$, and the two telescopes alternately to the *left* through $d\,b$, or twice the arc $a\,b$.

3225.



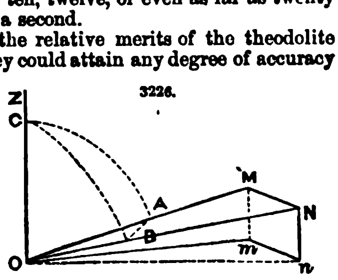
The advantages of this method are obvious. The errors of graduation may be diminished to any degree, and the errors of observation tend to destroy each other. The two circles which Delambre used were $0^m{\cdot}21$ and $0^m{\cdot}18$, or about 7 in. in diameter; and although the instruments were only graduated to minutes, yet by successively repeating the angle ten, twelve, or even as far as twenty times, he imagined that he could determine the angle within a second.

Various opinions have been entertained with respect to the relative merits of the theodolite and the repeating circle. The French have imagined that they could attain any degree of accuracy with the circle, and that all errors of division and errors of observation might be entirely annihilated by repetition.

When the angles are measured with a theodolite, no correction is required on account of the different altitudes of the signals, as it is the horizontal angle which is observed with the instrument; but when the sextant or repeating circle is employed, the oblique angles are observed, and these must be reduced to the plane of the horizon.

To reduce the oblique angles to the plane of the horizon.—Let O, Fig. 3226, be the place of the observer, M O N the angle observed between two signals M, N; M m, N n, two vertical lines meeting the horizontal plane m O n in the points m, n. Let O Z be a vertical line passing through O, and with the centre O and radius 1 conceive a sphere to be described, and let

3226.



the planes Z O M, Z O N, M O N, cut this sphere in the great circles C A, C B, A B. The angle observed with the repeating circle is the oblique angle M O N, which is measured by the arc A B, but the required angle is m O n, which is equal to the spherical angle C. The angles M O m, N O n, are known from observation, and therefore the complements of these angles, or the arcs C A, C B, are known. We have then, in the triangle C A B, the three sides C A, C B, A B, given to find the angle C.

Let h, h', be the altitudes of the two signals M and N, θ the angle between them; also, let the horizontal angle $= \theta + x$; then $CA = 90^\circ - h$, $CB = 90^\circ - h'$, $AB = \theta$, angle $C = \theta + x$. By Trig., $\cos. C = \dfrac{\cos. AB - \cos. CA \cos. CB}{\sin. CA \sin. CB}$, or $\cos. (\theta + x) = \dfrac{\cos. \theta - \sin. h \sin. h'}{\cos. h \cos. h'}$.

Now, in practice, h, h', are very small, and $\theta + x$ is nearly equal to θ, therefore x also is very small. Hence $\cos. (\theta + x) = \cos. \theta \cos. x - \sin. \theta \sin. x = \cos. \theta - x \sin. \theta$, nearly.

Also, $\cos. h \cos. h' = (1 - \frac{1}{2} h^2 + \&c.)(1 - \frac{1}{2} h'^2 + \&c.) = 1 - \frac{1}{2}(h^2 + h'^2) + \&c.$

$$\therefore \frac{1}{\cos. h \cos. h'} = \frac{1}{1 - \frac{1}{2}(h^2 + h'^2) + \&c.} = 1 + \tfrac{1}{2}(h^2 + h'^2), \text{ nearly, and } \sin. h \sin. h' = h h',$$

nearly. Substituting these values above, we have

$$\cos. \theta - x \sin. \theta = (\cos. \theta - h h') \{ 1 + \tfrac{1}{2}(h^2 + h'^2) \}.$$

$$\text{Hence} \quad x \sin \theta = h h' - \tfrac{1}{2}(h^2 + h'^2) \cos. \theta = \frac{(h + h')^2 - (h - h')^2}{4} - \frac{(h + h')^2 + (h - h')^2}{4} \cos. \theta;$$

$$\therefore x = \frac{(h + h')^2}{4} \; \frac{1 - \cos. \theta}{\sin. \theta} - \frac{(h - h')^2}{4} \; \frac{1 + \cos. \theta}{\sin. \theta} = \tfrac{1}{4}(h + h')^2 \tan. \tfrac{1}{2}\theta - \tfrac{1}{4}(h - h')^2 \cot. \tfrac{1}{2}\theta.$$

Here x is measured in parts of the radius; if it be measured in seconds we have $x = x'' \sin. 1''$; therefore

$$x'' = \frac{(h + h')^2}{4} \; \frac{\tan. \frac{1}{2}\theta}{\sin. 1''} - \frac{(h - h')^2}{4} \; \frac{\cot. \frac{1}{2}\theta}{\sin. 1''}. \qquad [4]$$

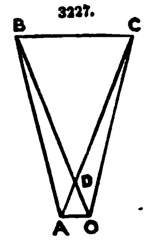
Example.—Let $\theta = 51^\circ 9' 29''\cdot774$, $h = 1^\circ 32' 45''$, $h' = 1^\circ 7' 10''$, then $\frac{1}{2}(h + h') = 4797''\cdot5$, $\frac{1}{2}(h - h') = 767''\cdot5$.

2 log. $\frac{1}{2}(h + h')$	7·362030	2 log. $\frac{1}{2}(h - h')$	5·770156
tan. $\frac{1}{2}\theta$	9·680038	cot. $\frac{1}{2}\theta$	0·319962
ar. co. log. sin. 1″..	4·685575	ar. co. log. sin. 1″..	4·685575
53″·413	1·727643	5″·966	0·775693

	°	′	″
Observed angle	51	9	29·744
		+	53·413
		−	5·966
Angle reduced to the horizon	51	10	17·191

It sometimes happens, when the steeple of a church or other remarkable object is selected as a signal, that the theodolite cannot be placed immediately over the point occupied by the axis of the signal. In this case the instrument must be removed to some convenient place near it, and a reduction is then applied to the observed angle in order to obtain the true angle at the centre.

3227.



It is required to determine the reduction to the centre.—Let A, Fig. 3227, be the situation of the axis of the signal observed from the stations B and C, O the place of the centre of the instrument. Put A, B, C, for the angles of the triangle A B C, and a, b, c, for the sides, respectively, opposite to them. Let $AO = m$, angle $AOB = \beta$, angle $AOC = \gamma$, angle $BOC = O$; also, angle $ABO = x$, $ACO = y$. Now, angle $A = BDC - x = O - x + y$; also, $\sin. x : \sin. \beta :: m : c$; $\sin. y : \sin. \gamma :: m : b$; $\therefore \sin. x = \dfrac{m \sin. \beta}{c}$, $\sin. y = \dfrac{m \sin. \gamma}{b}$. But since O is always near the station A, the angles x and y are very small, and therefore $\sin. x = x = x'' \sin. 1''$, $\sin. y = y = y'' \sin. 1''$, very nearly. Hence

$$A = O - \frac{m \sin. \beta}{c \sin. 1''} + \frac{m \sin. \gamma}{b \sin. 1''}. \qquad [5]$$

When all the angles have been observed and reduced to the plane of the horizon, if the triangle were a plane one, their sum ought to be equal to 180°, and thus the correctness of the observations might be verified. But in a spherical triangle the sum of the three angles exceeds 180° by a certain quantity, called the *spherical excess;* and as this can be easily calculated, we have the same means of verifying the operation in Spherical as in Plane Trigonometry.

It is required to determine the spherical excess in a small triangle measured on the surface of the earth.—Let A, B, C, be the three angles of a spherical triangle, r the radius of the sphere expressed in feet, x the area of the triangle in square feet, and ϵ the spherical excess given in seconds; we

have then $x : \pi r^2 :: A + B + C - 180° (= \epsilon'') : 180 \times 60 \times 60$ seconds; $\therefore \epsilon = \frac{x \times 648000''}{\pi r^2}$;

and, if we suppose the mean value of r to be 20,888,761 ft., the logarithm of $\frac{\pi r^2}{648000}$ is equal to 9·32540. The value of x may be calculated as if the triangle were a plane one, without any sensible error. Hence we have the following

Rule.—From the logarithm of the area of the triangle, taken as a plane one in feet, subtract the constant logarithm 9·32540, the remainder will be the logarithm of the spherical excess in seconds, nearly.

When the triangles are very large, a more correct value of r will be obtained by computing for the mean latitude of the three stations, the radius of curvature of the meridian, and of the arc perpendicular to the meridian, and taking the mean of the two for the value of r.

The following example is taken from the Encyclopædia Britannica. The triangle connects the west of Scotland with Ireland, and is one of the largest which occurs in the Trigonometrical Survey.

The three stations are Benlomond, in Stirlingshire (A), Cairnsmuir-on-Deugh, in Kirkcudbright (B), and Knocklayd, in the county of Antrim (C); the arc c is 352037·62 ft., and the angles are as follows;—

	A	B	C
	° ′ ″	° ′ ″	° ′ ″
	56 43 29·97	79 42 28·69	43 34 38·36
	27·04		35·43
	28·72		
Mean ..	56 43 28·58		43 34 36·89

We shall first compute approximate values of the two sides, a, b (which will be afterwards required), from the formulæ $a = \frac{c \sin. A}{\sin. C}$, $b = \frac{c \sin. B}{\sin. C}$; and then compute the area from the formula, area $= \frac{1}{2} b c \sin. A$.

log. c = 5·54659	log. c = 5·54659	log. c = 5·54659
log. sin. A = 9·92223	log. sin. B = 9·99295	log. b = 5·70112
log. cosec. C = 0·16158	log. cosec. C = 0·16158	log. sin. A = 9·92223
		ar. co. log. 2 = 9·69897
log. a = 5·63040	log. b = 5·70112	log. area = 10·86891
a = 426970	b = 502480	

The latitude of Benlomond (the most northern station) is 56° 11′; and that of Knocklayd (the most southern) is 55° 10′; the mean of the two is 55° 40′. The values of the radii of curvature are therefore r = 20924824 ft., r' = 20968900 ft., mean = 20946862 ft.

$$\begin{aligned} \log. \frac{180 \times 60 \times 60}{\pi} &= 5{\cdot}31443 \\ \log. r^2 &= 14{\cdot}64224 \\ \hline & 9{\cdot}32781 \\ \log.\ \text{area} &= 10{\cdot}86891 \\ \hline \epsilon = 34''{\cdot}76 \ \ldots\ldots & 1{\cdot}54110 \end{aligned}$$

The sum of the three angles of the triangle being found from observation = 180° 0′ 34″·16, and the true spherical excess being 34″·76, it appears that the errors of observation in the three angles are = − 0″·60. If there were no reason to suppose that one angle has been determined more accurately than another, the error should be equally divided among the three angles; but as it generally happens that some of the angles have been determined from a greater number of observations, or from observations made under more favourable circumstances than the others, this error should be distributed among the three angles in such a manner that the respective corrections may be inversely proportional to the relative goodness of the observations. For this purpose we have the following rule, given by Gauss, but which our limits will not permit us to demonstrate in this work.

To apportion the error among the different angles.—*Rule.*—Let l, l', l'', &c., be the seconds of reading in any angle A, n the number of observations, and let m be the mean or average of the whole: then $m - l$, $m - l'$, $m - l''$, &c., are the errors of the individual observations, and the *weight* of the determination, or of the average m, will be given from this equation,

$$x = \frac{\frac{1}{2} n^2}{(m-l)^2 + (m-l')^2 + (m-l'')^2 + \&c.} \qquad [6]$$

In like manner, the weights y and z are found for the angles B and C. This error in the sum of the three angles is then divided into three parts, proportional to $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$, which are to be added respectively to the three angles A, B, and C

To apply this to the last example, we have for the angle A, $l = 29''\cdot 97$, $l' = 27''\cdot 04$, $l'' = 28''\cdot 72\cdot$ therefore $n = 3$, $m = \frac{1}{3}(l + l' + l'') = 28''\cdot 58$. Hence

$$\frac{1}{x} = \frac{(1\cdot 39)^2 + (1\cdot 54)^2 + (0\cdot 14)^2}{\frac{1}{2} \times 9} = \cdot 961.$$

The angle B was given from one observation only. We may, therefore, assume the weight $y = \cdot 1$, and $\frac{1}{y} = 10$.

At C the reciprocal of the weight $\frac{1}{z} = \frac{(1\cdot 47)^2 + (1\cdot 46)^2}{\frac{1}{2} \times 4} = 2\cdot 146$.

Hence the error $- 0''\cdot 60$ is to be divided into three parts proportional to the numbers $\cdot 961$, 10, $2\cdot 416$; and consequently the corrections of the angles are, respectively, $+ 0''\cdot 04$, $+ 0''\cdot 46$, and $+ 0''\cdot 10$. The true spherical angles, therefore, are

$$A = 56^\circ\, 53'\, 28''\cdot 62;\ B = 79^\circ\, 42'\, 29''\cdot 15;\ C = 43^\circ\, 44'\, 36''\cdot 99.$$

The Calculation of the Sides of the Triangles.—The three spherical angles of the triangle being thus determined from observation, and corrected, and one of the sides being always known, either from actual measurement or calculation, it is necessary to show how the two other sides may be determined. The triangle may be considered as a spherical triangle, whose sides are very small, compared with the radius of the sphere; in which case three different methods have been employed for its solution;—1st. From the three given spherical angles, the angles formed by the chords are deduced, and from the given side of the triangle the corresponding chord is calculated. With these data the other chords are found by Plane Trigonometry, and from thence the arcs themselves. 2nd. A second method is by the theorem of Legendre, by which the spherical triangle is reduced to a plane triangle, whose sides are respectively equal in length to the sides of the triangle of the sphere. 3rd. The third method is to compute the sides by Spherical Trigonometry.

First Method.—To reduce the angle of a spherical triangle to the angle formed by the chords of the containing sides.—Let a, b, c, be the sides of the spherical triangle, and r the radius of the sphere, all measured in feet; also, let $\frac{a}{r} = \alpha$, $\frac{b}{r} = \beta$, $\frac{c}{r} = \gamma$, then will α, β, γ, be the sides of a similar triangle on a sphere whose radius is 1. Let A be the spherical angle opposite to the side a, and let $A - x$ be the corresponding angle formed by the chords. We have then

$$\begin{aligned}\cos. A &= \frac{\cos. \alpha - \cos. \beta \cos. \gamma}{\sin. \beta \sin. \gamma} \\ &= \frac{(1 - 2 \sin.^2 \frac{1}{2} \alpha) - (1 - 2 \sin.^2 \frac{1}{2} \beta)(1 - 2 \sin.^2 \frac{1}{2} \gamma)}{2 \sin. \frac{1}{2} \beta \cos. \frac{1}{2} \beta \times 2 \sin. \frac{1}{2} \gamma \cos. \gamma} \\ &= \frac{\sin.^2 \frac{1}{2} \beta + \sin.^2 \frac{1}{2} \gamma - \sin.^2 \frac{1}{2} \alpha}{2 \sin. \frac{1}{2} \beta \sin. \frac{1}{2} \gamma \times \cos. \frac{1}{2} \beta \cos. \frac{1}{2} \gamma} - \frac{\sin. \frac{1}{2} \beta \sin. \frac{1}{2} \gamma}{\cos. \frac{1}{2} \beta \cos. \frac{1}{2} \gamma}.\end{aligned}$$

Also, because chord $\alpha = 2 \sin. \frac{1}{2} \alpha$, chord $\beta = $ &c., we have, in the triangle formed by the chords,

$$\cos. (A - x) = \frac{\text{chord}^2 \beta + \text{chord}^2 \gamma - \text{chord}^2 \alpha}{2 \text{ chord } \beta \text{ chord } \gamma} = \frac{\sin.^2 \frac{1}{2} \beta + \sin.^2 \frac{1}{2} \gamma - \sin.^2 \frac{1}{2} \alpha}{2 \sin. \frac{1}{2} \beta \sin. \frac{1}{2} \gamma}.$$

Substituting this in the preceding equation, we get

$$\cos. A = \frac{\cos. (A - x)}{\cos. \frac{1}{2} \beta \cos. \frac{1}{2} \gamma} - \frac{\sin. \frac{1}{2} \beta \sin. \frac{1}{2} \gamma}{\cos. \frac{1}{2} \beta \cos. \frac{1}{2} \gamma}$$

$$\therefore\ \cos. (A - x) = \sin. \tfrac{1}{2} \beta \sin. \tfrac{1}{2} \gamma + \cos. \tfrac{1}{2} \beta \cos. \tfrac{1}{2} \gamma \cos. A. \qquad [7]$$

This expression is exact. But, because the three arcs, α, β, γ, are very small, $A - x$ is nearly equal to A, and therefore x is also very small. Hence

$$\cos. (A - x) = \cos. A \cos. x + \sin. x \sin. A = \cos. A + x \sin. A, \text{ nearly.}$$

Also, $\sin. \frac{1}{2} \beta = \frac{1}{2} \beta$, $\cos. \frac{1}{2} \beta = 1 - \frac{1}{8} \beta^2$, $\sin. \frac{1}{2} \gamma = $ &c., very nearly. Hence, substituting these values in equation [7], and reducing, we obtain

$$\begin{aligned}x \sin. A &= \tfrac{1}{4} \beta \gamma - \tfrac{1}{8} (\beta^2 + \gamma^2) \cos. A \\ &= \frac{(\beta + \gamma)^2 - (\beta - \gamma)^2}{16} - \frac{(\beta + \gamma)^2 + (\beta - \gamma)^2}{16} \cos. A; \\ \therefore\ x &= \frac{(\beta + \gamma)^2}{16} \frac{1 - \cos. A}{\sin. A} - \frac{(\beta - \gamma)^2}{16} \frac{1 + \cos. A}{\sin. A} \\ &= \left(\frac{b + c}{4 r}\right)^2 \tan. \tfrac{1}{2} A - \left(\frac{b - c}{4 r}\right)^2 \cot. \tfrac{1}{2} A;\end{aligned}$$

or, if x be estimated in seconds,

$$x'' = \left(\frac{b+c}{4r}\right)^2 \frac{\tan.\ \tfrac{1}{2} A}{\sin.\ 1''} - \left(\frac{b-c}{4r}\right)^2 \frac{\cot.\ \tfrac{1}{2} A}{\sin.\ 1''}. \qquad [8]$$

Having obtained the three reduced angles, we find the chords of the spherical arcs intercepted between the stations, from Plane Trigonometry, and from them we deduce the arcs themselves, by means of the following formula;—

$$\frac{a}{2} = \sin.\ \tfrac{1}{2} a + \frac{1}{2}\frac{(\sin.\ \tfrac{1}{2} a)^3}{3} + \frac{1.3}{2.4}\frac{(\sin.\ \tfrac{1}{2} a)^5}{5} + \&c.;$$

and because chord $a = 2 \sin.\ \tfrac{1}{2} a$, and a is very small, if we neglect the terms after the second, and multiply by 2, we get $a = \text{chord } a + \dfrac{(\text{chord } a)^3}{24}$; hence $\dfrac{a}{r} = \dfrac{\text{chord } a}{r} + \dfrac{(\text{chord } a)^3}{24 r^3}$;

$$\therefore\ \ a = \text{chord } a + \frac{(\text{chord } a)^3}{24 r^2}. \qquad [9]$$

Example.—As an example of this method of solution, we will take the following;—

log. r^2	14·64224	(1)	9·46807
16 sin. 1''	5·88969	$(b-c)^2$	10·35468
	0·53193	cot. ½ A	0·26765
(1)	9·46807	1''·231	0·09040
$(b+c)^2$	11·86342		+ 11·583
tan. A	9·73235		− 1·231
11''·583	1·06384		x = 10·352

In the same manner, the corrections for the angles B and C will be found to be 14''·684 and 9''·724 respectively. Hence the three angles formed by the chords are

$$A' = 56^\circ\ 43'\ 18''\cdot 27,\ B' = 79^\circ\ 42'\ 14''\cdot 47,\ C = 43^\circ\ 34'\ 27''\cdot 26,$$

and the sum of these = 180°, as it should be.

The chord c having been previously found equal to 352033·48 ft., we are enabled to find the lengths of the chords opposite A' and B' from the proportions

sin. C' : sin. A' :: chord c : chord a; sin. C' : sin. B' :: chord c : chord b.

cosec. C'	0·1615956	.. ..	0·1615956
sin. A'	9·9222144	sin. B'	9·9929499
chord c	5·5465840	.. ..	5·5465840
chord a	5·6303940	chord b	5·7011295

Hence chord a = 426966·69 ft., chord b = 352033·48 ft.

We have now to determine the lengths of the arcs a and b from the corresponding chords, from formula [9]. Making use of the logarithms already given in the preceding solution, we readily find

$$\frac{(\text{chord } a)^3}{24 r^2} = 7\cdot 39,\ \frac{(\text{chord } b)^3}{24 r^2} = 12\cdot 05,\ \frac{(\text{chord } c)^3}{24 r^2} = 4\cdot 14,$$

and therefore the lengths of the arcs are a = 426974·08, b = 502504·51, c = 352037·62.

Second Method.—Legendre's Theorem.—If the three sides of a plane triangle be equal to the three sides of a small spherical triangle, respectively, the difference between each of the angles of the plane triangle, and the corresponding angle of the spherical triangle, will be equal to one-third of the spherical excess.— As before, let a, b, c, be the three sides of the small spherical triangle, measured in feet, r the radius of the sphere, and $\dfrac{a}{r} = \alpha$, $\dfrac{b}{r} = \beta$, $\dfrac{c}{r} = \gamma$. Also, let A be the spherical angle opposite to the side a, and A' the corresponding angle in a plane triangle, whose sides are a, b, c. We have then, as before, $\cos.\ A = \dfrac{\cos.\ \alpha - \cos.\ \beta \cos.\ \gamma}{\sin.\ \beta \sin.\ \gamma}$.

If we now expand each of the quantities cos. α, cos. β, sin. β, &c., in a series, and arrange the terms according to the powers of α, β, γ, we shall find that the terms of the first order will be the same as if the triangle were rectilineal, and those of the second order will contain the fourth powers of the arc in the numerator, and the second powers in the denominator. Neglecting, therefore,

all powers higher than the fourth, we have $\cos. \alpha = 1 - \frac{1}{2}\alpha^2 + \frac{1}{24}\alpha^4$, $\sin. \beta = \beta - \frac{1}{6}\beta^3$, $\cos. \beta =$ &c.

Substituting these values in the preceding equation, it becomes

$$\cos. \mathrm{A} = \frac{\frac{1}{2}(\beta^2 + \gamma^2 - \alpha^2) + \frac{1}{24}(\alpha^4 - \beta^4 - \gamma^4) - \frac{1}{4}\beta^2\gamma^2}{\beta\gamma(1 - \frac{1}{6}\beta^2 - \frac{1}{6}\gamma^2)}.$$

And because $\frac{1}{1 - \frac{1}{6}(\beta^2 + \gamma^2)} = 1 + \frac{1}{6}(\beta^2 + \gamma^2) + \frac{1}{36}(\beta^2 + \gamma^2)^2 +$ &c., if we substitute this above, and neglect all terms containing powers higher than the fourth, we get

$$\cos. \mathrm{A} = \frac{\beta^2 + \gamma^2 - \alpha^2}{\beta\gamma} + \frac{\alpha^4 + \beta^4 + \gamma^4 - 2\alpha^2\beta^2 - 2\alpha^2\gamma^2 - 2\beta^2\gamma^2}{24\beta\gamma}$$

$$= \frac{b^2 + c^2 - a^2}{2bc} + \frac{a^4 + b^4 + c^4 - 2a^2c^2 - 2a^2c^2 - 2b^2c^2}{24bc \times r^2}.$$

But $\frac{b^2 + c^2 - a^2}{2bc} = \cos. \mathrm{A}'$ also $2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4 = (4 \text{ area})^2 = 16\,\mathrm{S}^2$;

$$\therefore \quad \cos. \mathrm{A} = \cos. \mathrm{A}' - \frac{2\,\mathrm{S}^2}{3bc \times r^2}. \qquad [10]$$

Let $\mathrm{A} = \mathrm{A}' + x$, then x is evidently a very small angle, consequently

$$\cos. \mathrm{A} = \cos. \mathrm{A}' \cos. x - \sin. x \sin. \mathrm{A}' = \cos. \mathrm{A}' - x \sin. \mathrm{A}', \text{ nearly.}$$

Comparing this value of cos. A with equation [10], we have

$$x = \frac{2\,\mathrm{S}^2}{3r^2 \times bc \sin. \mathrm{A}'} = \frac{\mathrm{S}}{3r^2}.$$

Hence $$\mathrm{A}' = \mathrm{A} - \frac{\mathrm{S}}{3r^2}. \qquad [11]$$

In like manner, $$\mathrm{B}' = \mathrm{B} - \frac{\mathrm{S}}{3r^2}, \quad \mathrm{C}' = \mathrm{C} - \frac{\mathrm{S}}{3r^2};$$

$$\mathrm{A}' + \mathrm{B}' + \mathrm{C}' = 180° = \mathrm{A} + \mathrm{B} + \mathrm{C} - \frac{\mathrm{S}}{r^2}.$$

Hence $\frac{\mathrm{S}}{r^2}$ is the excess of the three angles of the spherical triangle above two right angles, and each of the angles A, B, C, exceeds the corresponding angle of the plane triangle by one-third of this spherical excess.

Example.—Taking the same example as before, we find the spherical excess = 34″·76, and one-third of this excess = 11″·59. Hence A′ = 56° 43′ 17″·04, B′ = 79° 42′ 17″·56, C′ = 43° 34′ 25″·40. With these angles, and the given side c = 352037·62 ft., we then compute the other sides, a, b, by Plane Trigonometry, sin. C′ : sin. A′ :: c : a, and sin. C′ : sin. B′ :: c : b.

cosec. C	0·1615997	.. ..	0·1615997
sin. A′	9·9222127	sin. B′	9·9929511
c	5·5465891	.. ..	5·5465891
a	5·6304015	b	5·7011399

Hence a = 426974·06 ft., b = 502504·42 ft.

Third Method.—To compute the sides by Spherical Trigonometry.—By Trig.,

$$\sin. \mathrm{C} : \sin. \mathrm{A} :: \sin. c : \sin. a. \qquad [\alpha]$$

And since c and a are very small, compared with the radius of the sphere,

$$\frac{\sin. c}{r} = \frac{c}{r} - \frac{c^3}{6r^3}, \text{ nearly,} \qquad \therefore \sin. c = c\left(1 - \frac{c^2}{6r^2}\right)$$

$$\log. \sin. c = \log. c + \log. \left(1 - \frac{c^2}{6r^2}\right) = \log. c - \frac{\mathrm{M}}{6r^2}c^2, \qquad [12]$$

nearly, these logarithms being taken from the common tables, and M being the modulus of the system. Having found log. sin. c from this expression, we get log. sin. a from proportion [α]. We then obtain a from the equation

$$\log. a = \log. \sin. a + \frac{\mathrm{M}}{6r^2}a^2 = \log. \sin. a + \frac{\mathrm{M}}{6r^2}\sin.^2 a. \qquad [13]$$

Example.—To apply this to the last example,

log. r^2	14·64224	log. c	5·5465891
$\frac{1}{6}$ M	8·85963		·0000204
(1)	4·21739	sin. c	5·0465687
c^2	11·09318	sin. A	9·9222287
		cosec. C	0·1615740
·0000204	$\bar{5}$·31057	sin. a	5·6303714
sin. a	5·63037		·0000301
	5·63037		
(1)	4·21739	a	5·6304015
0000301	$\bar{5}$·47813		

As the logarithm of a is exactly the same as that which we obtained by Legendre's method, the arc itself will also be the same as before.

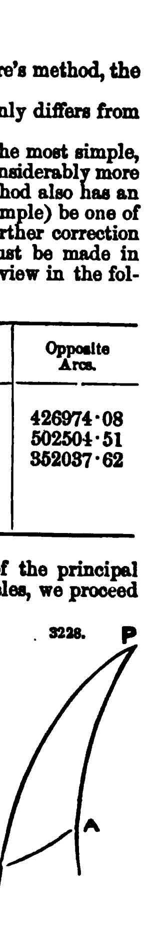
The logarithm of the side b is found in the same manner = 5·7011398, which only differs from the former logarithm by a unit in the last place of decimals.

In comparing these three methods together, Legendre's certainly appears to be the most simple, and the first method perhaps the most difficult. They are all, however, rendered considerably more easy in practice by means of auxiliary tables, previously calculated. The third method also has an advantage over the two others, in this respect, that if any of the angles (A for example) be one of the angles in another triangle, as in calculating the latitudes and azimuths, no further correction will be necessary; whereas, in the first and second methods, a new reduction must be made in order to obtain the angles for calculation. The whole may be brought under one view in the following Table;—

Stations.	Observed Angles.			Apportionment of Error.	Spherical Angles.	Chord Angles.	Mean Angles.	Opposite Chords.	Opposite Arcs.
	°	′	″	″	″	″	″		
A	56	43	28·58	+ 0·04	28·62	18·27	17·04	426966·69	426974·08
B	79	42	28·69	+ 0·46	29·15	14·47	17·56	502492·46	502504·51
C	43	34	36·89	+ 0·10	36·99	27·26	25·40	352033·48	352037·62
	180	0	34·16	0·60	34·76	0·00	0·00		

Calculation of the Latitudes, Longitudes, and Azimuths.—When all the sides of the principal triangles have been found, by one of the methods described in the preceding articles, we proceed to determine the latitudes and longitudes of the different stations, and the inclinations which the sides of the triangles make with the meridian. For this purpose it is necessary that the latitude of one of the stations and the azimuth of one of the sides should be found independently, by astronomical means; and from them we may determine the longitudes and latitudes of all the other stations, and the azimuths of the sides of the triangles. We shall first suppose the earth to be a sphere, and afterwards correct the error arising from this hypothesis.

3228.



Given the latitude of a station A, *the distance of* A *from another station* B, *and also the azimuth of* B *as seen from* A, *to determine the latitude of* B, *the earth being considered as a sphere.*—Let P, Fig. 3228, be the pole of the earth, P A, P B, the meridians of the stations A and B. Let the angle P A B = A, P B A = B, arc P A = 90° − l, P B = 90° − l', and $l - l' = \lambda$; also, let the arc A B measured in feet = D, and in parts of the radius = δ; and let the radius of the earth measured in feet = r. We have then, from Spherical Trigonometry, cos. P B = cos. P A cos. A B + sin. P A sin. A B cos. A, or

$$\sin. l' = \sin. l \cos \delta + \cos. l \sin. \delta \cos. A. \qquad [a]$$

But $\sin. l' = \sin. (l - \lambda) = \sin. l \cos. \lambda - \cos. l \sin. \lambda = \sin. l (1 - \frac{1}{2} \lambda^2) - \lambda \cos. l.$

Also, $\cos. \delta = - \frac{1}{2} \delta^2$, $\sin. \delta = \delta$, neglecting all the powers of δ and λ higher than the second.

Making these substitutions in equation [a], we have

$$\sin. l (1 - \tfrac{1}{2} \lambda^2) - \lambda \cos. l = \sin. l (1 - \tfrac{1}{2} \delta^2) - \delta \cos. l \cos. A$$

$$\therefore \lambda = \delta \cos. A + \tfrac{1}{2} (\delta^2 - \lambda^2) \tan l.$$

For a first approximation, we may neglect the second powers of δ and λ, and assume $\lambda = \delta \cos. A$, which is the same thing as if we supposed the meridians at A and B to be parallel. Substituting this first value of λ in the second member of the last equation, we obtain

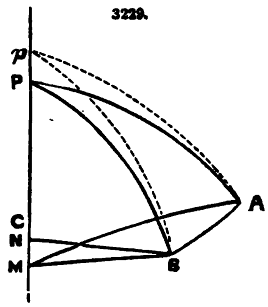
$$\lambda = \delta \cos. A + \tfrac{1}{2} \delta^2 \sin.^2 A \tan. l.$$

Here λ and δ are measured in parts of the radius. If λ'' be the number of seconds in λ, then $\lambda = \lambda''$ sin. 1″; also $\delta = \frac{D}{r}$. Making these substitutions, the last equation becomes

$$\lambda'' = \frac{D \cos. A}{r \sin. 1''} + \frac{D^2 \sin.^2 A \tan. l}{2 r^2 \sin. 1''}. \qquad [14]$$

To determine the same when the spheroidal figure of the earth is taken into consideration.—Let P A, P B, Fig. 3229, be the meridians of A and B, the earth being considered as a spheroid; let A M, B N, be the normals to the surface meeting the polar axis in M and N; join B M. Suppose A p B to be the surface of a sphere whose centre is M, and radius M A. Then, because the arc A B is very small, and A M is a normal to the spheroid, it is nearly equal to the radius of curvature at A, therefore the surface of the sphere will very nearly pass through B, and the difference between the arc A B on the sphere and on the spheroid will be altogether insensible. The spherical triangle p A B may be considered as that whose solution we have just given, and on this supposition B M p = 90° − l' is the colatitude of B. But the true colatitude of B is the angle B N P = 90° − L, which is greater than B M P by the angle M B N. Let $l - l' = \lambda$, $l' - L$ = M B N = ϕ; we have then, in the triangle B M N,

3229.



$$\sin. \phi = \frac{MN}{BM} \sin. BNM = \frac{CM - CN}{BM} \cos. L;$$

but C M = A M . e^2 sin. l, C N = B N . e^2 sin. L, therefore

$$\sin. \phi = e^2 \cos. L \left(\frac{AM}{BM} \sin. l - \frac{BN}{BM} \sin. L\right).$$

And since $\frac{AM}{BM}$ and $\frac{BN}{BM}$ differ from unity by a quantity of a very minute order, we have

$$\sin. \phi = e^2 \cos. L (\sin. l - \sin. L), \text{ very nearly.}$$

Now, sin. L = sin. $\{ l - (\lambda + \phi) \}$ = sin. $l - (\lambda + \phi) \cos. l$, nearly. Also, sin. $\phi = \phi$, very nearly; therefore $\phi = e^2 (\lambda + \phi) \cos. L \cos. l$. Hence, transposing and dividing,

$$\phi = \frac{e^2 \lambda \cos. L \cos. l}{1 - e^2 \cos. L \cos. l} = e^2 \lambda \cos. L \cos. l, \text{ nearly;}$$

$$\therefore \quad \phi = e^2 \lambda \cos.^2 l, \text{ nearly, and } \lambda + \phi = \lambda (1 + e^2 \cos.^2 l).$$

Hence, on the spheroid, the difference of latitude

$$l - L = \left\{\frac{D \cos. A}{r \sin. 1''} + \frac{D^2 \sin.^2 A \tan. l}{2 r^2 \sin. 1''}\right\} (1 + e^2 \cos.^2 l), \qquad [15]$$

where r = A M, the normal to the surface at the station A.

The same things being given, to find the difference of longitude.—The difference of longitude on the sphere is the angle A p B, which is equal to A P B, the difference of longitude on the spheroid. We have then, by Spherical Trigonometry, sin. Bp : sin. A :: sin. δ : sin. p :: δ : p. But sin. Bp = cos. l' = cos. L, very nearly, $\delta = D + r$, $p = P = P''$ sin. 1″, therefore

$$P'' = \frac{D \sin. A}{r \cos. L \sin. 1''}. \qquad [16]$$

To find the azimuth of A *as seen from* B.—In the spherical triangle A p B we have, from Napier's analogies, cos. ½ (p B + p A) : cos. ½ (Bp − p A) :: cot. ½p : tan. ½ (A + B).

Now, ½ (p B + p A) = ½ (90° − l') + ½ (90° − l) = 90° − ½ ($l + l'$), ½ (p B − p A) = ½ ($l - l'$), ½ (A + B) = 90° − ½ (180 − A − B).

Making these substitutions, this proportion becomes

$$\sin. \tfrac{1}{2} (l + l') : \cos. \tfrac{1}{2} (l - l') :: \cot. \tfrac{1}{2} p : \cot. \tfrac{1}{2} (180° - A - B)$$
$$:: \tan. \tfrac{1}{2} (180° - A - B) : \tan \tfrac{1}{2} p.$$

And because the distance A B is always very small, compared with the radius of the earth, A + B is nearly equal to 180°, and therefore 180° − A − B is a very small angle. Also, ½ p or ½ P is very small. We may therefore substitute the arcs for the tangents, and also L for l', without sensible error. Hence, forming an equation, we obtain

$$B = 180° - A - P \frac{\sin. \frac{1}{2} (l + L)}{\cos. \frac{1}{2} (l - L)}. \qquad [17]$$

The angle B, which we have calculated, is the spherical angle p B A, or the angle contained between the planes M B v M B A; but the true azimuth is the spheroidal angle contained between

the planes N B P, N B A; the difference, however, between these angles has been proved by Delambre to be so small as not to be sensible in practice.

In the trigonometrical survey the angles are measured either from the north or south to the east or west; but in the "base du système mètrique," the angles are measured from the south towards the west, entirely round the circle.

To determine the azimuth of one of the signals independently from astronomical observations.—The general principle of the method is this. The error of a clock or chronometer is found either by means of a transit instrument, or by observations of equal altitudes, or by single altitudes, if the latitude of the place be well known. The observer then takes the angle (θ) between the signal and the sun, or a star, when near the horizon, and notes the time when the observation was made. The azimuth of the heavenly body is also calculated for this time; the latitude and declination being known. Then the sum or difference of the angle θ and the azimuth of the heavenly body will give the azimuth of the signal required. The refraction will scarcely affect the result, but a small error in the time would produce a considerable error in the azimuth.

The method adopted in the trigonometrical survey was to take the mean of the two angles observed with the theodolite, between a flagstaff and the pole star at its greatest elongation east and west. But, from the great altitude of the pole star in our latitudes, any error in the adjustment of the cross axis of the theodolite to horizontality, would materially affect the resulting azimuth.

Example.—From the Trigonometrical Survey, vol. ii., p. 88, the distance of Black Down from Dunnose = 314397·5 ft., the latitude of Dunnose = 50° 37′ 7″·3 N., and azimuth of Black Down, as seen from Dunnose = 84° 54′ 52″·5 N.W. Required the latitude and longitude of Black Down, and the azimuth of Dunnose, as seen from Black Down.

To find the latitude.—The normal A M, which is equal to r the radius of the curvature at A, perpendicular to the meridian, is found = 20963000, nearly.

log. r	7·32145
sin. 1″	4·68557
	2·00702
ar. co.	7·99298
D	5·49735
cos. A	8·94763
$(1 + e^2 \cos.^2 l)$	0·00116
274″·87	2·43912

$2 r^2$ sin. 1″.	9·62950
ar. co.	0·37050
D^2	10·99470
$\sin.^2$ A	9·99658
tan. l	0·08573
$(1 + e^2 \cos.^2 l)$	0·00116
28″·10	1·44867

Hence $l - L = \times\ 274''\cdot 87 + 28''\cdot 10 = -\ 4'\ 6''\cdot 77$, and $L = l + 4'\ 6''\cdot 77 = 50°\ 41'\ 14''\cdot 07$.

To find the Difference of Longitude.

ar. co. log. r sin. 1″ ..	7·99298
D	5·49735
sin. A	9·99829
sec. L	0·19822
4862″·3	3·68684

Hence P = 1° 21′ 2″·3; and since the longitude of Dunnose was previously found = 1° 11′ 36″, therefore, the long. of Black Down = 2° 32′ 38″·3.

To find the Azimuth.

sin. $\frac{1}{2}(l + L)$	9·88836
cos. $\frac{1}{2}(L - l)$	0·00000
P	3·68684
3760″·12	3·57520

Hence

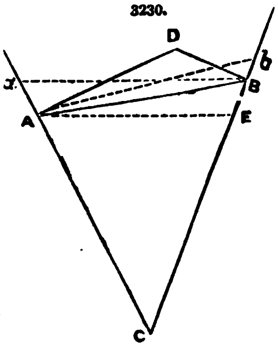
P B A = 180° − A − 1° 2′ 40″·12 = 94° 2′ 27″·38.

The observed angle P B A was 94° 2′ 22″·75.

In the survey, the value of P is found to be 1° 20′ 46″·4. The difference, 15″·9, arises from an erroneous assumption in the length of the perpendicular degree, which gives all the longitudes on the southern coast of England too small.

Heights of the Stations, and Terrestrial Refraction.—*To find the altitude of the station* B *above the station* A.—Let C, Fig. 3230, be the centre of the earth, supposed to be a sphere, and A and B two stations on its surface. Draw A D, B D, perpendicular to the radii C A, C B, respectively, in the plane C A B; and suppose a and b to be the apparent places of A and B as seen from each other, and elevated by refraction. If the rays of light proceeded in straight lines, the angle D A B would be the depression of B below the horizon of A, and D B A the depression of A below the horizon of B. And because D A C, D B C, are right angles, C + D = 180° = D A B + D B A + D, and ∴ C = D A B + D B A. Also, since the distance A B is known, and the radius of the earth (sufficiently near for this purpose), the angle C can easily be found.

3230.



Let α, β, be the observed depressions at A and B respectively, and ρ, ρ', the two refractions, then

$$D A B = \alpha + \rho,\ D B A = \beta + \rho',\ \text{and } (\alpha + \rho) + (\beta + \rho') = C;$$

$$\therefore \ \text{mean refraction } \tfrac{1}{2}(\rho + \rho') = \tfrac{1}{2}(C - \alpha - \beta). \qquad [18]$$

Let E be the point in C B which is on the same level with A, then C E = C A, and E B is the

altitude of B above A, which is to be determined. Join A E, then the angle $DAE = 90^\circ - CAE = \frac{1}{2} C$, therefore the angle

$$BAE = \phi = DAE - DAB = \tfrac{1}{2} C - (\alpha + \rho); \qquad [19]$$

and since the angle B A E is always very small, and B E A very nearly a right angle,

$$BE = AE \times \phi = D \times \phi'' \sin. 1''. \qquad [20]$$

If one of the stations, B for example, is elevated above the horizon of A, β must be considered negative. Also, each observation must be reduced, previously to the calculation, to the place of the axis of the instrument.

Example.—At Allington Knoll the top of the staff on Tenterden steeple was depressed 3′ 5″; and the axis of the instrument was $5\frac{1}{2}$ ft. above the ground: on Tenterden steeple the ground at Allington Knoll was depressed 3′ 35″, and the axis of the instrument was 3·1 ft. below the top of the staff. The distance between the stations being 61,777 ft., it is required to calculate the mean refraction, and also the height of Tenterden steeple above Allington Knoll. (Trig. Survey, vol. i., p. 176.)

The angle which a perpendicular height of 5·5 ft. subtends at the distance = 61777 ft. is $\dfrac{5 \cdot 5}{61777 + \sin. 1''} = 18'' \cdot 4$; and in like manner the angle which 3·1 ft. subtends is $10'' \cdot 4$. Hence

	′ ″
Depression of the top of the staff	3 51
Correction due to 3·1 ft.	+ 10·4
Depression of instrument	1 1·4
Depression of the ground	3 35
Correction due to $5\frac{1}{2}$ ft.	− 18·4
Depression of instrument	3 16·6

Length of perpendicular degree at Tenterden (vol. i., p. 168) = 61185 fathoms.

Fathoms.		Feet.		°		′ ″
61185	:	61777	::	1	:	10 6

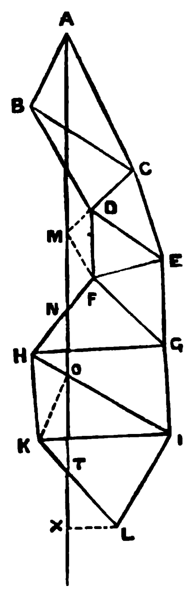
	′ ″
β	4 1·4
α	3 16·6
$\alpha + \beta$	7 18·0
C	10 6
$\rho \times \rho'$	2 48
Mean refraction	1 24

Hence $\phi'' = \frac{1}{2} C - (\alpha + \text{mean refr.}) = 22'' \cdot 4$, and $h = D \times \phi'' \sin. 1'' = 6 \cdot 7$ ft.

The vertical height of the axis at Allington Knoll had been previously found to be 329 ft., so that the height of the axis on Tenterden steeple was 322·3 ft.

To find the absolute altitudes it is necessary that the heights of one or more of the stations be ascertained by actually levelling down to the surface of the sea. The heights of all the intermediate stations are then determined by the reciprocal angles of elevation or depression, carried on from station to station, and it is obvious that a verification will be obtained for every three stations; for the difference of altitude between A and B, when found from direct observation, ought to be the same as when deduced from the difference of the heights of each of those stations and a third station C.

3231.



In the preceding example the effect of refraction is $\frac{1}{7}$ of the intercepted arc. In other cases the refraction varied from $\frac{1}{8}$ to $\frac{1}{17}$ of the contained arc. When reciprocal observations could not be obtained, $\frac{1}{14}$ of C was generally assumed as a mean value of ρ, in order to obtain the angle ϕ in equation [19].

Measurement of the Arcs of the Meridian, and the Arcs parallel to the Equator.—When a chain of triangles has been formed nearly in the direction of the arc of a meridian, and all the sides have been computed, according to the preceding rules, we are enabled to determine the length of the arc of the meridian intercepted between the parallels of the extreme stations. For this purpose two different methods have been adopted, which we shall briefly explain.

The Method of Oblique-angled Triangles.—To measure the arc of the meridian intercepted between the parallels of A *and* L. — Let A B C D . . ., Fig. 3231, be a chain of triangles lying nearly in the direction of the meridian A X. All the sides of the triangles are supposed to have been previously computed, and the angle C A X is given from observation. Produce C D to M, join F M; and, from the last station L, draw L X perpendicular to the meridian A X. The following spherical triangles will then be most easily solved, according to Legendre's method, by first computing the spherical excess in each case, and then deducting one-third of this excess from each of the spherical angles.

In the triangle A C M, there are given A C, $\angle$ A C M, $\angle$ C A M, to find A M, C M, and $\angle$ A M C.

Then $DM = CM - CD$, and $\angle MDF = 180^\circ - CDF$.

In the triangle D M F are given D F, D M, $\angle$ D, to find M F, $\angle$ D M F, $\angle$ D F M.

$$\angle FMN = 180^\circ - (AMC + DMF); \quad \angle MFN = DFN - DFM.$$

[illegible] N, ∠ M F N, to find M N, F N, and ∠ M N F.

[illegible] and ∠ F N M = H N O.

[illegible] O, ∠ N H O, to find N O, H O, and ∠ H O N.

[illegible] T, L X T, we find O T and T X. Hence we have, [illegible] + T X.

[illegible] X is not in the same parallel of latitude with L. [illegible] of X, and let L = the latitude of L, and L + x = [illegible] suppose X A produced to meet the meridian of L [illegible] cos. L X, or

$$[\text{illegible}]\ L + \sin. L \cos. x = \frac{\sin. L}{\cos. p}.$$

[illegible] + &c., $\cos. p = 1 - \frac{1}{2} p^2 + \&c.$, consequently we [illegible] = sin. L $(1 + \frac{1}{2} p^2 + \&c.)$;

[illegible] $(\frac{1}{2} p^2 + \frac{1}{2} x^2) + \&c.$

[illegible], and therefore the term involving x^2 being of the [illegible] tan. L. In this expression p and x are measured [illegible] be measured in feet, we must substitute $\frac{p}{r}$ and $\frac{x}{r}$ for [illegible] A X (the latitude of A being greater than that of

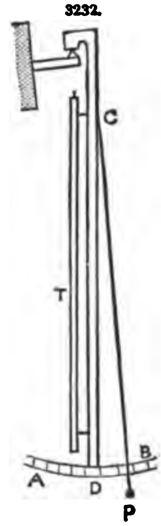
$$[\text{illegible}]\ \tan. L. \qquad [21]$$

[illegible] employed by Delambre to determine the length of the arc [illegible] meridian all the stations to the east, by means of [illegible] computed the distance between every two succeeding [illegible] these distances will give the entire length of the meri- [illegible] done the same for the stations to the west of the [illegible] same value for the length of [illegible] computations serve to verify [illegible] a mean should be taken [illegible] method, however, can only be [illegible] are previously known with

[illegible] of the extreme stations has [illegible] to determine the latitudes [illegible] corresponding celestial arc. This [illegible] The error of a single [illegible] equivalent to about 100 ft. on the [illegible] that an error in the latitude [illegible] affect the measurement of the [illegible] of the meridian. In the [illegible] were observed with a zenith

[illegible] A B, Fig. 3232, is an arc of [illegible] firmly fixed a telescope T of [illegible] vertically, and the telescope [illegible] plane of the meridian a few [illegible] to observe stars within a [illegible] suspended from the centre [illegible] B, shows the angle between [illegible] can be turned half round [illegible] the same stars in the two [illegible] graduation will be entirely [illegible] too great in the one case [illegible] of the sector used in

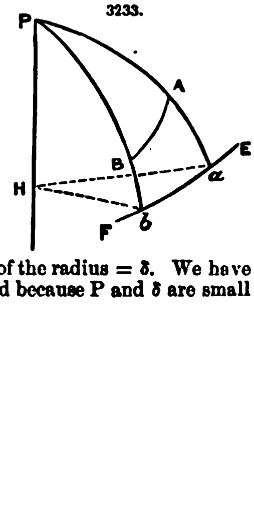
3232.



[illegible] described, was used [illegible] been doubted whether [illegible] determining so important an

[illegible] *the equator.*—Let A B, [illegible] triangles which lie in a di- [illegible], and let E F be the [illegible] are to be projected. [illegible] required to determine the [illegible] latitude of the parallel E F, [illegible], N = the normal at b, [illegible] feet = D, and in parts of the radius = δ. We have [illegible] : sin. x :: sin. δ : cos. l, and because P and δ are small

3233.



arcs, sin. $P = P - \frac{1}{6} P^3$, sin. $\delta = \delta - \frac{1}{6}\delta^3$, very nearly; therefore, making an equation and transposing, $P = (\delta - \frac{1}{6}\delta^3)\frac{\sin. z}{\cos. l} + \frac{1}{6} P^3$. As a first approximation, we have $P = \delta \frac{\sin. z}{\cos. l}$; substituting this value of P in the second member of the last equation, we get

$$P = \delta \frac{\sin. z}{\cos. l} - \frac{1}{6}\delta^3 \frac{\sin. z}{l}\left(1 - \frac{\sin.^2 z}{\cos.^2 l}\right). \qquad [a]$$

Let H be the centre of the circle E F, and let $a\,b$ measured in feet $= p$, then $p : b\,\mathrm{H} ::$ measure of the angle $a\,\mathrm{H}\,b$ or $\mathrm{P} : 1$; $\therefore p = \mathrm{P} \times b\,\mathrm{H} = \mathrm{P} \times \mathrm{N} \cos. \mathrm{L}$, equation [26]; also $\delta = \frac{\mathrm{D}}{\mathrm{N}'}$. Making these substitutions in equation [a], we get

$$p = \frac{\mathrm{N} \cos. \mathrm{L}}{\mathrm{N}' \cos. l}\left\{\mathrm{D} \sin. z - \frac{\mathrm{D}^3 \sin. z}{6\,\mathrm{N}'^2}\left(1 - \frac{\sin.^2 z}{\cos.^2 l}\right)\right\}. \qquad [22]$$

By applying this formula to all the sides of the triangles, the sum of these projections will give the required length of the total arc.

We have now to determine the astronomical difference of longitude from observation. In the Philosophical Transactions for 1824, an account is given of some experiments performed by Dr. Tiarks, for determining the differences of longitude of Dover and Falmouth. Twenty-four chronometers were transported by sea three several times from the one place to the other, by which means the difference of longitude was determined to be 6° 22′ 6″; and as the length of the parallel found from the survey was 1,474,672 ft., we have the length of a degree of parallel in latitude 50° 44′ 24″, equal to 231,563 ft. The difference of longitude of Marennes and Padua was determined by five signals, at five intermediate stations. The length of the parallel in feet was found, from triangulation, to be 1,010,996 mètres, or 3,316,976 English ft., and the difference of longitude was 12° 59′ 3″·75. This gives for the mean length of a degree in latitude 45° 43′ 12″, found from the whole arc between Marennes and Padua, 255,470 ft.; the length of the degree found from the partial arc between Marennes and Geneva was 255,546 ft. Both these results are greater than a degree in the same parallel of latitude on a regular spheroid, which most nearly represents the meridional arcs; but no great reliance can be placed on these numbers, as the determination of the longitudes was attended with considerable difficulty.

The Figure of the Earth.—If the earth were perfectly fluid, and had no motion of rotation about an axis, it would assume a spherical form; for in this case there would be no tendency in the fluid to run in any direction, and therefore it would be in a state of equilibrium. But if any portion of the surface were farther removed from the centre than the rest, the pressure arising from the protuberant would be greater than that from the less elevated parts, and therefore the equilibrium would be destroyed.

But since the earth revolves on its axis, every particle has a tendency to recede from that axis proportional to its distance; consequently its gravity will be diminished, and the columns of fluid at the equator being composed of parts that are lighter, must be extended in length in order to balance the columns in the direction of the axis. It has been proved by Maclaurin and succeeding writers, that a mass of homogeneous fluid will be in equilibrium if it be formed into an oblate spheroid, such that the polar diameter shall be to the equatorial diameter as the attraction at the equator, diminished by the centrifugal force there, is to the attraction at the pole. And as it appears from experiments on the vibration of pendulums that the centrifugal force is to the force of gravity at the equator as 1 to 289, it may be demonstrated that a homogeneous fluid of the same mean density as the earth would be in equilibrium if the ratio $\frac{a-b}{a} = \frac{5}{4}\,\frac{1}{289} = \frac{1}{231}$, nearly, a being the equatorial, and b the polar diameter; that is, if $b : a :: 230 : 231$.

If the fluid mass of the earth be supposed not to be homogeneous, but to be formed of strata that increase in density towards its centre, the solid of equilibrium will still be an elliptic spheroid, but less oblate than before. Now, as it appears, from experiments made on the density of the mountain Schehallien, in Scotland, and also from those of Cavendish, that the mean density of the earth is greater than the density at the surface, it follows, that if the earth be a solid of equilibrium, the ratio $\frac{a-b}{b}$ will be less than before, or less than $\frac{1}{230}$.

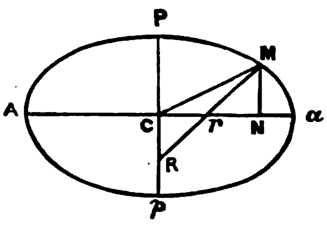
If the earth were homogeneous, the increase of gravity from the equator to the pole would be $\frac{1}{230}$ G, G being the gravity at the equator; and the gravity g, at any latitude l, would be represented by the equation $g = \mathrm{G}\,(1 + \frac{1}{230} \sin.^2 l)$. But if the density of the earth increase towards the centre, the ratio $\frac{a-b}{b}$, and the increase of gravity from the equator to the pole, divided by the gravity at the equator (γ), will no longer be expressed by the same fraction, but the sum of the two fractions is constant, and equal to twice the value of $\frac{a-b}{b}$, which the spheroid would have if it were homogeneous, that is,

$$\frac{a-b}{b} + \gamma = \frac{5}{2}\,\frac{1}{289}\ \cdot 00865, \text{ and } g = \mathrm{G}\,(1 + \gamma \sin.^2 l). \qquad [23]$$

This theorem was first given by Clairaut, and is of great importance in determining the figure of the earth from experiments with the pendulum.

With respect to the values given to g, see our article on GUNNERY. We shall now proceed to show how the figure of the earth is to be determined from geodetic operations. We shall, therefore, first consider the different properties of an oblate spheroid, and then compare them with the results deduced from observation.

3234.



Let A P a p, Fig. 3234, be an ellipse, which, by its revolution about its minor axis P p, generates an oblate spheroid. Let A C $= a$, C P $= b$, the eccentricity $= a\,e$, the ordinate M N $= y$, C N $= x$, the normal M $r = n$, M R $=$ N, the radius of curvature at M $= \rho$, and the latitude of M, or the angle M $r\,a = l$. Now, the equation to the ellipse is $a^2 y^2 + b^2 x^2 = a^2 b^2$.

Also $y = n \sin. l$, and $x = \frac{a^2}{b^2} \times \mathrm{N}\,r = \frac{a^2}{b^2}\, n \cos. l$; $\therefore\ a^2 n^2 \sin.^2 l + \frac{a^2}{b^2}\, n^2 \cos.^2 l = a^2 b^2$; consequently $n = \frac{b^2}{\sqrt{(a^2 \cos.^2 l + b^2 \sin.^2 l)}}$. And because $b^2 = a^2(1 - e^2)$, therefore

$$a^2 \cos.^2 l + b^2 \sin.^2 l = a^2(1 - e^2 \sin.^2 l);$$

hence

$$n = \frac{a(1 - e^2)}{\sqrt{(1 - e^2 \sin.^2 l)}}, \qquad [24]$$

$$x = \frac{a \cos. l}{\sqrt{(1 - e^2 \sin.^2 l)}}, \qquad y = \frac{a(1 - e^2) \sin. l}{\sqrt{(1 - e^2 \sin.^2 l)}}, \qquad [25]$$

$$\frac{x}{\cos. l} = \frac{a}{\sqrt{(1 - e^2 \sin.^2 l)}}, \qquad [26]$$

$$\mathrm{C}\,r = \frac{a\,e^2 \cos. l}{\sqrt{(1 - e^2 \sin.^2 l)}} = \mathrm{N}\,e^2 \cos. l, \qquad [27]$$

$$\mathrm{C\,R} = \frac{a\,e^2 \sin. l}{\sqrt{(1 - e^2 \sin.^2 l)}} = \mathrm{N}\,e^2 \sin. l, \qquad [28]$$

$$\mathrm{C\,M} = \sqrt{x^2 + y^2} = a\sqrt{\left(\frac{1 - (2e^2 - e^4)\sin.^2 l}{1 - e^2 \sin.^2 l}\right)}, \qquad [29]$$

$$\rho = \frac{a^2}{b^4}\, n^3 = \frac{a(1 - e^2)}{(1 - e^2 \sin.^2 l)^{\frac{3}{2}}}. \qquad [30]$$

The lengths of two degrees on the meridian in given latitudes being known from measurement, it is required to determine the polar and equatorial diameters.—Let D, D′, be the lengths of two degrees in feet; l, l', the latitudes of their middle points; ρ, ρ', the radii of curvature at those points; then, since the two arcs are very small compared with their radii, we may suppose them to be arcs of two circles whose radii are ρ, ρ', without sensible error. Hence $180° : 1° :: \pi\rho : \mathrm{D}$;

$$\therefore\ \rho = \frac{180}{\pi}\,\mathrm{D} = \mu\,\mathrm{D};\ \text{and}\ \rho' = \mu\,\mathrm{D}',$$

μ being substituted for $\frac{180}{\pi}$. Hence, therefore, expanding the value of ρ, and neglecting higher powers of e than the second, we have, from equation [30],

$$\mathrm{D} = \frac{\rho}{\mu} = \frac{a(1 - e^2)}{\mu}\left(1 + \tfrac{3}{2} e^2 \sin.^2 l\right); \qquad [31]$$

$$\mathrm{D}' = \frac{\rho'}{\mu} = \frac{a(1 - e^2)}{\mu}\left(1 + \tfrac{3}{2} e^2 \sin.^2 l'\right);$$

$$\therefore\ \frac{\mathrm{D}}{\mathrm{D}'} = \frac{1 + \frac{3}{2} e^2 \sin.^2 l}{1 + \frac{3}{2} e^2 \sin.^2 l'} = 1 + \tfrac{3}{2} e^2 \sin.^2 l - \tfrac{3}{2} e^2 \sin.^2 l';$$

$$\therefore\ e^2 = \frac{2}{3}\,\frac{\mathrm{D} - \mathrm{D}'}{\mathrm{D}'(\sin.^2 l - \sin.^2 l')} = \frac{2}{3}\,\frac{\mathrm{D} - \mathrm{D}'}{\mathrm{D}' \sin.(l + l') \sin.(l - l')}. \qquad [32]$$

If $l' = 0$, or the degree is at the equator, the length of the degree $\mathrm{D}' = \frac{a(1 - e^2)}{\mu}$. Hence it follows, that the excess of the degrees of the meridian above a degree of the meridian at the equator, is as the square of the sine of latitude.

The length of a degree parallel to the equator, and the length of a degree of the meridian, being known from measurement, to determine the polar and equatorial diameters.—Let Δ be the length of a degree parallel to the equator, at a place whose latitude $= \phi$. Then the radius of this circle $x = \frac{a \cos. l}{\sqrt{(1 - e^2 \sin.^2 l)}}$, equation [25]; therefore $\Delta = \frac{x}{\mu} = \frac{a \cos. l}{\mu \sqrt{(1 - e^2 \sin.^2 l)}}$. Expanding this expression, and neglecting the powers of e higher than the second,

$$\Delta = \frac{a \cos. l}{\mu}(1 + \tfrac{1}{2} e^2 \sin.^2 l). \qquad [33]$$

From this equation, and equation [31], we can determine the values of e^2 and a, when D and Δ are known.

We shall now give some examples of the geodetic measurements which have been executed in our own country and in India. They are part of those which M. Schmidt has selected as the best for the purpose of determining the magnitude and figure of the earth. With these data he has found $a = 20921665$ ft., $b = 20852394$ ft. Ellipticity $= \frac{a - b}{a} = \frac{1}{302 \cdot 03}$. Degree at the equator $= 362732$; degree in latitude $45° = 364543 \cdot 5$.

No.	Country.	Latitude of Middle Points.			Arc measured.			Length in Feet.	Length of a Degree.	Difference.
		°	′	″	°	′	″			
1	India	12	32	21	1	34	56·4	574,368	362,988	+ 83
2	"	9	34	43	2	50	10·5	1,029,171	362,863	+ 29
3	"	13	2	54	4	6	11·3	1,489,198	362,873	− 46
4	"	16	34	42	2	57	21·7	1,073,409	363,125	+ 96
5	"	19	34	34	3	2	35·9	1,105,499	363,257	+118
6	"	22	36	32	3	1	19·9	1,097,320	363,084	−184
7	England ..	51	25	18	1	36	20·0	586,319	364,952	+256
8	"	52	50	30	1	14	3·4	450,018	365,036	−411
9	"	54	0	56	1	6	49·7	406,516	365,109	−107

The last column in this Table is the difference between the length of a degree computed with the values of a and b, given above, and the length of a degree given by measurement. These differences must be supposed to arise either from errors in the observations, or from local irregularity of form or density. The most probable source of error is in determining the latitudes; for an error of a single second in the difference of latitude is equivalent to 100 ft. measured on the ground. On this account, the largest arcs may be considered the best; for the probable error is the same, whether the arcs be great or small.

To these examples we may add the results of four arcs of parallel, measured in different countries, and also their errors, compared with the degrees computed from formula [33].

No.	Country.	Latitude.			Measured Degree.	Difference.
		°	′	″		
1	Mouth of the Rhone..	43	31	50	266,345	+1191
2	Beachy Head to Dunnose	50	44	24	232,331	+ 789
3	Dover to Falmouth	50	44	24	231,579	+ 37
4	Padua to Marennes	45	43	12	255,480	+ 110

To determine the length of any arc of the meridian.—Let the arc a M, Fig. 3234, measured from the equator $= s$, then $ds = \sqrt{dx^2 + dy^2}$, and if we differentiate the values of x and y, given in formula [25], we shall readily find $ds = \frac{a(1 - e^2)\,dl}{(1 - e^2 \sin.^2 l)}^{\frac{3}{2}} = \rho\, dl$. Expanding this expression, and neglecting all powers of e higher than the fourth, we get

$$ds = a\,dl\,(1 - e^2)(+ \tfrac{3}{2} e^2 \sin.^2 l + \tfrac{15}{8} e^4 \sin.^4 l);$$

and since $\sin.^2 l = \frac{1}{2}(1 - \cos. 2l)$, $\sin.^4 l = \frac{1}{8}(3 - 4 \cos. 2l + \cos. 4l)$, this equation becomes $ds = a\,dl\,(1 - e^2)(\text{A} - \text{B} \cos. 2l + \text{C} \cos. 4l)$, where

$$\text{A} = 1 + \tfrac{3}{4} e^2 + \tfrac{45}{64} e^4, \quad \text{B} = \tfrac{3}{4} e^2 + \tfrac{15}{16} e^4, \quad \text{C} = \tfrac{15}{64} e^4.$$

And integrating

$$s = a(1 - e^2)(\text{A}\,l - \tfrac{1}{2} \text{B} \sin. 2l + \tfrac{1}{4} \text{C} \sin. 4l). \qquad [34]$$

No constant is necessary, because at the equator s and l vanish together.

The lengths of any two arcs of the meridian being given from measurement, to determine the polar and equatorial diameters.—If l and l' be the latitudes of the two extremities of the first arc, and s, s', their distances measured from the equator, then we have, from equation [34],

$$s = a(1 - e^2)(\text{A}\,l - \tfrac{1}{2} \text{B} \sin. 2l + \tfrac{1}{4} \text{C} \sin. 4l),$$
$$s' = a(1 - e^2)(\text{A}\,l' - \tfrac{1}{2} \text{B} \sin. 2l' + \tfrac{1}{4} \text{C} \sin. 4l').$$

Taking the difference of these equations, and putting $s - s' = \mathrm{S}$, $l - l' = \lambda$, $l + l' = \mathrm{L}$, we have, from Trigonometry, $\mathrm{S} = a(1 - e^2)(\mathrm{A}\lambda - \mathrm{B}\sin.\lambda\cos.\mathrm{L} + \frac{1}{2}\mathrm{C}\sin.2\lambda\cos.2\mathrm{L})$. In like manner we have, for the second arc,

$$\mathrm{S}' = a(1 - e^2)(\mathrm{A}\lambda' - \mathrm{B}\sin.\lambda'\cos.\mathrm{L}' + \tfrac{1}{2}\mathrm{C}\sin.2\lambda'\cos.2\mathrm{L}');$$

and since, in these two equations, the values of S, λ, L, S$'$, λ', L$'$, are all known from observation, the quantities a and e can easily be found, and the polar radius b from the expression $b = a\sqrt{(1 - e^2)}$.

If $b = a(1 - \alpha)$, the small fraction α is called the *ellipticity* of the spheroid. Hence

$$a(1 - \alpha) = a\sqrt{(1 - e^2)} = a(1 - \tfrac{1}{2}e^2 - \tfrac{1}{8}e^4);$$

$$\therefore\ \alpha = \tfrac{1}{2}e^2 + \tfrac{1}{8}e^4. \qquad [35]$$

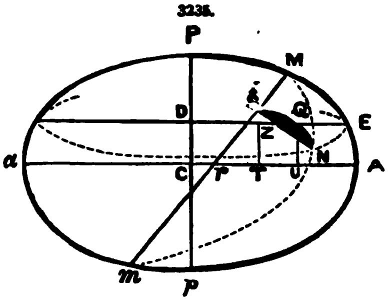
If Q be put for the elliptic quadrant, measured from the equator to the pole, we have $l = \frac{1}{2}\pi$ in equation [34]; therefore

$$\mathrm{Q} = \tfrac{1}{2}a(1 - e^2)\mathrm{A}\pi = \tfrac{1}{2}\pi a(1 - \tfrac{1}{4}e^2 - \tfrac{3}{64}e^4). \qquad [36]$$

If the earth be cut by a vertical plane perpendicular to the meridian, the radius of curvature of this section, at the point where it cuts the meridian, is equal to the normal M R, Fig. 3234.—For, since the earth is supposed to be a solid of revolution, the direction of gravity always passes through the axis of the earth. If therefore we conceive the plumb line to be carried over an indefinitely small arc perpendicular to the meridian, its direction will intersect the axis at the same point R as before; and therefore R is the centre, and M R the radius of curvature of this arc. The value of M R is given in formula [26].

To find the radius of the curvature at any place, when the earth is cut by a vertical plane making an angle θ with the meridian.—Let P A p, Fig. 3235, be an oblate spheroid, formed by the revolution of the ellipse P A p about its minor axis P p. Let P M A be the meridian of the given place M, M N m any section passing through the normal M r, making an angle θ with the meridian; then it is required to find the radius of curvature of the section M N m at the point M.

3235.



From any point N in the arc M N m draw N S perpendicular to M m, and S Q also perpendicular to M m in the plane P A p. Let the plane N S Q cut the plane D N E drawn through N, parallel to the equator in the line Q N. Because M S is perpendicular to S N and S Q, it is perpendicular to the plane N S Q, and therefore the plane M A m passing through M S is perpendicular to the plane N S Q. And because the planes N S Q, D E N, are perpendicular to the plane M A m, their common intersection Q N is perpendicular to this plane; therefore N Q S, N Q D, are right angles. Let $r\mathrm{S} = x$, $\mathrm{SN} = y$, $\angle \mathrm{NSQ} = \mathrm{AMN} = \theta$, $\angle \mathrm{S}r\mathrm{A} = \mathrm{QSZ} = l$, then will

$$\mathrm{SQ} = y\cos.\theta,\ \mathrm{SZ} = \mathrm{SQ}\cos.\mathrm{QSZ} = y\cos.\theta\cos.l,\ \mathrm{QZ} = \mathrm{SQ}\sin.\mathrm{QSZ} = y\cos.\theta\sin.l.$$

And because D E = D N, we have from the ellipse

$$a^2b^2 = a^2.\mathrm{CD}^2 + b^2.\mathrm{DE}^2 = a^2.\mathrm{CD}^2 + b^2(\mathrm{DQ}^2 + \mathrm{QN}^2). \qquad [a]$$

But

$$\begin{aligned}\mathrm{CD} &= \mathrm{ST} - \mathrm{SZ} = x\sin.l - y\cos.\theta\cos.l.\\ \mathrm{DQ} &= \mathrm{C}r + r\mathrm{T} + \mathrm{TU} = c + x\cos.l + y\cos.\theta\sin l.\\ \mathrm{QN} &= y\sin.\theta.\end{aligned}$$

Making these substitutions in equation [a], it will be of the form

$$\mathrm{A}x^2 + \mathrm{B}xy + \mathrm{C}y^2 + \mathrm{D}x + \mathrm{E}y + \mathrm{F} = 0, \qquad [37]$$

where

$$\begin{aligned}\mathrm{A} &= a^2\sin.^2 l + b^2\cos.^2 l. & \mathrm{D} &= 2b^2c\cos.l.\\ \mathrm{B} &= -2(a^2 - b^2)\sin.l\cos.l\cos.\theta. & \mathrm{E} &= 2b^2c\cos.\theta\sin.l.\\ \mathrm{C} &= b^2 + (a^2 - b^2)\cos.^2 l\cos.^2\theta. & \mathrm{F} &= -(a^2 - c^2)b^2.\end{aligned}$$

This is the equation to the ellipse, and we shall find the radius of curvature from the expression $\rho = \dfrac{ds^3}{d^2x\,dy}$, dy being considered constant. Now, at the point M, $x = n$, $y = 0$, $\dfrac{dx}{dy} = 0$, $\dfrac{ds}{dy} = -1$; therefore $\rho = -\dfrac{dy^2}{d^2x}$. Hence, differentiating equation [37] twice, we have

$$2\mathrm{A}x\frac{dx}{dy} + \mathrm{B}y\frac{dx}{dy} + \mathrm{B}x + 2\mathrm{C}y + \mathrm{D}\frac{dx}{dy} + \mathrm{E} = 0,$$

$$2\mathrm{A}x\frac{d^2x}{dy^2} + 2\mathrm{A}\frac{dx^2}{dy^2} + \mathrm{B}y\frac{d^2x}{dy^2} + 2\mathrm{B}\frac{dx}{dy} + 2\mathrm{C} + \mathrm{D}\frac{d^2x}{dy^2} = 0;$$

and because $\frac{dx}{dy} = 0$, $y = 0$, we get $-\frac{d^2 x}{dy^2} = \frac{2\,C}{2\,A\,x + D}$ and $\rho = \frac{2\,A\,x + D}{2\,C}$; and since

$$A = a^2 \sin.^2 l + b^2 \cos.^2 l = a^2 (1 - e^2 \cos.^2 l),$$

$$x = M\,r = \frac{a(1 - e^2)}{\sqrt{(1 - e^2 \sin.^2 l)}},\ b^2 = a^2 (1 - e^2),\ c = \frac{a e^2 \cos.\, l}{\sqrt{(1 - e^2 \sin.^2 l)}};$$

$$\therefore\ 2\,A\,x + D = \frac{2\,a^2 (1 - e^2)}{\sqrt{(1 - e^2 \sin.^2 l)}},$$

$$\text{and}\quad C = b^2 + (a^2 - b^2) \cos.^2 \theta \cos.^2 l = a^2 (1 - e^2 + e^2 \cos.^2 \theta \cos.^2 l);$$

$$\therefore\ \rho = \frac{a(1 - e^2)}{\sqrt{1 - e^2 \sin.^2 l}\,(1 - e^2 + e^2 \cos.^2 \theta \cos.^2 l)}. \qquad [38]$$

Cor. Because

$$\begin{aligned} &1 - e^2 + e^2 \cos.^2 \theta \cos.^2 l \\ &= (1 - e^2)(\sin.^2 \theta + \cos.^2 \theta) + e^2 \cos.^2 \theta \cos.^2 l \\ &= (1 - e^2) \sin.^2 \theta + (1 - e^2 \sin.^2 l) \cos.^2 \theta, \end{aligned}$$

We obtain from formula [38], $\frac{1}{\rho} = \frac{\sqrt{(1 - e^2 \sin.^2 l)}}{a(1 - e^2)}\,[(1 - e^2) \sin.^2 \theta + (1 - e^2 \sin.^2 l) \cos.^2 \theta]$.

And if r be the radius of curvature of the meridian at the point M, and r' the radius of curvature of a section perpendicular to the meridian, we have

$$r = \frac{a(1 - e^2)}{(1 - e^2 \sin.^2 l)^{\frac{3}{2}}}, \qquad r' = \frac{a}{\sqrt{(1 - e^2 \sin.^2 l)}}.$$

Hence it follows that $\frac{1}{\rho} = \frac{\sin.^2 \theta}{r'} + \frac{\cos.^2 \theta}{r} = \frac{r \sin.^2 \theta + r' \cos.^2 \theta}{r\,r'}$;

$$\therefore\ \rho = \frac{r\,r'}{r \sin.^2 \theta + r' \cos.^2 \theta}, \qquad [39]$$

an elegant expression, first given by Oliver Byrne, which may be proved by the differential calculus to be true of all surfaces, when r and r' are the radii of greatest and least curvature of all sections passing through the normal at the point M.

To determine the figure of the earth from the vibration of pendulums.—This method, which is now very generally practised on account of its great facility, may be thus briefly explained. It appears from Mechanics that the time of vibration of a simple pendulum in a vacuum, when the arcs are indefinitely small, is determined by the equation $t = \pi \sqrt{\frac{L}{g}}$. If, therefore, t and L be given, the value of g may easily be found. Let G represent the force of gravity at the equator, and g the force of gravity in any latitude l; then we have from Clairaut's theorem,

$$\frac{a - b}{b} + \gamma = \frac{5}{2} \cdot \frac{1}{289};\ \text{and}\ g = G(1 + \gamma \sin.^2 l). \qquad [23]$$

Suppose now that a *pendulum*, of either of the forms described, is made to vibrate, and its vibrations are compared with those of the *pendulum* of a clock, as explained in the article PENDULUM, then if n be the number of vibrations which the clock pendulum makes between two successive coincidences, the experimental pendulum will make $n \pm 2$ vibrations. Let τ be the rate of the clock in seconds, or its gain in twenty-four hours, then the number of vibrations which the clock makes in a day is $24 \times 60 \times 60 + \tau = 86400 + \tau$. If therefore N be the number of vibrations made by the experimental pendulum in a day, we have, manifestly, $n : n \pm 2 :: 86400 + \tau : N$; therefore

$$N = \frac{n \pm 2}{n}(86400 + \tau) = 86400 + \tau \pm \frac{172800 + 2\,\tau}{n}. \qquad [40]$$

Let N' be the number of vibrations which the same pendulum makes in any other latitude l' and g' the force of gravity at this place. We have then

$$\frac{N^2}{N'^2} = \frac{g}{g'} = \frac{G(1 + \gamma \sin.^2 l)}{G(1 + \gamma \sin.^2 l')} = 1 + \gamma(\sin.^2 l - \sin.^2 l'),$$

nearly, γ being a very small quantity; therefore

$$\gamma = \frac{N^2 - N'^2}{N'^2 (\sin.^2 l - \sin.^2 l')} \qquad [41]$$

The value of γ being determined in this manner from experiment, the ratio of a to b will be found from the first of equations [23].

In this investigation several corrections have been omitted which must be taken into consideration when great accuracy is required.

(1). *Correction for the amplitude of the arc of vibration.*—In the expression given for the time of vibration (see PENDULUM), the arc is supposed to be indefinitely small. Let t be the observed time of vibration, ϕ the amplitude or semiarc of vibration, and t_1 the time of vibration, when the arc is indefinitely small; then we have $t = \pi\sqrt{\frac{L}{g}}\left(1+\frac{\phi^2}{16}\right) = t_1\left(1+\frac{\phi^2}{16}\right)$. Hence if N be the observed number of vibrations made in a day, and N_1 the number in an indefinitely small arc, $\phi\, N_1\, t_1 = 24$ hours $= N\, t$, therefore

$$N_1 = N\frac{t}{t_1} = N\left(1+\frac{\phi^2}{16}\right). \qquad [42]$$

If therefore ϕ remains nearly constant during the time of observation, the number of vibrations N must be multiplied by the quantity $1 + \frac{1}{16}\phi^2$. But as the amplitude is continually diminishing on account of friction and the resistance of the air, it is necessary to make an allowance for this change. Now it is proved, both by theory and experiment, that the arcs decrease very nearly in geometrical progression. Let therefore ϕ be the first arc, ϕ' the last, and m the number of terms, which is always a very large number. Also, let q be the ratio of the square of each arc to the square of the preceding arc; then the whole time of vibration will be represented by the equation

$$T = \pi\sqrt{\frac{L}{g}}\left\{m + \frac{\phi^2}{16}(1+q^2+q^4\ldots\ldots+q^{m-1})\right\}$$

$$= \pi\sqrt{\frac{L}{g}}\left(m+\frac{\phi^2}{16}\,\frac{1-q^m}{1-q}\right).$$

Let $q = 1 - x$, then x is a very small quantity, and

$$\log.(1-x) = M(-x - \tfrac{1}{2}x^2 - \&c.) = -M\,x, \text{ nearly,}$$

M being the modulus in the common system of logarithms; hence

$$x = -\frac{\log.(1-x)}{M}, \quad \text{or, } 1-q = -\frac{\log. q}{M};$$

and since $q^{m-1}\phi^2 = \phi'^2$, we have

$$-\log. q = \frac{2\log.\phi - 2\log.\phi'}{m-1} = \frac{2(\log.\phi - \log.\phi')}{m}, \text{ nearly;}$$

$$\therefore\ 1-q = \frac{2(\log.\phi-\log.\phi')}{M\,m}.$$

Also, $$\phi^2(1-q^m) = \phi^2 - q\,\phi'^2 = \phi^2 - \phi'^2, \text{ very nearly.}$$

Making these substitutions in the expression for T given above, we have

$$T = \pi\sqrt{\frac{L}{g}}\left\{m + \frac{M\,m}{32}\,\frac{\phi^2-\phi'^2}{\log.\phi-\log.\phi'}\right\},$$

and therefore the mean time of one vibration is

$$t = \pi\sqrt{\frac{L}{g}}\left(1+\frac{M}{32}\,\frac{\phi^2-\phi'^2}{\log.\phi-\log.\phi'}\right), \text{ or } t = t_1\left(1+\frac{M}{32}\,\frac{\phi^2-\phi'^2}{\log.\phi-\log.\phi'}\right).$$

Hence if ν_1 be the correction to be added to the observed number of vibrations N in a day, we manifestly have

$$\nu_1 = N\frac{M\sin.^2 1^\circ}{32}\,\frac{\phi^2-\phi'^2}{\log.\phi-\log.\phi'}, \qquad [43]$$

the arcs ϕ and ϕ' being estimated in degrees.

(2). *Correction for temperature.*—When a pendulum is made to vibrate at different times, its length will vary with the temperature, and therefore the time of vibration will also vary; hence it is necessary to reduce the number of vibrations to a given standard (62°). Let T be the mean height of all the thermometers employed during the experiments, and e the rate of expansion of the metal for 1° of Fahrenheit, then if L, L′, be the lengths of the pendulum at the temperature of T°, and 62°, and N, N_2 be the corresponding numbers of vibrations in a day, we shall have $L = L'[1 + e(T^\circ - 62^\circ)]$, and consequently

$$\frac{N_2}{N} = \sqrt{\frac{L}{L'}} = \sqrt{1+e(T-62)} = 1 + \tfrac{1}{2}e(T^\circ - 62^\circ), \text{ nearly.}$$

Hence if ν_2 be the correction to be added on account of the increase of temperature,

$$\nu_2 = \tfrac{1}{2}N\,e(T^\circ - 62^\circ). \qquad [44]$$

(3). *Correction for the buoyancy of the atmosphere.*—When a body moves in a fluid its weight is diminished by the weight of an equal bulk of fluid, and therefore the accelerating force is diminished in the same proportion. Let N be the number of vibrations made in a day in air, N_3 ditto in

a vacuum; g the force of gravity in air, g' ditto in a vacuum; σ the specific gravity of air, S that of the pendulum during the experiments; then

$$\frac{g'}{g} = \frac{S}{S-\sigma} = \left(1 + \frac{\sigma}{S-\sigma}\right), \text{ also } \frac{N_s}{N} = \sqrt{\frac{g'}{g}} = \sqrt{\left(1 + \frac{\sigma}{S-\sigma}\right)} = 1 + \tfrac{1}{2}\frac{\sigma}{S-\sigma}, \text{ nearly.}$$

Let h be the height of the barometer, and T the temperature of the air during the experiments; also let σ' be the specific gravity of the air at the temperature of 32°, when the barometer stands at a given altitude H, and h' the height of the same weight of mercury reduced to the temperature T. It appears, then, from hydrostatics that the specific gravity of the air

$$= \frac{p}{k\,[1 + \alpha\,(T^\circ - 32^\circ)]},$$

when p is the pressure on a unit of surface, α is the expansion of air for 1° of temperature, and k is a constant quantity; hence $\sigma : \sigma' :: \frac{h}{1 + \alpha\,(T^\circ - 32^\circ)} : h'$. Also, if μ be the expansion of mercury for 1° of temperature, $h' = H\,[1 + \mu\,(T^\circ - 32^\circ)]$. Substituting this value of h' in the proportion above, and forming an equation, we get $\sigma = \sigma' \frac{h}{H}\,\frac{1}{1 + (\alpha + \mu)\,(T^\circ - 32^\circ)}$, very nearly.

According to MM. Arago and Biot, when $H = 29{\cdot}9218$, and the temperature is 32°, σ' is equal to $\frac{1}{770}$, therefore $\tfrac{1}{2}\frac{\sigma'}{H} = {\cdot}0000217$. Also, $\alpha = \frac{1}{450} = {\cdot}00222$, $\mu = {\cdot}0001$, and therefore $\alpha + \mu = {\cdot}0023$. Hence $\tfrac{1}{2}\,\sigma = \frac{{\cdot}0000217}{1 + {\cdot}0023\,(T - 32)}$, and $\frac{N_s}{N} = 1 + \frac{{\cdot}0000217\,h}{(S - \sigma)\,[1 + {\cdot}0023\,(T^\circ - 32^\circ)]}$; or if we put $\frac{1}{770}$ $(= {\cdot}0013)$ for σ in the denominator, we shall have, for the correction to be added to N,

$$\nu_s = N\,\frac{{\cdot}0000217}{S - {\cdot}0013}\,\frac{h}{1 + {\cdot}0023\,(T^\circ - 32^\circ)}. \qquad [45]$$

See Algebraic Signs. Barometer. Distances. Gravity. Gunnery. Pendulum. Surveying. Thermometer.

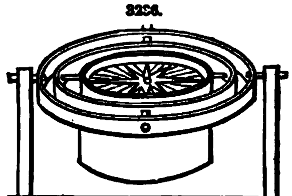
Works relating to Geodesy:—Delambre et Legendre, 'Méthode Analytique pour la Détermination d'un Arc du Méridien,' 4to, Paris, 1798. 'Mudge and Dalby's Account of the Operations for Conducting a Trigonometrical Survey of England and Wales,' 3 vols., 4to, 1799. Benoit, 'Cours complet de Topographie et de Géodésie,' 8vo, Paris, 1825. Salneuve, 'Cours de Topographie et de Géodésie,' 8vo, 1841. Puissant, 'Traité de Géodésie,' 2 vols., 4to, Paris, 1842. J. B. Williams, 'Practical Geodesy,' 8vo, 1846. G. Everest, 'Measurement of the Meridional Arc of India,' 2 vols., 4to, 1847. Captain Yolland, 'An Account of the Measurement of the Lough Foyle Base, Ireland,' 4to, 1847. Clark and James, 'Account of the Ordnance Survey of Great Britain and Ireland,' 2 vols., 4to, cloth, 1858. Sir H. James, 'Abstracts of the Principal Lines of Spirit Levelling in England, Wales, and Scotland, 4 vols., 4to, 1861. Colonel Frome, 'Outline of a Method of Conducting a Trigonometrical Survey,' 8vo, 1862. Sir H. James, 'Extension of the Triangulation of the Ordnance Survey into France and Belgium,' 4to, 1863. Borda, 'Description et Usage du Circle de Réflexions,' Paris, 4to. 'How to Measure the Earth, with the assistance of Railways,' by Oliver Byrne, the editor of this Dictionary.

GERMAN SILVER. Fr., *Argentan*; Ger., *Neusilber*; Ital., *Packfong*; Span., *Plata alemana*.
See Alloys.

GIMBALS. Fr., *Balanciers du compas ou de la lampe*; Ger., *Bügel des Compasses order des Nachthauses*; Ital., *Snodo universale*; Span., *Aparato de suspension*.

A *gimbals*, Fig. 3236, is a contrivance for securing free motion in suspension, or for suspending anything, as a chronometer, ship's compass, marine barometer, &c., so that it may keep a constant position, or remain in equilibrium unaffected by the motion of connected bodies, or by the motion of a ship. It consists of a ring Fig. 3236, within which the suspended body turns on an axis through the diameter, while the ring itself turns on another axis at right angles to the first, by means of pivots resting on an outer ring or other means of support. See Compass.

3236.



GIN. Fr., *Manége à malettes*; Ger., *Pferdegöpel*; Span., *Manija*.

A gin is a machine or instrument by which the mechanical powers are employed in aid of human strength; especially a machine consisting of a tripod formed of poles united at the top, one of them being longer than the rest and called the *pry-pole*, with a windlass, pulleys, ropes, &c., for raising or moving heavy weights, lifting ore from mines, hauling cannon, and like purposes.

A *gin* is also a machine for separating the seeds from cotton, called hence a *cotton gin*.

GIN, Carpentry. Fr., *Chèvre*; Ger., *Hebezeug*; Ital., *Capra*; Span., *Cábria de carpinteros*, or *Borriquete*.
See Lifts, Hoists, and Elevators.

GIN, Cotton Gin. Fr., *Machine égreneuse*; Ger., *Egrenirmaschine*; Ital., *Sgranatore*; Span., *Máquina para desmotar el algodon*.
See Cotton Machinery. Gin.

GLAND, of a Stuffing Box. Fr., *Chapeau d'une boîte à etoupes*, *Couronne de la presse-étoupe*; Ger., *Stopfbüchse*; Span., *Sombrerete de una caja de estopas*.

A gland is the cover of a stuffing box; sometimes called a follower. A cross-piece or clutch for engaging and disengaging machinery moved by belts or bands is also called a *gland*.

GLASS FURNACE. Fr., *Four de verrerie*; Ger., *Glasofen*; Ital., *Vetriera*; Span., *Horno de vidrio*.

See Glass Machinery.

GLASS MACHINERY. Fr., *Machines de verrerie*; Ger., *Maschinen zur Anfertigung des Glases*; Ital., *Macchine da lavorare il vetro*; Span., *Maquinaria para la fabricacion de vidrio*.

Machinery for the Manufacture of Plate Glass.—G. H. Daglish, in the P. I. M. E., 1863, observed that within the last ten years the production of plate glass in England has been quadrupled, whilst in the same time the price has been diminished fully one-half. The present extent of the manufacture in this country is about 85,000 sq. ft. per week, whilst about 12,000 sq. ft. per week of foreign plate glass is imported. The foreign glass has obtained a preference from its superior lightness of colour, which arises from the greater purity of the materials that it is made of, particularly with regard to the sand, of which the foreign makers have an abundant supply, of great purity and light colour.

Under the influence of competition, the English manufacturers have lately commenced an extensive course of experiments with the view of improving the quality of the plate glass made in this country, and also reducing the cost of manufacture; and in some instances very decided success has thus far been the result. In order to accomplish these objects, the sand employed at the British Plate-Glass Works at Ravenhead, near St. Helen's, is now imported from France; and every precaution is adopted to ensure as far as possible the chemical purity of the other ingredients of the glass. Under these altered circumstances the glass now manufactured is equal in every respect to the best samples of the French production.

After the materials have undergone the process of melting in the furnace and are considered in a fit state for casting, the pot containing the melted mass is taken to the casting table, and its contents poured out on one end of the table, in front of a large cast-iron roller; the material is then spread over the surface of the table by passing the roller over it, the thickness of the plate of glass being regulated by strips of iron placed along each side of the table, on which the ends of the roller run. As soon as the plate of glass is sufficiently solidified to bear removal, it is introduced into an annealing oven, there to be gradually reduced in temperature or annealed, until it is fit to be exposed to the atmosphere without risk of fracture. This process of annealing used formerly to occupy upwards of a fortnight, but from the improved arrangement and construction of the annealing oven it is now completed in four days; thus three times the quantity of glass can now be annealed in each oven compared with what was formerly considered possible; and consequently a large outlay in building and in space has been saved, since only one layer of plates can be placed in the oven at one time, no method of piling the plates being considered practicable or even safe. The chemical difficulties and manipulation in producing the raw material have thus been very satisfactorily overcome; but the problem of carrying out the necessary improvements in the subsequent mechanical operations has not perhaps been so completely solved.

The plates of glass when taken from the annealing ovens are exceedingly irregular, particularly on the surface which has been uppermost in the process of casting, that surface being undulated or wavy after the passage of the roller over it whilst in a semi-fluid state; the lower side too is affected by any irregularities on the surface of the casting table, and also to some extent by the floor of the annealing oven; and both sides of the plates are also covered with a hard skin, semi-opaque. The plates vary in size, the largest being about 17 ft. long by 9½ ft. wide; and the thickness varies according to the size from ⅜ to ⅝ in. The first process to which the plates are submitted is that of grinding, to take off the hard skin and reduce the surface to a uniform plane, which is performed by the application of sand and water. The second process is that of smoothing, which is a continuation of the first process, but performed with emery of seven different degrees of fineness, so as to prepare the surface of the glass for the final process of polishing. This last process is effected by the use of oxide of iron employed in a moist state.

The machine in general use for grinding is that which was originally employed at the commencement of the glass manufacture, and is believed to have been designed by James Watt. It is known by the name of the fly-frame machine, and is shown in side elevation and plan in Figs. 3237, 3238. It consists of two benches of stone A A, sufficiently large to hold a plate of glass, and placed about 12 ft. apart: on these benches the plates of glass are fixed by plaster of Paris, as shown by the black line in Fig. 3237. Each bench has a runner-frame B made of wood, about 8 ft. long by 4½ ft. wide, shod on the under-side with plates of iron about 4 in. broad and ½ in. thick, and provided with a strong wrought-iron stud on the upper side, by which it is moved about over the surface of the glass. The gearing for driving these two runner-frames B is placed between the two benches, and consists of the square cast-iron fly-frame C, with two flat bars D hinged to it on opposite sides, extending over each bench A, and suspended from the roof by long chains, as shown by the dotted lines in Fig. 3237, so as to allow them to radiate freely in every direction; this is called the fly-frame from the peculiar motion given to it, and each of the runner-frames is connected to it by the central stud B, Fig. 3237, working loosely in the slot between the bars D. The fly-frame receives its motion from an upright spindle E, which is driven from the main line of shafting by a pair of bevel-wheels with a friction clutch for throwing in and out of gear. On the top of the spindle E is a wrought-iron arm or crank carrying a movable stud, which works in a bush in the centre of the fly-frame C. Round the centre spindle E are also four other spindles F, equidistant from the centre spindle and from one another, each carrying on the top a wrought-iron arm or crank with movable stud similar to the centre one; these studs severally work in bushes at each corner of the fly-frame. Hence when motion is given to the centre spindle E, the fly-frame C is carried round by the stud on the crank-arm, while its sides are always kept parallel to their original position by the four corner cranks F. The two runner-frames B, being connected by their central stud to the arms D of the fly-frame, receive the same circular motion as the fly-frame; but at the same time they are left free to revolve round their own centres, which they do in a greater or less degree according to the varying friction of the grinding surfaces. The grinding

constantly applied, until the surface of the glass is …cts; the sand is then washed off the glass, and the …d on the same machine by substituting the coarser … plate of glass is then removed from the bench, …mitted to the same process on the other side. The …forty revolutions per minute. It will be seen that …ly large to act upon the entire surface of a large …ry to divide the operation and shift the position of …serting the centre stud of the runner-frame into a …e bars D.

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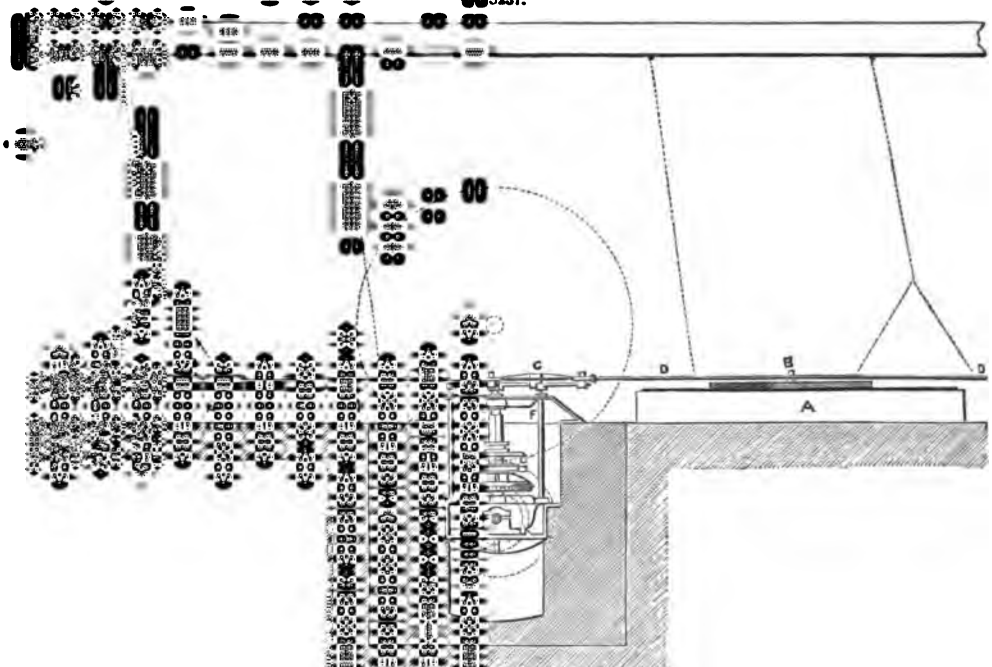
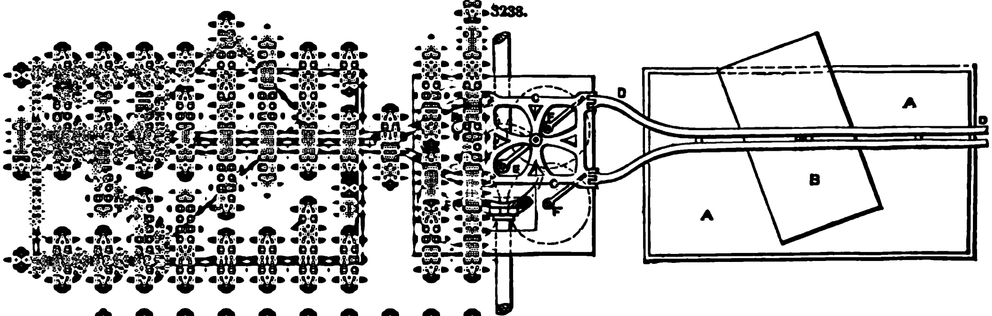


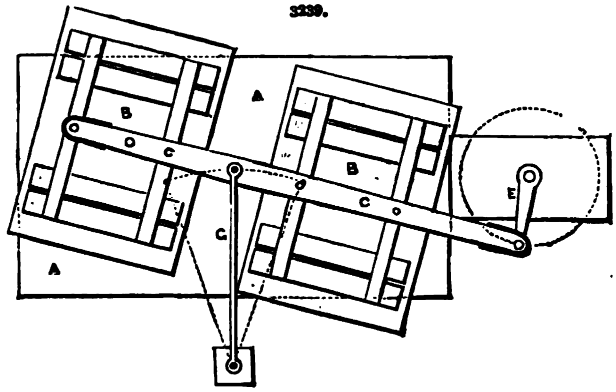
3238.



… of the operation of smoothing was effected by manual …bbing two pieces of glass together, and applying …equisite as the work approaches completion that no … account that hand labour is considered absolutely … scratch being immediately felt by a practised hand, …chine would spoil the whole surface before it was … a machine for smoothing the plates of glass, … hand touch is only required for the final part of …n in plan in Fig. 3239, is exceedingly simple and … connected at one end to a crank E on an upright …A, on which the plate of glass is laid; two runner- …d on the under-side of each frame is fixed another …ss on the bench. In this case the runner-frames B

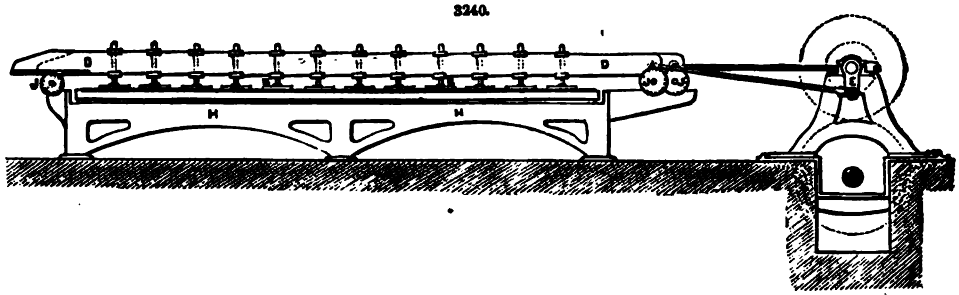
are only allowed to partake of the motion given to them by the bar, and are not left free to revolve round their own centres as in the grinding operation previously described. The centre of the bar C between the two runner-frames is kept in position by a radius rod G secured to a fixed bracket on one side of the bench, at right angles to the direction of the bar. The crank E being set in motion, the bar and runner-frames receive a movement somewhat similar to the figure 8, which is very similar to the motion given in manual labour. One advantage of this machine is that two surfaces of glass are finished at one operation. The space between the two runner-frames B is found very convenient for applying the emery, and also ascertaining the progress of the work, without having to stop the machine.

3239.

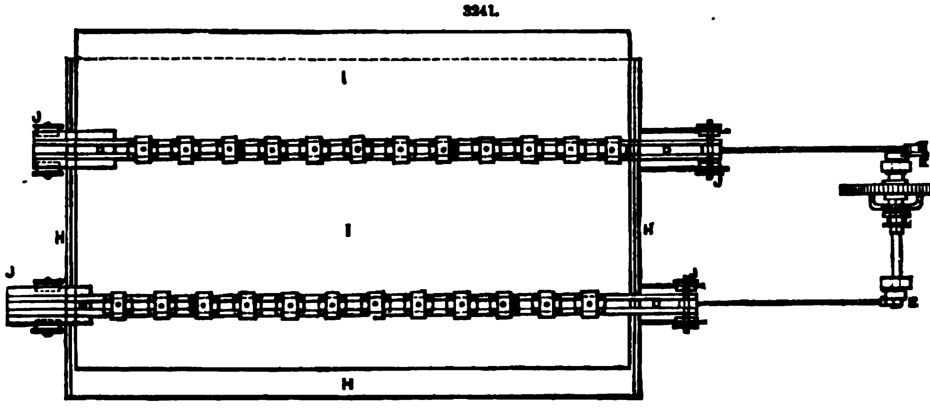


The machinery used in the polishing process remains the same in principle as that originally constructed for the purpose. Each machine consists of a strong cast-iron frame H, Figs. 3240, 2341, about 18 ft. long by 10 ft. wide, containing a series of small rollers, upon which is placed a

3240.



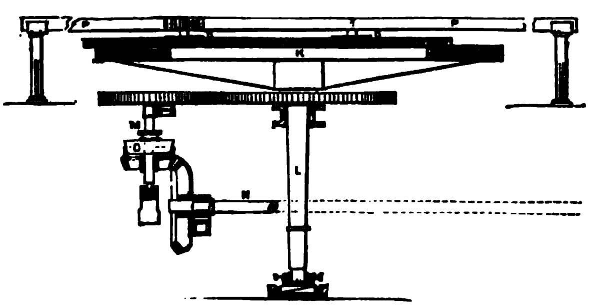
3241.



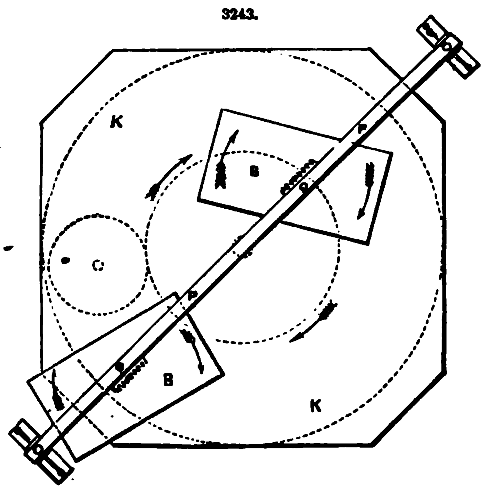
wooden table I, with two racks on the under-side; suitable gearing is connected to these racks, to give the table a slow alternate lateral motion so as to bring every part of the plate of glass under the action of the rubbers B. The plates of glass are fixed upon the table I by plaster of Paris, and the ends of the table move between slide-blocks secured to the main frame H, so as to prevent the action of the rubbers from displacing it. The rubber-blocks B are pieces of wood covered with felt, and provided with a central spindle and adjustable weights to regulate the amount of friction. A number of these blocks are secured to two movable bars D, running on rollers J J at each end of the table I, and driven by a short shaft E, with cranks at the ends set at right angles to each other. The rubber-blocks are thus worked transversely to the motion of the table; and by applying the polishing powder in a liquid state the surface of the glass is gradually brought up to the requisite degree of polish, both sides of the plate successively being subjected to the same operation.

About 1857 experiments were commenced at the British Plate-Glass Works at Ravenhead, with an entirely different class of machinery for grinding and smoothing plate glass, with the object of increasing the production, reducing the cost, and also completing the process of smoothing upon the same machine on which the glass is ground, so as to obviate the necessity of a separate machine for smoothing, and also save the expense and loss of time in removing and refixing the plates of glass. The new grinding and smoothing machine is shown in Figs. 3242, 3243, and consists of a revolving table K, 20 ft. diameter, fixed upon a strong cast-iron spindle L, and running at an average speed of twenty-five revolutions per minute, driven through an intermediate upright shaft M, from the main line of shafting N, by a pair of bevel-wheels, and friction cone O for throwing in and out of gear. This arrangement of gearing for driving the table was made by G. H. Daglish, and was adopted in order to obtain a long spindle L for the table, of a length equal to the semi-diameter of the table, and at the same time to keep the main line of shafting N continuous, for driving a series of tables in one room. Over the top of the table a strong timber bar P is fixed, about 10 in. from its surface; and on the two opposite sides of this bar are bolted two notched plates of cast iron, Q, one on each side of the centre of the table. The notches are for receiving the centre studs of the runner-frames B, which are very similar to those used on the old class of machinery; and the runners can thus readily be moved nearer to or farther from the centre of the table, as circumstances require, by shifting the stud into a different notch. The only motion which these runner-frames have is round their own centres, and this is given to them by the excess of friction on the side farthest from the centre of the table over that on the side nearest to the centre, this excess being caused by the greater velocity of the portion of the table farther from the centre. It is evident that the amount of grinding action is considerably greater on this machine than upon the old one, both from the increased velocity of the runner-frames themselves, and also from the double amount of movement obtained by the revolution of the table K and the runner-frames B. The idea of driving the runner-frames themselves, as well as the table, was conceived at an early stage of the experiments; but on being put to the test, it was found that the unaided movement of the runner-frames adapted itself to the work to be performed far better than any compulsory motion could do. It has also the advantage of leaving the surface of the table free and unencumbered with any machinery, and consequently facilitates the operation of laying and removing the plates of glass: the whole of the driving machinery is also covered over, and thus protected from the injurious effects of the sand and water thrown off from the edge of the table in working.

3242.



3243.



This machine has been found to answer equally well for smoothing as for grinding; and this is perhaps its most successful feature in a commercial and economical point of view. Both these processes are now completed on it at the Ravenhead Glass Works, the finishing portion of the smoothing operation alone being effected by manual labour for the reasons before stated. The plates of glass being generally oblong in form, it was found that the machine in its original shape, having a circular table K for carrying the glass, as shown by the dotted circle in Fig. 3243, entailed considerable waste in filling up the area of each table for grinding; and it was then determined to alter the shape to that of an unequal-sided octagon, or square with the corners taken off, as shown in the plan, Fig. 3243. No difficulty has been experienced in the process of grinding from this alteration in form, whilst the amount of waste in making up the tables has been considerably reduced, and greater facilities are obtained for grinding large plates. The amount of wear and tear on this machine has been found to be very small in comparison with the old machines, owing to the small number of working parts, the large extent of bearing surface, the smoothness of the motion, and the complete balancing of the table. The quantity of glass finished upon one of these machines a week is from 1200 to 1500 sq. ft., which is about one-third more than the old machines are capable of doing, due allowance being made for the difference of area.

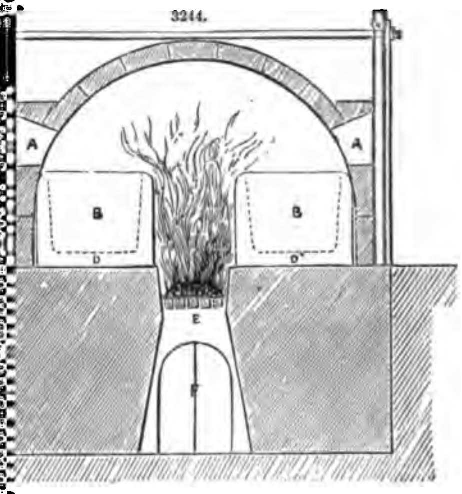
The Mechanical Appliances employed in the Manufacture of Polished Sheet Glass.—Richard Pilkington, jun., observed, in the Proceedings of Inst. M. E., 1863, that the manufacture of British sheet

…1832, by Messrs. Chance Brothers, of Birming- …having almost superseded crown glass, in conse- …ge squares at present required for windows, and … so much distorted in crown glass. The average …ired it can be made much larger; whilst with … square as large as 34 in. by 22 in. Sheet glass, … appearance when viewed from the outside of a …e, an eyesore partially obviated by the improved …e glass is polished. When polished it is known …n British plate. This polished sheet plate has …ier and more difficult to scratch, besides taking

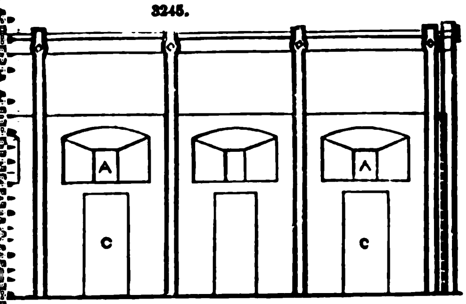
… of the three following processes;—1st, melting

…uired, one for melting the materials or frit, and … it into a cylindrical form. The melting furnace …g a high temperature with great uniformity and …m the fuel. The furnace, an eight-pot one, is …ring holes, or working holes A A on each side

3244.



3245.

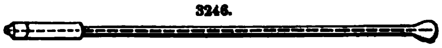


…ts is a matter of special importance, and they are …hich, when thoroughly tempered, is formed into …r upon layer into a solid mass, free from cavities …eight inside, 5 ft. diameter at top, and about 4½ ft. …d about 25 cwt., and containing about 22 cwt. of … any particles of foreign matter or dirt from getting …ot would not last its time, but would most likely …rature. After being made, a pot remains in the …aintained at 60° Fahr., and it is then removed to a … of 90° until it is wanted. When required for use, … heat is gradually increased to that of the melting …as quickly as possible, by means of a carriage or a … fire-grate. This operation is repeated until all … furnace ends are now closed, with the exception of … cullet, or broken glass, is put into each pot, and

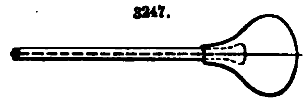
when melted is ladled so as to run down over the interior surface of each pot, after which the heat is increased for a short time. The pots are thereby glazed, and are now ready to receive the material to be melted.

The quantity of raw material, or frit, allotted to each pot is filled into it in three or four charges, allowing a sufficient interval of time to elapse between each charge to ensure the previous one being melted. About sixteen hours of intense heat are required to melt the entire quantity, during which time the fluid metal boils violently, and before it can be worked requires cooling, which takes about eight hours. Whilst cooling, the small bubbles of air arising from the boiling of the metal ascend and pass away, leaving the metal clear, excepting the surface, which is coated with impurities from the frit, from the roof of the furnace, and from the dust of the fuel, all of which must be removed before commencing work. Inside each pot, and floating upon the surface of the metal, is an annular ring, made of fire-clay, 2 in. thick, having an internal diameter of 18 in.: this inner space of 18 in. diameter is cleaned, instead of the entire surface of the metal, thereby saving both time and material. The cleaning or skimming is performed by means of a light iron rod, chisel pointed, which being warmed the metal adheres to it; and this process is repeated whenever any impurities are perceived upon the surface of the metal. The surface of the melted metal being cleaned, the workman dips into it the blow-pipe, Fig. 3246, having previously warmed the nose end of the pipe. Withdrawing 2 or 3 lbs. of the metal, he allows it to cool to a dull red, and then dips the pipe again; collecting by degrees in this way, as shown in Fig. 3247, a sufficient quantity to produce a given-sized sheet of glass, which on the average would weigh about 20 lbs. Then, while cooling the pipe he continually turns it round, drawing it towards himself, and in so doing forces the metal beyond the nose end of the pipe by means of the forked rest in which the pipe revolves, as shown in Fig. 3248, leaving as little metal as possible upon the pipe. The blower now takes the pipe, and places the red-hot mass in a hollowed wooden block upon the ground, Fig. 3249, keeping the pipe in a horizontal position whilst revolving it, thereby producing a solid

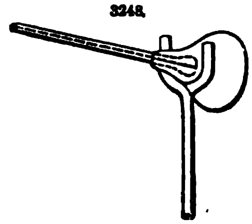
3246.



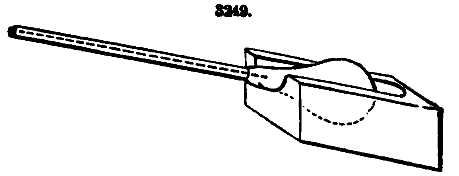
3247.



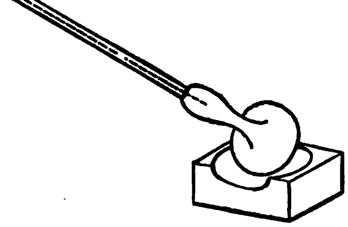
3248.



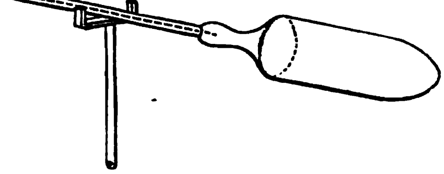
3249.



3250.

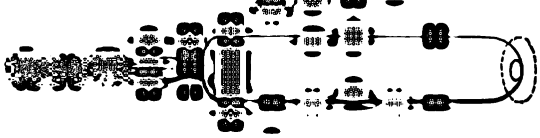


3251.

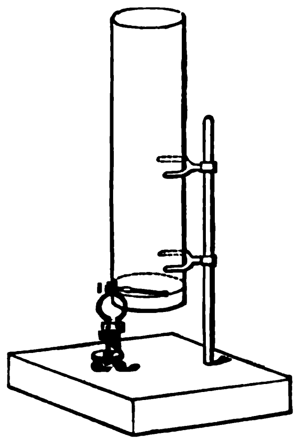


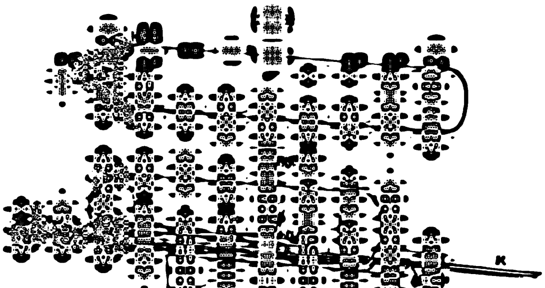
cylindrical mass of metal. During this process his assistant allows a fine stream of cold water to run into the block from a sponge, keeping the wood from being burnt, and giving a brilliant surface to the glass. He next raises the pipe to an angle of about 75°, and blows until he has produced a hollow pear-shaped mass, Fig. 3250, with its largest diameter the same as that of the finished cylinder. During this operation his assistant keeps the block wet, and a second block is generally used when commencing the blowing. The glass now requires reheating, which is done at a furnace built of ordinary brickwork in an oblong form, its dimensions being determined by the number of blowers intended to work at it, generally four, five, or six at each side. The ground at each side of this furnace is excavated to a depth of about 7 ft., a width of about 16 ft., and the same length as the furnace; and over each of these spaces four, five, or six wooden stages are erected, at distances of about 2 ft. apart. Having reheated the glass, the blower repeatedly blows to maintain the cylinder of equal diameter throughout, whilst lengthening it by swinging it backwards and forwards in the 2-ft. space, and occasionally swinging it round over his head, until a cylindrical piece of glass is produced, Fig. 3251, about 11 in. diameter and about 50 in. long, closed at one end, and having the blow-pipe attached to the other end. The blower first opens the closed end as follows: enclosing as much air as possible within the cylinder, and stopping the mouthpiece of the pipe with his hand, he exposes the end of the cylinder to the heat of the furnace, which, whilst softening the glass at the end, expands the contained air to such an extent that a small hole is burst in the glass, as in

… the pipe quickly, and when flashed the end of … by keeping the pipe in a vertical position for a … its shape. The cylinder is then placed upon … cold iron the pear-shaped neck near the pipe-nose … ntinued round the neck by gently striking the … in Fig. 3253. The cylinder has now one end of … 3 in. diameter, Fig. 3253, and must therefore be … cylinder having become cold whilst remaining on … of metal upon the end of an iron rod, and draws it … ans of a pair of pincers. This thread he passes … 253, and after it has remained on a few moments … heated part, and the sudden contraction causes … cylinder about 45 in. long and 11 in. diameter.



3254.

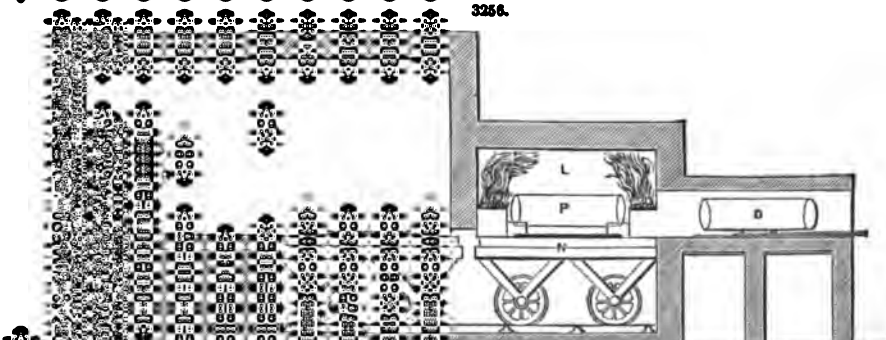




… from the cylinder thus obtained forms the second … accomplished as follows;—The end of the cylinder … meter, and the thickness of metal much reduced, it … n that end. For this purpose the cylinder is sup… e, as shown in Fig. 3254, over a small horizontal … uced between the jaws of the small cutting instru… ting diamond is pressed by a spring against the … ly pushing the instrument forwards round the … ls, the end of the cylinder is cut off perfectly true. … lly, which is accomplished by placing it in a hori… Fig. 3255, and a diamond fixed in the cleft of a … from end to end, guided by the straight-edge K, a … pposite directions, at the diamond cut, to complete …

… kiln, Fig. 3256, which consists of two furnaces built … r, M, for annealing, the former being maintained …

3256.



… A portion of the bottom of the flattening kiln L, … e flattened, is supported upon a carriage N, which … he annealing kiln M, this plan being a very great

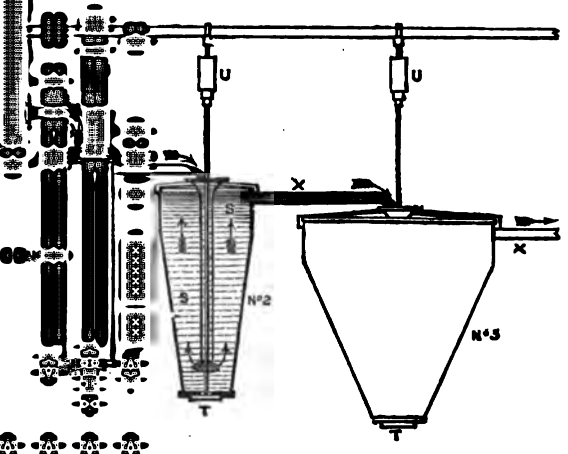
improvement over the old method of pushing the flattened sheet whilst in a soft state. The movable bed N is either of clay or stone, and by careful work is made as true as possible; upon this a sheet of glass is first flattened and left there to flatten others upon, in order to obtain sheet glass with as true a surface as possible. The split cylinder to be flattened is gradually introduced into the flattening kiln, being placed first at O and then at P, and when sufficiently warmed is placed upon the glass bed N, with its split side uppermost; the heat soon softens it, so that with a slight assistance from the workman it lies down nearly flat on the bed N, and the sheet is afterwards carefully rubbed as flat as possible with a piece of wood fixed to the end of an iron rod. The movable bed N is now pushed forwards into the annealing kiln M, as shown by the dotted lines, and after placing another cylinder to warm at O and P, the workman removes the flattened sheet from the carriage N by means of a tool like a fork, and places it upon a prepared part of the floor of the annealing kiln M, to stiffen previous to piling it. The carriage N is now returned to the flattening kiln L, and the flattening operation repeated till the carriage again appears in the annealing kiln M. The previously-flattened sheet is first piled on its end against one side of the kiln at R, and then the last flattened sheet is removed off the carriage N, and left to cool on the floor of the annealing kiln, like the previous sheet. This flattening process is continued until the annealing kiln M is filled, when it is closed up, and allowed to cool, generally from twenty-four to thirty-six hours, the time being regulated by the thickness of the glass. On the completion of the cooling, the kiln M is opened, and the sheets of glass are taken to the warehouse, where they are sorted to suit various purposes, a very large portion being packed and sent away without undergoing any further process.

3. *Polishing.*—The sheets intended to be polished are now selected, and pass through the third process of the manufacture to produce polished sheet plate. Two processes are necessary for this purpose, smoothing and polishing.

Smoothing consists in working two sheets of glass one upon the other, by hand, with emery and water between them; and as their surfaces become obscured, finer and finer *emery* is used until the surfaces are smoothed free from all defects. The apparatus used consists of a wooden bench, one half of which is 6 in. higher than the other; upon the former is placed a slab of slate about 1¼ in. thick, larger than the sheet of glass, having as true a surface as possible. Upon this slab a sheet of glass is laid, with a piece of wet calico between the surfaces of the glass and the slab; by exerting a gentle pressure upon the glass the air is expelled from between them, and the sheet of glass is consequently held down upon the slab by the whole atmospheric pressure upon its surface, which holds it so firmly that when the sheets have to be raised from the slab many are broken, even by experienced workmen. The wet calico is used in this case instead of plaster of Paris for bedding the sheet of glass upon the table. In consequence of the close adhesion caused by the atmospheric pressure when the surfaces of the two sheets of glass get so true as to become closely in contact, it is impossible to work two large sheets one upon the other with the finest emeries, and it therefore becomes necessary to perform the latter portion of the rubbing process with a small piece of glass, say about 10 in. by 5 in., until the process is completed. Both sides of the sheet of glass having been smoothed in this manner, and after a careful examination found free from defect, the sheet is then handed over to the polishing machine.

The perfection of the smoothing process is entirely dependent upon the purity of the emery, and the perfect uniformity of the grain in each successive quantity employed; and consequently a very perfect process of cleansing and sorting the emery is requisite. The ordinary ground emery contains, besides numerous degrees of fineness of grain, many impurities, which must be removed, and the good emery must also be accurately sorted into portions varying in size of grain from coarse to the finest. For every degree of fineness a separating vessel or cylinder is required; and taking No. 1 as the coarsest quality, that cylinder is made the smallest in the series, No. 2 cylinder about twice the capacity of No. 1, and No. 3 twice the capacity of No. 2; and so on throughout the required number of cylinders. The emery-sorting apparatus is shown in Fig. 3257, and consists of the required number of cylinders, fixed so that No. 1 cylinder is about 3 in. higher than No. 2, and No. 2 the same height above No. 3, and so on. The cylinders are made of copper, and inside each is fixed a copper funnel S, long enough to reach within 3 or 4 in. of the bottom of the cylinder; and in the bottom of the cylinder is a hole closed by a wooden plug, or a valve T, of about 3 or 4 in. diameter, which is held up by the rod and spring balance U. The action of the apparatus is as follows;—A supply of water being maintained by the cistern V, a constant stream is delivered by means of the tap W into the funnel of No. 1 cylinder; the water descends through this funnel to the bottom, and ascends through the annular space to the top of the cylinder, whence it is conveyed by the spout X and poured down the funnel of No. 2 cylinder, ascending in the annular space of No. 2, and passing by the spout to No. 3 funnel; this is repeated as often as there are cylinders, and from the last and largest cylinder the overflow is carried to a drain. When the stream of water is running through all the cylinders, and also passing away at the overflow, the powdered *emery* to be cleansed and sorted is sprinkled into the funnel of No. 1 cylinder, and this is continued until enough has been fed to fill up to within ½ in. of the bottom of the funnel. No. 1 being the smallest cylinder, the current of water through it will be the fastest, and the grains of emery left behind in this cylinder will consequently be the coarsest. The feeding of the emery is then stopped for a short time, and the stream allowed to continue until the water is running quite clear into the funnel of No. 2 cylinder. The valve T at the bottom of No. 1 cylinder is now opened, allowing the emery and water to fall into a vessel placed beneath to receive it; and as soon as the stream of water is again running through all the cylinders and passing away at the overflow, more emery is again sprinkled into No. 1 funnel. The succeeding cylinders are emptied in the same way, as they respectively become filled with the finer sorts of emery. The beauty of this process is the simplicity of apparatus required, and the certainty of always obtaining an exact repetition of the several degrees of fineness in the respective cylinders. It will be observed that, in consequence of the cylinders increasing successively in capacity, the current of water ascending in the annular

...tion; consequently, the emery deposited in each
...deposited in the previous one.



... the polishing blocks, and working lengthways
... the sheet of glass is laid, which is made to travel
... bars. The polishing blocks are worked at about
... ying them are supported upon rollers at a height
... le is worked similarly to the table of a planing
... by a reversing motion similar to that of a planing-
... obtain a good polished surface the polishing blocks
... e surface. Upon the moving table are fastened
... tes, upon which the sheets of glass to be polished
... as been polished, the glass is taken up and relaid.
... ks are about 5 in. square, covered with felt, and
... er used in polishing is red oxide of iron, obtained
... furnace to a dark red when cold, and it is then
... cutting grain of this material is about the hardest
... examination by the microscope.
... ostances endowed with a certain degree of trans-
... cture called *vitreous*. In this point of view, many
... allize easily, such as phosphoric and boracic acids,
... ion parlance the name *glass* is exclusively applied
... d when hot by blowing, and which are unchange-
... double silicate of lime and potassa or soda. In
... ates are partly replaced by very fusible metallic
... xide of lead is also substituted for the lime. This

... ion of the various kinds of glass used in the arts,
... have hitherto done, the properties of the simple

... he manufacture of glass are the silicates of potassa
... eir degrees of fusibility greatly varying, however,
... s clearly the composition of the simple or multiple
... f the silicic acid and that of the united bases is
... he quantities of oxygen contained in the several
... times its weight of potassa or soda, a substance is
... red heat, and completely soluble in cold water.
... r soda, also produces a homogeneous substance,
... in water. As the proportion of alkali diminishes,
...: an alkaline silicate, in which the oxygen of the
... only at the highest temperature of a forge-fire.
... melting together, in an earthen crucible, 15 parts
... rcoal. This substance, treated with cold water,
... d with the carbonate of potassa, but is itself com-
... of boiling water. It has been proposed to use this
... trical decorations, incombustible. In Germany,
... *ias*, a large manufactory of which is at Prague, is
... e wooden work of buildings incombustible, and
... position (rotting). In England it is used for the

same purpose, made up with various pigments, as *silica colours*. It probably would also make an excellent artificial marble, capable of being moulded into architectural ornaments, or spread as a plaster on walls, when made up with proper proportions of porcelain clay, or, perhaps, even chalk or plaster of Paris, with a slight admixture of borax. It was first obtained by Fuchs, at Munich. In fact, if a coat of this solution be applied to any stuff, it remains covered, after drying, with a transparent and fusible varnish, which preserves it from the air; and it burns with difficulty, because the silicate prevents the access of the air. The stuff merely carbonizes, and does not favour the progress of the fire, as would be the case if its surface were free. Many fusible and non-efflorescent salts, among which are the phosphate and borate of ammonia, would produce the same effect. The silicates of potassa and soda are distinguished by the property of not crystallizing on cooling after fusion, owing to their passing from the state of perfect liquidity to that of a solid, not suddenly, but through all the intermediate doughy conditions. This property accompanies the alkaline silicates in their combination with the other metallic silicates, and is very important, as it facilitates the working of these multiple silicates by blowing; and, moreover, the substance retains its transparency after cooling.

Silicates of Lime.—The silicates of lime melt at only very high temperatures. The most fusible compound is that resulting from the union of silicic acid with lime, in such proportions that the oxygen of the lime is to that of the silicic acid as 1 : 3; this silicate melts in a strong forge-fire, and becomes crystalline on cooling. The silicates of lime, having a ratio of 1 : 4 or 1 : 1 between the oxygen of the base and that of the acid, do not fuse completely, only softening in the highest heat that can be produced in a forge-fire.

Silicates of Magnesia.—The silicates of magnesia are as difficult of fusion as those of lime. The most fusible is that of which the formula is MgO, SiO_2; it melts in a strong forge-fire.

Silicates of Alumina.—The silicates of alumina are still more infusible than those of lime and magnesia. The silicate $Al_2O_3, 3SiO_2$, which appears the most fusible, merely softens in a forge-fire. All these silicates melt easily in the oxyhydrogen blow-pipe; for we know that alumina and silex melt separately in the powerful heat produced by this apparatus.

Silicates of the Protoxide of Iron and Manganese.—These silicates, which enter into the composition of some kinds of glass, melt much more readily than the silicates of the earths and those of the alkaline earths. The silicates FeO, SiO_2 and MnO, SiO_2 may be melted in the common furnaces of our laboratories; they all crystallize easily by slow cooling.

Silicates of Lead.—The silicates of lead are fusible in proportion to the quantity of oxide of lead they contain; that showing the composition PbO, SiO_2 melts at a strong red-heat. The silicates of lead crystallize with difficulty; the cooling must take place very slowly, in order to obtain any indices of crystallization in the mass.

Multiple Silicates, formed by the Alkalies, the Alkaline Earths, the Earths, and Metallic Oxides.—Several multiple silicates, in the form of beautiful crystals, are found in nature. We know that feldspar is a double silicate of alumina and potassa, of the formula $KO, SiO_2 + Al_2O_3, 3SiO_2$. This mineral melts in a forge-fire, and does not crystallize during the very slow cooling of a porcelain furnace; but crystals of this compound have been found in the fissures of iron blast-furnaces, showing the same form as those of native feldspar. When the alkaline silicates are melted with other metallic silicates, vitreous substances are generally obtained after cooling, which appear homogeneous, and crystallize only when the cooling is extremely slow. But it is difficult to decide whether these substances are formed by a homogeneous chemical combination, or whether they merely result from a solution of various silicates in each other; a solution which has set in mass, without crystallizing during the process of cooling. The temperature at which a multiple silicate fuses is almost always below the medium temperature of fusion of the various simple silicates which compose it; sometimes it is even below that of the most fusible silicate entering into the combination. Thus, the simple silicates of alumina and lime are nearly infusible in our forge-fires, but they form, when combined, double silicates which readily melt in these fires. By adding to a silicate which crystallizes easily on cooling one which has not this tendency, for example, an alkaline silicate, double silicates are obtained, which crystallize with great difficulty, and preserve their vitreous appearance after cooling. Thus, the double silicates of potassa or soda, combined with those of lime or oxide of iron, do not crystallize after fusion. Silicate of alumina likewise opposes the crystallization of the multiple silicates into which it enters, although less effectually than the alkaline silicates. The silicates of potassa and soda lose by volatilization a large proportion of their bases. Thus, it may be explained how the multiple silicates containing alkaline silicates become less and less fusible as these are allowed to remain for a longer time in furnaces at a very high temperature, and acquire, with time, the property of crystallizing by slow cooling, at the same time losing their vitreous appearance.

We have seen that the alkaline silicates which contain a large proportion of alkali are soluble in water. When they contain more silex, they are not attacked by this fluid, but they may be by powerful acids; but when they are still richer in silex, even acids do not affect them. The silicates of lime, alumina, and oxide of lead are attacked by acids when they contain a large proportion of base, but they are intangible when rich in silex. Fluohydric acid, however, decomposes every silicate, whatever proportion of silicic acid it may contain, for it attacks quartz itself. By combining the alkaline silicates with silicate of lime, double silicates are obtained sufficiently fusible to be worked by blowing, and nevertheless containing enough silicic acid to resist the action of acids.

We shall divide the various kinds of glass into three grand classes:—

1st. Common colourless glass, which is a double silicate of lime and potassa or soda.

2nd. Common coloured glass, or bottle glass, a multiple silicate of lime, oxide of iron, alumina, and potassa or soda.

3rd. Crystal, which is a double silicate of potassa and oxide of lead.

1st. *Colourless Glass.*—Common colourless or white glass, which is used for making tumblers,

window glass, and looking-glasses, is a double silicate of lime and potassa or soda, either of these being preferred according to its price. Carbonate of soda being much cheaper in France than carbonate of potassa, is almost exclusively employed in the manufacture of white glass; in Germany and the north of Europe the potassa, being cheaper, is preferred. The selection of these bases is not a matter of indifference. Soda yields a more fusible and easily-worked glass, but it is always more or less coloured by a greenish-yellow tinge, not perceptible when the glass is very thin, but very decided when it is thicker, as, for example, in a window-pane.

The most beautiful glass having a base of potassa and lime is the Bohemian. This glass, made with the utmost care from choice materials, is remarkable for its lightness, its brilliant transparency, and permanency. The ratio between the oxygen of the silicic acid and that of the bases is as 4 : 1, sometimes rising to 6 : 1; the oxygen of the lime is to that of the potassa as 1 : $\frac{3}{8}$ in the most esteemed tumbler-glass of Bohemia. This proportion is as 1 : 1 in the glass used for mirrors, in which great fusibility is required. The proportion of silex is increased in order to make hard and infusible glass; in this way the Bohemian glass tubes for chemical purposes are made, as they are much less fusible than the French glass, and therefore preferable for organic analysis. The silex used in Bohemia is the hyalin quartz of the old rocks, found in the form of large pebbles in the fields or the beds of the mountain streams. This quartz is heated to a strong red-heat in a reverberatory furnace, and then thrown into cold water, by which it becomes very friable, and is then, without difficulty, finely powdered by stampers, or ground by edge-stones. The carbonate of potassa used in the manufacture of Bohemian glass is the refined carbonate; nevertheless, this salt is never pure, some carbonate of soda always being mixed with it. The crude potashes are carefully selected and refined by solution: the crude potash, on being treated with one-half its weight of water, leaves the foreign salts, as well as a considerable quantity of carbonate of potassa, as a residue. The solution yields, when evaporated, potassa for the manufacture of first-quality glass, while the remainder serves for that of an inferior quality. The lime is obtained by subjecting a very pure and often perfectly white saccharoid carbonate of lime to calcination in a reverberatory furnace.

When these materials, however carefully they may have been selected, contain a small quantity of protoxide of iron, a greenish tinge, which greatly lessens its commercial value, is imparted to the glass. This discoloration is remarkably destroyed by adding to the mixture a small quantity of peroxide of manganese. The protoxide of iron imparts a deep green colour to glass, when present in any quantity; but, if converted into a sesquioxide, it gives a scarcely perceptible yellow tinge. Sesquioxide of manganese colours the glass violet; but a corresponding quantity of protoxide scarcely produces a sensible change. If, therefore, to a mixture to which protoxide of iron would give a high colour, a quantity of peroxide of manganese sufficient to transform the protoxide of iron into a sesquioxide, by passing itself into the state of a protoxide of manganese, is added, a nearly white glass is obtained; for the colour it then has is due only to the sesquioxide of iron, which produces a scarcely perceptible yellow tinge, the protoxide of manganese effecting no colouring at all. But it is important not to use an excess of peroxide of manganese, because the glass would have a violet shade, owing to the formation of sesquioxide of manganese. Peroxide of manganese, on account of this special use, is called the *glass-maker's soap*. Frequently, also, a small quantity of arsenious acid is added to the mixture: as this acid is completely volatilized during the melting of the glass, none of it remains in the objects manufactured: its object is merely to render the mixture more homogeneous, or to facilitate the *refining* of the glass. By volatilizing at a high temperature, it forms bubbles of gas, which, on traversing the fluid mass, mix its several particles together, and precipitate the solid material scattered through it.

The fuel used in Bohemia is a resinous wood, burning with a bright flame, and causing a very rapid fusion. The air of the furnace being always oxidizing, no alteration of the glass need be feared by the carbonaceous dust or other particles contained in the smoke. An admixture of carbon would considerably injure the quality of the glass, and discolour it; but when it exists in small quantity, the glass assumes a beautiful yellow colour. These coloured glasses are often made expressly. When it is present in somewhat greater quantity, the glass assumes a purple-red colour. Peroxide of manganese opposes also this discoloration of glass by carbon, an accident which frequently happens when the furnace has no proper draught. In some glass-houses, it is prevented by the addition of a small quantity of nitrate of potassa.

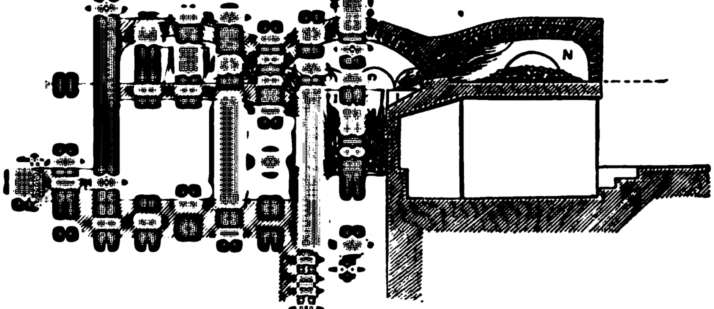
A white glass of first quality is made by melting together 110 parts of pulverized quartz, 64 parts of refined carbonate of potassa, and 24 parts of caustic lime.

In other glass factories in Bohemia beautiful tumbler-glass is made of a mixture of 120 parts of pulverized quartz, 60 parts of refined carbonate of potassa, 25 parts of caustic lime, $\frac{1}{2}$ part of arsenious acid, 2 parts of peroxide of manganese, and 2 parts of nitre.

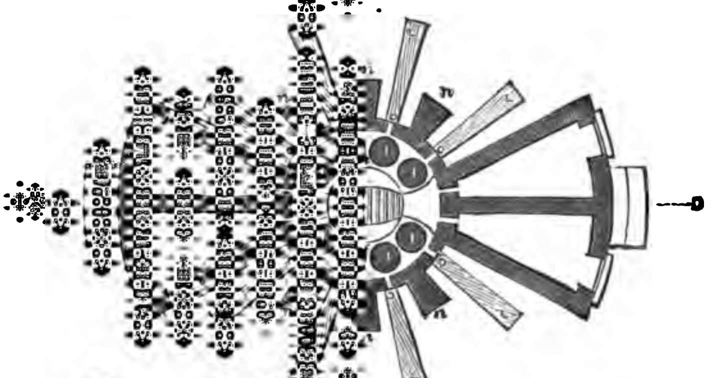
First-quality white glass is made in France of white quartzose sand, artificial soda, quicklime, and a certain proportion of fragments of glass: in this glass the ratio of the oxygen of the silicic acid to that of the united bases is ordinarily as 4 : 1. This composition gives an easily fusible but slightly tender glass. When a harder glass is desired, the proportion of silicic acid is increased. A fine sand, as white as possible, is selected, and sometimes made more friable by heating it to redness, and throwing it in that state into cold water. The sands from Aumont, near Senlis, from Etampes and Fontainebleau, are highly esteemed, and are exclusively used in the glass factories in the environs of Paris. The lime is obtained from a limestone as pure as possible, and previously calcined in an oven to drive off the carbonic acid; it is then exposed to the air, and falls to dust. It is sometimes used in the state of carbonate of lime, finely powdered. Very white chalk, as that from Bougival, near Paris, is perfectly adapted to this purpose. For first-quality white glass, the carbonate of soda obtained in the manufacture of artificial soda is used. For the inferior qualities, sulphate of soda, which is cheaper than the carbonate, is substituted; but as the sulphate of soda is decomposed by silicic acid only at a very high temperature, at which the crucibles would soon be destroyed, a certain quantity of charcoal is added: this facilitates its decomposition, by abstracting

...us causing it to pass into the state of sulphurous
...feeble. One part of charcoal is generally mixed

...preliminary calcination called *frit*, before being
...the combination, and at the same time to allow
...pots already heated to redness. The breaking of
...fusion is more rapid.
...r the manufacture of window glass. Fig. 3259
...line A B of Fig. 3258; Fig. 3258 represents a
...line C D of Fig. 3259. The oven is composed



3259.



...he grate G above the ash-hole. On each side of
...k, on which the pots I I are placed; the pots are
...wall of the oven, which are subsequently closed
...rved above each pot, large enough to allow the
...the oven the object to be manufactured. The
...oven M; it is then conducted by openings into
...preliminary preparation is made, the *frit* of the
...e kept for a long while before introducing them
...repare them to bear the high temperature of the
...nd smoke, having passed through the ovens N,
...workmen, a master glass-blower and an assistant.
...bridge L, raised from 1 to 1¼ mètre above the
...d handle the pieces he is about to blow. Small
...pot, in order that the blower may not be incon-
...hole.

...f the melting pots; only the most infusible clays
...ASTING AND FOUNDING. They are generally $0^{m} \cdot 7$
...500 kilogrammes of melted material. The pots
...n hot rooms, so as to dry slowly. They are then
...rature of which is not very high, and are gradually
...arch where the heat is greatest. They are intro-
...ving been subjected to a very high temperature.
...rarely necessary to replace all the pots of a furnace
...ear out, and a sufficient supply should always be
...stroyed.

...omposed of 100 parts of sand; 35 to 40 parts of
...an equivalent quantity of a mixture of sulphate

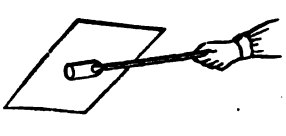
of soda and charcoal; and 50 to 150 parts of broken glass, or *cullet*. These materials, intimately mixed, are set to frit in an arch of the furnace, where they are turned from time to time, in order to render the mixture more uniform. The fire on the grate is made to burn actively after the working holes of the furnace have been closed. The workman deposits the frit in the pots, removing it red hot from the arch with a shovel; after the addition of each shovelful he waits until the material is melted before adding another, and so on until the pot is filled. He then leaves it to itself for several hours, in order to clear it of bubbles of air and foreign substances which rise to the surface. These substances, called *glassgall* (also called *sandiver* or, commonly, *salts*), are formed by alkaline salts in excess, which have not been decomposed by the silicic acid; they are particularly numerous when impure carbonate of soda has been used, or when a mixture of sulphate of soda and charcoal has been substituted for it. The workman generally removes them with an iron ladle. From time to time he extracts a small quantity of melted glass, and judges of its quality by its appearance after solidification.

When the glass is sufficiently fused, the temperature of the furnace is lowered, in order to bring the glass to a consistency fit for working. We shall not attempt to describe the processes of glass blowing in detail, but merely that adopted in France for the making of window glass.

The *pipe*, Fig. 3260, is the principal tool of the master-blower. It is an iron tube, $1^{m}\cdot50$ in length, having a perforation through its long axis of 3 millimètres in diameter; it is covered externally, to a distance of about 35 centimètres, by a wooden tube *c d*, to protect the workman's hand from the intense heat. At the end of each bridge L, Fig. 3259, is a small platform, of the height of $0^{m}\cdot65$, protected by an iron plate, called the *marver*, on which the workman moulds the doughy glass, Fig. 3261, adhering to the end of the pipe into the proper shape for blowing. Near the *marver* is a wooden block, containing several hemispherical or pear-shaped cavities, which are kept constantly moist. The pipes are heated in a small opening at the base of the furnace. The workman, taking one, dips it into the glass, collects a certain quantity, withdraws it, and turns it so that the fluid glass may not separate, then collects an additional quantity, and hands the pipe thus charged to the master-blower. The latter, having received it, rests it on the iron platform, always turning it, dips it again into the pot, and then returns quickly to the platform with a mass of red-hot glass, and rests it, still keeping up the rotary motion, in the water which fills the cavity of the block. He then draws the greater portion of the glass which envelops the sides toward the end of the pipe, by means of a sheet-iron blade, Fig. 3262. The mass of glass, cooled by the water, but adhering to the end of the pipe, is carried back to the working hole to be softened. When the workman thinks it is soft enough, he withdraws the pipe, and recommences the same manipulation in the water, but at the same time blows in the pipe, so as to give the glass the shape of a sphere

3260.

3261.



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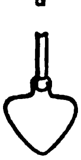

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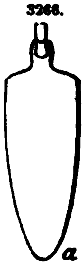

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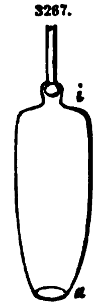

of about 3 decimètres in diameter, Fig. 3263, and then suddenly lifts the pipe into the air, and blows the sphere above his head. The upper part of the sphere then sinks by its own weight, and the bulb spreads horizontally, Fig. 3264. By suddenly dipping the pipe, the sphere assumes the shape of Fig. 3265. The workman then swings the pipe backward and forward, like the pendulum of a clock, blowing from time to time through the pipe while making this movement, so that, by the simultaneous action of weight and blowing, the glass balloon elongates and assumes the shape of a cylinder, Fig. 3266. The glass cylinder can rarely be brought to the proper dimensions by one operation, but generally must be heated several times in the oven. When the cylinder is finished the master-blower rests the pipe on a portable hook which the assistant arranges in the direction of the working hole; and introducing the cylinder into the furnace so that its end becomes excessively heated, blows through the pipe with the whole force of his lungs, until the cylinder is pierced. The piercing of the cylinder is also often effected in another manner. The assistant fastens, by means of a pipe, a small quantity of very hot glass to the extremity *o* of the cylinder; this end the workman dips into the oven, and blows forcibly through the pipe, or simply stops its orifice with his finger. The pressure of the internal air bursts the end *o*, where the glass has been softened by the drop of hot glass, Fig. 3267. The workman then removes the cylinder from the

nvexity of the cylinder, so as to open it entirely,
great rapidity, either by swinging it or causing it
glass rapidly, at the
becoming mis-shaped.
to the assistant, who,
drop of water, taken
the pipe and cylinder,
detaches the cylinder.
ndow glass; but, being
this end must also be
e workman applies to
the size of the pane is
from the pot, with an
wing out; by applying
b of the cylinder, Fig.
division is immediately

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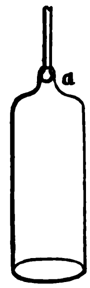

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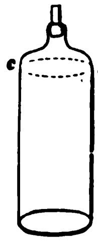
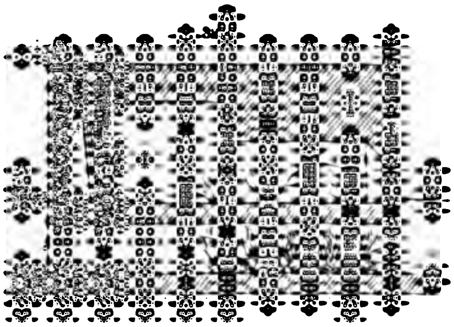


lattening furnace, Figs.
s V, U, separated only by a very small thin brick
this partition wall is an opening *i i*, of 1 mètre in
ving for the passage of the panes, which having
ated and slowly cooled in the chamber U. Both
e cylinders to be flattened are laid on a table; a



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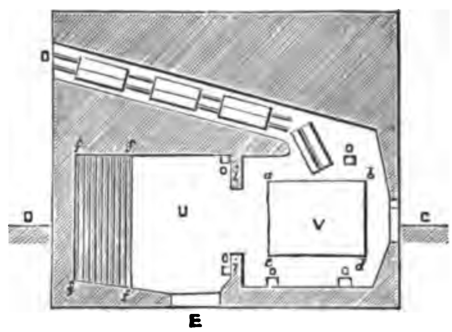
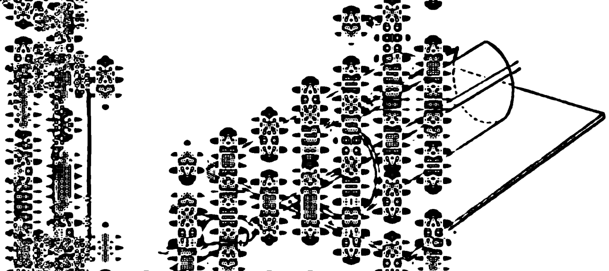


g. 3272, followed by a red-hot iron, which effects a
r this the cylinders are presented to the opening
dually introduced into it by means of two grooves,
too sudden heating, which might crack them.
about bending on themselves, he takes the hottest
middle of the furnace, near the flatting plate V,
ometimes of thick plate glass, dusted with a little
mmediately in front of the longitudinal opening *i i*,
baking furnace U; its upper surface should also
ce, so that the pane of glass may meet with no
reached the plate, the workman, armed with his
sides, which yield readily to the weight of the
Fig. 3274, terminating in a highly-polished piece,
sses it rapidly over the surface, so as to flatten it



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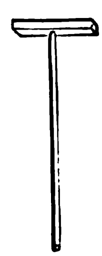

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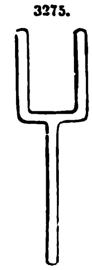

gh the longitudinal opening *i i* into the second
lower; a workman passes beneath it a thin iron
g the pane, which is already firm enough not to
iron bar *f f*, Fig. 3271, which passes through the

whole length of the oven. A number of panes are thus heaped on each other, until the workman deems it sufficient. A second horizontal bar is then arranged, on which additional panes are disposed, and so on until the compartment U is nearly filled. The furnace is then allowed to cool: and the glass, when withdrawn, is ready for sale. Clock-shades, decanters, tumblers, &c., are made of the same glass. Inferior glass articles, such as common window glass, apothecaries' phials, &c., are made of less pure materials; they are commonly coloured green by protoxide of iron.

In France, the base of plate glass is a mixture of soda and lime, and the oxygen of the silicic acid is to that of the united bases as 6 : 1. For the same quantity of lime, a quantity of carbonate of soda is added double of that contained in window glass, because it is necessary to give greater fusibility to plate glass. In the plate glass factory of Saint-Gobain, which is the largest in France, the mixture is made of 300 parts of very white quartzose sand, 100 parts of dried carbonate of soda, 43 parts of lime slaked in the air (fallen lime), and 300 parts of cullet. The materials are most carefully selected and purified; for it is essential to obtain as white and perfect a glass as possible. Melting furnaces similar to those described are used, but they are always heated by wood. The material passes successively into two pots; it is first melted in a conical one, into which it is gradually poured until the pot is nearly filled. This fusion requires fifteen to sixteen hours; it is then allowed to fine, by rest, at a high temperature. Workmen then remove the liquid glass with copper ladles, and transfer it to smaller square pots, called *cuvettes*, placed in the furnace on the same shelf and alongside of the melting pots. When the transfer has been effected, the working holes are closed, to restore fluidity to the glass; the cuvettes are then removed on a peculiar kind of cart, and brought above a very smooth bronze table, previously heated by red-hot coals laid thereon. The fluid glass is poured on this table, spread out, and smoothed by means of a cylinder or roller; when cooled it is placed in a furnace and again heated, in order that it may easily bear changes of temperature. It is then divided into pieces of the requisite size, leaving out the defective portions, and *polished*, by fixing the glass on a stone table with plaster, and rubbing it with quartzose sand, by means of a second piece of glass smaller than the first. In making large glasses, several pieces, set in motion by a machine, are used at once. The surface of the glass thus becomes perfectly smooth, and is *rough-ground*, but as yet unpolished. The final *polish* is given by rubbing the surface first with finer emery, diluted with water, and then rubbing it with colcothar, also diluted with water, by means of heavy polishers covered with felt.

2. *Bottle Glass.*—Bottles are made of cheap materials, because it is important that their price should be low, and the peculiar colour is not a matter of much importance. The most ochreous sands are frequently preferred, because the oxide of iron they contain imparts fusibility to the glass. Pure alkaline carbonates being too expensive, the alkaline material is furnished by the crude sea-soda and wood ashes. A considerable portion of washed ashes, called *spent ashes*, is added, which introduces the silicates of alumina and potassa. Lastly, a large quantity of cullet is poured into the mixture. In bottle glass, the oxygen of the silicic acid is double or treble that of the united bases. The following is the composition of a mixture used for bottle glass;—

Ochreous sand	100	Spent ashes	150 to 180
Soda from seaweed ..	40 to 60	Ochreous clay	80 „ 100
Fresh ashes	30 „ 40	Cullet	100 „ 150

Bottle glass is of various colours. That of French bottles is a deep green, owing to protoxide of iron; those made in certain parts of Germany have a brownish-yellow hue, produced by a mixture of the sesquioxides of iron and manganese. Bottle-glass furnaces generally contain six pots of the largest size. The fusion should be rapid, to economize the fuel. The pots being entirely filled with the mixture, the fire is stirred up to effect the fusion, and when the material is liquid, a fresh quantity is added; seven or eight hours are required thus to fill the pots with melted glass, after which the work is begun immediately, the sandiver first being removed. The furnace is allowed to cool until the material has acquired the degree of consistency proper for working.

The pipes having been heated in the holes at the bottom of the furnace, an assistant dips one into the melted glass, collecting as much of it as he can, and withdraws it by a continuous rotary motion. When the glass has become sufficiently consistent not to bend on itself, he collects some more, and so on; when he has gathered enough to finish a bottle, he passes it to the blower, who applies the glass to the left face of the marver, turning the pipe constantly, in order to fashion the neck of the bottle; at the same time he compresses the glass at the end of the pipe by means of the sheet-iron plate, Fig. 3262, and then blows through the pipe, so as to give the glass an egg-like form, Fig. 3276. He then rests the glass against the edge of the marver, marks the neck of the bottle, heats the piece in the furnace, withdraws it, and blows it, after having introduced it into a bronze or earthen mould of the proper size. When the bottle is formed, the blower withdraws it from the mould, and by a see-saw motion raises it on high, Fig. 3277, and indents the bottom of the bottle, by means of an instrument, Fig. 3278, called the *punty* or *pontil*, consisting of a small square piece of sheet iron, the angle of which rests on the centre of the bottom of the bottle, while it revolves on the pipe. Then, taking a drop of water with the punty, he applies it to the neck of the bottle, which is immediately carried to a small cavity in the side of the furnace, and separated from the pipe by a dextrous jerk. The bottle being thus prepared, the blower turns it and fastening the pipe to its base, Fig. 3279, extracts from the pot with another pipe a small quantity

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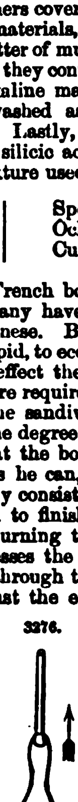

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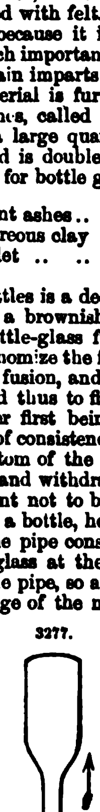

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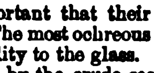

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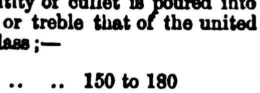

of melted glass, which elongates like a thread; the end of this he brings to the neck of the bottle, and by a rotary motion surrounds the mouth with a small glass cord; he then introduces the neck into the working hole, and finishes the mouth with pincers. The bottle being completed, an assistant takes it from the hands of the master-workman, carries it to the annealing furnace, and detaches the pipe by a dextrous blow. The bottles are arranged in rows, upon each other, in the annealing furnace, the heat of which should be kept below a dull red. When it is filled, the working holes are closed, and it is allowed to cool. Modern annealing furnaces are composed of a long gallery, heated by a furnace in the centre, and terminating by doors at either end. This longitudinal furnace is traversed by an endless iron chain, to which iron carts are attached containing the objects to be annealed. They enter at one end, and are withdrawn at the other, after having remained in the furnace long enough to be properly annealed.

3. *Crystal* is a kind of glass used only for the fabrication of articles of luxury: it must therefore be very transparent, perfectly homogeneous and colourless, and the greatest care must be exercised in the selection of the materials for its composition. Crystal is a double silicate of potassa and oxide of lead, the composition varying greatly in the different factories; the proportion of the oxygen of the silicic acid to that of the united bases ranges from 6 : 1 to 9 : 1. The ratio of the oxygen of the potassa to that of the oxide of lead ranges between still wider limits, namely, from 1 : 1 to 1 : 2·5. By increasing the proportion of oxide of lead, greater density and higher refracting and dispersing powers are imparted to the crystal, which produce in cut glass the beautiful effects of colour by transmitted light. But the proportion of the oxide of lead cannot be increased indefinitely, because the crystal, in that case, acquires a yellowish tinge. The finest and purest sand is chosen for the manufacture of crystal; the carbonate of potassa employed is refined, and the ordinary oxide of lead or *litharge* is not used, because it always contains some particles of metallic lead, which would be scattered through and injure the glass. Minium, an oxide of lead of a degree of oxidation superior to the protoxide, only is used: this oxide cannot contain metallic lead, and the oxygen it evolves when heated prevents the reduction of any lead by the carbonaceous dust or particles of other substances which may fall into the pot. The ordinary proportions for tumblers, decanters, &c., are 300 parts of pure sand, 200 parts of minium, and 100 parts of purified carbonate of potassa.

Crystal-glass furnaces are generally heated with wood; in some, however, coal is burned, but in that case the shape of the pots must be changed. Coal produces a very fuliginous smoke, the deoxidizing action of which it would be very difficult to prevent, if the glass were melted in open pots; peculiarly shaped pots, Fig. 3280, called *covered crucibles*, or *muffles*, are therefore used; their vertical opening is placed in front of the working hole of the furnace.

Many articles are made of crystal by blowing, but it is also cast in great quantities in bronze or wooden moulds, which latter are kept moist, so as not to carbonize too rapidly.

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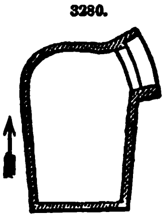

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The glass tubes used by chemists, and also thermometer tubes are made by a particular process, which we shall briefly describe. The workman gathers on the end of his pipe a certain quantity of glass prepared as usual; he then blows it into the shape of a pear, Fig. 3281, which he makes larger or smaller, thicker or thinner, according to the size and thickness of the tube required. Another workman has also gathered some melted glass on the end of a pipe, and applies it to the bottom of the bottle, Fig. 3283; the two workmen then recede rapidly from each other. The glass pear is then drawn out, as seen in Figs. 3283, 3284, and is converted into a tube terminating into two swollen extremities. Tubes of 30 or 45 mètres in length are thus made; they are laid on a wooden floor, and divided into lengths of 1 mètre each. It will be seen that the external diameter of these tubes is not equal throughout its whole length, being generally smallest toward the centre; neither is the internal calibre more regular, and it is rare to find a tube possessing the same internal diameter throughout its whole length.

Manufacture of Glass for Optical Purposes.—Crown Glass and Flint Glass.—Two kinds of glass are used for optical instruments; one, called *crown glass*, is analogous in its composition to Bohemian glass, while the other, called *flint glass*, is a species of crystal. This glass must be as colourless as possible, and perfectly homogeneous; great care is therefore required in the choice of the materials entering into its composition, and they must be refined expressly. Ordinary flint glass is manufactured of 100 parts of white sand, 100 parts of minium, and 30 parts of very pure carbonate of potassa. The density of this flint is about 3·5. A more refracting flint, but one slightly coloured yellow, is made of 225 parts of white sand, 225 parts of minium, 52 parts of carbonate of potassa, 4 parts of borax, 3 parts of nitre, 1 part of peroxide of manganese, 1 part of arsenious acid, and 89 parts of cullet of the preceding flint.

The melting furnace, Fig. 32°5, contains only one covered crucible or pot, into which the mixture is gradually introduced by small portions at a time, always waiting until the preceding charge has become perfectly fluid. Eight or ten hours are required for the whole charge of a pot. A strong blast is then applied, and kept up for four hours, to render the mixture perfectly fluid. When this is effected, a hollow cylinder *a b*, made of fire-clay, previously heated to redness, and

t of its greater lightness, is introduced into the
bar $f\,e$ is passed, the end of which is heated to
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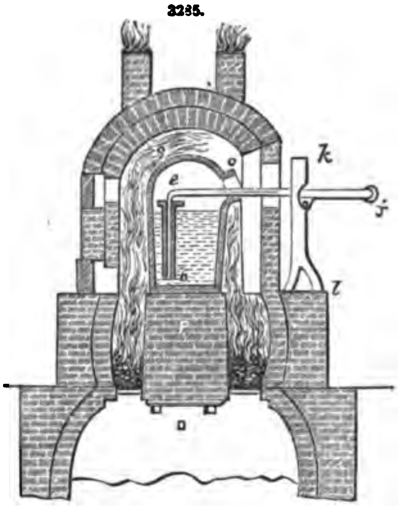

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y of
ays,
s of
the edges of which are free from the coloured
simple lenses. This property, however, is very
ry far from the axis of the lens.
es made, very dense and refracting, resembling
colouring this glass with various metallic oxides
stones are obtained. This crystal, called *strass*,
uires great care in fining: generally, a certain
f artificial jewels has in modern times reached

ecies of glass rendered opaque by an addition of
nic acid is generally used for this purpose: how-
niate of oxide of antimony may also be employed.
al. An alloy of 15 parts of tin and 100 parts of
hich a stannate of oxide of lead is formed, which
eous stannate are then mixed with 100 parts of
ss. An addition of small quantities of certain
mels.

ss is subject.—We have seen that objects made of
red heat, and then allowed to cool slowly. This
ion, for glass cooled suddenly after blowing is so
that common tumblers, which are imperfectly
temperature; such glass sometimes, also, is frac-
open door. This property is highly developed in
These are drops of glass suddenly cooled, and
o cold water; they thus become suddenly solid, in
ng tail; and as the outer surface solidifies while
ains nearly the shape it had in the
n abnormal condition by those of the
e surface particles be removed, at only
nd falls into dust. This occurs, for
similar effect is produced in a small
phial, a kind of glass tube, thick, and
ntly makes them on his pipe, when
e, a small ball, for example, be dropped

3286.

shock is sufficient to reduce the phial to dust.
k in a given direction when touched with a cold
r to crack the glass in any direction required.
o a high temperature, it loses, by volatilization,
less and less fusible, at the same time acquiring
ing. Thus masses of glass of a crystalline struc-
ve been for a long time in the furnace, and cooled
veloped only in some parts of it, the remainder
ining more alkali than that rendered opaque by
place not only at its fusing point, but also at a
several days in a furnace, at a degree of heat
the glass, it entirely loses its transparency, and
tered, *devitrified*, is much less fusible than when

transparent. A peculiar art was attempted to be founded on this property, which consisted in making objects of blown glass, and then destroying their fusibility by devitrification. This devitrified glass was called *Réaumur's porcelain;* but the manfacture of it has been abandoned.

Glass containing a large proportion of alkali changes by exposure to moist air, its surface becoming rugose and cracked. Frequently an excessively thin pellicle of altered glass forms on it, producing the same play of colours as a soap-bubble, or a drop of oil on a large surface of water; an alteration produced by the surface of the glass parting, after a long time, with a portion of its alkali. It is particularly remarkable in pieces of glass which have remained buried for years in a damp soil. These pieces are sometimes found to have entirely lost their transparency, to be swollen, and cleavable into very thin lamellæ: then they exhibit the same play of colours as mother-of-pearl.

Various small objects are made of the glass tubes of commerce. For this purpose an oil lamp, generally made of tin, Fig. 3287, fed by a bellows, and called an *enameller's lamp*, is used. The wick is of cotton, and does not project very high. The bellows is worked with the foot: the blast of air is conveyed by a pipe which can be turned in various directions. By properly arranging the wick, and modifying the inclination of the pipe, and adapting a proper aperture to it, a flame of any size may be obtained at pleasure. When working with a plumbeous glass, or crystal, the flame must be made oxidizing by admitting a greater quantity of air; for if the flame were reducing, oxide of lead would be brought to the surface of the glass in the state of metallic lead, and the glass would be blackened. It is important not to heat the glass too suddenly, lest it should break; it is therefore first held for a few moments before the flame, and brought by degrees into the hottest part.

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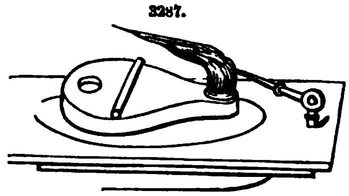

In order to bend a glass tube, it is heated to the distance of 3 or 4 centimètres on each side of the point of flexion, turning it constantly, so that its whole periphery may be uniformly heated. As soon as the tube is sufficiently soft to yield to a slight force, it is bent; but it is important not to make the curve too short, because the tube would be mis-shaped and brittle. The tube is therefore not heated at the point where it was begun to be bent, but the flame is directed on the adjacent part, so as to make a small arc of a circle. Tubes can be bent in an alcohol lamp even more readily than in an enameller's lamp, for it is better not to have the glass too hot.

In order to close a tube at one end, a longer tube is heated in the enameller's lamp, at the point of closure, turning it constantly in the flame. As soon as it is perfectly soft, both ends are gently drawn out, still turning it. The tube thus takes the shape of Fig. 3288. The point of the flame is then directed to the point *a* of the narrow part, and the two halves of the tube are separated, each of which will furnish a tube closed at one end; the ends are then rounded, and made more uniform in thickness. To do this, the end is again heated in the lamp, blowing into it occasionally, to round it. Lastly, a *border* is only required to complete it, which is made by simply heating the sharp edges until they are rounded by fusion. If the edges are to be widened, or a mouth made to pour liquids, it is done by applying an iron wire against the softened edges, by which means the aperture can be fashioned at will, Fig. 3289. When the end is to be closed, this end is heated in the lamp, and the heated end of another tube applied to it. The two tubes are soldered together, and the operation is then continued as just described.

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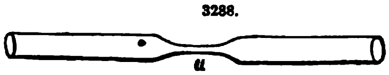

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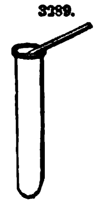

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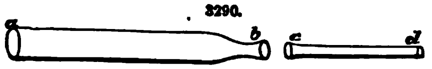

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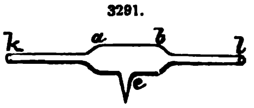

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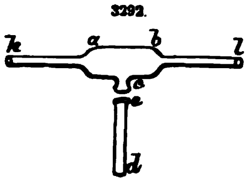

It is frequently necessary to solder a smaller tube *c d*, Fig. 3290, to the end of a larger one *a b*. The larger tube is then drawn out, in the lamp, till it is of the size of the smaller one, and the tube *a b* is next closed at the point *b* of the narrow part, by placing the part *b* in the point of the flame, and turning the tube between the fingers. Then, after having heated the closed end to soften it, a very thin sphere, which bursts by blowing through the opening *a*, is formed at the end *b*. By means of a file the glass is separated so as to leave only a widened edge-border at the end *b*. The same is done to the end *c* of the small tube; the ends *b* and *c* are then exposed to the flame, opposite to each other, turning them constantly, after having previously closed the end *a* with a cork. When these ends are sufficiently softened, they are pressed firmly against each other, the joint is equally heated throughout, and from time to time the operator blows through the small tube, in order to prevent the solder from forming a ring. Lastly, it is drawn out slightly, so that no swelling may exist at the point of union.

If a narrow tube *c d*, Fig. 3292, is to be soldered to the side of a larger tube *a b*, the point of the flame, after having rendered it as sharp as possible by a proper arrangement of the pipe and lamp-wick, is directed on the point *e*, Fig. 3291, of the tube *a b*. When it is sufficiently softened, the

end of a glass point, also heated, is fastened and drawn quickly forward: thus a point *e f* is formed on the tube *a b*. This point is closed in the lamp; then, having stopped the end *k* with wax, the point *e f* is again introduced into the flame, and when it is in fusion, a very thin sphere, which bursts, is formed by blowing through the open end *l*. A portion of the glass is filed off, the edges of the aperture are melted, Fig. 3292, and after having closed the end *l* with wax, the end *ė* of the small tube, also heated, is brought in contact with the opening *e*. The joint is formed by gradually heating all its parts, and blowing from time to time through the opening *d*.

If a globe is to be blown at the end of a tube, the tube is closed in the lamp, and by continuing the action of the flame, a mass of glass, large enough to make the globe required, is collected at this end. This mass of glass being very soft, the tube is gradually extended by blowing gently into it. It is then heated again uniformly, and afterward, by constantly turning the tube and blowing gently, a globe of any size may be produced at pleasure. When the globe is to be large, and still be at the end of a narrow and thin tube, it is better to blow the globe separately on a larger tube, and then solder it to the narrow one. To do this, the larger tube is drawn out between two points, Fig. 3293, by the process before stated; one end, *a*, is closed in the lamp, and then the part A heated in the flame, so as to soften it completely. Lastly, the operator blows through the end *b*, turning it constantly, until the globe has attained the size required. The globe is then soldered to the tube. But as the globe is still terminated by a point, the latter is placed in the flame, and by blowing gently after having softened this part of the globe, it is distended so as to cause the small piece of glass to disappear. The bottles which are to contain the volatile liquids intended for analysis are blown in the same way.

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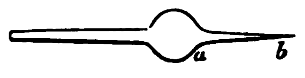

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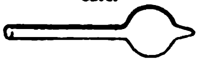

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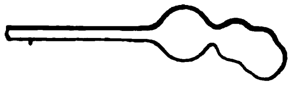

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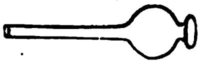

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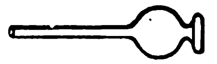


In order to fashion a funnel at the end of a tube, as, for example, on safety-tubes, a globe drawn out between two points, Fig. 3294, is soldered to the end of the tube, and then the point *a b* is detached, Fig. 3295. The part *a*, as well as the end of the globe, is heated, and when they are very soft, a smart blow of wind through the tube is given; thus a second irregular and very thin globe, Fig. 3296, fastened to the first, is produced; this is broken and the glass detached by means of a file, Fig. 3297, so as to leave only an edge, which is melted in the lamp, and properly widened by an iron rod, Fig. 3298.

In order to break a glass tube at any given point, a mark is made on it with a gun-flint or a very sharp three-edged file; the tube is then pulled in the direction of its length, and it separates at the mark. If the tube be large, it must be slightly bent at the same time. In order to separate thicker and larger portions, as, for example, to shorten the neck of a retort or flask, a mark with a file is made at the proper point, and followed with a point of red-hot iron; it then cracks in the direction of the mark.

A red-hot coal, held with a forceps, carried round the intended line of separation, answers the same purpose; care must be taken to blow away the ashes as soon as they form by contact with the cold glass, so as always to present a red-hot point to the surface of the glass.

The process of dividing a tube by friction, described in Hare's Chemistry, is so much superior to that adopted by previous operators, that the Editor has not hesitated to substitute it for the French mode;—"Some years ago, Isaiah Lukens showed us that a small phial or tube might be separated into two parts, if subjected to cold water, after having been heated by the friction of a cord made to circulate about it, by two persons alternately pulling in opposite directions. We were subsequently enabled to employ this process for dividing large vessels of 4 or 5 in. in diameter; and likewise to render it in every case more easy and certain, by means of a piece of plank forked like a boot-jack, and also having a kerf or slit cut by a saw, parallel to and nearly equidistant from the principal surface of the plank, and at right angles to the incision forming the fork. By means of the fork, the glass is held steady by the hand of the operator. By means of the kerf, the string, while circulating about the glass, is confined to the part where the separation is desired. As soon as the cord smokes, the glass is plunged into water, or if too large to be easily immersed, the water must be thrown upon it. This method is always preferable when the glass vessel is so open that, on being immersed, the water can reach the inner surface. As plunging is the most effectual method of employing the water, we usually, in the case of a tube, close the end which is to be sunk in the water, so as to restrict the refrigeration to the outside."—Hare's Compendium, ed. 4th, p. 60.

Coloured Glass and Painting on Glass.—Glass dissolves the greater part of the metallic oxides, and while it preserves its transparency, is often tinged with the most beautiful hues; on this property the manufacture of coloured glass is founded. It suffices to mix intimately with the metal of which the glass is to be made, a given quantity of the metallic oxide, to produce coloured melted glass; with certain metallic oxides, however, peculiar care is required. Protoxide of iron FeO imparts to glass a deep or bottle-green colour, while the sesquioxide Fe_2O_3 produces a yellow tinge. Oxide of copper CuO and oxide of chrome Cr_2O_3 yield a beautiful green, but of different shades. Oxide of cobalt CoO gives a brilliant blue; sesquioxide of manganese Mn_2O_3 a violet. A mixture of equal parts of oxide of cobalt and oxide of iron colours the glass black. Protoxide of copper Cu_2O

yields a very beautiful red colour, but so intense that the glass nearly loses its transparency if the oxide be in the proportion of a few hundredths. A fine purple is obtained by mixing a certain quantity of oxide of tin with finely-powdered crystal, soaking the mass in a solution of chloride of gold, and melting it, when dried, in a crucible. When the metallic oxide to be used as a colouring agent can be deoxidized in the furnace, as, for example, the sesquioxide of manganese can be, a small quantity of nitre is added to the mixture. A beautiful yellow glass is made by adding lampblack to a mixture which would produce common white glass. By varying the proportion of lampblack, several intermediate shades between a bright and a purple yellow can be produced.

When it is wished to make glass of clear colours with metallic oxides which possess powerful colouring action, it is difficult to obtain the shade desired by adding the proper quantity of the colouring oxide to the mixture in the pot. Glass is then set in layers (*verre plaqué*), so that it is formed of white glass throughout the greater part of its thickness, and has one face only formed by a thin layer of coloured glass; and in order to vary at will the intensity of the colour, the layers are made of suitable thickness. This kind of glass is made as follows;—Two pots are placed in the oven, one filled with white and the other with coloured glass. The workman first takes up with his pipe a certain quantity of white glass; then, when it begins to assume the proper degree of consistency, he dips it into the coloured glass, and thus fastens a layer of this on the white mass. He then blows the whole into cylinders, in order to make muffs for flatting. The inside of the cylinder is necessarily white, the layer of coloured glass being only external.

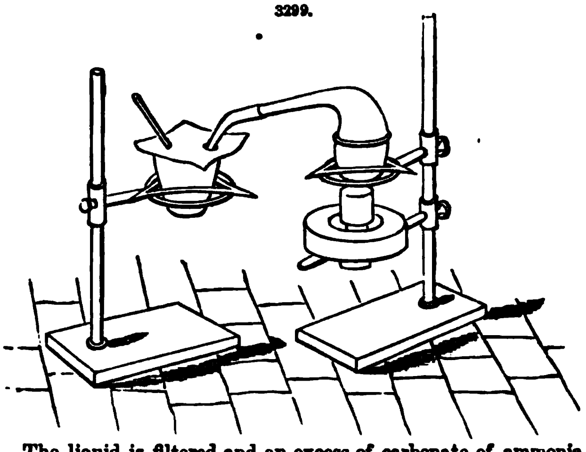
Painting on glass is done with very fusible and finely-powdered coloured glass. The composition of this glass varies with the nature of the colouring oxide; for the majority of them, a mixture of 2 parts of quartz, 2½ of oxide of lead, and 1 of bismuth is used; but as certain colouring oxides are altered by the oxides of lead and bismuth, in this case a mixture of 2 parts of quartz, 1½ of melted borax, ½ of nitre, and ½ of carbonate of lime is used. The colouring oxide is added to these mixtures, and they are melted in a muffle furnace; the glass obtained is reduced to an impalpable powder, ground in turpentine, and the paint thus prepared is applied with a pencil. The painted glass is then heated in a muffle, at a temperature sufficient to melt the coloured glass, but not to affect the object on which the painting has been made. In order to form the groundwork of the picture, glass coloured in the paste is generally used, the outlines and shades being painted on one of the surfaces. The various pieces of glass are then dextrously fitted together by means of small sheets of lead, each small pane harmonizing with the outlines and shades of the figures designed. The painted surface of the glass is placed outside, so that the picture is seen through the coloured glass. The numbers and divisions marked on enamel dial-plates are applied in the same way

Analysis of Glass.—We will suppose that the glass to be analyzed contains, or may contain, silex, potassa, soda, lime, manganese, alumina, oxide of iron, oxide of manganese, and oxide of lead. Five grammes of the glass, reduced to an impalpable powder, are intimately mixed with about three times its weight of pure carbonate of soda; the mixture having been weighed in a platinum crucible, the latter is covered with its lid, and heated in an alcohol lamp having a double current of air, so as to completely melt the carbonate of soda. For this purpose it is well to surround the crucible with a small sheet-iron chimney, extending a few centimètres beyond it: the chimney, at the same time increasing the draught, forces the flame completely to envelop the crucible. The carbonate of soda is kept melted for at least twenty minutes, and then allowed to cool. By using a thin crucible, the alkaline cup may be detached by the pressure of the fingers, and is received in a porcelain saucer, containing a certain quantity of water, and covered by an inverted funnel. Water, acidulated with nitric acid, being poured into the platinum crucible, and then into the saucer, the alkaline cup dissolves with effervescence, the funnel preventing any loss of substance, by the projection of the small liquid pellicles surrounding the gaseous bubbles which burst on the surface of the fluid. Toward the close the liquid is acidulated with an excess of nitric acid, and evaporated to dryness at a moderate heat. Hot water, acidulated with nitric acid, is poured on the dried matter; it is allowed to digest for some time hot, and then diluted with water; all the metallic oxides then dissolving, leave the silex alone as an insoluble residue. It is collected on a filter, calcined after being well washed, and weighed. A current of sulphuretted hydrogen is passed through the liquid, which precipitates only the lead in the state of a sulphuret; and finally, the liquid is heated to ebullition, still keeping up the current of sulphuretted hydrogen, in order to facilitate the deposit of sulphur. The sulphuret of lead is collected on a filter, and, after having washed it, the filter is burned in a platinum crucible, and the substance sprinkled with nitric acid, mixed with a small quantity of sulphuric, in order to convert it into sulphate of lead; lastly, it is calcined to redness. The weight of the oxide of lead is deduced by calculation from the weight of the sulphate of lead obtained. Sulphydrate of ammonia is then poured into the liquid to precipitate the alumina and the sulphurets of iron and manganese; the wet precipitate is redissolved in chlorohydric acid, and by the separation of the two oxides. The liquid, which then contains only lime, magnesia, and the alkaline salts, is boiled to drive off the excess of sulphydrate of ammonia, and chlorohydric acid added to decompose that which still remains. Lastly, it is supersaturated with ammonia, and the lime precipitated in the state of oxalate of lime by oxalate of ammonia; the presence of ammoniacal salts in the liquid keeping all the magnesia in solution. The solution is then concentrated by evaporation, an excess of carbonate of soda added, and it is evaporated to dryness, to decompose the ammoniacal salts, and drive off the ammonia as carbonate; it is then treated with water, which leaves the magnesia in the state of insoluble carbonate.

In the analysis just described the proportions of all the various components of the glass have been ascertained successively, with the exception of those of the alkalies, which must be found by a particular process. The glass is first dissolved in fluohydric acid. As this acid is difficult of preservation, it is better to prepare it freshly for each analysis, which is done in the following manner;—Into a small platinum retort, Fig. 3299, made of two pieces, very finely powdered fluor-spar is introduced and sulphuric acid added; on the other hand, 5 grammes of glass in impalpable powder are placed in a large platinum crucible, with a certain quantity of water, and covered with a sheet

of platinum pierced with two openings. The neck of the platinum retort passes through one of those openings; the other, much smaller, is traversed by a platinum wire, flattened into a spoon at its end, and used for stirring the material in the crucible. On gently heating the retort the fluohydric acid dissolves in the water of the crucible, attacks the vitreous matter, and a large quantity of fluoride of silicum is disengaged. The material is stirred from time to time with the platinum spoon, and when the glass is entirely dissolved, the crucible is gently heated, to drive off the excess of acid and evaporate the water; sulphuric acid is then poured upon the residue, completely to expel the fluohydric acid and convert all the oxides into sulphates. When the greater part of the sulphuric acid has been driven off by heat, the substance is treated with water, which leaves the silex and sulphate of lead as a residue. The liquid is filtered and an excess of carbonate of ammonia added, which precipitates the alumina, the lime, the oxide of iron, a part of the oxide of manganese, and the magnesia; an addition of a small quantity of sulphydrate of ammonia completes the precipitation of the manganese. The liquid, when filtered, contains only the alkaline salts, a small quantity of magnesia, and salts of ammonia; it is evaporated to dryness, the residue calcined at a strong red heat, and the alkaline bases are weighed in the state of sulphates. The magnesia is overlooked for the moment, until the termination of the analysis; the potassa is separated by perchloride of platinum, and the soda is determined by calculation from the difference obtained. The magnesia must be sought in the solution remaining after the precipitation of the double chloride of potassium and platinum. The platinum is then precipitated by sulphydrate of ammonia, and the liquid, filtered with an excess of carbonate of soda, is evaporated; the carbonate of magnesia is then separated by treatment with water. This base may also be precipitated by phosphate of ammonia. A much better method of separating the magnesia from the alkalies is the following, when the bases can easily be obtained as chlorides;—The liquid containing magnesia and the alkalies is evaporated to dryness in a platinum crucible, after having condensed its volume by evaporation in a porcelain capsule, out of which the very concentrated solution is carefully washed, with as little water as possible, into the platinum vessel; a small quantity of pure red oxide of mercury is then added, and the crucible subjected to a strong white heat over a spirit lamp, until all the mercury is volatilized. Care must be taken not to inhale the fumes. The magnesia, then all remaining as insoluble caustic magnesia, is separated by filtration from the alkalies, which then may be determined by weighing them together, determining the potassa by precipitation with chloride of platinum, and finding the weight of the soda by the difference. Phosphate of soda, with the addition of some ammonia, effects the precipitation of magnesia much more perfectly than phosphate of ammonia.

3299.



GOLD. Fr., *Or;* Ger., *Gold;* Ital., *Oro;* Span., *Oro.*

Gold is found almost over the whole globe, but in most cases in small quantities compared with other metals. At the present time California affords the largest amount of this metal in the world. Gold is chiefly found in its native condition, in a metallic state, alloyed with silver, and sometimes with tellurium, as is the case in Virginia and North Carolina. In California it is found chiefly in alluvial ground, bedded upon rock in most cases; it is also found enclosed in quartz rock, apparently in veins ramifying the rocks of an extensive mountain range. This California gold is obtained chiefly in large grains, and often in lumps of several pounds weight. In the other States of the Union the gold is in very minute fragments, often invisible to the eye if not aided by a lens, only to be detected by crushing and grinding the rock, and washing off the débris. This gold is apparently derived from the decomposition of iron and copper pyrites, chiefly the first; which assertion cannot be objected to, because it is founded in principle that almost all iron pyrites contain gold, that the gold ores of that region are rocks which are coloured by iron, and that this iron is evidently derived from the decomposition of the pyrites. Pyritous ores of this kind are worked which contain no visible gold, or which do not yield gold at the first crushing and washing, but which furnish gold in a succession of amalgamations, performed after regular intervals of exposure to the air in a fine powder.

A splendid yellow colour and brilliant metallic lustre characterizes gold distinctly from other metals; its specific gravity being 19·3 to water, is another quality easily appreciated by the senses. It is pre-eminently ductile, which qualifies it for an extensive use in the arts. One grain of gold may be drawn into a wire 500 ft. long: silver may be coated with gold, of which the thickness is only the twelve-millionth part of an inch, and still the microscope cannot detect the slightest indication of an interruption of the gold coating. Pure gold requires more heat for melting than either silver or copper, but as all native gold is alloyed with some other metal, it may be considered more fusible than those metals. If, in cupelling gold, the hot globule shines with a greenish light, we may consider the gold not much adulterated; if it contains 10 per cent., or from there to one-third of silver, the colour of the gold is in the hot cupel white as silver. Pure gold is not very

volatile, and may be exposed to a strong heat for a long time without loss of metal; but if gold is alloyed with volatile metal, such as lead, zinc, and antimony, it is liable to be carried off by their vapours. Gold has a considerable cohesion, which inclines it to crystallization. Its crystal form is an octahedron; it is often found in fragments of crystals imbedded in quartz. In melting gold along with pure borax it assumes a whitish colour, as if adulterated with silver; in melting it again with saltpetre, or common salt, it recovers its rich yellow colour.

The geological position of gold is in the primitive rock. It is found in granite, disseminated in grains and spangles through the mass of rock. In the United States gold is chiefly found in the stratified transition series. Most of the gold, the California gold exclusively, is found in alluvial soil. In the Southern gold region this source is much exhausted, and the gold is here obtained from regular, well-developed veins, running parallel with the general direction of the rock strata, south-west by north-east. The plane of inclination of these veins is also parallel with the plane of inclination of the general formation. It appears from this that the gold-bearing veins are of a simultaneous origin with the rock; at least, they have been introduced when the rock was in a plastic condition. In Virginia and North Carolina the gold-bearing veins are a ferruginous talcose slate, often inclined to mica slate. In North Carolina this slate is found to be very hard in many instances, showing a compact solid mass of rock, apparently the same slate; but having been under the influence of a considerable heat, it is hardened. In Virginia this slate is more soft, the fissures open more readily, and the whole vein shows the appearance of soft slate. This slate is impregnated with small quartz veins, from $\frac{1}{4}$ to $\frac{1}{2}$ an inch, and often 2 in. thick. Where these quartz veins are thin and in great numbers, the ore is always found to be richest in gold. The veinstone of the gold-bearing veins is strongly impregnated with oxide of iron, showing evidences that this iron is derived from pyrites, because the oxide appears in dots or flowers, and groups of dots. Many of these veins have been traced to that depth where the pyrites are not oxidized; here they appear in their perfect crystal form, and are profusely distributed through the slate. The oxidation of these pyrites appears to depend on the penetrability of the rock by atmospheric agents; where the slate is soft we find it oxidized to the depth of from 50 to 150 ft.; where the slate is hard, as is the case at the Sawyer Mine, North Carolina, the oxidation reaches hardly 10 or 20 ft. deep, and is in many places, such as bluffs, not developed at all. At the latter spots the pyrites are in their original form, untouched by oxygen. Where the pyrites are not oxidized, the extraction of gold is connected with considerably more expense than it is from soft slate and oxidized pyrites. The crushing of the hard slate is in the first place more expensive; the sulphur of the pyrites destroys a large portion of quicksilver in amalgamation, and the gold cannot be all extracted; the largest portion of it remains enclosed by the sulphuret of iron, which can only be liberated by destroying that envelope.

There is, however, one drawback to the rapid extraction of gold from deposits—the ores are all, without exception, pyritous in greater depth, and to work these sulphurets to advantage no progress has been made up to this time. Various experiments tending to accomplish this purpose, and affording means of extraction, have been tried, but none of these succeeded so far as to work the poorer class of ores. At Goldhill, N.C., where the ores yield from \$1.50 to \$3 of gold in 100 lbs. or one bushel of ore, the pyritous ores are ground, amalgamated, and a certain portion of gold extracted. The crushed ore, now a fine sand, is exposed to the influence of the atmosphere for one year, after which the process of grinding and amalgamating is repeated, and another portion of gold, almost equal to the first, is extracted. An exposure of another year furnishes another crop of gold, which operation may be repeated four or five times without extracting all the metal from the sand. This way of working is tedious, expensive, and will not answer where the ores yield but 25 cents to the bushel. The process of roasting these ores by artificial fire is too expensive, and all processes which require much labour are out of the question.

The extraction of gold is performed in California, and also in some parts of the Southern States, simply by washing the alluvial soil, removing the sand, clay, and débris of rock; after these operations the gold, as specifically the heaviest matter, will remain in the vessel in which the washing has been performed. This washing may be done to advantage in a tin pan or a sheet-iron pan. Such a pan is filled with sand containing the gold, and immersed in water; in stirring it gently by hand the clay and light sand flow off, and, after some of the earthy matter is removed, the pan is shaken so as to bring the heavier gold to the bottom of it; the superstratum of sand is now removed, and the gold found in the bottom of the pan (see p. 265).

Gold enclosed in rocky matter cannot be washed with success in the foregoing described manner; the rock must be crushed, and is, in this operation, transformed into more or less fine sand. The bulk of this sand is removed by washing, and the rest, with the gold, reserved for amalgamation.

The crushing is performed in the stamp-mill (p. 272).

After the crushing is performed, the sand, including gold, is conducted over hides, which retain the gold, and the sand is floated away. The gold and sand from the hides are removed, when the latter are filled, to an amalgamating machine, which combines the gold with quicksilver, and admits the sand to flow off. Instead of hides, woollen blankets are also used for gathering the gold, and there is a diversity of opinions as to the merits of either. Blankets, it is contended, are more expensive than hides, but they have the advantage of working more uniformly. Hides are cheaper, but they lose their hairs or wool very soon, and are then not fit to do good work. Hides of short, curly wool are selected; these are spread on the ground, and over these the water, sand, and gold are led in a broad sheet. In other instances, shaking tables are suspended at the discharge of the stampers, which gather the gold and some sand. Shaking tables are wooden platforms of 8 or 10 ft. long, and from 3 to 4 ft. wide, made of 2-in. plank well joined together, and the whole smoothly planed. Around the edges of the table are projecting ribs, which prevent the water from flowing over the edges. In suspending this table, a little inclined to the horizontal, leading the sand and water over it in a broad sheet, and applying a gentle shaking motion to it, the gold will sink to the bottom and move gently down the plane; it is arrested at the lowest end of the table

cases the gold is brought to the amalgamating

vided with Chilian mills for crushing the ore.
rolina, which work by four or five runners or

is in operation at Goldhill. It is a cast-iron
and 6 in. deep; the trough is firmly fixed upon
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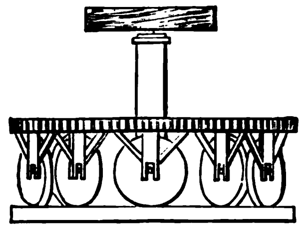
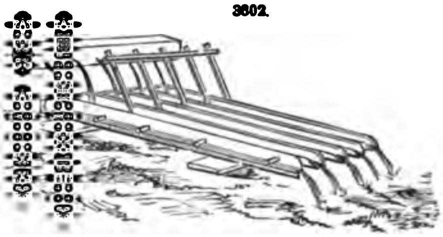


ended, in which a rake moves around with a
may settle from the passing water. It retains
van bowls (a small machine which derived its
ther the heavy parts which may escape the

vertical wooden shaft, of about 18 in. long and
vessel or bowl B, about 2 in. deep and 18 in. in
n and sheet-iron periphery. The bowl receives
ircumference, and discharges at the centre. By
on, caused by a crank connected with the same.
it is an expensive machine, because it works but
he work of a small establishment.

3302.



h some sand and other impurities, is carried to
no other machine for doing that work. This is
s losses in quicksilver and gold. In most cases
North Carolina the cradle is generally employed.
ed out so as to form a round trough, closed at one
3302.
as many as that are frequently connected and
ft. long, hollowed out of a trunk of at least 24 in.
sides. The first cradle in the drawing shows a
d in the bottom; in each of these grooves from
end sand is shovelled in and water led upon it,
charge. A gentle current of water will have a
the trough, the trough being, in the mean time,
washing off everything. The quicksilver in the
avy granules of gold gliding down on the bottom
nd, &c., are not attracted, and pass over the mer-
k slow, and lose much of the fine suspended gold.
put in operation; their efficacy is, however, not
orth Carolina the German barrel amalgamation
are not informed of the results. In Virginia,
been tried, but we are not acquainted with their

mmon evil—they cannot work all the water, as it
In all instances half the golden contents of the
ondition of the ore, which clay encloses particles
division of the gold in the ores of these regions,
ision causes the gold to be suspended in water,
urrent. A good amalgamating apparatus, which
achines, rub off clay and other matter from the

particles of gold, so as to make it adhere to the quicksilver, and which does not lose any quicksilver, is still a desideratum in the Southern gold-mining districts.

Gold gathered by quicksilver forms a white amalgam. In the amalgamating machines a surplus of quicksilver is used to secure the fluidity of the mercury; for if it gets slimy, or, still worse, plastic, like clay, it will not absorb any more gold with facility. The fluid amalgam is pressed through a soft leather or a piece of close canvas, to remove the superfluous mercury, after which a solid amalgam, called quick, remains in the bag. The quicksilver which passes through the bag retains always some gold in solution, the quantity of which varies according to the stuff through which it has been squeezed. The amalgam thus obtained contains from 30 to 70 per cent. of gold, according to the mode of working and the quality of the ore. The quick from the Chilian mills generally contains but from 30 to 40 per cent. of gold, while that from stampers contains seldom less than 40, and in most cases from 50 to 60 per cent. of gold. This circumstance appears to speak in favour of the stamps; the difference in the contents of gold, in the amalgam, is owing to its division; the finer the gold, the less of it the amalgam contains. The dry amalgam is distilled in an iron retort, lined with clay; a red heat will drive off the mercury, which is condensed by leading it into cold water. The gold remains in the retort in the form of a powder, which is collected, melted in a crucible along with some saltpetre, and cast into iron moulds, forming square bars of about 1 lb. weight each. One pennyweight of gold of the Virginia mines is generally worth from 90 to 92 cents. North Carolina gold contains more silver than the first, and a pennyweight is seldom more than 90, and in the majority of cases from 80 to 90 cents to the pennyweight. California gold ranges from 75 to 90 cents.

The gold in gold coin and jewellery is never pure, being alloyed with a certain quantity of copper and frequently of silver, to give it a greater degree of hardness. In order to obtain pure gold, gold coin is dissolved in aqua regia, and the solution being evaporated to dryness, by gentle heat, to drive off the excess of acid, the residue is treated with water, by which means the silver is separated as insoluble chloride. An excess of protosulphate of iron, which precipitates the gold in the metallic state, in the form of brown powder, is then poured into the liquid, the reaction ensuing according to the following equation;—$Au_2Cl_3+6(FeO,SO_3)=2Au+2(Fe_2O_3,3SO_3)+Fe_2Cl_3$.

The precipitate is digested with weak chlorohydric acid, and, after being well washed, is fused in an earthen crucible with a small quantity of borax and saltpetre. The protosulphate of iron may be replaced by sesquichloride of antimony Sb_2Cl_3 dissolved in an excess of chlorohydric acid; the sesquichloride of antimony being converted into the perchloride Sb_2Cl_5, while the gold is precipitated in the metallic state.

Gold has a characteristic yellow colour, and its density is 19·5. It fuses at a strong white heat, or at about 2200° of the air thermometer, giving off sensible vapours at a very high temperature. A gold wire is converted into vapour when traversed by the current of a powerful electric battery: and if this take place over a sheet of paper placed at a small distance, the paper becomes coloured of a purplish brown, by the very finely divided gold which is precipitated on it. A blade of silver substituted for the paper soon becomes gilded. A globule of gold gives off vapour very copiously when held between two pieces of charcoal terminating the conductors of a powerful galvanic battery.

Gold is the most malleable of all the metals, and when beaten into very thin leaves is transparent, the transmitted light appearing of a beautiful green colour. Gold may be crystallized by fusion, when it assumes the shape of cubes modified by other facets of the regular system. Native gold is sometimes found in well-defined crystals presenting the same form.

When precipitated in a metallic state from its solutions, gold forms a brown powder, which by burnishing soon recovers the metallic lustre and characteristic colour of malleable gold, and which aggregates by percussion. If the mass be heated to redness before being hammered, a perfectly aggregated metal can be obtained without having heated it to fusion.

Gold does not combine directly with oxygen at any temperature. Chlorohydric, nitric, and sulphuric acids do not affect it, while aqua regia, on the contrary, readily dissolves it in the state of sesquichloride, Au_2Cl_3. Gold is also dissolved by chlorohydric acid when a substance capable of disengaging chlorine is added, such as peroxide of manganese, chromic acid, &c. Chlorine and bromine also attack gold, even when cold, while iodine acts on it but feebly.

Sulphur does not attack gold at any temperature, nor does the metal decompose sulphydric acid; but by fusing it with the alkaline polysulphides it is powerfully acted on, a double sulphide being formed, in which the sulphide of gold Au_2S_3 acts the part of a sulphacid. Arsenic when assisted by heat combines with gold, and forms a very brittle alloy.

Gold is attacked neither by the alkalies nor the alkaline carbonates or nitrates.

Compounds of Gold with Oxygen.—Two combinations of gold with oxygen are known;—

1. A suboxide Au_2O,
2. A sesquioxide Au_2O_3,

neither of which forms salts with the oxides.

The suboxide Au_2O is obtained by decomposing the chloride Au_2Cl by a dilute solution of potassa, in the shape of a deep violet-coloured powder, which decomposes at about 77°, disengaging oxygen. The oxacids exert no action on this substance, while chlorohydric acid decomposes it, forming sesquichloride of gold Au_2Cl_3, while metallic gold is separated.

Sesquioxide of gold (often called *auric acid* on account of its property of combining with bases) is prepared by digesting a hot solution of sesquichloride of gold with magnesia, when aurate of magnesia is formed, which remains mixed with the free magnesia. The deposit is boiled with nitric acid, which dissolves the magnesia and leaves hydrated sesquioxide of gold. Auric acid may also be obtained by saturating a solution of sesquichloride of gold by carbonate of soda, and then boiling the liquid, when a large proportion of the gold is precipitated in the state of sesquioxide, while the other portion remains in solution, but may be precipitated by successively adding to the liquid an excess of caustic potassa and acetic acid.

Hydrated auric acid is a yellow or brown powder, which loses its water at a low temperature and becomes anhydrous, while at about 482° it decomposes into gold and oxygen, which reaction is also effected by the solar light. Deoxidizing substances, such as the organic acids, or boiling alcohol, reduce it to the metallic state; while chlorohydric acid dissolves it and produces the sesquichloride Au_2Cl_3. The most energetic oxacids do not form definite compounds with sesquioxide of gold, while the latter dissolves, on the contrary, readily in cold alkaline solutions, producing alkaline aurates which crystallize by evaporation.

By adding a small quantity of ammonia to a solution of sesquichloride of gold, a fulminating substance is produced, which contains, at the same time, oxide of gold, ammonia, and chloride, and which, by digesting with an excess of ammonia, furnishes a bright brown powder of still higher detonating properties than the first, and which is a simple combination of sesquioxide of gold with ammonia $Au_2O_3+2NH_3+HO$.

Compounds of Gold with Sulphur.—Although sulphur does not combine directly with gold, two sulphides corresponding to the two oxides are obtained by decomposing the sesquioxide of gold by sulphydric acid, which, on being passed through a cold solution of sesquichloride of gold, yields a brownish-yellow precipitate, which is the sulphide Au_2S_3, readily soluble in the alkaline sulphides. If the solution of the chloride is boiling, a sulphide Au_2S, of a deep-brown colour, is precipitated, while sulphuric and chlorohydric acids are formed $2Au_2Cl_3+3HS+3HO=2Au_2S+6HCl+SO_3$.

Compounds of Gold with Chlorine.—By dissolving gold in aqua regia a yellow solution of sesquichloride of gold Au_2Cl_3 is obtained, which, when allowed to evaporate slowly in dry air, deposits yellow crystals of a compound of sesquichloride of gold and chlorohydric acid. If the solution be evaporated to drive off the excess of acid, the substance assumes a brown colour, and a deliquescent crystalline mass remains, which dissolves readily in alcohol and in ether. Sesquichloride of gold dissolves even more rapidly in ether than in water; for, if an aqueous solution of the chloride be shaken with ether and water, the supernatant of ether contains nearly all the chloride of gold in solution. The solution of sesquichloride of gold in ether was formerly used in medicine under the name of *aurum potabile.*

Sesquichloride of gold forms with other metallic chlorides double crystallizable chlorides, in order to obtain which it is sufficient to mix and evaporate the solutions of the two chlorides. The formula of the double chloride of gold and potassium, which is deliquescent, is $KCl+Au_2Cl_3+5HO$, while the formula of that of gold and sodium is $NaCl+Au_2Cl_3+4HO$, and that of the double chloride of gold and ammonia is $NH_4HCl+Au_2Cl_3+2HO$. Compounds of chloride of gold with the chlorides of barium, calcium, manganese, iron, zinc, &c., are also known.

Subchloride of gold Au_2Cl is prepared by heating the sesquichloride of gold Au_2Cl_3 to a temperature of about 400°, when chlorine is disengaged, while a greenish insoluble powder remains.

Compound of Gold with Cyanogen.—By adding a solution of cyanide of potassium to a concentrated hot solution of perchloride of gold, until the liquid loses its colour, a solution is obtained, which, on cooling, deposits prismatic crystals of a double cyanide of gold and potassium of the formula $KCy+Au_2Cy_3$. The crystals, which are efflorescent and very soluble, disengage cyanogen when subjected to moderate heat; and when treated with water, a solution is obtained, which, on cooling, deposits a double cyanide of the formula $KCy+Au_2Cy$.

The name of *purple of Cassius* is given to a precipitate containing gold, tin, and oxygen, which is used by painters on porcelain and glass, and is prepared in various ways. Its composition not being always uniform, chemists are not yet agreed upon its nature. It is generally obtained by pouring into a sufficiently dilute solution of sesquichloride of gold, a mixture of protochloride and bichloride of tin, the precipitate showing a beautiful purple hue when it is of small bulk, while it assumes a brown colour when more copious.

A purple of Cassius of uniform composition is prepared by dissolving 20 grammes of gold in 100 grammes of aqua regia made of 20 parts of nitric and 80 of chlorohydric acid; driving off the excess of acid by evaporation in a water-bath and dissolving the residue in 7 or 8 decilitres of water. Some pieces of tin being placed in the liquid, a purple precipitate of the formula $Au_2O,SnO_2+SnO,SnO_2+4HO$ is formed, but which may also be considered as $2Au+3SnO_2+4HO$. The substance, on being subjected to heat, evolves water alone and no oxygen, while the calcined residue presents all the characters of a mixture of metallic gold and stannic acid. But as before calcination the substance will not give off gold to mercury, it is evident that the gold did not exist in it in the metallic state.

A beautiful purple of Cassius is obtained by heating suboxide of gold Au_2O with a solution of stannate of potassa.

Lastly, purple of Cassius is obtained by fusing together in a crucible 1 part of gold, ½ part of tin, and 4 or 5 of silver, forming a ternary alloy, from which nitric acid extracts the silver, while the gold and tin are precipitated in combination with oxygen, and a brilliant purple is formed, the shades of which can be changed by altering the relative proportions of gold and tin.

A solution of sesquichloride of gold stains linen of a purple colour, as it also does the skin and the organic tissues generally, which colouring is probably owing to suboxide of gold, as friction does not restore a metallic lustre to the spots, although they acquire it in a short time when exposed to solar light in a bottle filled with hydrogen gas.

Determination of Gold, and its separation from other metals.—Gold is always determined in the metallic state, and is precipitated from its solutions by means of protosulphate of iron, after having added chlorohydric acid to the liquid in order to maintain the sesquioxide of iron which forms during the reaction in solution. But it is important, in order to completely precipitate the gold, that the liquid should contain no nitric acid; in which case it must be previously evaporated with chlorohydric acid. The gold, when collected on a filter, is calcined to redness before being weighed.

In order to separate gold from the metals previously described, the insolubility of the metal in nitric acid is sometimes relied on, while at other times all the metals are dissolved in aqua regia, and the gold is precipitated by protosulphate of iron, or, better still, by heating the solution with

a certain quantity of oxalic acid; which latter method has the advantage of not introducing a new metal into the liquid. Gold is sometimes also separated by precipitating it in the state of sulphide, by sulphydric acid gas, the sulphide leaving metallic gold after calcination.

Metallurgy of Gold.—Gold is almost always found in the native state, being sometimes pure, but more generally alloyed with certain quantities of silver. It occurs in three kinds of bearings;—

1. In veins, generally quartziferous, which contain other metallic minerals, as ores of copper, lead, silver, and pyrites; the veins usually traversing the primitive rocks.

2. In small veins scattered through rocks situated at the separation of the crystalline and stratified rocks.

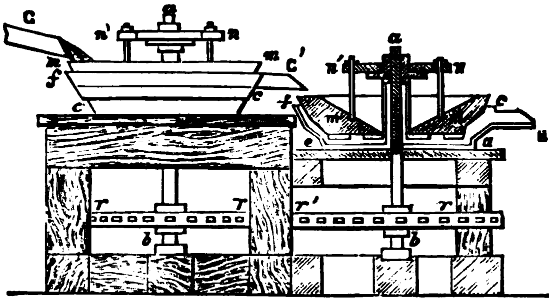
3. In disaggregated quartzose sands, often extensively seen in alluvial formations, and owing their presence to the disintegration of auriferous crystalline rocks which exist in the vicinity. The greater specific gravity of the gold prevents its particles from being carried as far as those of the other minerals with which it is mixed, and its resistance to the action of the greater part of chemical agents preserves it in the state of spangles. Alluvious soils containing gold chiefly occur in open valleys between primitive mountains, where gold is frequently found in place. The principal localities of auriferous sands are in California, Australia, Brazil, Mexico, Chili, Africa, the Ural and Altai Mountains in Siberia—the quantity of gold annually extracted from all of which amounted, in 1851, to 178 tons, of which California alone produced 110. Gold is generally found in the sands in the form of spangles, or shapeless and rounded grains, which, when they are of any considerable size, are called *river* or *wash gold* (pépites). Grains are sometimes found of the size of a hazel-nut, and pieces weighing several kilogrammes have been met with; one lump weighing 36 kilogrammes was found in the Ural. Gold exists in the drift-sand of all rivers which arise from or flow over a large extent of primitive rocks; and several auriferous alluviæ are known in France, such as those of the Ariége in the Pyrenees, of the Gardon in Cevennes, the Garonne, and the Rhine, near Strasburg. It is found in too small quantity to be worked to advantage; but the inhabitants look for it when they would otherwise be idle, and are then called *gold-finders*. The spangles of gold scattered through the river-sand are generally so excessively small that more than twenty are often required to make a milligramme. In Siberia, sands containing only 0·000001 of gold are not considered worthy of being worked; and the Rhenish sands contain, on an average, about $\frac{1}{4}$ of this quantity. Gold exists also, combined with tellurium, in certain mines of Transylvania. An alloy of gold with silver and palladium, in the form of small crystalline grains, occurs in Brazil, and is called *auro-powder* or *auro-dust*. Lastly, all pyrites in primitive rocks contain a small quantity of gold, and are often rich enough to be worked to advantage.

When gold exists in veins which contain other metals, as lead, copper, or silver, those metals in which the gold is concentrated are first extracted from the ores, and the gold is then separated by *refining*, a process presently to be described. The ore is frequently first subjected to amalgamation, as in the case of silver ores, when the gold dissolves in the mercury, and, after the liquid amalgam has been filtered, a more solid amalgam is obtained, from which the gold is separated by distillation. The ore is then smelted, so as to obtain a matt from which a certain quantity of gold can still be extracted.

Auriferous sands are washed in the most simple manner, either in wooden tubs, or on inclined planes over which a current of water flows, and they are then treated by amalgamation. In the Ural, the auriferous sand is poured into boxes, the sheet-iron bottom of which is provided with openings of 2 centimètres in diameter, and, while a stream of water flows through the boxes, the workman stirs the sand constantly with a shovel, when the finer portions fall through the holes and are collected on large sleeping tables covered with muslin. The sand is frequently swept toward the head of the table, where the gold remains with the heavier minerals; and the sand, being enriched by this washing, is again more carefully washed on smaller tables. The titanic iron and magnetic oxide of iron being separated by a magnet, the material is fused in large graphite crucibles, at the bottom of which the gold collects, while the upper part is filled by a slag containing a quantity of unmelted grains of gold. The slag being stamped and washed, the rich schlich thus obtained is smelted, yielding an auriferous lead, from which the gold is separated by cupellation.

In Tyrol a certain quantity of gold is extracted from pyrites by amalgamating them in mills resembling that represented in Fig. 3303, several mills being generally placed above each other. (The figure gives an external view of the upper mill and a section of the lower one.) The pyrites, in the state of an impalpable powder, is suspended in water, and conveyed into the upper mill by the conduit G, whence it flows into the second mill by the sluice G′. The bed of each mill is made of a cast-iron vessel *c d e f*, securely fastened on a strong wooden table; and in the centre of the vessel is a tubulure traversed by an axis of rotation *a b*, set in motion by the cog-wheel *r r′*. The runner-stone *m m′* of each mill is of wood, and resembling the shape of the bed; but being about 2 centimètres smaller, is furnished with several sheet-iron teeth projecting about 1 centimètre. The upper surface of the runner-stone is shaped like a funnel, into which is poured the liquid mud, which passes between the stones and flows out by the conduit G′. The stones make about

3303.



fifteen or twenty revolutions a minute; and 25 kilogrammes of mercury are placed at the bottom of each, making a layer of about 1 centimètre in thickness, against which the teeth of the wheel constantly strike, while at the same time they stir up the ore. The gold is dissolved by the mercury, and, after continuing this process for four weeks, it is withdrawn and filtered through a chamois skin, which retains a solid amalgam containing nearly one-third of its weight of gold, which is then separated from the other metals by cupellation.

Alloys of Gold.—Gold is rarely used in a state of purity, as it is too soft, and its hardness must be increased by the addition of a small quantity of silver or copper, forming more fusible alloys than pure gold.

The standard of French pure gold coin is $\frac{900}{1000}$, the law allowing a variation of $\frac{2}{1000}$ above and $\frac{2}{1000}$ below; while medals contain 0·916 per cent. of gold, with the same variation. There are three legal standards for jewellery, the most common of which is $\frac{750}{1000}$, while those of $\frac{840}{1000}$ and $\frac{920}{1000}$ are rarely used; and the legal variation is $\frac{3}{1000}$ below the standard, no superior limit being fixed.

Gold is soldered with an alloy called *red gold*, of 5 parts of gold and 1 of copper; an alloy made of 4 parts of gold, 1 of copper, and 1 of silver also being used.

The clear colour of gold is given to jewellery by dissolving the copper which exists in the superficial layer; to effect which the articles are heated to a dull red heat, and dipped, after cooling, into a weak solution of nitric acid, which dissolves the copper. A thicker coating of pure gold is obtained by allowing them to remain for fifteen minutes in a paste formed of saltpetre, common salt, alum, and water; the chlorine set free by the action of the sulphuric acid on the salt and saltpetre dissolving the copper, silver, and gold, while the latter metal is again deposited on the article. The surfaces are then burnished.

Separation of Gold and Silver.—The separation of gold and silver, more generally called the *refining of the precious metals*, is now done by treating the alloy by concentrated hot sulphuric acid, which dissolves the silver only. But in order that the alloy may be completely acted on, it should neither contain more than 20 per cent. of gold, nor than 10 per cent. of copper, because sulphate of copper is but slightly soluble in concentrated sulphuric acid. The alloys are fused in crucibles, and when they are too rich in gold, a certain quantity of silver is added—silver containing a small quantity of gold being preferred. The fused alloy is granulated by being poured into water, and then placed in a large kettle with 2½ times its weight of concentrated sulphuric acid marking 66° on the areometer, the kettle being covered with a lid furnished with a disengaging tube. The acid, being heated to boiling, is partly decomposed, and sulphates of silver and copper are formed, while sulphurous acid is disengaged, which is sometimes passed into the leaden chambers where sulphuric acid is manufactured. When gold coin is to be refined, it is merely roasted.

After four hours, when the alloy is completely destroyed, there is introduced into the kettle a certain quantity of sulphuric acid marking 58°, and obtained by the concentration of the acid mother liquid of the sulphate of copper obtained in refining, as will presently be explained. After having boiled the liquid for fifteen minutes, the kettle is taken from the fire and allowed to rest, when the greater part of the gold collects at the bottom of the vessel, from which the nearly boiling liquid is decanted off into leaden boilers containing the mother liquid arising from the purification of the sulphate of copper by crystallization. The boilers are heated by steam; and after the sulphate of copper at first deposited is redissolved, the liquid is allowed to rest for some time, when the whole of the gold is deposited. The clear liquid is then drawn off by a siphon, and passed into other boilers heated by steam, and containing blades of copper, which precipitate the silver in the form of small crystalline grains; the metal being in a short time so perfectly precipitated that the liquid is not clouded by common salt. The precipitated silver is carefully washed, and then compressed by an hydraulic press into compact prisms, which, after being dried, are melted in earthen crucibles, furnishing a metal which contains only a few thousandths of copper.

As the gold arising from the first action of the sulphuric acid still contains a certain quantity of silver, it is heated anew, in a platinum crucible, with concentrated sulphuric acid, which abstracts the balance of the silver; a third treatment with sulphuric acid being often required. The gold dust, after being well washed and fused, contains 995 thousandths of pure gold.

The acid solution of sulphate of copper which arises from the precipitation of the silver by copper is evaporated in leaden kettles until it marks 40° on the areometer; a large proportion of the sulphate of copper being deposited in small crystals during the cooling. After another evaporation, the mother liquid yields an additional quantity of crystals: and the last liquid, which refuses to crystallize, is used as a solution of sulphuric acid, and poured into the cast-iron boiler, after this action on the alloy. The sulphate of copper is purified by recrystallization.

When the quantity of gold and silver contained in an alloy does not exceed 0·200 or 0·300, the granular material is first heated in a reverberatory furnace, when a portion of the copper is converted into oxide, which is dissolved by treating the roasted substance with weak sulphuric acid; and the alloy, being thus brought to the medium standard of 0·500 or 0·600, may be refined by the ordinary process. The process of refining gold pursued at the United States Mint, in Philadelphia, is similar to the method formerly called quartation, and consists in melting gold with silver, and then extracting the silver with pure nitric acid. The deposit of grains of native gold is first melted with borax and saltpetre, occasionally with soda to remove quartz, and being cast into a bar, is carefully weighed, accurately assayed to $\frac{1}{2000}$ for gold, and from the assay and weight the value of the deposit is calculated. Although a million of dollars may be deposited in a day, upon an arrival from California, yet such is the expedition of the assay department, that in a few days the deposits are all paid off. As soon as the gold is assayed, each lb. of it is melted with 2 lbs. of pure silver, and the mixture, after stirring, poured into cold water, by which it is *granulated*, divided into small irregular fragments, presenting a large surface to the subsequent action of the acid. The granulations are then put into large porcelain jars of 50 gallons each, of which there are about seventy in use, and nitric acid poured in them. The jars being placed in leaden-lined wooden troughs, containing water, are heated by a steam coil in the water, causing the nitric acid

to dissolve out the larger proportion of silver. A steam heat is given during several hours, and the liquid allowed to repose until the following morning, when the solution of nitrate of silver is drawn off by a gold siphon, and transferred to a large vat of 1200 gallons, containing a saturated solution of common salt. Fresh acid is then added to the gold in the pots, already nearly parted, steam heat applied again for several hours, and the whole left again to repose. On the following morning the acid liquid of one of the pots being drawn off and the fine gold removed to its filter, fresh granulations of gold and silver are introduced, and the acid liquid of the adjoining pot, containing only a small quantity of nitrate of silver, poured over it. A fresh charge of granulated metal is thus first worked by the yet strong acid, which acted on the nearly fine gold of the previous charge. A charge of $800,000 or more is easily worked off, *refined*, in two days, by 4½ lbs. of parting acid to every lb. of gold. The gold is washed thoroughly on a filter by hot water, pressed in a hydraulic press, further dried, melted with copper, and cast into bars, about 2400 ozs. troy constituting a melt. After being assayed, they are then remelted with the calculated quantities of copper or fine gold requisite to bring them to a standard of 900 thousandths fine, and cast into ingots. Upon their proving correct in the assay, usually to within $\frac{1}{1000}$ of the standard, they are delivered to be coined. The chloride of silver, accurately precipitated with a slight excess of salt, is filtered and washed thoroughly on large filters, of 3 ft. by 5 ft., and 1½ in. deep. It is then transferred to lead-lined wooden vats, reduced to metallic silver by granulated zinc, and, the excess of zinc being removed by sulphuric acid, washed, pressed in the hydraulic press, dried by heat, and remelted with a new portion of gold.

This method of parting formerly required 3 parts of silver to 1 part of gold, and the latter constituting a fourth part of the alloy, the process was termed *quartation*. We have, however, found that 2 parts of silver to 1 part of gold are quite sufficient; and if the metal be well granulated, the acid will not leave 10 thousandths of silver in the gold, which is sufficient to prevent the too darkening effect of copper in the coin.

Analysis and Assaying of Alloys of Gold.—Alloys of gold and copper may be analyzed by cupelling them with lead, and following exactly the same process as described for the cupellation of alloys of silver and copper. If the alloy contains no silver, the weight of the lump obtained represents pretty exactly the quantity of pure gold which existed in the alloy; but if, as more frequently happens, the alloy contains a certain proportion of silver, this latter metal remains alloyed with the gold after the cupellation. However, the process of direct cupellation is attended with surpluses and losses which sometimes reach 3 thousandths. When the temperature of the muffle is very great, there is a small loss arising from the absorption of a small quantity of gold by the cupel; and when the heat is too low, the gold retains a small quantity of copper and lead; although gold loses less by volatilizing than silver.

In order to determine exactly the quantity of gold existing in a ternary alloy of gold, silver, and copper, it is cupelled at a moderate heat with a certain quantity of silver and lead, in order to obtain an alloy of silver and gold, from which the latter can be perfectly separated by means of an excess of nitric acid, which dissolves the silver and leaves the gold pure. In order, however, to ensure exact results, there must be a certain ratio between the quantities of gold and silver; because, if the proportion of silver be too small, the nitric acid does not dissolve it entirely; and if, on the contrary, the quantity of silver be too great, the silver and copper are completely dissolved, while the gold separates in the form of powder, which it is difficult to collect without loss. Experience has shown that the most favourable conditions for the assay, commonly called the parting (*départ*), consist in reducing the alloy to ¼ of gold and ¾ of silver, in which case it is completely acted on, while the separated gold preserves the form of the original alloy, and does not become divided, if the operation be carefully conducted. This operation has received the name of *quartation*.

The proportion of lead to be added, which varies with the standard of the alloy, is indicated in the following Table;—

Standard of gold alloyed with copper.	Quantity of lead necessary to be added to entirely remove the copper by cupellation.
1000 thousandths	1 part.
900 "	10 parts.
800 "	16 "
700 "	22 "
600 "	24 "
500 "	26 "
400, 300, 200, 100 "	34 "

Let us suppose that the standard of a piece of coin is to be determined, the legal standard of which, which may be regarded as its approximate standard, is $\frac{900}{1000}$. The quantity of alloy usually operated on being 0·500 gramme, containing according to the legal standard 0·450 gramme of gold, therefore 1·350 gramme of silver and 5 grammes of lead must be added. But if an alloy is to be assayed, the legal standard of which is entirely unknown, the first step is to ascertain the latter by approximation, by means of the *assay by the touch-needle*, about to be described, after which the process is continued as usual.

The lead is first placed in the heated cupel, and when it is in fusion, the mixture of gold and silver is introduced, having been previously weighed and wrapped in a piece of paper. The cupellation is allowed to go on as usual, and requires less care than the cupellation of silver, because silver alloyed with gold is not liable to blister; but the cupel should be removed immediately after the lighting, to avoid loss by volatilization. The lump is removed after cooling,

flattened under a hammer, annealed for a few moments, and then rolled between cylinders; after which the sheet thus obtained is rolled into a spiral form, and subjected to the action of nitric acid in a small assayer's flask, Fig. 3304, into which 30 grammes of nitric acid of 22°, Baumè's *Hydrometer*, are poured, and boiled for twenty minutes. The acid is then decanted and replaced by 30 grammes of pure concentrated nitric acid marking 32°, which is boiled for ten minutes; when the acid is decanted, and the gold, which has preserved the shape of the alloy, washed several times. The flask being afterward completely filled with water, its mouth is closed with the thumb, and it is inverted, when the spiral sheet of gold falls slowly through the liquid column, and is received in a small earthen crucible, after which the water is poured off, and the crucible heated to redness in the muffle.

3304.

The acid should not be too concentrated, because the gold might be divided. When the assay has been made with the precautions indicated, the gold remains in the form of a spongy, brown, and very friable mass, of nearly the same volume as the original alloy; but it contracts considerably when heated in the small crucible, becoming harder and assuming the lustre and colour of malleable gold. The calcined gold being exactly weighed, the standard of the alloy is thus obtained within nearly 1 thousandth.

Direct assays made on known alloys of gold and silver have shown that the operation, when carefully performed, as just described, can give rise only to the following errors;—

True standards of the alloy.	Standards found.	Differences.
900	900·25	+0·25
800	800·50	+0·50
700	700·00	0·00
600	600·00	0·00
500	499·50	−0·50
400	399·50	−0·50
300	299·50	−0·50
200	199·50	−0·50
100	99·50	−0·50

The assay just described cannot be applied to fine jewellery, because the article would be destroyed by the process, and gold jewellery is therefore subjected to a test called the assay by the touch-needle, which does not injure it, and yet enables a skilful assayer to determine its standard within nearly 1 thousandth. The method consists in rubbing the object against a very hard black stone, on which it leaves marks, from the colour of which, and their behaviour when moistened with a mixture of nitric acid of a density of 1·34 with 2 per cent. of chlorohydric acid, the assayer forms an approximate opinion of the standard of the alloy. The black stone used, called *touch-stone*, is a kind of quartz, coloured with bitumen, which formerly was imported from Lydia, but has likewise been found in Bohemia, Saxony, and Silesia. The conditions essential to a good touch-stone are;—An intense black colour, incapability of being acted on by acids, hardness, and a sufficient degree of roughness to retain some of the gold.

The assayer is provided with a series of small blades, called *touch-needles*, consisting of alloys of copper and gold, the standard of each of which is exactly known, which enable him to compare the marks they leave on the touch-stone, before and after the action of the acid, with that of the alloys to be assayed.

No regard should be paid to the first marks left by the articles on the touch-stone, as they are made by the superficial layer, and always show a higher standard, because the surface consists of pure gold; and several marks should therefore be made, the last of which only is examined. Alongside of these marks others are made with that touch-needle the composition of which approaches nearest to that of the article; when a glass rod, dipped in the acid, is drawn over both, after which the colour of each mark and the manner of action of the acid are examined.

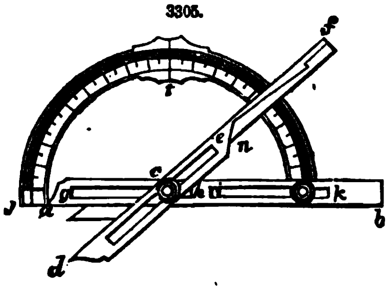
See Alloys. Amalgamating Machine. Amalgamation Pan. Assaying. Atomic Weights. Batea. Battery. Boring and Blasting. Buddle. Drainage. Electro-metallurgy. Founding and Casting, p. 1551. Furnace.

Works on Gold: — J. Calvert, 'The Gold Rocks of Great Britain and Ireland,' 8vo, 1853. S. Davison's 'Gold Deposits in Australia,' 8vo, cloth, 1861. J. Arthur Phillips, 'The Mining and Metallurgy of Gold and Silver,' royal 8vo, 1867. Silversmith, 'Handbook for Miners,' 12mo, New York, 1868. R. B. Smyth, 'The Gold Fields of Victoria,' 4to, Melbourne, 1869. W. P. Blake, 'The Production of the Precious Metals,' 8vo, New York, 1869. Von Cotta, 'Treatise on Ore Deposits,' by Prime, 8vo, New York, 1870. P. M. Randall, 'The Quartz Operator's Handbook,' 12mo, 1871.

GONIOMETER. Fr., *Goniomètre;* Ger., *Goniometer;* Span., *Goniómetro.*

Various instruments termed goniometers are employed in the measurement of the angles of crystals. Two kinds are in use—the common or contact goniometer, and the reflecting goniometer. The first class only of instrument is here described, as it will sufficiently answer every purpose of the mining mineralogist. The most simple form of instrument, Fig. 3305, consists of a graduated brass semicircle, on which two metallic cross-blades are fixed. One of these cross-blades, *a b*, is fixed at the zero of the division; the other, *d f*, is movable, and denotes on the circle the angle of the crystal. In order to measure a dihedral angle, one of its faces is

3305.

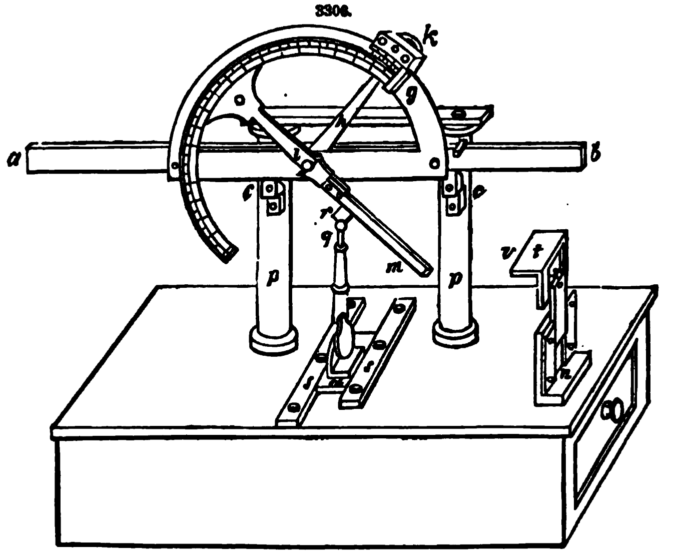


applied to the fixed cross-blade $a b$, in such a manner that the edge of the angle is perpendicular to the plane of the circle; the movable cross-blade is then adjusted until its prolongation rests upon the outer face of the angle. It is evident that the angle comprised between the two cross-blades, and which is directly indicated on the circle, is the measure of the angle sought.

The two cross-blades $a d$, $d f$, slide in the grooves i K, $g h$, $d n$, so as to admit of the ends $c a$ and $c d$ being made as short as is required. This condition is indispensable, as it is often necessary to measure very small crystals, which can only be introduced easily between the two cross-blades when their free ends can be very much shortened.

This form, however, of the common goniometer has many inconveniences. The observations are rendered difficult from the fact that the crystal under examination has to be held with one hand, and the instrument with the other. Moreover, in holding it before the eye, to ascertain if the cross-blade is in perfect contact with the crystal, continual vacillations and disturbances are produced, which render anything like a correct observation very difficult. These inconveniences are overcome by the use of a fixed instrument. The crystal under examination is also fixed on a support, so that both hands are at liberty. This instrument, Fig. 3306, consists of a semicircle

3306.



fixed on a rod $a b$, supported by columns $p p$. The rod $a b$ can be moved horizontally, from right to left, in the grooves $c c$, in which are placed small friction rollers, so as to render the movement as easy as possible. The fixed semicircle carries another, $f g$, movable on the centre o, and divided into degrees; $h i k$ is a vernier which also moves on the centre, but behind the movable semicircle between it and the fixed, to which it can be at any time fastened, and in any required position, by the thumb-screw k; this vernier gives the minutes. $l m$ is a blade whose movement carries round the circle $f g$; q is a small stem, the function of which is to support the crystal r, which is firmly fastened with wax. It is so arranged that it can be lengthened or shortened, be inclined either from or towards the operator, and capable of turning on itself. It is supported on a small movable platform u, running between the rods $s s$, which form a groove. The piece $t v$, seen on the side of the apparatus, is a sight, which, applied against one of the rods s, when the platform is drawn sufficiently forward, enables the operator to judge if the edge formed by the two faces of the crystal is exactly horizontal, and if it be perpendicular to the plane of the circle.

To measure a crystal it must be firmly fixed on r, and the movable platform brought forward; the sight must now be placed against the rod s, and the upper part raised or lowered as needed; looking from above, it can be seen whether the edge of the crystal is parallel to the edge v, in which case it is perpendicular to the plane of the circle. If the parallelism be not perfect, the rod q is turned on its axis until the proper position is attained. The crystal must then be viewed through the opening x, and the same angle adjusted horizontally, which can be effected by inclining the rod either one way or the other as required

When the crystal is properly adjusted, the movable platform is pushed under the circle. The blade $l m$ is now to be moved, and at the same time the rod $a b$ is to be pushed either to the right or left as may be found necessary, so that the edge of the blade may be in perfect juxtaposition with the face of the crystal; when this has been accomplished, the vernier is carried to the end of the movable semicircle, where a small cleat stops it exactly at zero; it is then fixed by the screw k.

This done, the platform is drawn from under the circle, and the blade passed in the contrary direction to that which it before occupied; the platform replaced, and the blade brought into juxtaposition with the other face of the crystal; this accurately done, the stem and crystal are removed.

By this second application of the blade to the crystal the semicircle has turned, and the point where it stops indicates the measure of the angle, which is read on it in degrees; the vernier furnishes the minutes.

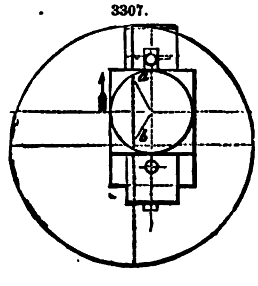
GOUGE. Fr., *Gouge*; Ger., *Hohlmeissel*; Ital., *Sgorbia*; Span., *Gubia*.
See Augers. Hand-Tools.

GOVERNOR, Steam-Engine. Fr., *Régulateur*; Ger., *Regulator*; Ital., *Regolatore*; Span., *Regulador*.

How the action of centrifugal force is utilized in the construction of steam-engine governors is fully investigated in our article, Angular Motion, p. 101.

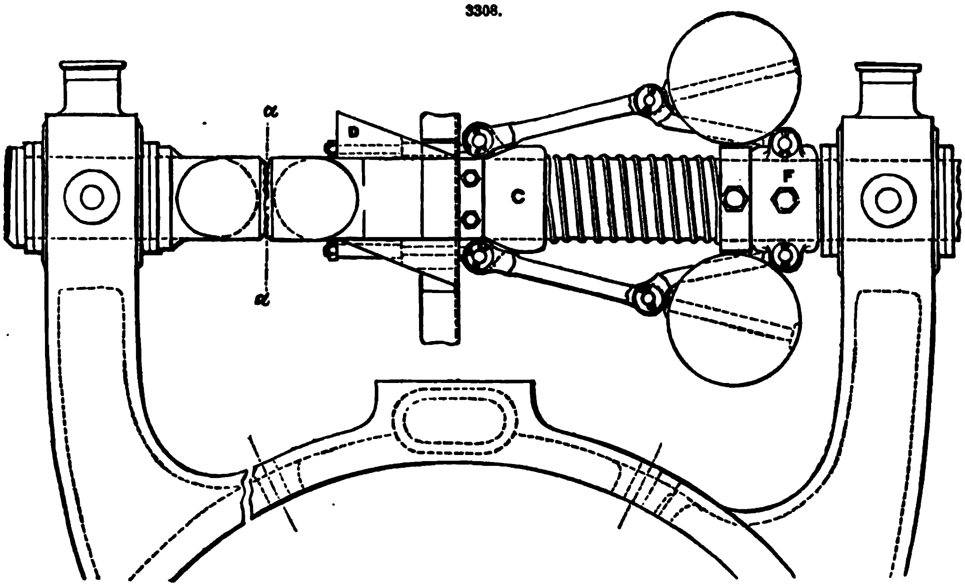
Roby and Richardson's Governor, Figs. 3307, 3308.—This invention consists in mounting the governor on the crank-shaft, with which it revolves, and causing it to act directly upon the slide-valve eccentric, so as to regulate the quantity of steam that shall be admitted during the stroke according to the work done by the engine; and as no throttle-valve is needed, steam can at all times be taken into the cylinder at nearly boiler-pressure, and thus do the same work; a much earlier cut-off is attainable and many advantages gained.

3307.



The eccentric A, Fig. 3307, has a rectangular slot cut in it parallel to a line connecting its two centres of forward and backward motions. The slot *a b* fits over a square part of the crank-shaft, upon which it slides at right angles to the crank by which it is driven. It is held in position on this square by two wedges D E, shown in Fig. 3308. Fig. 3308 also shows the position of the governor on crank-shaft; the boss F is fast on the shaft, while G is free to slide towards F. When the balls expand to this slide G the wedges D E are fixed. When the balls expand by their centrifugal force the wedges are drawn out, and the eccentric slides upward in the direction of the arrow, Fig. 3307. The travel of the valve is reduced, the angle of the eccentric with the crank is altered so as to make the cut-off earlier while the lead remains constant.

3308.



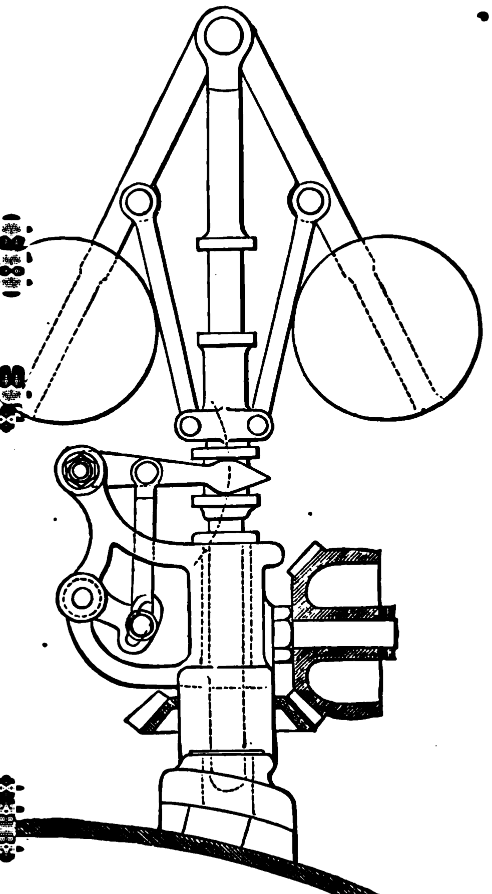
Clayton and Shuttleworth's Governor, Fig. 3309. This is a simple form of centrifugal governor employed both to portable and stationary engines, but especially to the former class of engine. Motion is imparted to the vertical spindle, through the mitre-wheels and pulley by a plain leather belt, from the crank-shaft of the engine. As the speed increases the centrifugal force causes the balls to expand, and through a proper arrangement of links and sliding sleeve, the balls raise the forked lever, which in turn, by means of the link and quadrant, closes the throttle-valve in the steam-pipe, thus instantly checking the speed of the engine. The speed of the engine being checked, the speed of the governor is also checked, and the throttle-valve opened to a corresponding extent. As in ordinary governors, by a proper adjustment of the quadrant, the throttle-valve is set so that the speed of the engine may be rendered uniform under any variation of work.

N. P. Burgh, in his useful work on Marine Engineering, observes;—" The changes of speed being so sudden, it is obvious that in designing a governor for a marine engine, such an arrangement should be adopted as would have an action extremely sensitive, powerful, and prompt in affecting the valve. Modifications of the old two-ball governors were first tried, and afterwards balanced four-ball governors. The first failed, as the governing action of the balls was destroyed by the motion of the vessel. With the last, in which the balls are arranged to balance each other in such a way that the action of the instrument is not affected by the same cause, it is apparent that when the sudden acceleration of speed in the engine takes place, the inertness of the balls resisting the sudden motion may prevent the prompt action on the valve.

" Attention being drawn to the subject by this conclusion, a governor, consisting of a fly-wheel

...ose. There are several varieties of these governors ...o governing principle on which the action of most

3309.



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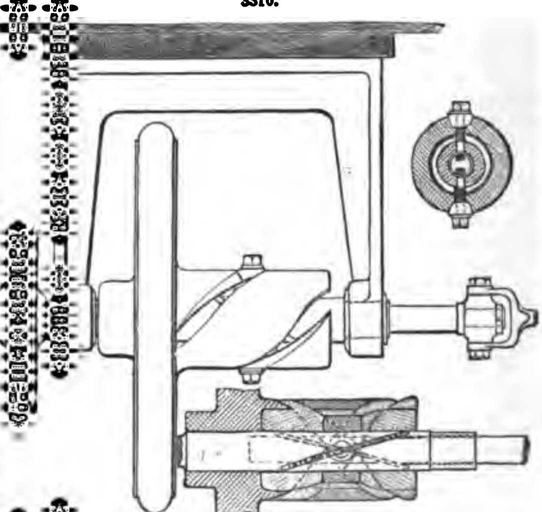
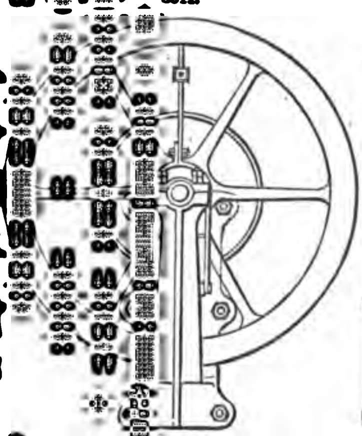


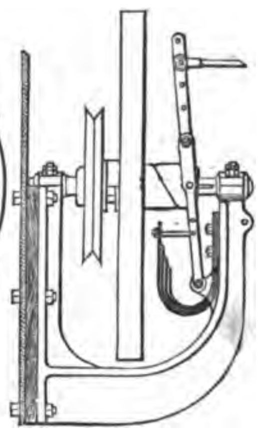
...spindle attached to a lever for working the valve, ...it in such a manner that the ends project and form ...in the inner surface of a ring, which ring has also

...o pins or studs within the circle of the ring pass ...e short spindle connected with the lever working ...of the ring pass into the spiral guides in the

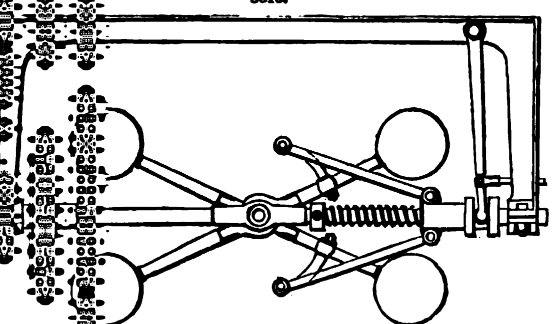
3311.



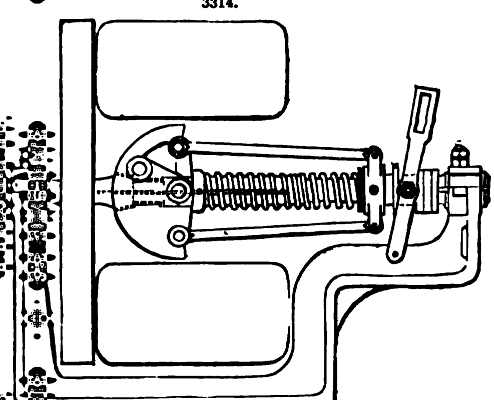
3312.



3313.



3314.



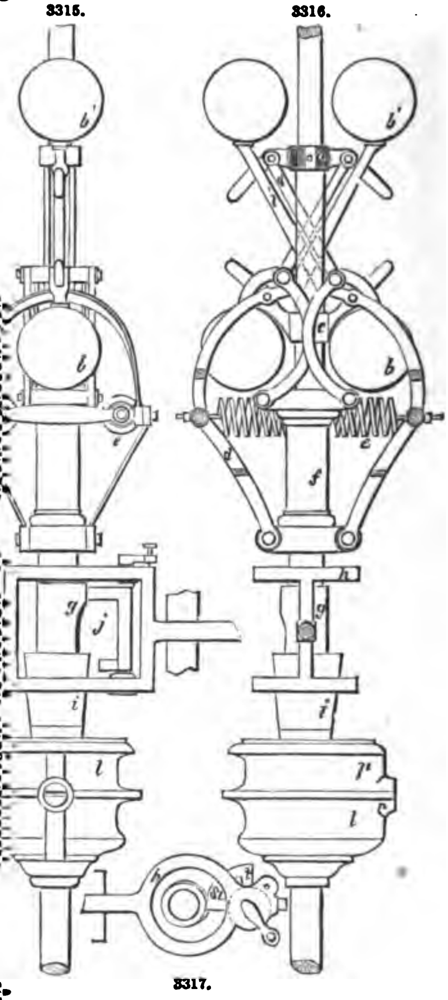
...rks loosely on the spindle and gears into two ... a cross-head made fast to and supported by the ...d, as they are linked by the rods to the sliding ...y the lever and rod is certain.

...al regulator is turned by the engine to which it ...arried on the fixed cross-head, being geared into ...dency to turn round on this pinion; but as they ...y pull inwards this collar, and so compress the ...ar, and consequently on the toothed sectors, serves ...anes on the momentum-wheel balance the action ...ers to their progress through it. As the leverage

[illegible] -wheel pinion increases, as the spring becomes [illegible] action of the spring in propelling the momentum- [illegible] uch only is required as will carry round the [illegible] speed, and overcome the friction of working [illegible]. When the momentum-wheel is in motion, it [illegible], at a velocity proportioned to that at which it is [illegible] from the usual causes may attempt to vary this [illegible] it leaves it free to act upon the sliding collar, [illegible] time closing the throttle-valve by its action in [illegible] time opening the throttle-valve by its action [illegible] of the engine. It will now be evident that the [illegible] great indeed, having for its agent a momentum- [illegible]; and from the powerful resisting tendency of [illegible] eness of action must also be very great, and in [illegible] vary its speed; and the engine itself being the [illegible] follows that the inert power of the momentum- [illegible] to the rapidity with which the engine varies its

[illegible] 3317.—This governor has four balls, $b\,b$, $b^1\,b^1$, [illegible] being, in their turn, connected by the rods $c\,c^1$

3315. 3316.



3317.

[illegible] amount of opening given to the throttle-valve of [illegible] of the cam which bears against the tappet t, this [illegible] revolution of the governor, so long as the balls [illegible] ment of the governor-balls is prevented by a piston [illegible] in the air-cylinder l, this cylinder being furnished [illegible] other. The sectional area of this passage, and [illegible] transferred from one side of the piston to the other,

[illegible] means the speed with which an alteration in [illegible] gulated.

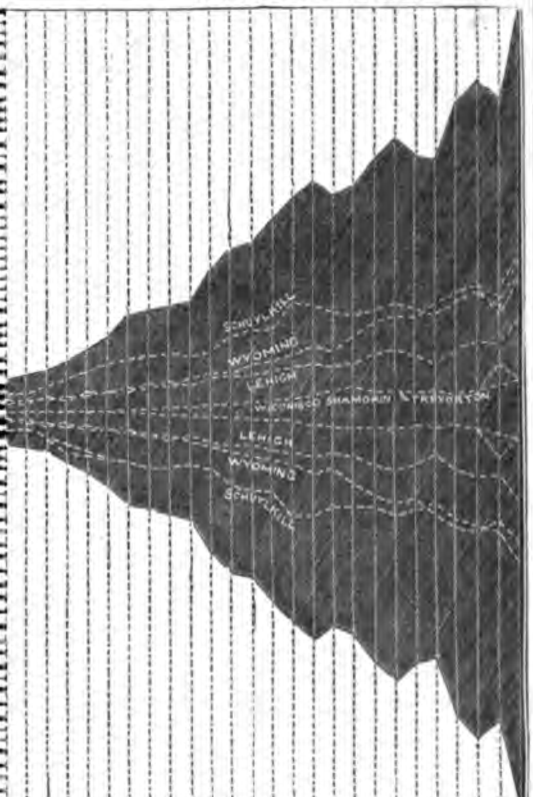
[illegible] TAILS OF ENGINES. ENGINES, VARIETIES OF.

[illegible] ique; GER., *Graphisches Diagram*; ITAL., *Dia-*

[illegible] ion, in which the relations or laws involved in [illegible] ns of *diagrams*, is termed the graphic method; [illegible] led graphic diagrams. It is necessary here to [illegible] hen employed to show the daily changes of the [illegible] hours of the day, and the ordinates, the corre-

[illegible] P. W. Sheafer to trace the progress of the [illegible] ntion.

1842	272,546	541,504	252,6[illegible]	[illegible]	1,1[illegible]
1843	267,793	677,312	285,605	10,000	1,263,598
1844	371,002	840,378	365,911	13,087	1,630,850
1845	429,453	1 083,796	451,836	10,000	2,013,013
1846	517,116	1,236,582	518,389	12,572	2,344,005
1847	633,507	1,583,374	583,067	14,904	2,882,309
1848	670,321	1,652,835	685,196	19,356	3,089,238
1849	781,656	1,605,126	732,910	45,075	3,242,966
1850	690,456	1,712,007	827,823	57,684	3,358,899
1851	964,224	2,229,426	1,156,167	99,099	4,403,730
1852	1,072,136	2,450,950	1,284,500	119,342	4,933,471
1853	1,054,309	2,470,913	1,475,732	113,007	5,195,151
1854	1,207,186	2,895,208	1,603,478	234,000	6,002,334
1855	1,284,113	3,318,555	1,771,511	240,338	6,604,318
1856	1,351,970	3,258,356	1,972,581	313,444	6,927,580
1857	1,318,541	2,985,541	1,952,603	388,255	6,663,828
1858	1,380,030	2,886,449	2,186,094	370,424	6,759,369
1859	1,628,311	3,004,953	2,731,236	443,755	7,780,518
1860	1,821,674	3,270,516	2,941,817	479,116	8,412,946
1861	1,738,377	2,697,489	3,055,140	463,308	7,843,211
1862	1,351,054	2,890,593	3,145,770	481,990	7,739,899
1863	1,894,713	3,433,265	3,759,610	478,418	9,631,101
1864	2,054,669	3,642,218	3,960,836	519,752	10,184,320
1865	2,040,913	3,735,802	3,254,519	621,157	9,652,391
1866	2,179,364	4,957,180	4,736,616	830,722	12,703,882



GRATE. Fr., *Grille*; Ger., *Rost*; Ital., *Grata*; Span., *Reja*.

See Boiler.

GRAVING DOCK. Fr., *Forme de radoub*; Ger., *Trockendock*; Ital., *Bacino a secco*; Span., *Astillero*.

See Dock.

GRAVITY. Fr., *Gravité*; Ger., *Schwerkraft*; Ital., *Gravità*; Span., *Gravedad*.

Centre of Gravity.—The centre of gravity is the centre of the parallel forces due to weight. The weight of each molecule of a solid body is a vertical force acting in a downward direction. The resultant of these forces is the total weight of the body; and the centre of the parallel forces takes the name of *centre of gravity*. As a matter of fact, the common direction of the forces cannot be made to vary here; but we may, which amounts to the same thing, vary the position of the body with respect to the vertical; and the centre of gravity is the point through which the resultant of the weight of all the molecules constantly passes, whatever the position of the body may be. If the centre of gravity is one of the points in a solid body, we may conceive the weight of all the molecules replaced by a single vertical force equal to their sum and applied to the centre of gravity. But if the centre of gravity is situate outside of the body, we can conceive this substitution only by supposing the point to be invariably fixed to the system of bodies, a supposition which is made use of for the purpose of simplifying demonstrations, data, or formulæ, but to which no idea of reality can be attached. It is for this same purpose of simplifying enunciations and formulæ that we sometimes extend the notion of a centre of gravity to a system which is not solid. There exists in this case no force capable of producing by itself the effect of the weight of the various molecules which make up the system; but it is often convenient to introduce into calculations the resultant which these forces would have if the system were to become instantaneously solid, and consequently to consider the point through which the resultant would constantly pass if the system, without changing its form, changed its position with respect to the vertical, in other words, the centre of gravity of this system. In the remarks which follow we shall assume that the body in question is solid.

Bodies are, in reality, composed of molecules separated from each other; but, in seeking the centre of gravity, we consider them as formed of a continuous matter. The effect of this mode of viewing the subject is merely to misplace, by qualities infinitely small, the points of application of the vertical forces considered, an error which in no way affects the result.

I. If $p, p', p'' \ldots$ &c., denote the weight of the elements of the volume of a body, $x, y, z, x', y', z', x'', y'', z''$, &c., their co-ordinates with respect to three rectangular axes, P the total weight of the system, and X, Y, Z, the co-ordinates of the centre of gravity, we have, by taking the moments successively with respect to the three co-ordinate planes,

$$\begin{aligned} \mathrm{P\,X} &= p\,x + p'x' + p''x'' + \ldots = \Sigma\, p\,x, \\ \mathrm{P\,Y} &= p\,y + p'\,y' + p''y'' + \ldots = \Sigma\, p\,y, \\ \mathrm{P\,Z} &= p\,z + p'\,z' + p''z'' + \ldots = \Sigma\, p\,z, \end{aligned}$$

whence we deduce

$$\mathrm{X} = \frac{\Sigma\, p\,x}{\mathrm{P}}, \qquad \mathrm{Y} = \frac{\Sigma\, p\,y}{\mathrm{P}}, \qquad \mathrm{Z} = \frac{\Sigma\, p\,z}{\mathrm{P}}, \qquad [1]$$

formulæ which determine the centre of gravity when we know the total weight of the system, and know also how to calculate the sums which appear as the numerators of these fractions.

II. When the body under consideration is *homogeneous*, that is, when its parts, however small we may suppose them, have a weight proportional to their volume, the position of the centre of gravity in the body becomes independent of the nature of this body, and its discovery is merely a matter of geometry. If we call the volumes of the various elements of the body v, v', v'', &c., the total volume V, and the weight of the unit of matter of which its body is composed Π, we shall have $p = \Pi\, v$, $p' = \Pi\, v'$, $p'' = \Pi\, v'' \ldots$, $\mathrm{P} = \Pi\, \mathrm{V}$, and the formulæ [1] will become, by cancelling in the numerators and the denominators the common factor Π,

$$\mathrm{X} = \frac{\Sigma\, v\,x}{\mathrm{V}}, \qquad \mathrm{Y} = \frac{\Sigma\, v\,y}{\mathrm{V}}, \qquad \mathrm{Z} = \frac{\Sigma\, v\,z}{\mathrm{V}}, \qquad [2]$$

formulæ which no longer depend upon the nature of the body, but only upon its geometrical form.

If one of the dimensions of the body were infinitely small with respect to the other two, so that the body was reduced to a surface, the quantities v, v', v'', &c., would denote the elements of this surface, and V its total area. If two dimensions were infinitely small with respect to the third, so that the body was reduced to a line, v, v', v'', &c., would represent the elements of this line, and V its total length.

The formulæ [2] would still hold if v, v', v'', &c., instead of representing the infinitely small elements of the volume, area, or length, expressed by V, represented the finite parts of this volume, area, or length, provided that x, y, z, were then the co-ordinates of the centre of gravity of v; x', y', z', the co-ordinates of the centre of gravity of v', and so on; for the weight of each part may be considered as a vertical force applied to its centre of gravity. Consequently, the moment of the total weight is equal to the sum of the moments of the partial weights; and in the case of homogeneous bodies, *the moment of the total volume is equal to the sum of the moments of the partial volumes*, if by the moment of a volume with respect to a plane, we understand the product of this volume by the distance from its centre of gravity to this plane. Applying this theorem successively to the three co-ordinate planes, and dividing by the total volume V, we fall again upon the equations [2].

III. To simplify the labour of finding the centre of gravity in homogeneous bodies, recourse may be had to the following propositions;—

1. *If the body can be decomposed into a number of parts having their centres of gravity in the same plane, or on the same straight line, the centre of gravity of the whole body will likewise be in this same plane and upon this same straight line.* For we may suppose the weight of each part applied to the centre of gravity of this part; we have then to compose a system of forces parallel and in the same direction, whose points of application are situate, by the hypothesis, in the same plane or on the same straight line. But according to the construction which determines the centre of parallel forces, this point will itself be situate in this plane or on this straight line; and this point is the centre of gravity of the whole body.

2. *If the body has a plane of symmetry, its centre of gravity is in this plane.* It is evident that the centres of gravity of the two parts of the body separated by the plane of symmetry are symmetrically placed with respect to this plane. But these centres of gravity may be considered as the points of application of the weights of the two parts, that is, of two equal forces, parallel and in the same direction. The resultant of these two forces passes therefore through the middle of the straight line which joins their planes of application, and that, whatever the position of the body with respect to the vertical may be; this middle is therefore the centre of gravity of the body. It is evident, besides, that this middle is in the plane of symmetry.

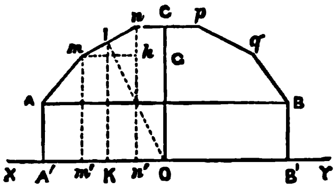
3. *If the body has an axis of symmetry, its centre of gravity is upon this axis.* For an axis of symmetry is always the intersection of at least two planes of symmetry.

4. *If the body has a centre of shape, its centre of gravity is in this point.* For a centre of shape is the intersection of at least two axes of symmetry.

5. *We may substitute for the elements of volume, area, or length, which make up the system under consideration, other elements of volume, area, or length, proportional to them, provided they have their centres of gravity in the same points.* For the centre of the parallel forces is not changed by substituting for the given forces, other forces proportional to them and applied to the same points.

IV. We will now pass on to the consideration of the methods of finding the centre of gravity of the principal figures, and we will begin with lines.

3319.



The Straight Line.—The centre of gravity of a straight line is in its middle; for this middle is in the plane of symmetry perpendicular to the straight line.

A Regular Broken Line.—Let A $m\,n\,p\,q$ B, Fig. 3319, be a regular broken line, A B its chord, O the centre of the inscribed circle, and O C its axis of symmetry. Here the centre of gravity sought, G, will be upon O C; we have to find the distance G O. Draw through the centre X V parallel to the chord A B; and project upon this parallel line the summits of the broken line, by means of the perpendiculars A A', $m\,m'$, $n\,n'$ . . . B B'. Let I be the middle of any part $m\,n$; join I O. Draw I K perpendicular to X Y, and $m\,h$ parallel to this line. Let x be the distance I K from the centre of gravity I of the part $m\,n$, to the straight line X Y, and X the distance O G. If we take the moments with respect to a plane drawn through X Y perpendicularly to the plane of the broken line, we shall have, from the formulæ [2], $X = \frac{\Sigma\, v\, x}{V}$ or $X = \frac{\Sigma\, .\, m\, n\, .\, IK}{V}$.

The similar triangles I K O and $m\,h\,n$ give $m\,n : IO = m\,h : IK$, whence $m\,n\,.\,IK = IO\,.\,m\,h$; consequently, $X = \frac{\Sigma\,.\,IO\,.\,m\,h}{V}$; or, as I O, the radius of the inscribed circle, is a factor common to all the terms of the numerator, and as $m'\,n'$ may be substituted for $m\,h$, $X = \frac{IO\,.\,\Sigma\, m'\,n'}{V}$. But $\Sigma\, m'\,n'$, or the sum of the projections of the sides or parts of the broken line upon X Y is A' B', which is equal to A B; and V is here the length A $m\,n\,p\,q$ B of the broken line. We may therefore write $OG = \frac{IO\,.\,AB}{A\,m\,n\,p\,q\,B}$, that is, *the distance from the centre of gravity of a broken line to its centre is a fourth proportional to the length of this broken line, its chord, and the radius of the inscribed circle.*

Circular Arcs.—The preceding proposition is independent of the number of parts in the broken line; it is therefore true if this number becomes infinitely great, that is, if the broken line becomes an arc of a circle calling the length of the developed arc L, its chord C, and the radius of the inscribed circle, which in this case will be confounded with the arc itself, R, we have $X = \frac{R\,.\,C}{L}$. We may therefore affirm that *the centre of gravity of an arc of a circle is upon its axis of symmetry, and its distance from the centre is a fourth proportional to the arc, its chord, and the radius.* For a half circle we should have $X = \frac{R\,.\,2\,R}{\pi\, R} = \frac{2}{\pi} R$.

Curves.—If the curve is plane, let $y = f(x)$ be its equation with respect to two rectangular axes traced in its plane, and let $d\,s$ be an element of the curve, corresponding to the co-ordinates x and y; the equations of the moments will give, calling s the developed length of the curve from the point the abscissa of which is x_0 to the point the abscissa of which is x_1,

$$s\,.\,X = \int_{x_0}^{x_1} x\, d\, s, \text{ and } s\, Y = \int_{x}^{x_1} y\, d\, s.$$

But we have $ds = dx\sqrt{1+[f'(x)]^2}$; consequently, deducing the values of X and Y, it will become

$$X = \frac{\int_{x_0}^{x_1} x\,dx\sqrt{1+[f'(x)]^2}}{\int_{x_0}^{x_1} dx\sqrt{1+[f'(x)]^2}}, \quad \text{and} \quad Y = \frac{\int_{x_0}^{x_1} f(x)\,dx\sqrt{1+[f'(x)]^2}}{\int_{x_0}^{x_1} dx\sqrt{1+[f'(x)]^2}}.$$

If the curve has a double curvature, let $x = \phi(z)$ and $y = \psi(z)$, be its equations with respect to three rectangular axes; let s be again the developed length of the curve from $z = z_0$ to $z = z_1$. We shall have by the theorem of the moments,

$$s\,X = \int_{z_0}^{z_1} x\,ds, \quad s\,Y = \int_{z_0}^{z_1} y\,ds, \quad s\,Z = \int_{z_0}^{z_1} z\,ds;$$

but here $ds = dz\sqrt{1+[\phi'(z)]^2 = [\psi'(z)]^2}$. Deducing the values of X Y, Z, we have

$$X = \frac{\int_{z_0}^{z_1} \phi(z)\,dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}}{\int_{z_0}^{z_1} dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}},$$

$$Y = \frac{\int_{z_0}^{z_1} \psi(z)\,dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}}{\int_{z_0}^{z_1} dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}},$$

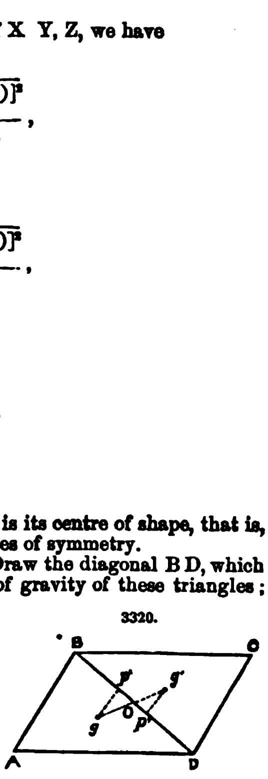
$$Z = \frac{\int_{z_1}^{z_0} z\,dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}}{\int_{z_1}^{z_0} dz\sqrt{1+[\phi'(z)]^2+[\psi'(z)]^2}}.$$

V. We will now consider the centre of gravity of plane figures.

Rectangular Figures.—The centre of gravity of a rectangular figure is its centre of shape, that is, the point of intersection of its median or bisecting lines, which are axes of symmetry.

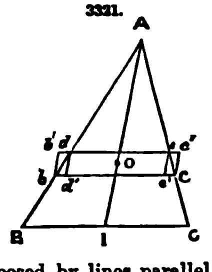
The Parallelogram.—Let A B C D, Fig. 3320, be a parallelogram. Draw the diagonal B D, which will divide it into two equal triangles. Let g and g' be the centres of gravity of these triangles; draw $g\,p$ and $g'\,p'$ perpendicular to B D, and let O be the middle of the diagonal B D. Without knowing the position of the points g and g' with respect to the two triangles to which they belong, we may admit that as these triangles are equal, their centres of gravity would coincide if the triangles were placed one upon the other. We have therefore $g\,p = g'\,p'$ and $O\,p = O\,p'$, consequently the angles $p\,O\,g$ and $p'\,O\,g'$ are equal, and $g\,O = g'\,O$. Therefore $g\,O\,g'$ is a straight line, and the point O is the middle of it. But the weights of the triangles A B D and B D C may be considered as applied to g and g'; the point of application of their resultant, that is, the centre of gravity of the parallelogram, is therefore situate in the middle of the straight line $g\,g'$, that is, in the point O, which is the middle of the diagonal B D. Thus *the centre of gravity of a parallelogram is in the middle of the diagonals*, which is also their point of intersection.

3320.



The Triangle.—Let A B C, Fig. 3321, be a triangle. Bisect B C and join A I. Draw $b\,c$ and $d\,e$ parallel to B C; and $b\,b'$, $c\,c'$, $d\,d'$, $e\,e'$ parallel to A I, which passes through the middle of $b\,c$ and $d\,e$, and also of $b'\,c'$ and $d'\,e'$, since $b'\,d = b\,d'$ and $e\,c = e'\,c$. The centre of gravity of the parallelogram $b\,b'\,c'\,c$ is therefore situate upon the straight line A I, in the middle O of the portion of this straight line included between $b\,c$ and $d\,e$. Likewise the centre of gravity of the parallelogram $d'\,d\,e'\,e$ is situate upon A I, in the middle of the portion of this straight line included between $b\,c$ and $d\,e$, that is, in the same point O. But the nearer the straight lines $b\,c$ and $d\,e$ are together, the smaller is the difference of the parallelograms $b\,b'\,c'\,c$ and $d'\,d\,e\,e'$ with respect to each other; and consequently the more they tend to be confounded with each other, and with the trapezium $b\,d\,e\,c$ included between them. Therefore, when the distance between the straight lines $b\,c$ and $d\,e$ is infinitely small, we may consider the trapezium $b\,d\,e\,c$ as confounded with one of these parallelograms, and we have consequently its centre of gravity in the same point O upon A I. It follows from this that if we conceive the triangle decomposed by lines parallel to B C into trapeziums infinitely narrow, all of these trapeziums may be considered as having their

3321.

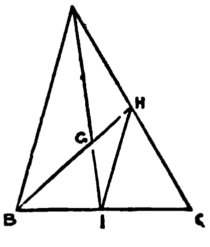


centres of gravity upon A I. Therefore, in virtue of the principle I established above, the centre of gravity of the triangle A B C is situate upon A I.

This being proved, and we might prove the same for any other bisecting line, taking another side as a base, it follows that *the centre of gravity of a triangle is in the point in which the three bisecting lines intersect each other.*

Let A I and B H, Fig. 3322, be two of these bisecting lines, and G their point of intersection. Join I H. The triangles I G H and A G B being similar, we shall have the proportion I G : A G = I H : A B. But the triangles I C H and B C A being also similar, we shall have I H : A B = I C : B C = 1 : 2, therefore, by reason of the common relation, I G : A G = 1 : 2, whence we deduce

3322.



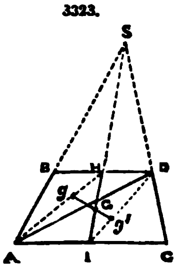
$$IG : IG + AG = 1 : 1 + 2, \text{ or } IG : A = 1 : 3,$$

that is, I G is a third of A I. Therefore *the centre of gravity of a triangle is situate upon the straight line which joins the summit to the middle of the base, and one-third of this line distant from the base.*

The centre of gravity of a triangle possesses a property which is worthy of being known. Suppose applied to the three summits, three equal forces, parallel and in the same direction, the common intensity of which we will represent by P. Required the point in the triangle through which the resultant of the three forces passes. The law for the composition of forces will give us, first, for the two forces P applied in B and C, a force 2 P applied in the middle I of B C; and for this force 2 P applied in I and the force P applied in A, the resultant is found by dividing the distance A I in the inverse proportion of these forces, that is, in the inverse proportion of the numbers 2 and 1, which will give the point G. Consequently the centre of gravity of a triangle may be regarded as the point of application of the resultant of three equal forces, parallel and in the same direction applied to the three summits respectively.

Trapezium.—Let A B D C, Fig. 3323, be a trapezium. Produce the sides that are not parallel till they meet in S; join this point to the middle I of the base A C. The line S I will pass through the middle H of the base B D. It may be shown from the principles employed above that the centre of gravity of the trapezium must be upon the straight line I H. But if we draw the diagonal A D, and determine the centres of gravity g and g' of the two triangles A B D and A D C into which the trapezium has been resolved, the centre of gravity of the trapezium must also be upon the straight line $g g'$, since the weight of the trapezium is the resultant of the weights of the two triangles. The centre of gravity required is therefore in G the point of intersection of the straight lines I H and $g g'$.

3323.



It may be remarked that the line $g g'$ is divided at the point G in the inverse proportion of the weights, or of the surfaces of the two triangles. But these triangles are equal in height; the line $g g'$ is therefore divided in the inverse proportion of the bases A D and B C. It may sometimes be required to know the proportion of the segments G I and G H of the bisecting line I H. To ascertain this, proceed as follows. The proportion required is the same as the proportion of the distances from the point G to the two bases. Let x and y be these distances, and $h = x + y$ the height of the trapezium. Denote A C by B and B D by b. Apply to the weight of the trapezium and to the weights of the two triangles B A D and A C D the theorem of the moments, taking first as the plane of the moments a plane upon the line A C perpendicular to the plane of the trapezium. The distances from the centres of gravity g and g' to the plane of the moments, or, which amounts to the same thing, to A C, will be $\frac{1}{3} h$ for the triangle D A C and $\frac{2}{3} h$ for the triangle B A D. We shall have therefore, substituting for the weights the areas which correspond to them,

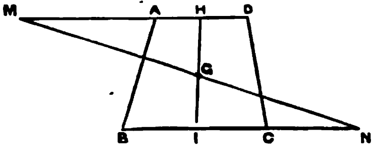
$$ABDC \,.\, x = DAC \cdot \frac{1}{3} h + BAD \cdot \frac{2}{3} h, \quad \text{or } ABDC \,.\, x = \frac{1}{6} h^2 (B + 2 b).$$

Taking the moments with respect to a plane upon B D perpendicular to the plane of the trapezium, and observing that the distance from the points g and g' to B D is $\frac{1}{3} h$ for the first, and $\frac{2}{3} h$ for the second, we shall have likewise,

$$ABDC \,.\, y = DAC \,.\, \frac{2}{3} h + BAD \,.\, \frac{1}{3} h, \quad \text{or } ABDC \,.\, y = \frac{1}{6} h^2 (2 B + b).$$

Dividing member by member the two equations thus obtained, we find $\frac{x}{y}$ or $\frac{GI}{GH} = \frac{B + 2b}{2B + b}$.

3324.



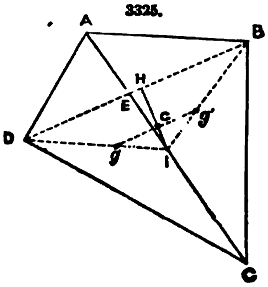
This formula leads us to the following construction;—Produce D A, Fig. 3824, by a quantity A M equal to B C; produce B C in the contrary direction by a quantity C N equal to A D; join M N, which will cut I H in the centre of gravity G. Accordingly we have $\frac{GI}{GH} = \frac{IN}{MH} = \frac{\frac{1}{2} B + b}{B + \frac{1}{2} b} = \frac{B + 2b}{2B + b}$, as the formula requires.

If the bases B and b differed infinitely little from each other, B + 2 b would be sensibly equal

to $2B + b$, and we should have sensibly $GI = GH$, that is, the point G would be in the middle of the bisecting line I H. This is what happens in the case of the elementary trapeziums considered in the demonstration relative to the centre of gravity of the triangle.

Any Quadrilateral.—Let A B C D, Fig. 3325, be a quadrilateral. Draw the two diagonals, which will intersect each other in the point E. Let I be the middle of the diagonal A C; join D I and B I. Take upon these straight lines the points g and g' at a distance of one-third of their length from the point I; these points will be the centres of gravity of the triangles A D C and A B C. Consequently, if we join them by a straight line $g\,g'$, the centre of gravity of the quadrilateral will be upon this line, and will divide it in inverse proportion to the surfaces of the triangles. But these triangles, which have the same base A C, are to each other as their heights, or as the lines D E and B E which are proportional to them. We ought therefore to have, if G is the point sought, $Gg : Gg' = BE : DE$.

3325.



To fulfil this condition, it is sufficient to take B H, equal to D E, and to join the point H to the point I by a straight line cutting $g\,g'$ in the required point G. For we shall have

$$Gg : Gg' = DH : BH = BE : DE.$$

It may be remarked that we have also $IG : IH = Ig : ID$, and that consequently I G is a third of I H. Hence the following construction;—Draw the two diagonals A C and B D meeting each other in E; take upon one of them the length B H equal to the segment D E; join the point H thus found to the middle I of the other diagonal, and take upon I H a third from the point I. The point G thus found will be the centre of gravity of the quadrilateral.

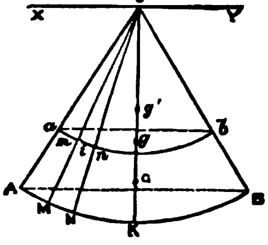
The Polygon.—To find the centre of gravity of any polygon, divide it into triangles; determine the area and the centre of gravity of each one of them, and apply the construction, which gives *the centre of parallel forces.*

A Regular Polygon.—The centre of gravity of a regular polygon is its centre of shape.

The Circle.—The centre of gravity of a circle is its centre.

The Circular Sector.—Let A O B, Fig. 3326, be a circular sector. Conceive the arc A B which forms the base divided into a large number of equal parts, and radii drawn to all the points of division. The surface of the sector is then divided into a large number of equal elementary sectors, as M O N; and as the arcs, such as M N, are supposed to be very small, these sectors may be considered as rectilinear triangles. From the point O as a centre with a radius equal to ⅔ of the radius O A, describe the arc $a\,b$; this arc will be divided by the radii, such as O M and O N, drawn to the points of division of the arc A B, into the same number of equal parts, as $m\,n$; which may be considered as straight lines parallel to the corresponding elements of the arc A B. The centre of gravity of the triangle M O N is in the middle i of the straight line $m\,n$ drawn parallel to the base at ⅔ of the distance between the summit and this base; for this point i is on the bisecting line that would be drawn from the point O, and at a distance from the summit of ⅔ of this line. But the point i is also the middle of $m\,n$; and the same may be said of the other elementary triangles. In virtue of the principle V established above, we may therefore substitute for the superficial elements, as M O N, the linear elements as $m\,n$, since they are proportional to them, and have their centres of gravity in the same points. It follows from this that *the centre of gravity of the circular sector is the same as that of the arc a b described from the centre* O *with ⅔ of the radius.* This centre of gravity is therefore upon the line which bisects the angle A O B, at a distance ρ from the centre indicated by the expression $\rho = \dfrac{Oa \,.\, ab}{amb}$ But $Oa = \dfrac{2}{3}\,OA$, $ab = \dfrac{2}{3}\,AB$, $amb = \dfrac{2}{3}\,AMB$; we may therefore write $\rho = \dfrac{2}{3} \cdot \dfrac{OA \,.\, AB}{AMB}$, that is, *to find the centre of gravity of a circular sector, find the centre of gravity of the arc forming the base, join this point to the centre, and take two-thirds of the joining line, reckoning from the centre.*

3326.



For a semicircle with a radius R, we should have $\rho = \dfrac{2}{3} \cdot \dfrac{R \,.\, 2R}{\pi R}$ or $\rho = \dfrac{4}{3\pi} \,.\, R$.

Circular Trapezium.—Let A B b a, Fig. 3326, be a circular trapezium. Denote the radii O A and O a by R and r, and the angle A O B by α. The centre of gravity sought G will be upon the bisecting line O K: for the centre of gravity g of the sector a O b and the centre of gravity g' of the sector A O B are upon this line, and the weight of the sector A O B is the resultant of the weight of the sector a O b and that of the trapezium A B b a. Through the point O draw a plane perpendicular to O K; and let X V represent this plane upon the plane of the trapezium; taking the moments with respect to this plane, we have $AOB \,.\, Og = aOb \,.\, Og' + ABba \,.\, OG$, whence

$$OG = \frac{AOB \,.\, Og - aOb \,.\, Og'}{AOB - aOb}.$$

But, $AOB = \dfrac{1}{2}R^2\alpha$, $aOb = \dfrac{1}{2}r^2\alpha$, $Og = \dfrac{2}{3} \cdot \dfrac{R \,.\, 2R \sin. \frac{1}{2}\alpha}{R\alpha}$ and $Og' = \dfrac{2}{3} \cdot \dfrac{r \,.\, 2r \sin. \frac{1}{2}\alpha}{r\alpha}$;

$$\ldots = \frac{2}{3} \cdot \frac{R^2 + R r + r^2}{R + r} \cdot \frac{\sin. \frac{1}{2} \alpha}{\frac{1}{2} \alpha}.$$

... ρ and half their difference by e, which gives ... ession under the form

$$\ldots \cdot \frac{\sin. \frac{1}{2} \alpha}{\frac{1}{2} \alpha}$$

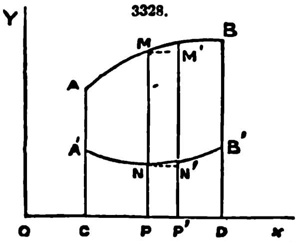
... ing the centre of gravity of the voussoirs of a ...

... the first place, a trapezium A B D C, Fig. 3327, ... is given by the axis of the x's and by two ordi- ... a and b. This trapezium may be considered as ... ving as their height the ordinate M P or y, and ... abscissa, or $d x$. The distance of the centre of ... is equal to $x + \frac{1}{2} d x$, and its distance from the ... alling the area of trapezium A B D C A, and

$$\ldots \mathrm{A} \cdot \mathrm{Y} = \int_a^b \frac{1}{2} y^2 \cdot d x, \text{ besides } \mathrm{A} = \int_a^b y \, d x.$$

... have the co-ordinates X and Y of the centre of ...

3328.



... ider the area included between two curves A B ... given, and the ordinates corresponding to the ... r the proposed area as made up of an infinite ... se the infinitely small increase P P′, or $d x$, from ... tween the ordinates M P $= y$ and N P $= y'$ of the ... The distance of the centre of gravity of this ... $+ \frac{1}{2} d x$, or simply x; and its distance from the ...). We shall have therefore, calling the area

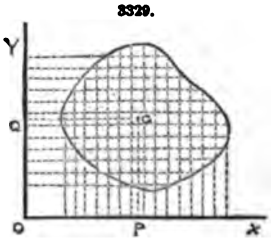
$$\ldots \mathrm{Y} = \int_a^b \frac{1}{2} (y + y') \cdot (y - y') \, d x; \text{ besides}$$

$$\ldots (y - y') \, d x.$$

... rating, we obtain the co-ordinates X and Y. This ... which the curve A′ B′ is a second branch of the ... n of the centre ... the equation

... an irregular ... we may make ... el to the axes, ... s being small ... contour included ... ctilinear. The ... ular trapeziums ... and centres of ... oments of these ... mming, we get ... area is the sum of the partial areas, by dividing ... the co-ordinates of the centre of gravity with an ... the parallels decreases.

3329.



VI. We have now to determine the centre of gravity of curved surfaces.

Surfaces of Revolution.—Let O X, Fig. 3327, be the axis of revolution, and A B the generating line, or generatrix, whose equation is supposed to be given; and let O C $= a$ and O D $= b$ be the abscissæ of the planes perpendicular to the axis O X serving as limits to the surface. Divide this surface by planes M P, M P′, perpendicular to the axis of revolution, into infinitely small zones, which may be considered as surfaces of frusta of cones. Let x and y be the co-ordinates of the point M, and s the arc A M of the generatrix. We shall have as the expression of the surface of the frustum generated by the element M M′ or $d\,s$, $\frac{1}{2}\Big(2\,\pi\,y + 2\,\pi\,(y + d\,y)\Big)\,d\,s$ or $2\,\pi\,y\,d\,s$, by neglecting the infinitely small $d\,y$ before the finite quantity y.

The centre of gravity of this elementary zone is situate upon the axis of revolution between the points P and P′, and consequently its distance from the point O is expressed by $x + \epsilon\,d\,x$, ϵ denoting a fraction. The centre of gravity of the whole surface is likewise situate upon the axis, and, calling its distance from the point O, X, and the area of the surface S, we have, by the theorem of the moments, $S.X = \int_a^b 2\,\pi\,y\,.\,d\,s\,(x + \epsilon\,d\,x) = 2\,\pi \int_a^b x\,y\,d\,s$, by neglecting $\epsilon\,d\,x$ before x. We have besides, $S = 2\,\pi \int_a^b y\,d\,s$. Therefore, putting for $d\,s$ its value $d\,x\sqrt{1 + y'^2}$, replacing the ordinate y and its derivative y' by their values in x and integrating, we obtain the distance X.

We should thus find that *the centre of gravity of the surface of a* CONE OF REVOLUTION *is situate upon its axis, at a distance of one-third of its length from the base.* We should see in like manner that the centre of gravity of the surface of a frustum of a cone is situate upon its axis of revolution, and that it divides it into two portions x and y, the expression of whose ratio is $\frac{x}{y} = \frac{2\,R + r}{R + 2\,r}$, where R denotes the radius of the larger base and r the radius of the smaller.

Spherical Zone.—In the case of a spherical zone we have $y = \sqrt{R^2 - x^2}$, whence $y' = \frac{x}{\sqrt{R^2 - x^2}}$, and $\sqrt{1 + y'^2} = \frac{R}{\sqrt{R^2 - x^2}}$. It follows from this that $y\,d\,s = R\,d\,x$, and

$$S = 2\,\pi \int_a^b R\,d\,x = 2\,\pi\,R\,(b - a).$$

We have further, $S\,X = 2\,\pi \int_a^b R\,x\,d\,y = 2\,\pi\,R\,\frac{(b^2 - a^2)}{2}$; consequently $X = \frac{1}{2}\,(b + a)$.

This value is the abscissa of the middle of the axis; consequently *the centre of gravity of a zone is the middle of its axis.*

Any Surface whatever.—Let $z = \phi\,(x\,,y)$ the equation of the surface, and S the area included between the limits assigned, a and a' for x, b and b' for y. The expression of the element of surface is $d\,S = d\,x\,d\,y\sqrt{1 + [\phi'_x\,(x,y)]^2 + [\phi'_y\,(x,y)]^2}$, and the whole surface is expressed by

$$S = \int_a^{a'} \int_b^{b'} d\,x\,d\,y\sqrt{1 + [\phi'_x\,(x,y)]^2 + [\phi'_y\,(x,y)]^2}.$$

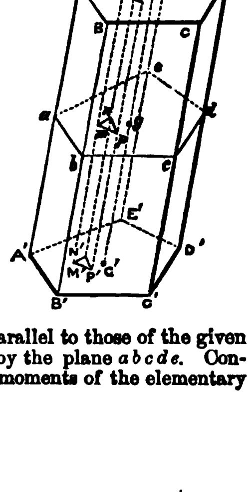
We have further, by the theorem of the moments,

$$S\,X = \iint x\,d\,S, \qquad S\,Y = \iint y\,d\,S, \qquad S\,Z = \iint z\,d\,S.$$

Substituting for $d\,S$ and z their values in x and y, and integrating between the limits indicated, we obtain the values of the co-ordinates X, Y, Z, of the centre of gravity required.

VII. It remains for us now to consider the centre of gravity of volumes.

3330.



The Prism.—Let A B C D E A′ B′ C′ D′ E′, Fig. 3330, be any prism. We may conceive this prism divided, by planes parallel to the bases, into equal and infinitely thin sections. These sections will have their centres of gravity similarly placed, since they are equal to each other. All their centres of gravity will therefore be upon the same straight line G G′ parallel to the lateral edges; consequently the centre of gravity of the whole prism will be upon this line. Again, it will be in the middle g of this line; for the weight of these sections will be equal and parallel forces applied in equidistant points of G G′, and consequently, as we may consider the line G G′ loaded with weights uniformly distributed throughout its length, the point of application of their resultant is in the middle of this length. The centre of gravity g of the prism is therefore situate in the section $a\,b\,c\,d\,e$ parallel to the bases and equally distant from them. It is also the centre of gravity of this section. Suppose the whole prism decomposed into infinitely small triangular prisms, as M N P M′ N′ P′, having their edges parallel to those of the given prism; and let $m\,n\,p$ be the section of one of these elementary prisms by the plane $a\,b\,c\,d\,e$. Conceive a plane P perpendicular to the bases of the prism, and take the moments of the elementary

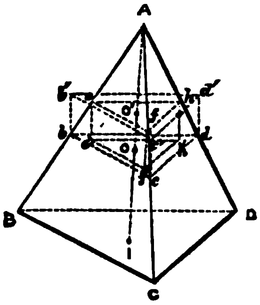
prisms and of the whole prism with respect to this plane. Denote the height of the prism by h, the area mnp by ω, the distance of the centre of gravity of the elementary prism from the plane P by x, the distance of the centre of gravity of the triangle mnp from the same plane by x'; the distance of the centre of gravity of the whole prism by X, and that of the centre of gravity of the polygon $abcde$ by X'. It may be remarked that x and x' can differ only by an infinitely small quantity ϵ, since the centres of gravity of the elementary prism and of the mean section are both situate in the infinitely small triangle mnp, say $x' = x + \epsilon$. Representing the area of the polygon $abcde$ by Ω, we have $h\,\Omega\,.\,X = \Sigma\,h\,\omega\,.\,x$, whence $\Omega\,X = \Sigma\,\omega\,x$, and $\Omega\,X' = \Sigma\,\omega\,(x + \epsilon)$, or, neglecting the infinitely small ϵ before the finite quantity x, $\Omega\,X' = \Sigma\,\omega\,x = \Omega\,X$; whence $X' = X$.

Thus the centre of gravity of the given prism and that of the section $abcde$ are at the same distance from the plane P; and as this plane is any plane perpendicular to the bases, it follows that the two centres of gravity coincide. Consequently *the centre of gravity of a prism is that of the section parallel to the bases and equally distant from those bases.*

The same demonstration applies to right and oblique cylinders.

The Tetrahedron.—A demonstration analogous to the above may be applied to the tetrahedron, and generally to the pyramid and the cone. But on account of the importance which the research for the centre of gravity of the tetrahedron possesses, it will be well to apply to it a special geometrical method. Let A B C D, Fig. 3331, be the given tetrahedron. Join the point A to the centre of gravity I of the opposite face. Draw the planes bcd, efh, parallel to B C D; and the straight lines bb', cc', dd', ee', ff', hh', parallel to A I, and terminating in these planes; join $b'c'$, $c'd'$, $b'd'$, and $e'f'$, $f'h'$, $e'h'$.

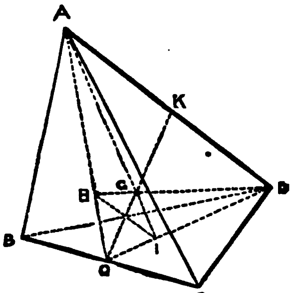
3331.



The straight line A I being drawn to the centre of gravity of the base B C D, passes through the centres of gravity o and o' of the sections bcd and efh; for the point A is their common centre of similitude. The triangular prisms bcd, $b'c'd'$, efh, $e'f'h'$, the lateral edges of which are parallel to A I, have therefore both of them their centre of gravity in the middle of oo'. But the nearer the sections bcd, efh, are together, the more will the truncated pyramid $bcdefh$, included between the two prisms, tend to confound itself with each of them. Therefore, when the distance oo' is infinitely small, we may consider the truncated pyramid as confounded with one or the other of these prisms, and that consequently it has its centre of gravity in the same point upon the line A I. It follows from this that if we conceive the tetrahedron decomposed, by planes parallel to B C D, into infinitely thin truncated pyramids, all these truncated pyramids may be considered as having their centres of gravity upon the line A I. Therefore, in virtue of principle I, the centre of gravity of the tetrahedron A B C D is situate upon the same line A I. As the same result would be arrived at if we took another face as a base, it follows that *the centre of gravity of a tetrahedron is in the point of intersection of the straight lines drawn from each summit to the centre of gravity of the opposite face.*

This being established, let O, Fig. 3332, be the middle of the edge B C; draw A O and D O, take O H equal to a third of A O, and O I equal to a third of O D. The points H and I will be respectively the centres of gravity of the forces A B C and B C D. Draw A I and D H. These st a ght lines which are both in the plane A O D, will meet in a point G, which will be the centre of gravity of the tetrahedron. But, if we draw I H, the similar triangles I G H and A G D will give the proportion I G : G A = I H : A D; but the similar triangles I O H and A O D also give I H : A D = O I : O D = 1 : 3; therefore, on account of the common proportion, I G : G A = 1 : 3, whence I G : I G + G A = 1 : 1 + 3, or I G : I A = 1 : 4, that is, I G is a fourth of A I. Thus *the centre of gravity of a tetrahedron is situate upon the straight line which joins the summit to the centre of gravity of the base, at a distance of one-fourth of this line from the base.*

3332.



It may be remarked that if through the point G we draw a plane parallel to the base B C D of the tetrahedron, this point will be the centre of gravity of the section determined by this plane; so that *the centre of gravity of a tetrahedron is that of the section parallel to its base, at a quarter of the distance between this base and the opposite summit.*

Suppose the points A, B, C, D, to be the points of application of four equal and parallel forces the common intensity of which we will represent by P. To compose these four forces, we may first compose the two forces P applied to the points B and C, which will give a force 2 P applied to the middle O of B C. We shall have, further, to compose this force 2 P applied in O with the force P applied in D; to do this we must divide the distance O D in the inverse ratio of these forces, that is, in the inverse ratio of the numbers 2 and 1, which will give the point I, the centre of gravity of the base B C D. Lastly, we shall have to compose the force 3 P applied in I with the force P applied in A; to do this we must divide A I in the inverse ratio of the numbers 3 and 1, which will give exactly the point G.

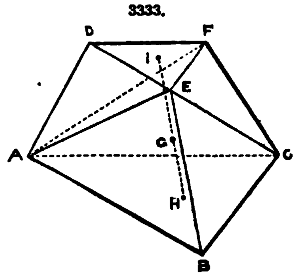
Consequently *the centre of gravity of a tetrahedron is the point of application of the resultant of four equal forces, parallel and in the same direction applied to the four summits respectively.*

The four forces P may be composed in another way. We may first compose the forces P applied in B and C into a single force 2 P applied in the middle O of B C. We may then compose the two

other forces P applied in A and D into a single force 2 P applied in the middle K of A D. It will remain to compose the force 2 P applied in O with the force 2 P applied in K, which will give a force 4 P applied in the middle G of the straight line O K. The point G found in this way must evidently be the same as that which has been found by another method of composition. Therefore *the centre of gravity of a tetrahedron is in the middle of the straight line which joins the middles of two opposite edges.* As there are three analogous right lines joining the middles of two opposite edges, it follows from what we have just said that these three right lines cut each other in the middle; which is indeed a known theorem in geometry.

Truncated Tetrahedron.—Let A B C D E F, Fig. 3333, be the given frustum. It may be demonstrated, as in the case of the tetrahedron, that the centre of gravity must be upon the straight line I H which joins the centres of gravity of the two bases. Let G be this point; it remains for us to determine the ratio of the lengths G I and G H, or, which amounts to the same thing, the ratio of the distances from the point G to the planes of the two bases. Let x and y be these distances; then $x + y = h$ the height of the frustum. Denote the base A B C by B, and the base D E F by b. Decompose the frustum into three pyramids by the planes A E C and A E F, as would be done in finding its volume; and take successively the moments with respect to the two bases, noting that the distances from the centres of gravity of these partial pyramids to the bases D E F and A B C are respectively $\frac{1}{4}h$ and $\frac{3}{4}h$ for the pyramid A D E F, $\frac{3}{4}h$ and $\frac{1}{4}h$ for the pyramid E A B C, and $\frac{1}{2}h$ and $\frac{1}{2}h$ for the pyramid E A F C, as its centre of gravity is in the middle of the straight line which would join the middles of the opposite edges E F and A C. We shall have therefore, by first taking the moments with respect to the base D E F,

3333.



$$\mathrm{ABCDEF} \,.\, x = \mathrm{ADEF} \,.\, \frac{1}{4}h + \mathrm{EABC} \,.\, \frac{3}{4}h + \mathrm{EFAC} \,.\, \frac{1}{2}h,$$

or $$\mathrm{ABCDEF} \,.\, x = \frac{1}{12}h^2\left(b + 3\mathrm{B} + 2\sqrt{\mathrm{B}b}\right).$$

Then taking the moments with respect to the base A B C, we shall have in like manner

$$\mathrm{ABCDEF} \,.\, y = \mathrm{ADEF} \,.\, \frac{3}{4}h + \mathrm{EABC} \,.\, \frac{1}{4}h + \mathrm{EFAC} \,.\, \frac{1}{2}h,$$

or $$\mathrm{ABCDEF} \,.\, y = \frac{1}{12}h^2\left(3b + \mathrm{B} + 2\sqrt{\mathrm{B}b}\right).$$

Dividing the two equalities member by member, and simplifying, we find

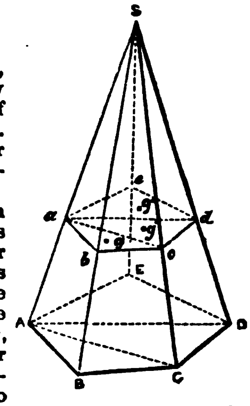
$$\frac{x}{y} = \frac{\mathrm{GI}}{\mathrm{HG}} = \frac{b + 3\mathrm{B} + 2\sqrt{\mathrm{B}b}}{3b + \mathrm{B} + 2\sqrt{\mathrm{B}b}}. \qquad [A]$$

The bases B and b may be replaced by the squares of their homologous edges, since they are proportional to them; calling these edges A and a, we get

$$\frac{x}{y} = \frac{\mathrm{GI}}{\mathrm{HG}} = \frac{a^2 + 3\mathrm{A}^2 + 2\mathrm{A}a}{3a^2 + \mathrm{A}^2 + 2\mathrm{A}a}.$$

It may be remarked that when the two bases are infinitely near, they differ infinitely little from each other, and that G I is then sensibly equal to G H, that is, the centre of gravity is sensibly in the middle of the straight line which joins the centres of gravity of the two bases. This was what occurred in the case of the elementary sections under consideration when we were seeking the centre of gravity of the tetrahedron.

3334.



Any Pyramid.—Let S A B C D E, Fig. 3334, be a pyramid with any base. Decompose it into tetrahedrons by the diagonal planes A S C and A S D. At a distance from the base equal to a quarter of the height of the pyramid, draw a plane $abcde$ parallel to this base. This plane will contain the centres of gravity g, g', g'', of the partial tetrahedrons, and consequently the centre of gravity of the whole pyramid (Principle I). But the tetrahedrons S A B C, S A C D, S A D E, having the same height, are to each other as their bases, or as the triangles abc, acd, ade, proportional to these bases. Therefore, if we suppose applied to the points g, g', g'', weights equal to those of the corresponding tetrahedrons, these weights would be at the same time proportional to the areas of the triangles abc, acd, ade. Hence it follows that the point of application of the resultant of these weights is no other than the centre of gravity of the polygon $abcde$. But it

may be easily seen by simple similitudes of triangles, that the straight line which joins the summit S to the centre of gravity of the base A B C D E of the pyramid, passes through the centres of gravity of all the sections, as $a\,b\,c\,d\,e$, parallel to this base. Therefore *the centre of gravity of any pyramid is upon the straight line which joins the summit to the centre of gravity of the base, at a distance of one-fourth of this line from the base.*

This theorem extends to a cone, whether right or oblique, and with any base, since such a body is a pyramid whose base is a polygon with an infinite number of infinitely small sides.

Truncated Pyramid.—If the frustrum of the pyramid be decomposed into frusta of tetrahedrons, their upper bases will be proportional to the lower, and generally to the sections made by the same plane parallel to the bases. Hence it follows that the ratio of the distances from their centres of gravity to the two bases will be the same for each of them, and that consequently their centres of gravity will be in the same plane parallel to the bases, and determined by the formula [A] given above for the frustum of tetrahedron. The centre of gravity of the whole frustum of the pyramid will therefore be also in this plane. Again, the partial truncated tetrahedrons having the same height, and proportional bases, are to each other as these bases, or as the sections made by the plane containing the centres of gravity of the partial tetrahedrons. Therefore *the centre of gravity of a frustum of the pyramid is upon the straight line which joins the centres of gravity of the two bases, and it divides this line in the proportion expressed by the formula* [A] *relative to the tetrahedron*, the letters B and b denoting in this case the bases of the frustum of the pyramid.

This proposition extends to the frustum of the cone; and the bases of B and b being in this case proportional to the squares of their Radii R and r, we have, still denoting by x and y the segments determined by the centre of gravity sought upon the straight line which joins the centres of gravity of the two bases, $\frac{x}{y} = \frac{r^2 + 3\,R^2 + 2\,R\,r}{3\,r^2 + R^2 + 2\,R\,r}$.

The Sphere.—The centre of gravity of a sphere is its centre of shape.

A Spherical Sector.—We may conceive the sector divided up into elementary pyramids, all of them having their summits in the centre of the sphere. The centre of gravity of each of them will be upon the radius drawn to the centre of gravity of the element of spherical surface which serves as its base at a distance of $\frac{3}{4}$ of this radius from the centre. Suppose an auxiliary spherical surface described, with a radius equal to $\frac{3}{4}$ that of the sphere, and terminated in the cone which limits the sector, which spherical surface will be similar to that which forms the base of the sector. This auxiliary surface will cut all the pyramids, and the section obtained in each of them will have its centre of gravity at the same point as the pyramid. Hence it follows (Principle V) that the centre of gravity of the sector is the same as that of the auxiliary surface, and that consequently it is in the middle of the axis of this auxiliary zone. If R is the radius of the sphere and h the height of the spherical surface which forms the base of the sector, $\frac{3}{4}\,R$ and $\frac{3}{4}\,h$ will be the radius and the height of the auxiliary portion. Calling the distance from the centre of gravity of the sector to the centre X, we have therefore $X = \frac{3}{4}\left(R - \frac{1}{2}\,h\right)$. If the sector is half a sphere, we have $h = R$, and consequently $X = \frac{3}{8}\,R$.

A Body terminated by a Surface of Revolution.—Let O X, Fig. 3327, be the axis of revolution, and $y = f(x)$ the equation of the generating line A B. We may regard the whole volume as composed of elementary cylinders, as M P P′ I, having as a radius M P $= y$, and a height P P′ $= dx$. The expression of one of these elementary cylinders is $\pi\,y^2\,dx$. If therefore we put O C $= a$, and O D $= b$, the abscissæ of the planes perpendicular to the axis serving as limits to the body under consideration, we shall have first, $V = \pi \int_a^b y^2\,dx$, V being the volume of the body. The centre of gravity is upon the axis of revolution at a distance X from the origin, which will be given, in virtue of the theorem of the moments, by the relation $V\,X = \pi \int_a^b y^2\,x\,dx$. Replacing y by its value and integrating, we obtain the unknown distance X.

A Body terminating in any Surface.—Suppose the body included between the two given surfaces $z_1 = F(x, y)$ and $z_2 = f(x, y)$, the planes $x = a$, $x = a'$, and the planes $y = b$, $y = b'$. The element of the volume is the rectangular parallelopiped $dx\,dy\,dz$; the total volume V is therefore expressed by the relation

$$V = \int_a^{a'} \int_b^{b'} \int_{z_2}^{z_1} dx\,dy\,dz = \int_a^{a'} \int_b^{b'} [F(x,y) - f(x,y)]\,dx\,dy.$$

We have further, taking the moments of these elements with respect to the three co-ordinate planes,

$$V\,X = \int_a^{a'} \int_b^{b'} \int_{z_2}^{z_1} dx\,dy\,dz = \int_a^{a'} \int_b^{b'} [F(x,y) - f(x,y)]\,x\,dx\,.\,dy.$$

$$V\,Y = \int_a^{a'} \int_b^{b'} \int_{z_2}^{z_1} y\,dx\,dy\,dz = \int_a^{a'} \int_b^{b'} [F(x,y) - f(x,y)]\,dx\,.\,y\,dy.$$

$$V\,Z = \int_a^{a'} \int_b^{b'} \int_{z_2}^{z_1} z\,dx\,dy\,dz = \int_a^{a'} \int_b^{b'} \frac{1}{2}\,\{[F(x,y)]^2 - [f(x,y)]^2\}\,dx\,dy.$$

Any Volume.—In certain cases, in earthwork for example, it may be required to find the centre of gravity of a wholly irregular figure. We will suppose the case of a mound given by the projections of the curves of its level. The first step is to compute the area included under each of these curves. Let h be the distance of the consecutive planes of these curves. If this distance is not too great, and the curves do not vary too abruptly, we may consider each section included between two consecutive planes as a truncated pyramid, the volume of which may be determined by the known rule. Find the centres of gravity of the two bases; the centre of gravity of the section will be upon the straight line which joins these two centres, and it will divide this line in the proportion expressed by the formula [A]. Knowing thus the volume and the centre of gravity of each section, take the moments with respect to a horizontal plane and with respect to two rectangular vertical planes, and the rectangular co-ordinates of the centre of gravity of the mound will be obtained.

VIII. The centre of gravity possesses various properties, the most important of which is expressed by *Guldin's Theorem.*

1. *The volume of a truncated cylinder is equal to the product of its right section by the distance between the centres of gravity of its two bases.* Suppose, in the first place, the lower base to be the right section itself; take it as the plane of the $x\,y$'s, and let θ be the angle which the upper base makes with this plane. If Ω denote the total area of the upper base, and ω an element of this area, $\Omega \cos.\theta$ and $\omega \cos.\theta$ will denote the total area of the lower base and the element of this base corresponding to the element ω. Call the ordinate of the element ω, z. The volume of the cylinder which projects ω upon the plane of the base will be expressed by $\omega \cos.\theta.z$ within an infinitesimal of a superior order, and the volume of the truncated cylinder will consequently be expressed by $V = \Sigma\,\omega \cos.\theta.z = \cos.\theta.\Sigma\,\omega\,z$. But if Z is the ordinate of the centre of gravity of the upper base, we have, by the theorem of the moments, $\Omega\,Z = \Sigma\,\omega\,z$; therefore $V = \cos.\theta.\Omega\,Z$, that is, the volume sought is the product of the lower base $\Omega \cos.\theta$ by the ordinate Z of the centre of gravity of the upper base. But the foot of this ordinate is precisely the centre of gravity of the lower base, for if X denote the distance from the centre of gravity of the upper base to the plane of the $y\,z$'s and X′ the distance from the centre of gravity of the lower base to this same plane, we shall have, to determine these two distances by, the equations $\Omega\,X = \Sigma\,\omega\,x$ and $\Omega \cos.\theta.X' = \Sigma.\omega \cos.\theta.x$; the second may be reduced to $\Omega\,X' = \Sigma\,\omega\,x = \Omega\,X$, whence $X = X'$.

It may be seen in the same way that these two centres of gravity are at the same distance from the plane of the $x\,z$'s; therefore they are upon the same line parallel to the axis of the z's, and the second is the foot of the ordinate of the first. The theorem is thus demonstrated for the case under consideration.

If the planes of the two bases are of any kind, we may divide the truncated cylinder, by a plane perpendicular to its edges, into two truncated cylinders which will come under the first case; and, by summing, we shall see that the measure of the volume is the right section multiplied by the sum of the ordinates of the centres of gravity of the two bases with respect to this right section, the foot of both of which ordinates is the centre of gravity of this section; this expression amounts therefore to the product of the right section by the distance between the centres of gravity of the two bases.

2. We will now consider any number of bodies the weights of which are p, p', p'', &c. We shall determine the centre of gravity of this system as if it were solid. Let ρ, ρ', ρ'', &c., be the distances from the respective centres of gravity of these bodies to the origin, R the distance from the centre of gravity of the system to this same origin; $\alpha, \beta, \gamma, \alpha', \beta', \gamma', \alpha'', \beta'', \gamma''$, &c., a, b, c, the angles which the straight lines upon which these distances are measured make with the three axes. Putting P for the total weight, we have, by the theorem of the moments,

$$P\,R \cos.a = \Sigma p\,\rho \cos.\alpha,\quad P\,R \cos.b = \Sigma p\,\rho \cos.\beta,\quad P\,R \cos.c = \Sigma p\,\rho \cos.\gamma. \quad [1]$$

These relations express that *if we apply to the origin, forces proportional to the products $p\,\rho, p'\rho', p''\rho''$, &c., and respectively directed towards the centres of gravity of the partial bodies, they will have as a resultant a force proportional to the product* P R *and directed towards the centre of gravity of the system.* If the origin were the centre of gravity itself, the forces $p\,\rho, p'\rho', p''\rho''$, &c., would hold each other in equilibrium in this point, since the resultant would be nil.

3. If we square both members of the equations [1] and add them together member by member, we get $P^2 R^2 = \Sigma p^2 \rho^2 + \Sigma.2\,p\,p'\rho\,\rho' (\cos.\alpha \cos.\alpha' + \cos.\beta \cos.\beta' + \cos.\gamma \cos.\gamma')$, or, calling the angle of ρ with ρ' $(\rho\,\rho')$, $P^2 R^2 = \Sigma p^2 \rho^2 + \Sigma\,2\,p\,p'\rho\,\rho'.\cos.(\rho\,\rho')$. If r denote the distance of the centres of gravity of the bodies p and p', we have $r^2 = \rho^2 + \rho'^2 - 2\,\rho\,\rho' \cos.(\rho\,\rho')$, whence $2\,\rho\,\rho' \cos.(\rho\,\rho') = \rho^2 + \rho'^2 - r^2$, consequently $P^2 R^2 = \Sigma p^2 \rho^2 + \Sigma p\,p' (\rho^2 + \rho'^2 - r^2)$. Collecting all the terms in ρ^2, we have $\rho^2 (p^2 + p\,p' + p\,p'' + \ldots)$ or $\rho^2.P\,p$; analogous terms would be found by collecting all those containing ρ'^2, then those containing ρ''^2, and so on. We may therefore write

$$P^2 R^2 = P\,\Sigma p\,\rho^2 - \Sigma p\,p'\,r^2. \quad [2]$$

We conclude from this relation that *if the system be moved without changing its form, and in such a way that its centre of gravity remains always at the same distance from a fixed point (the origin), the sum of the products of the weights of the different bodies by the square of their distance from this fixed point, will remain constant.* For R being constant, as well as the distances represented by r, the term $P\,\Sigma p\,\rho^2$ must be constant, and consequently the same is true of $\Sigma p\,\rho^2$

The relation [1] may be written $\Sigma p\,\rho^2 = P\,R^2 + \dfrac{\Sigma p\,p'\,r^2}{P}$; under this form, that the system retaining its form, that is, r, r', r'', &c., remaining constant, $\Sigma p\,\rho^2$ will be as small as possible when R is equal to zero; in other words, the centre of gravity possesses this property, namely, that *the*

sum of the products of the weights of the different bodies by the square of the distance from their partial centres of gravity to the centre of gravity of the system, is a minimum.

IX. If all the material points forming a part of the system under consideration are in the same place, where the value of the acceleration g due to the weight may be regarded as constant, we may, in the equations of the moments, substitute the masses for the weights, and write

$$MX = \Sigma m x, \quad MY = \Sigma m y, \quad MZ = \Sigma m z.$$

If these different points are far enough from each other to make g vary sensibly, these equations cannot be deduced from the equations of the moments. But they define, nevertheless, the co-ordinates X, Y, Z, of a certain point in space, which plays an important part in the mechanics of free bodies, and particularly in astronomy. Euler proposed for this point the name of *centre of inertia;* other writers have proposed to call it the *centre of mass;* the name of *centre of gravity* has however predominated, though gravity is foreign to the determination of this point. Care must be taken, in order to avoid confusion, to distinguish the case in which the weights are proportional to the masses, from that in which this proportion has no existence.

Movement of the Centre of Gravity.—When a material system is in motion, its centre of gravity is generally in motion too; and this motion may be determined when that of each of the material points which make up the system is known. The determination of this is the object of a theorem known as the *Principle of the motion of the centre of gravity*, which we will now establish.

I. Let p, p', p'', &c., be the weights of the material points of which the material point under consideration is composed, x, x', x'', &c., their distances from a plane of comparison, P the total weight of the system, and X the distance of its centre of gravity from the same plane. We shall have by the theorem of the moments of parallel forces,

$$p x + p' x' + p'' x'' + \ldots = P X. \qquad [1]$$

But if all the points of the system are in a space so limited that the acceleration g due to the weight is the same for all these points, we may, dividing all the terms of the relation [1] by g, substitute the masses for the weights, and write

$$m x + m' x' + m'' x'' + \ldots = M X. \qquad [2]$$

In this relation, the quantities x, x', x'', &c., X vary with the time, and may be considered as functions of this variable. Differentiating with respect to the time, we have

$$m \frac{d x}{d t} + m' \frac{d x'}{d t} + m'' \frac{d x''}{d t} + \ldots = M \frac{d X}{d t}.$$

But $\frac{d x}{d t}$ is the component, perpendicular to the plane of comparison, of the velocity of the point whose mass is m; we will represent it by v_x. Also $\frac{d x'}{d t}$ is the component, in the same direction, of the velocity of the point whose mass is m'; we will represent it by v'_x, and so on with the others. Similarly $\frac{d X}{d t}$ is the component, perpendicular to the plane of comparison, of the velocity of the centre of gravity; we will represent it by V_x. By means of these notations, the above relation may be written, $m v_x + m' v'_x + m'' v''_x + \ldots = M V_x$, or, abridging the expression,

$$\Sigma m v_x = M V_x, \qquad [3]$$

that is, *the sum of the quantities of movement of the whole system, projected upon an axis perpendicular to the plane of comparison, is equal to the quantity of movement of the centre of gravity, projected upon the same axis* (if we attribute to the centre of gravity a mass equal to the total mass of the system). If we consider the system with reference to three rectangular axes, of the x's, of the y's, and of the z's, and project the quantities of movement successively upon these three axes, we shall obtain, for the axes of the y's and the z's, two other equations analogous to the equation [3], namely,

$$\Sigma m v_y = M V_y, \qquad [4]$$

$$\text{and} \quad \Sigma m v_z = M V_z. \qquad [5]$$

The equations [3], [4], and [5] will determine the velocity V of the centre of gravity; for we deduce first, $V_x = \frac{\Sigma m v_x}{M}$, $V_y = \frac{\Sigma m v_y}{M}$, $V_z = \frac{\Sigma m v_z}{M}$. We shall have further

$$V = \sqrt{V^2_x + V^2_y + V^2_z},$$

and if α, β, γ, denote the angles which this velocity makes with the axes,

$$\cos. \alpha = \frac{V_x}{V}, \quad \cos. \beta = \frac{V_y}{V}, \quad \cos. \gamma = \frac{V_z}{V}.$$

II. The equations [3], [4], and [5] are, besides, susceptible of a remarkable interpretation. The quantity of movement of a material point is a number of kilogrammes; we may therefore always conceive a force which has the same direction as the velocity of the body in motion, and whose intensity is expressed by its quantity of motion. Let ϕ, ϕ', ϕ'', &c., be the forces which would thus represent the quantities of motion of the various material points of the system, Φ the force

which, in like manner, would represent the quantity of motion of the centre of gravity; let ϕ_x, ϕ'_x, ϕ''_x, &c., Φ_x, be the projections of these forces upon the axis of the x's, respectively equivalent to the quantities of motion projected upon the same axis, or to $m v_x$, $m' v'_x$, $m'' v''_x$, &c., $M V_x$. In virtue of the equation [3] we shall have $\Sigma \phi_x = \Phi_x$. We should have likewise for the other two axes $\Sigma \phi_y = \Phi_y$ and $\Sigma \phi_z = \Phi_z$. But these last three equations denote that the force Φ is the resultant of the forces ϕ, ϕ', ϕ'', &c. The equations [3], [4], and [5] denote that *the quantity of motion of the centre of gravity is the resultant of the quantities of motion of the various points of the system transferred parallel to each other to this point* (supposing the quantities of motion to be composed like the forces).

III. This relation exists at any instant during the motion, and consequently also at the instant of initial motion. So that if we denote by the index *zero* the initial velocities, we shall have, in virtue of the equations [3], [4], [5] themselves,

$$\Sigma m v_{0x} = M V_{0x}, \qquad [6]$$
$$\Sigma m v_{0y} = M V_{0y}, \qquad [7]$$
$$\Sigma m v_{0z} = M V_{0z}. \qquad [8]$$

Subtracting member by member the equations [6], [7], [8] from the equations [3], [4], [5], we obtain

$$\Sigma m v_x - \Sigma m v_{0x} = M V_x - M V_{0x}, \qquad [9]$$
$$\Sigma m v_y - \Sigma m v_{0y} = M V_y - M V_{0y}, \qquad [10]$$
$$\Sigma m v_z - \Sigma m v_{0z} = M V_z - M V_{0z}. \qquad [11]$$

But in virtue of *the principle of the quantities of motion, or of the effect of impulse*, we have

$$\Sigma m v_x - \Sigma m v_{0x} = \int_o^t R_x \, dt,$$

$$\Sigma m v_y - \Sigma m v_{0y} = \int_o^t R_y \, dt,$$

$$\Sigma m v_z - \Sigma m v_{0z} = \int_o^t R_z \, dt,$$

R denoting the *resultant of translation* of the external forces soliciting the system; we may therefore write

$$\left.\begin{aligned} M V_x - M V_{0x} &= \int_o^t R_x \, dt \\ M V_y - M V_{0y} &= \int_o^t R_y \, dt \\ M V_z - M V_{0z} &= \int_o^t R_z \, dt \end{aligned}\right\} \qquad [12]$$

But these equations are those of the motion of a material point whose mass is M, whose initial velocity is V_0, and which is subjected to a force R. Therefore we may say, *the centre of gravity of a material system moves as if the whole mass of the system were concentrated in it, as if the resultant of translation of all the external forces were applied to it, and as if all the quantities of initial motion had been transferred to it parallel to each other and composed like forces.* Such is the principle of the movement of the centre of gravity.

IV. This principle does not depend upon the *mutual* forces which are exerted between the various material points of which the system is composed. From this observation, several consequences are deduced;—

1. Suppose a spherical bomb thrown into space; its centre will describe a trajectory in the vertical plane passing through the direction of the initial velocity. Suppose also that at a certain instant the bomb bursts; as the explosion is due merely to the interior mutual forces which are developed, these forces will not alter the motion of the centre of gravity; and if it were possible to determine at each instant the centre of gravity of the system formed by the fragments of the bomb, we should see that this point continues to describe the trajectory which the centre of the whole bomb was describing before the explosion occurred.

2. The equations [12] explain also the effects of the *recoil* in fire-arms. Let us take as an example a piece of cannon standing upon a horizontal soil with its carriage. Previous to the explosion the system was subjected merely to its own weight, and to the reactions of the ground, producing a resultant equal and contrary to this weight; and these forces passing through the centre of gravity gave a total resultant equal to zero. The explosion being due solely to mutual molecular forces, the resultant of translation R remains nil; in virtue of the equations [12], which in this case are reduced to one. If we take as the axis the horizontal direction of the shot, the final quantity of motion is equal to the initial quantity of motion; but this was nil; the final quantity of motion is therefore nil also. Denoting the mass of the ball by m, its velocity by v, the mass of the piece and its carriage by M, and the velocity of the recoil by u, we have $m v - M u = 0$,

the recoil would be a fraction of the initial
by the ratio between the mass of the ball and
elocity of the recoil is a little less, on account
of the wheels upon their bearings, and that of

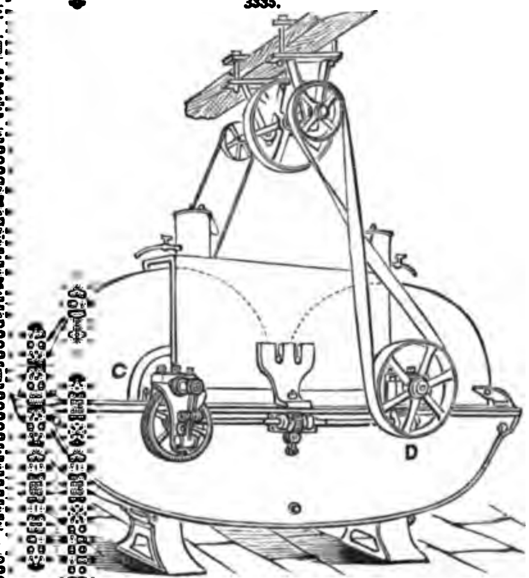
es, or when the external forces give a resultant
he equations [12] are nil, and the final quantity
initial quantity. If the centre of gravity were
sessed a certain velocity, it would retain that
otion.
reactions exerted by the bodies with which it
t with no kind of body, it would vainly exert

a *couple*. The two equal and contrary forces
equal to zero; consequently the couple has no
If this point were originally at rest, it would
ple can be merely one of rotation about an axis

GER., *Schleifstein*; ITAL., *Mola*; SPAN., *Piedra*

other Articles.—This arrangement, Figs. 3335,
the same trough, and in supporting the shafts

3335.

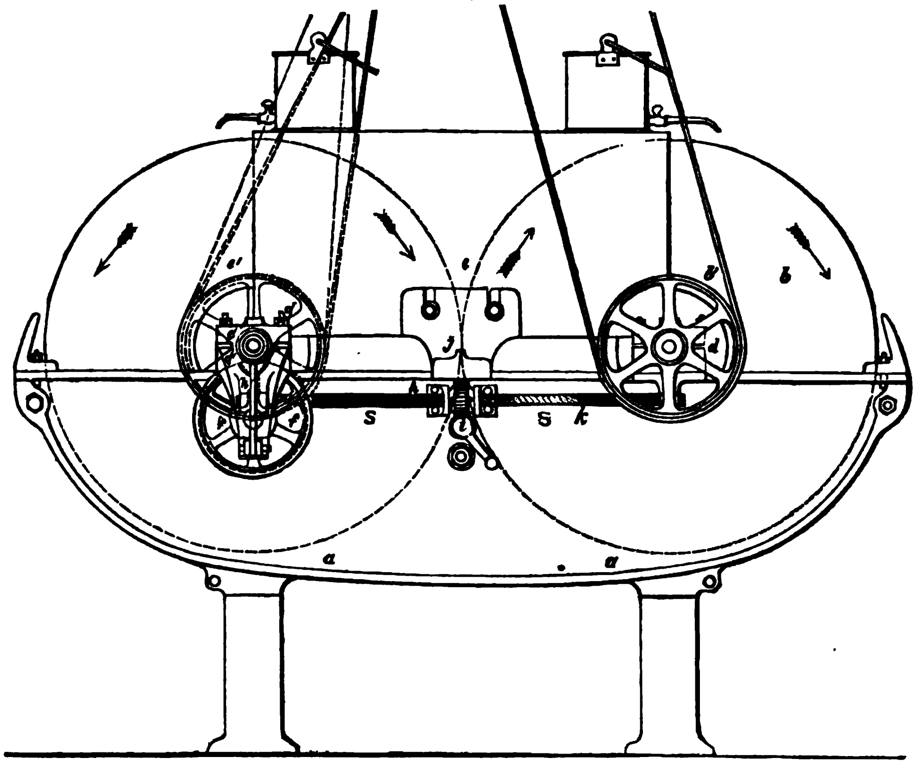


ween the peripheries of the grindstones. The
her in repair, and thereby obviating the neces-
they become uneven, owing to the inequalities

longitudinal, of one of Muir's grindstones. In
cast iron or other suitable material; *b* and *c* are
d, bolted to the flanches of the trough *a*. The
point *e*. One of these grindstones is driven by
the other by a crossed strap passing round the
pulleys are geared. When the grindstones are
ifferent diameters, in order that there may be a
ndstones. This rubbing action has a tendency to
ndrical; but in order to produce this result more
re given to one or both of the grindstones, in the
one *c* is fixed a spur-pinion c^1, which gears into
ocket d^1, bolted to one of the pedestals *d*; to the
in this groove a stud, projecting from the link *g*,
d its outer end is jointed to the lower end of the
bracket d^1; its upper end is forked, and takes
xed on the shaft of the grindstone *c*. By this
ral motions of the grindstone are simultaneous;
lving moves the link *g* to and fro. This motion

is communicated to the lever *h*, and by it to the grindstone. When the grindstones diminish in diameter, the bearings *d* in which they revolve are gradually brought closer together, so as to keep the peripheries in contact, by the attendant turning the shaft *i*. This shaft is furnished with two

3336.



worms taking into the wheels *j*, fixed on the screws *k*; each of these screws is made with a right-handed thread at one end, and a left-handed thread at the other; consequently, when the shaft *i* is turned in one direction it causes the pedestals *d* to approach, and when turned in the other direction it causes them to recede from each other; the rubbing produced by the peripheries of the grindstones moving at different velocities, and by the lateral motion, causes the inequalities on the peripheries of one stone to be removed by the other, thereby keeping them both in working condition. Instead of making the peripheries of the grindstones revolve at different velocities, and of giving a lateral to-and-fro motion to one or both of them, in some cases we may introduce a flat piece of stone, or other suitable material, between the peripheries of the grindstones at the point *e* in Fig. 3336, and give a to-and-fro motion to this piece of stone, the action of which on the peripheries of the grindstones would keep them true. The same object may also be obtained by causing one or both of the grindstones to swivel partly round, so as to produce a rubbing action on their peripheries. The stones may also be driven so that they revolve in the same direction; the peripheries at *e* in Fig. 3336 would then run in opposite directions, as shown by the arrows.

GRIST-MILL. Fr., *Moulin à drêche;* Ger., *Schrotmühle;* Span., *Molino para trigo.*

See Mills.

GROUND-AUGER. Fr., *Tarière à fond;* Ger., *Grundbohrer;* Ital., *Trivella;* Span., *Sonda.*

See Augers.

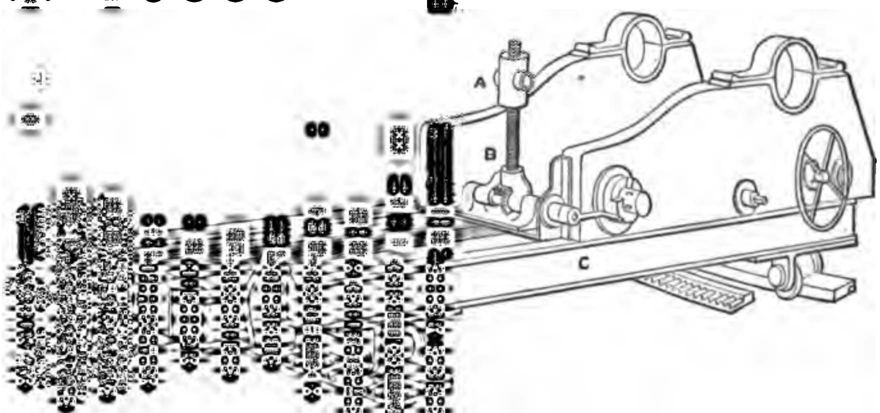
GUDGEON. Fr., *Tourillon;* Ger., *Drehzapfen;* Ital., *Perno;* Span., *Pivote.*

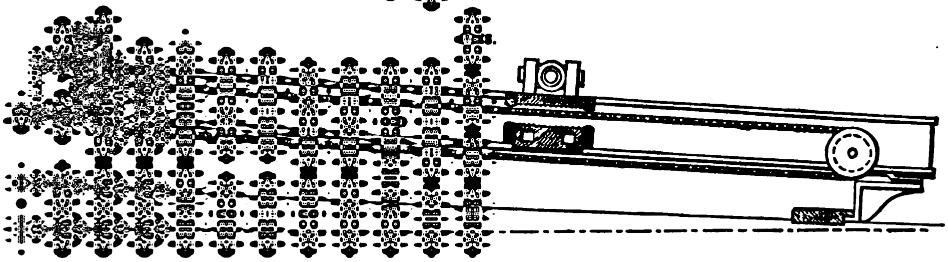
A *gudgeon* is the piece of iron in the end of a wooden shaft, on which it turns in a collar or on a gudgeon-block; formerly the part of any horizontal shaft on which it runs.

GUN-CARRIAGE. Fr., *Affût de canon;* Ger., *Geschütz Rampert;* Ital., *Affusto;* Span., *Cureña.*

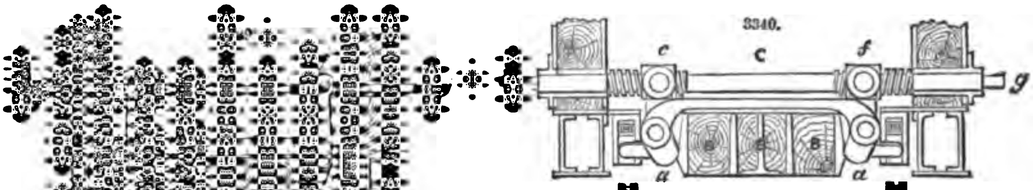
Captain Scott's Gun-Carriage for Heavy Naval Ordnance.—Simple as the invention of a suitable gun-carriage for heavy naval ordnance may appear, yet, in attempting this apparently easy task, before R. A. E. Scott succeeded in accomplishing it, many ingenious inventors failed. Fig. 3337 represents Scott's 300-pounder carriage and slide. The running-in-and-out gear is shown in Fig. 3338, and consists of two endless chains, stretched over two pitch-wheels on each side of the slides, with a screw arrangement for tightening the chains. When the gun is required to be run in, the outside part, or toes, of the compressor-levers are pressed against the lower part of the box, as shown in Figs. 3338 and 3340, which is serrated on its upper edge, so as to fit between the pins of the chain links, and press them up against the serrated edge of the upper box. By this means

...eld by the whole force of the compressor $a\,a$, ...ch them are attached to the carriage, it is only ...he slides to draw the gun in or out. When either ...r is turned, and the inner part, or heel, of the ...ssing. Thus one compressor performs the double ...g the chains on both sides of the slide together ...ing always in place, and out of the way of every ...s previously experienced in working heavy guns ...rs, under the idea—however powerful and strong ...upon a single part.





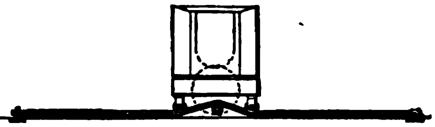
3340.



...sing the beams B, B, B, together, and retaining ...n the aft compressor shown in Fig. 3340. The ... the other, and hence the upper arms $a\,c$, $a\,f$, are ...Fig. 3340. The shaft $b\,g$ upon which these arms ...d projects beyond; it is provided with a right and ...rning a handle shipped upon the shaft at either ...lever-arms. The fore compressor is applied when ...n additional means of readily checking the gun ...plan of raised racers was adopted in H.M.'s ships ...h the latter ships the centres of the racers were ...ends raised so as to admit the guns to be trained ...the ship's side of the front racers were slotted to ...e slide securely down to the deck, and rendered ... well as in the Research, the front of the slide ...rking in a grooved racer; but after the trial of ... the roll in the Minotaur without this fastening, ... a pivot-bar, or flap, of the same strength as the ...o the pivot-point in the port as a preventer. In ...31° each way; but no other gun with the V-flap

and fixed pivot obtained so much as 29°. A similar result was observed in the Bellerophon, where, although the port was closed up from 2 ft. 9 in. to 2 ft. 1 in., the training was still 31° each way. The training of the other guns in 2 ft. 9 in. apertures was only 30°. Fig. 3341 shows how the Minotaur's port was closed on the lower side 14 in., the corners being rounded and made much higher.

3341.



The application of this important feature of Scott's system of mounting had a similar effect to throwing the lower port-sill a mean of 16 in. higher out of the water, thus adding greatly to a vessel's capabilities of fighting in a sea-way. Had a small half-port been fitted up, $7\frac{1}{2}°$ of depression could have been obtained in the Bellerophon, and 9° in the Minotaur, when wanted. This woodwork rendered protection to the loaders against the spray of the sea; nor was any disadvantage found, but the contrary, from closing up the port, the rapidity of the fire in the Minotaur with the 150-lb. ball being more than double that previously obtained, and the quickness in the Bellerophon, with the 250-lb. rifled shot, being equally unmatched. The elevating gear A B C, Fig. 3337, consists of a screw worked through a box, fitting inside another box which is fastened to the gun. These boxes have a washer interposed between their surfaces, and the outer box is open at the bottom; hence, when the gun is fired with its muzzle above the upper port-sill, the outer box is lifted several inches up from its resting place on the inside box, when the muzzle dipped under the port-sill, and then dropped easily down again upon the washer on the top of the inner box. By this contrivance the weight of the breech of the gun is received without any damaging shock, and the jar of the discharge is absorbed likewise. Any fixed elevation can also be given and maintained with certainty in bombarding. Motion is communicated to the screw through two bevelled wheels, shown at B, Fig. 3337, suitably supported upon the bottom of the carriage. By means of handles worked upon each side, a rapid touch may be given in elevating the gun; in case the captain should find it rolling up or down, his sights would not come on with the object to be fired at.

Running-in-and-out Eccentrics.—For Scott's carriage, eccentrics, which had to stop to allow them to pass the centre, and remain fixed in that position, were devised. This prevented the necessity of having the men to hold on to them in running the gun in or out; the arrangement also allows the crew on each side to hold on by the ropes which were attached to the ends of the levers of the eccentrics, if required, and so keep the eccentrics ready to drop the carriage off its rear rollers or trucks. The levers which work the eccentrics are upon the sides of the carriage, and so fitted that the screw is prevented from injury in case the gun should go off in being run out. Should the eccentrics be slacked up, the carriage would drop upon the slides, with a surface of wood everywhere touching a surface of iron, as dropping the rear of the carriage lifts the front trucks off the slides; and as both these surfaces are rough, there would then be an absence of sliding sufficient to keep the carriage and gun from moving in a roll. The lever-handles are fitted with bands round the drumhead of the eccentrics, which hold them securely when the levers are let drop out of use. In consequence of these arrangements, every part of the mounting is in place, ready for use: the man who is termed No. 7, having no mechanical labour to perform, can give his whole attention to keeping the gun pointed upon the object, which can be done by means of the rack and pinion, so steadily as not to interfere with the loading. The requisite elevation in case of a change in the heel of the ship can also be given with the same ease and steadiness; and all these operations can be performed simultaneously.

The rear compressor, Fig. 3340, being on a lower level than the fore one, Fig. 3339, and being also considerably below the level of the proposed height for the lower port-sill, would probably escape injury should the front compressor be hit. Although the rear compressor is less powerful, it is more important than the other, being employed to catch the chains in running the gun in and out, and being also the principal working compressor for holding the gun on being fired. The only addition made to this compressor, to enable it to perform also the duty of clutching the chain, consists of small pieces, or toes, on the outside of the lever-arms *o a*, *f a*, Fig. 3340. These toes are shown at *a a*, Fig. 3340, in which the rear compressor is shown in section, compressing the balks B, B, B, preparatory to firing, and consequently with the toe-pieces at *a a* clear of the lower box. The balks of wood B, B, B, upon which the compressors act are slightly tapered longitudinally towards the front of the slide, and are very much tapered in their depth.

The Moncrieff System of Gun-Carriage.—We shall in this place only explain the Moncrieff system of working artillery as far as it relates to coast defence.

This system is based on sound philosophical principles, and may be investigated under the three following heads;—

1st. The mechanical principle of the gun-carriages.

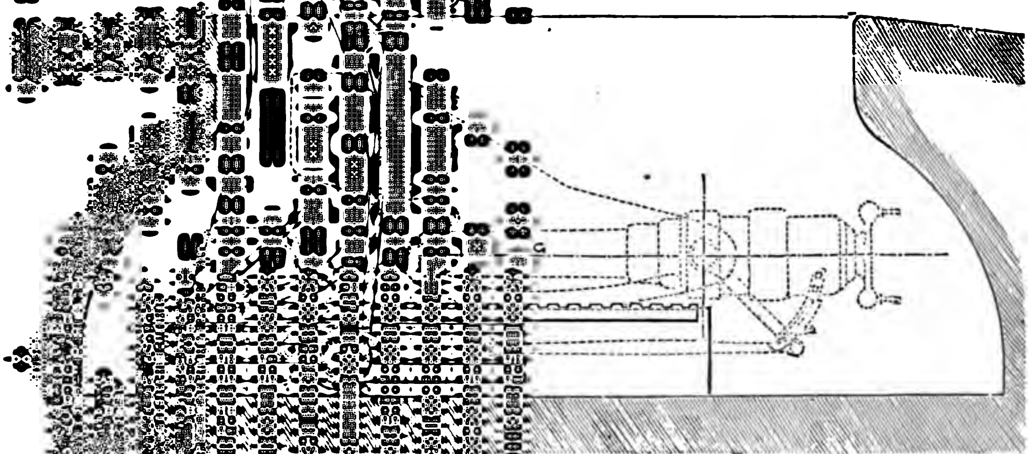
2nd. The form internal and external of the batteries.

3rd. The selection of ground for placing the batteries, and the arrangement for working them to the greatest effect; or, in other words, the *tactics* of defence for positions where the system is employed.

The principle on which the carriage, Fig. 3342, is constructed is the first and most important part of the new system, because on it depends the possibility of applying the other parts. This principle may be shortly stated as that of utilizing the force of the recoil in order to lower the whole gun below the level of the crest of the parapet, so that it can be loaded out of sight and out of exposure, while retaining enough of the force above referred to to bring the gun up again into the firing or fighting position. This principle belongs to all the carriages; but the forms of these carriages, as well as the method in which this principle is applied, vary in each case. For instance, in siege guns, where weight is an element of importance, the recoil is not met by counterpoise.

With heavy garrison guns, on the other hand, which when once mounted remain permanent in

that case, therefore, the force of gravity is used
me, easily managed, and not likely to go wrong;
owerful guns, it is a great advantage to have the



enormous and hitherto destructive force of the
out the manner in which that difficulty is over-

the elevator may be spoken of and treated as a
end of the power-arm, and the centre of gravity
ght-arm, there being between them a moving

e fulcrum on which this lever rests is almost
nter-weight C, and when the gun is fired the
e fulcrum, or point of support, travels away from
power-arm, or, in other words, it passes from the

s on the upper or flat part of a cycloid. It is
t of the structure, because the counter-weight
the recoil goes on, however, the case changes
ds the gun, making the weight-arm longer and
the recoil, least at first, goes on in an increasing
the recoil it is seized by a self-acting pawl or

out any sudden strain, and its force is retained
e gun to the firing position at any moment they
however violent at first, does not put injurious
Moncrieff's experiments at Edinburgh with a
tion on the platform caused by firing, that the
at experiment, and which were only secured in
sition; nor even when heavy charges or double
ved tops. See p. 1716.
el of a 95-cwt. gun, the model was fired on the ice
stationary.
ot be appreciated fully without referring to the
now felt, in getting pivots, platforms, &c., on
ew artillery, where the recoil is stopped by fric-
ally called *compressors* attached to the platform.

Schiessbaumwolle; ITAL., *Pirosillina*; SPAN., *Algodon-*

Kanonenmetall; ITAL., *Bronzo*; SPAN., *Metal de*

GUNNERY. FR., *Science de l'artillerie;* GER., *Artillerie Wissenschaft;* ITAL., *Artiglieria;* SPAN., *Ciencia del artillero.*

Gunnery is that department of military science which comprehends the theory of projectiles and the manner of employing *ordnance.*

When great minds conspire to perpetuate a fallacy, it has always been a difficult matter to clear that fallacy away. We know of no subject capable of being submitted to mathematical investigation that has received a greater amount of fallacious treatment, and that too, by great minds, than the motion of projectiles. Besides, school-taught pedants, thimble-rigging with mathematical symbols, reduced this branch of military science to a deplorable state of uncertainty, and left the artillerist to play a game of blindman's-buff with his guns. Initial velocities have been little more than guessed at, the resistance of the air overrated, and the force of gravity misstated. It is well known that General Anstruther's physical courage is great, but, with these facts before him, his moral courage must be as great as his physical, to propound and develop a new system of gunnery; but "his heart is in his work, and the heart giveth grace unto every art." The system introduced by General Anstruther, which is practical and easily applied, must give correct results within the range of his experiments, without offering any special theory about initial velocities, the resistance of the air, or the force of gravity; indeed, in Anstruther's system are collected all these elements.

In the following fifty-eight paragraphs Major-General P. Anstruther lays the foundation, and illustrates the practical application of his system.

In the first paragraph he denies the difficulty of drawing the trajectory of a projectile.

2. Defines what it is that we want to do.

3. States our want of data for the purpose.

4. Expresses a wish that we may get them.

5. Describes our intended demonstration.

6. Shows how Colonel Boxer says it is to be done, algebraically.

7. Admits his demonstration, but requires it in numerals.

8. Names an elevation and time of flight, 45° and 27·1 seconds.

9. Gives the ascent, descent, and range, in a vacuum.

10. Defines Fig. 3343, the triangle for the given elevation.

11. Graduates the ascent of this triangle, unresisted.

12. Graduates the descent of this triangle, unresisted.

13. Proposes comparison with recorded fact.

14. States the recorded range for elevation 45° in 27·1 seconds.

15. Shows the reduction produced by the resistance of the air.

16. Infers the power of measuring the resistance.

17. Shows the value of $\frac{1}{2}g$ for 27·1 seconds of time.

18. Shows the varying value of $\frac{1}{2}g$, as printed years ago.

19. Shows where this may be had, printed, *in extenso.*

20. Assumes that we now know the true law of gravity.

21. Defines Fig. 3344, a parallelogram on Fig. 3343.

22. Defines the two lines added.

23. Applies the law of the composition and resolution of forces.

24. Why applied to our question.

25. Requires the graduation of the vertical descent.

26. Shows the graduation of the descent the same for all elevations.

27. The graduation of the vertical ascent varying with elevation.

28. At 90° elevation the two coincide exactly.

29. Proposes to apply this to our example.

30. Shows place of ball at end of 27·1 seconds, elevation 90°.

31. Shows additional time to be required for descent.

32. Shows that this will equally increase time of ascent.

33. Tries an addition of 6·9 seconds, it is too much.

34. Tries an addition of 6·8 seconds, which will do.

35. Therefore 27·1 + 6·8 = 33·9 seconds is the time for elevation 90°.

36. Therefore 761 ft. per second is the initial velocity.

37. Shows graduation of descent, for 0·9. 1·9, 2·9, 3·9, &c., &c., to 33·9.

38. Shows graduation of ascent, the inversion of the descent.

39. Describes Table A.

40. Shows how to draw the trajectory.

41. Shows the French Table of Ranges for 45° elevation with velocities.

42. Selects one for comparison with our theory, 10,699 ft.

43. Deduces the time of flight, and shows the oblique ascent.

44. Shows that 35·32 seconds is the time for elevation 90°.

45. Shows that 777 ft. a second is the velocity, compared with 784 ft. a second.

46. Shows that Table B gives ranges at 45° with velocities.

47. Shows the application of the instrument, Fig. 3345.

48. Supposes an example. Elevation 5°, range 1000 yds.

49. Works it out by the instrument, velocity 777 ft. a second.

50. Describes the method of working it out.

51. Shows the limits beyond which our data will not carry us.

52. Quotes a range from a Text-Book, a French range.

53. Puts it into English feet.

54. Shows the vertical descent and oblique ascent and time.

55. Finds the mean velocity.

56. Refers to Table C.

57. Finds the time for the mean velocity.

58. Deduces the final velocity and initial velocity.

1. There would be no difficulty whatever in determining the trajectory of any projectile, if the artillery officers could be persuaded to deduce the laws of their own science, gunnery, from the

recorded results of their own practice, instead of intrusting this, their first, and most important of all duties, to the professors of mathematics.

2. These men, however able and eminent, do not know what it is that the artillery require; they teach us how to calculate the trajectory for given elevation with given initial velocity; what we want to know is, how to determine the initial velocity from given range and elevation.

3. And this we could easily do if we had the data; but no book, either in the French or English language, affords reliable record of range for elevation with time of flight exceeding 34 seconds; we cannot therefore determine the trajectory for any initial velocity exceeding 800 ft. per second; but within that limit we can do it without any difficulty.

4. We shall offer one example, fully worked out, in the hope that the official advisers of governments may yet be induced to recommend the few, and comparatively cheap, additions to our practice tables, which are required to complete our professional knowledge; the results of private practice, under any circumstances, fail to carry with them the weight of authority requisite to establish the laws of science; were it otherwise, the experiments required should have been furnished long ago.

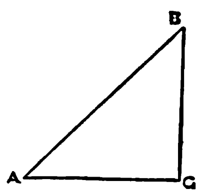
5. In working out the example selected, we shall show what would be the trajectory of a projectile, if not resisted by the atmosphere, and then we shall compare that calculation with the recorded results of actual practice with the same elevation, in the same time of flight.

6. To find the trajectory of a projectile in a vacuum, Colonel Boxer, in his treatise on Artillery, tells us, on p. 87, that we have only to "compound the motion produced by gravity, which, by the second law of motion, is the same as it would produce upon a body at rest, with the uniform motion in the line" of direction, "in order to obtain the actual motion of the shot upon the hypothesis assumed," that hypothesis being the leaving out of consideration for the present the resistance of the atmosphere.

7. Colonel Boxer proceeds to prove, algebraically, that the curve resulting is a true parabola. We accept his demonstration, but for our present purpose it is necessary to show, by the use of numerals, the application of this theory to some specified elevation and time of flight, to enable us to draw a comparison between the curve of calculation and the recorded results of actual practice.

8. For this purpose let us suppose a ball fired at 45° elevation, seen to strike the plane at the expiration of 27·1 seconds of time of flight: putting out of consideration for the present the resistance of the atmosphere, we are to draw the trajectory of this ball; we know that it is a parabola, but we require to show its measurements.

3343.



9. The time of flight being 27·1 seconds, the fall by gravity, unopposed by the atmosphere, will be equal to $27 \cdot 1^2 \times 16\frac{1}{12} = 11811 \cdot 76$ ft., and as the elevation is 45°, the horizontal range will be equal to the vertical descent, which is the fall by gravity, therefore the horizontal range is also 11811·76 ft., and the oblique ascent is the square root of the sum of the squares of these two, it is 16704·35 ft.

10. Let A B G, Fig. 3343, be a right-angled triangle, in which the sides are respectively 11811·76 ft., 11811·76 ft., and 16704·35 ft.; within these three lines we are to inscribe the trajectory of a ball fired from A, towards B, seen to strike G at the expiration of 27·1 seconds of time; leaving out of consideration for the present, the resistance of the atmosphere.

11. We divide the length of A B by the time of flight, the quotient $\frac{16704 \cdot 35}{27 \cdot 1} = 616 \cdot 396775$ ft. a second, is the uniform velocity of the oblique ascent; we therefore lay off upon A B, in succession from A, 27 equal spaces, each 616·396775 ft., to show the uniform motion in the line of direction, with which, as Colonel Boxer tells us, we are to compound the motion produced by gravity.

12. From each of the points so marked, in succession, we let fall a perpendicular, denoting by its length the fall by gravity, $t^2 \times 16\frac{1}{12}$ ft., in the number of seconds of time, t, elapsed since the ball left A on its passage towards B; a line joining the lower ends of all these perpendiculars is the trajectory required, the parabola.

13. We are now to compare this with recorded fact.

14. The 13-in. sea-service iron mortar, at elevation 45°, with a charge which gave 27·1 seconds time of flight, had a range of only 3327 yds., or 9981 ft.; if we apply this to Fig. 3343, we have A G = 9981 ft., B G = 9981 ft., and A B = 14115·26 ft.

15. Each side has been reduced by the resistance of the atmosphere in the proportion of 11811·76 to 9981 ft. in 27·1 seconds.

16. Such a reduction in the magnitude of B G, the fall by gravity in the time of flight, affords us a ready measure of the effect of the resistance of the atmosphere.

17. The fall by gravity in 27·1 seconds of time would be, in a vacuum, equal to $27 \cdot 1^2 \times 16 \cdot 083333 = 11811 \cdot 76083333$, and in the atmosphere it is $27 \cdot 1^2 \times 13 \cdot 590501 = 9980 \cdot 99983941$; this last we shall call $27 \cdot 1^2 \times 13 \cdot 59$.

18. It is some years since General Anstruther offered to the service Table D, showing the fall by gravity as modified by the resistance of the atmosphere, in which the varying value of the multiple of the square of the time was deduced from the measure of the fall in 27·1 seconds as follows, namely;—

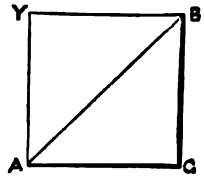
For 3	seconds of time	the fall is	3^2	× 16
13	"	"	13^2	× 15
23	"	"	23^2	× 14
24	"	"	24^2	× 13·9
25	"	"	25^2	× 13·8
26	"	"	26^2	× 13·7
27	"	"	27^2	× 13·6
27·1	"	"	$27 \cdot 1^2$	× 13·59, as above.

19. Table D shows the fall by gravity, together with the velocity which a ball would acquire by the fall, for the tenth parts of seconds of time from one-tenth of one second to fifty seconds.

20. We shall now suppose the true graduation of B G, the vertical descent in Fig. 3343, to be known to us; we shall deduce from it the graduation of the oblique ascent A B, the first step of which, when found, is the measure of the initial velocity of the ball.

21. To deduce the graduation of the oblique ascent from that of the vertical descent, we again draw the triangle A B G exactly the same in all respects as that in Fig. 3343; but we now add a line A Y, parallel to and equal to B G, and we join B Y, so that A G B Y in Fig. 3344 is a parallelogram, of which A B is the diagonal, and in which we know the magnitude of every line.

3344.



22. The line A Y now added represents the vertical ascent of the ball during the time of flight; this line and the line B G are added for the purpose of bringing the question within the scope of the law of the composition and resolution of forces.

23. That law teaches us that the force which produces the motion represented by the diagonal A B, is the resultant or equivalent of two forces producing motions represented both in magnitude and direction by the two sides A Y, A G, of the parallelogram, of which A B is the diagonal.

24. If therefore we could determine the graduation for time of the line A Y, we could at once find that of A B; and we could then, to use Colonel Boxer's words once more, "compound the motion produced by gravity with the motion in the line" of direction, "in order to obtain the actual motion of the shot."

25. We desire to determine the graduation of A Y, and we know that it is equal in magnitude to B G; it is 9981 ft.; the duration of the motion which it represents is the same as in B G, 27·1 seconds of time, and still it is quite certain that the graduation of A Y cannot be that of B G read in inverse order of succession.

26. The graduation of B G is the same for any one number of seconds of time of flight, whatever be the elevation, as it is the fall by gravity in the time of flight t; the graduation of this line is always an increasing series or progression, it must always commence in the same manner; if the time of flight is four seconds, the graduation of the descent will always be 1, 3, 5, 7, total 16 spaces of $\frac{1}{2} g$, whether the elevation be 5° or 85°.

27. But the graduation of A Y varies with every change of elevation; the motion represented by this line is necessarily exactly equal in magnitude and in duration to the motion represented by the parallel B G, but it will always be differently graduated; in four seconds' time of flight the ascent will always be 16 spaces, but they will be divided into 7, 5, 3, 1, total 16, for elevation 89° 59′ 58″; and into 4, 4, 4, 4, total 16, for elevation 0° 00′ 02″.

28. At elevation 90° the oblique ascent and the vertical ascent become merged in one, and, as we have just seen, the graduation is the inverted reading of the descent; we are therefore enabled to determine its graduation by referring to the Table described in our paragraph 19, which we shall suppose to be in the hands of our reader.

29. We now return to our selected example, the range 9981 ft. at elevation 45°; we showed in paragraph 14 that the oblique ascent was 14115·26 ft., the simultaneous vertical descent 9981 ft.

30. If we now change the elevation from 45° to 90°, the ascent in 27·1 seconds will again be equal to 14,115 ft., the descent again 9981 ft., therefore at the expiration of 27·1 seconds of time the ball will be at a height of 4134·26 ft. vertically over the point A from which it was projected.

31. To enable this ball to reach the ground, addition must be made to the time, and as the ascent and descent are simultaneous motions, additions to the time of either bring equal addition to the time of the other.

32. But equal addition to the time by no means brings equal addition to the magnitude; the addition of one second will bring an increase of 16 ft. to the ascent and of 671 ft. to the descent; a very few such additions will bring the two to equality of magnitude, and we proceed to try how many will do it.

33. We try an addition of 6·9 seconds, making the time of flight $27·1 + 6·9 = 34$ seconds, then we find in our Table that $34^2 \times 12·9 =$ 14912·4
We also find that $6·9^2 \times 15·61 =$ 743·1921
which we add to the ascent, 14115·26

making the ascent 14858·4521 = 14858·4521

so that the descent now exceeds the ascent by 53·9479 ft., and we must try a less addition.

34. We try an addition of 6·8 seconds, making the time of flight $27·1 + 6·8 = 33·9$ seconds, and we find in our Table that $33·9^2 \times 12·91 =$ 14836·3011
We also find that $6·8^2 \times 15·62 =$ 722·2688
which we add to the ascent 14115·26

making the whole ascent 14837·5288 = 14837·5288

the ascent now exceeds the descent by 1·2277 ft., but we are satisfied.

35. The fall in 34 seconds we have seen to be 14912·4
and " 33·9 " " 14836·3011

the differences 0·1 second of time and 76·0989 ft. indicate a velocity of 760·989, say 761 ft. a second.

36. Therefore the initial velocity of the ball, which at 45° elevation ranged 3327 yds., was 761 ft. a second.

37. We can now give the graduation of the ascent from our Table; we read in it that the fall in

29·9 seconds is	$29·9^2 \times 13·31 = 11899·2731$	difference	713·767
30·9 ,,	$30·9^2 \times 13·21 = 12613·0401$	,,	727·827
31·9 ,,	$31·9^2 \times 13·11 = 13340·8671$	,,	741·287
32·9 ,,	$32·9^2 \times 13·01 = 14082·1541$	,,	754·147
33·9 ,,	$33·9^2 \times 12·91 = 14836·3011$		

38. The inverted reading of this Table is the graduation of the ascent; it commences thus, in

	Cumulative.	Gradual.						
1 second	754·147	754·147						
2 seconds	1495·434	741·287	1st difference	12·86				
3 ,,	2223·261	727·827	,,	13·46	2nd difference	0·6		
4 ,,	2937·028	713·767	,,	14·06	,,	0·6	3rd difference	0·0
5 ,,	3636·135	699·107	,,	14·66	,,	0·6	,,	0·0

39. We give, Table A, the graduation of the ascent for even seconds of time, for initial velocity 761 ft. a second, and the simultaneous descent; we also give thirty-three different ranges, together with the angles of elevation, by the use of which these ranges would be obtained.

40. In any right-angled triangle whatever, if the reader will mark off upon the hypothenuse the distances given as ascents, and let fall perpendiculars to denote the fall by gravity, then a line joining the lower extremities of all these perpendiculars is the trajectory, which we said in paragraph 3 we could draw without any difficulty.

41. In the Aide Mémoire a l'usage des Officiers d'Artillerie, p. 431, we find a Table containing thirty different ranges for elevation 45°, with the initial velocity for each, but not the time of flight.

42. The second of these ranges is 3261 mètres, its initial velocity 239 mètres; reducing these to English measures, we have 10,699 ft. of range, with 784·136 ft. velocity.

43. Here the oblique ascent is $10699 \sqrt{2} = 15130·7$, and the vertical descent, or fall by gravity, is 10,699 ft., which indicates a time of flight of 28·17 seconds for $28·17^2 \times 13·483 = 10699·4198187$.

44. An ascent of 15130·7 ft. in 28·17 seconds is what we have to graduate; we find that a descent of 35·32 seconds will be as follows, namely;—

$$35·32^2 \times 12·768 = 15928·1106432 \text{ ft., the time being } 28·17,$$

subtract and add 28·17   2·817

the differences $7·15^2 \times 15·585 = 796·7430625$

show a fall in 28·17 seconds of 15131·3675807 ft.,

which is only 8 in. in excess of the ascent.

45. The velocity acquired by a fall of this duration is thus found;—

from $35·32^2 \times 12·768 = 15928·1106432$

deduct $35·31^2 \times 12·769 = 15920·3394009$

the differences being $-0·01$, 0·001, and 7·7712423,

indicate a velocity of 777·124 ft. a second, to compare with 784·136 ft. a second, as given in the Aide Mémoire.

46. As no book in the English language gives us any record of initial velocity for range and elevation, we give, in Table B, thirty ranges for 45° elevation, with the time of flight calculated, and the initial velocity deduced; and in the same page, for convenience of comparison, we give Table C, named in paragraph 41.

47. We give, Fig. 3345, a drawing of a very simple instrument, by which, when made to a larger scale, any rifleman may at once draw the trajectory of his bullet, and read off its initial velocity, and the angle of its descent.

48. For instance, suppose a ball fired at elevation 5°, its range measured is 1000 yds. exactly. Then the range being 3000 ft., the oblique ascent is 3011·4 ft., the vertical descent or fall by gravity is 262·466 ft., therefore the time of flight is 4·06 seconds, very nearly.

49. Dividing the oblique ascent by the time of flight, we have $\frac{3011·4}{4·06} = 741·7$ ft. a second, the mean velocity of the ascent. The Table described in our paragraph 19, of which the instrument described in paragraph 47 and shown in Fig. 3345 is a portable epitome, shows that a velocity of 741·7 ft. is the result of a fall of 32·85 seconds;

to this we add half the time of flight 2·5 seconds,

and the sum of the two, 35·35 seconds, is the time of flight for this velocity at 90° elevation; the Table shows us that the velocity would be 777 ft. a second, roughly.

50. For all elevations usually employed with rifles or field artillery, the instrument shown in Fig. 3345 would enable the student to draw the trajectory at once; supposing his instrument to be made on a sufficiently large scale, he lays the mean velocity of the ascent, as found by dividing its magnitude by the time of flight, exactly against the centre of the hypothenuse of the triangle, calculated in paragraph 48, and marks off all the fifty spaces, twenty-five on each side, which will form the graduation of the hypothenuse, lets fall forty-nine perpendiculars to denote the fall by gravity, and joins the fifty points by a curve line, which is the true trajectory to the tenths of seconds of time.

51. But the instrument will not serve for any velocities beyond 800 ft. a second, because, as said in paragraph 4, we have not the data, and private experiments are not authoritative.

52. The want of such an instrument is very strikingly shown by an error contained in the clever little Text-Book for officers, sent to the Schools of Musketry, published by authority.

53. We read, in p. 8, that it was found in France that the range of the common percussion musket, with the regulation charge, at an angle of from 4° to 5°, was 640 yds., or 1920 ft., and that the usual velocity was "some 500 yds. per second," or, in our usual mode of expressing it, 1500 ft. per second.

54. We shall show how far from correct this is. Taking the elevation as 4°, then we multiply the range by the tangent of the elevation, 1920 × tan. 4° = 134·26 ft., for the fall by gravity in the time of flight, which is therefore 2·9 seconds ($2{\cdot}896^2 \times 16{\cdot}0104 = 134{\cdot}276$) as the time of flight, and we find the oblique ascent, 1920 × secant 4° = 1924·7 ft.

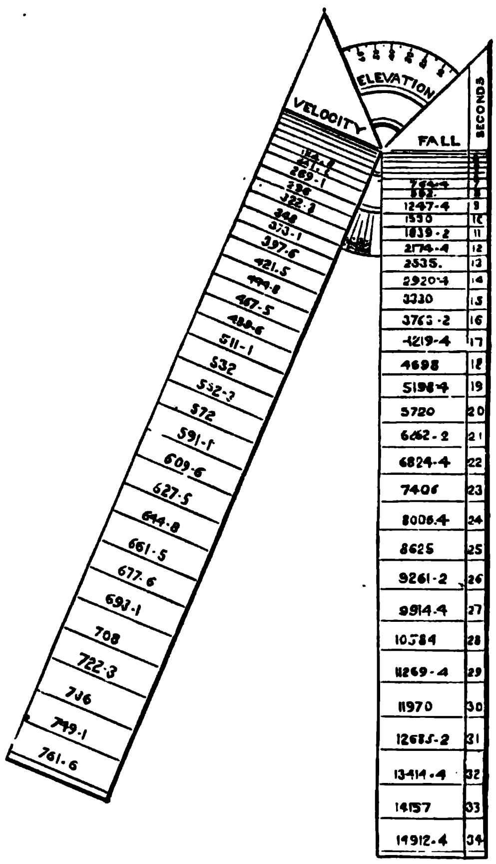
55. Dividing the oblique ascent by the time of flight, we have 663·7 ft. a second as the mean velocity of the ascent.

56. Our instrument is only graduated for seconds, as shown in Fig. 3345, but the Table quoted in paragraphs 18 and 19 shows how it would be read if graduated to the tenth parts of seconds, which we shall suppose to be done.

57. We find that a fall of 27·15 seconds would give a velocity of 663·9 ft. a second, then if we add half the time of flight, $\frac{2{\cdot}9}{2} = 1{\cdot}45$, to the time for the mean velocity, 27·15 seconds, we have 28·6 seconds as the time for the initial velocity, and the difference between the same two is the time for the final velocity, 25·7 seconds.

58. Our Table D shows us that the velocity for 28·6 seconds is 687 ft., that for 25·7 seconds is 639·67 ft. a second; these contrast strongly with the 1500 ft. velocity quoted by the Text-Book from the French.

3345.



Instrument to facilitate the drawing of the trajectory of any ball at any elevation with any velocity.

TABLE A.—ASCENT, DESCENT, RANGES, AND ELEVATIONS FOR INITIAL VELOCITY, 761 FEET A SECOND.

Seconds of Time.	Ascent.				Descent.	Range.	Elevation.
	Cumulative.	Gradual.	Differences.				
	feet.	feet.	feet.		feet.	feet.	° ′ ″
1	754·147	754·147	..		16·2	754·0	1 13 51
2	1495·434	741·287	12·86		64·4	1494·0	2 28 5
3	2223·261	727·827	13·46		144·0	2218·6	3 42 47
4	2937·028	713·767	14·06		254·4	2926·0	4 58 9
5	3636·135	699·107	14·66		395·0	3614·6	6 14 11
6	4319·982	683·847	15·26		565·2	4282·8	7 31 4
7	4987·969	667·987	15·86		764·4	4929·0	8 48 55
8	5639·496	651·527	16·46	0·6	992·0	5551·5	10 7 52
9	6273·963	634·467	17·06		1247·4	6229·5	11 28 4
10	6890·77	616·807	17·66		1530·0	6718·7	12 49 43
11	7489·317	598·547	18·26		1839·2	7260·0	14 12 55
12	8069·004	579·687	18·86		2174·4	7770·5	15 37 59
13	8629·231	560·227	19·46		2535·0	8248·5	17 5 1
14	9169·398	540·167	20·06		2920·4	8692·0	18 34 19
15	9688·905	519·507	20·66		3330·0	9098·6	20 6 7
16	10187·152	498·247	21·26		3763·2	9466·6	21 40 44

TABLE A—*continued.*

Seconds of Time.	Ascent. Cumulative.	Ascent. Gradual.	Ascent. Differences.	Descent.	Range.	Elevation.
	feet.	feet.	feet.	feet.	feet.	° ′ ″
17	10663·539	476·387	21·86	4219·4	9793·3	23 18 31
18	11117·466	453·927	22·46	4698·0	10076·0	24 59 50
19	11548·333	430·867	23·06	5198·4	10312·2	26 44 46
20	11955·54	407·207	23·66	5720·0	10498·4	28 35 0
21	12338·487	382·947	24·26	6262·2	10631·2	30 29 0
22	12696·574	358·087	24·86	6824·4	10706·5	32 30 49
23	13029·201	332·627	25·46	7406·0	10719·7	34 38 23
24	13335·768	306·567	26·06	8006·4	10664·9	36 53 47
25	13615·675	279·907	26·66 0·6	8625·0	10535·4	39 18 21
26	13868·322	252·647	27·26	9261·2	10322·8	41 53 50
27	14093·109	224·787	27·86	9914·4	10016·0	44 42 28
28	14289·436	196·327	28·46	10584·0	9600·	47 47 24
29	14456·703	167·267	29·06	11269·4	9055·0	51 13 2
30	14594·31	137·607	29·66	11970·0	8349·4	55 6 11
31	14701·657	107·347	30·26	12685·2	7431·3	59 38 14
32	14778·144	76·487	30·86	13414·4	6200·0	65 11 31
33	14823·171	45·027	31·46	14157·0	4393·8	72 45 27
33·9	14836·3011	12·967	32·06	14836·3	0·0	90 0 0

TABLE B.—RANGES AT 45° ELEVATION, WITH THE TIMES OF FLIGHT DEDUCED, AND THE INITIAL VELOCITY DETERMINED.

Range and Fall.		Time of Flight.	Oblique Ascent.	Unexpired Time of Ascent.	Time of Flight at Elevation 90°.	Vertical Ascent.	Initial Velocity
yards.	feet.						
100	300	4·3485	424·26	0·903	5·253	435·29	162·87
200	600	6·2	848·53	1·31	7·51	876·96	228·0
300	900	7·61	1272·79	1·64	9·25	1316·52	275·8
400	1200	8·82	1697·06	1·93	10·75	1759·54	315·2
500	1500	9·9	2121·32	2·16	12·06	2195·33	350·0
600	1800	10·88	2545·58	2·39	13·27	2636·64	379·0
700	2100	11·785	2969·84	2·615	14·4	3081·37	407·0
800	2400	12·635	3394·11	2·815	15·45	3522·05	432·0
900	2700	13·436	3818·37	3·004	16·44	3961·13	454·8
1000	3000	14·21	5242·63	3·19	17·4	4408·19	476·4
1100	3300	14·93	4666·89	3·37	18·3	4845·86	496·0
1200	3600	15·63	5091·16	3·53	19·16	5280·45	514·3
1300	3900	16·305	5515·42	3·725	20·03	5736·0	532·7
1400	4200	16·96	5939·68	3·9	20·86	6185·0	550·0
1500	4500	17·59	6363·96	4·06	21·65	6625·39	565·0
1600	4800	18·207	6788·22	4·22	22·427	7070·4	580·0
1700	5100	18·81	7212·48	4·38	23·19	7518·65	595·0
1800	5400	19·39	7636·75	4·53	23·92	7957·79	608·0
1900	5700	19·964	8061·01	4·7	24·664	8412·6	621·0
2000	6000	20·521	8485·28	4·88	25·41	8883·0	634·0
2100	6300	21·0695	8909·55	5·0	26·0695	9209·32	646·0
2200	6600	21·605	9333·81	5·15	26·75	9749·54	657·3
2300	6900	22·13	9758·07	5·3	27·43	10200·35	668·5
2400	7200	22·65	10182·33	5·45	28·1	10651·84	679·0
2500	7500	23·16	10606·6	5·6	28·76	11103·48	689·5
2600	7800	23·66	11030·86	5·74	29·4	11547·85	700·0
2700	8100	24·15	11455·13	5·9	30·05	12005·42	708·7
2800	8400	24·64	11879·39	6·04	30·68	12454·79	717·6
2900	8700	25·12	12303·66	6·18	31·3	12902·52	726·4
3000	9000	25·593	12727·92	3·331	31·93	13362·91	735·0
3100	9300	26·06	13152·18	6·48	32·54	13813·77	743·0
3200	9600	26·521	13576·44	6·619	33·14	14262·0	751·0
3300	9900	26·979	14000·71	6·761	33·74	14714·8	758·2
3327	9981	27·1	14115·76	6·8	33·9	14836·3	761·0

Table C.—Ranges at 45° Elevation, with the Times of Flight deduced, with the Initial Velocities determined by M. Lombard.

From the Aide Mémoire a l'usage des Officiers d'Artillerie, p. 431.
Reduced to English Measures.

Velocity.	Range.	Time.	Velocity.	Range.	Time.	Velocity.	Range.	Time.
892·4	12431·34	30·65	679·15	8787·26	25·26	439·64	4252·05	17·06
784·14	10699·0	28·16	606·97	7565·76	23·26	364·18	3625·4	15·69
695·55	9225·9	25·95	544·63	6483·07	21·4	351·34	3064·36·	14·35
·623·37	7949·6	23·9	488·85	5538·17	19·65	318·25	2552·54	13·05
557·75	6807·9	21·96	439·64	4675·29	17·95	282·16	2103·06	11·8
501·98	5813·76	20·175	393·71	3891·15	16·28	249·35	1692·95	10·54
449·48	4911·5	18·43	347·78	3202·16	14·7	216·54	1328·77	9·3
400·27	4084·7	16·7	308·41	3238·25	14·78	285·44	1010·52	8·03
357·62	3369·5	15·1	269·03	2027·6	11·58	..	..	..
314·97	2710·0	13·46	229·66	1542·02	10·04	..	..	..
275·6	2129·3	11·86	..	..	..	..	..	..
226·26	1617·5	10·3	..	..	..	..	..	..

Table D.—The Fall by Gravity as modified by the Resistance of the Atmosphere.

The square of the time of falling × by the substitute for 16$\frac{1}{12}$.	Fall by gravity in the whole time.	Fall in each tenth of a second.	Velocity acquired.	The square of the time of falling × by the substitute for 16$\frac{1}{12}$.	Fall by gravity in the whole time.	Fall in each tenth of a second.	Velocity acquired.
0·1² × 16·29	0·1629	0·4883	3·756	4·6² × 15·84	335·1744	14·5103	143·611
2 8	0·6512	·8131	7·007	7 3	49·6847	·8081	6·592
3 7	1·4643	1·1373	10·252	8 2	64·4928	15·1053	9·467
4 6	2·6016	·4609	13·491	9 1	79·5981	·4019	152·536
5 5	4·0625	·7839	16·724	5·0 ·80	95·	·6979	5·5
6 4	5·8464	2·1063	19·951	1 ·79	410·6979	·9933	8·456
7 3	7·9527	·4281	22·172	2 8	26·6912	16·2881	161·407
8 2	10·3808	·7493	25·387	3 7	42·9793	·5823	4·352
9 1	13·1301	3·0699	28·596	4 6	59·5616	·8759	7·291
1·0 ·20	16·2	·3899	31·8	5 5	76·4375	17·1689	170·224
1 ·19	19·5899	·7093	34·996	6 4	93·6064	·4613	3·151
2 8	23·2992	4·0281	38·187	7 3	511·0677	·7531	6·072
3 7	27·3273	·3463	41·372	8 2	28·8208	18·0443	8·987
4 6	31·6736	·6639	44·551	9 1	46·8651	·3349	181·896
5 5	36·3375	·9809	47·724	6·0 ·70	65·2	·6219	4·8
6 4	41·3184	5·2973	50·892	1 ·69	83·8249	·9143	7·696
7 3	46·6157	·6131	54·052	2 8	602·7392	19·2031	190·587
8 2	52·2288	·9283	57·207	3 7	21·9423	·4913	3·472
9 1	58·1571	6·2429	60·956	4 6	41·4336	·7789	6·351
2·0 ·10	64·4	·5569	64·1	5 5	61·2125	20·0659	9·224
1 ·09	70·9569	·8703	67·236	6 4	81·2784	·3523	202·091
2 ·08	77·8272	7·1831	70·367	7 3	701·6307	·6381	4·952
3 ·07	85·0103	·4953	73·492	8 2	22·2688	·9233	7·807
4 ·06	92·5056	·8069	76·611	9 1	43·1921	21·2079	210·656
5 ·05	100·3125	8·1179	79·724	7·0 ·60	64·4	·4919	3·5
6 ·04	108·4304	·4283	82·831	1 ·59	85·8919	·7753	6·336
7 ·03	116·8587	·7381	85·932	2 8	807·6672	22·0581	9·167
8 ·02	125·5968	9·0473	88·927	3 7	29·7253	·3403	221·992
9 ·01	134·6441	·3559	92·016	4 6	52·0656	·6219	4·811
3·0 ·00	144·	·6639	95·1	5 5	74·6875	·9029	7·624
1² × 15·99	153·6639	·9713	8·176	6 4	97·5904	23·1833	230·431
2 8	163·6352	10·2781	101·247	7 3	920·7737	·4631	3·232
3 7	173·9138	·5848	4·312	8 2	44·2368	·7423	6·027
4 6	184·4976	·8899	7·371	9 1	67·9791	24·0209	8·816
5 5	195·3875	11·1949	110·424	8·0 ·50	92·	·2989	241·6
6 4	206·5824	·4993	3·471	1 ·49	1016·2989	·5763	4·376
7 3	18·0817	·8031	6·512	2 8	40·8752	·8531	7·147
8 2	29·8848	12·1063	9·547	3 7	65·7283	25·1293	9·912
9 1	41·9911	·4089	122·576	4 6	90·8576	·4049	252·671
4·0 ·90	54·4	·7109	5·6	5 5	1116·2625	·6799	5·424
1 ·89	67·1109	13·0123	8·616	6 4	41·9424	·9543	8·171
2 8	80·1232	·3131	131·627	7 3	67·8967	26·2281	260·912
3 7	93·4363	·6133	4·632	8 2	94·1248	·5013	3·647
4 6	307·0496	·9129	7·631	9 1	1220·6261	·7739	6·376
5 5	20·9625	14·2119	140·624	9·0 ·40	47·4	27·0459	9·1

TABLE D—*continued.*

$T^2 \times g^1$.	Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.	$T^2 \times g^1$.	Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.
9·1² × 15·39	1274·4459	27·3173	271·816	16·1² × 14·69	3807·7949	44·8243	447·096
2 8	1301·7632	·5881	4·527	2 8	52·6192	45·0531	9·387
3 7	29·3513	·8583	7·232	3 7	97·6723	·2813	451·672
4 6	57·2096	28·1279	9·931	4 6	3942·9536	·5089	3·951
5 5	85·3375	·3969	282·624	5 5	88·4625	·7359	6·224
6 4	1413·7344	·6653	5·311	6 4	4034·7984	·9623	8·491
7 3	42·3997	·9331	7·992	7 3	80·1607	46·1881	460·752
8 2	71·3328	29·2003	290·667	8 2	4126·3488	·4133	3·007
9 1	1500·5331	·4669	3·336	9 1	72·7621	·6379	5·256
10·0 ·30	30·	·7329	6·	17·0 ·60	4219·4	·8619	7·5
1 ·29	59·7329	·9983	8·656	1 ·59	66·2619	47·0853	9·736
2 8	89·7312	30·2631	301·307	2 8	4313·3472	·3081	471·967
3 7	1619·9943	·5273	3·952	3 7	60·6553	·5303	4·192
4 6	50·5216	·7907	6·591	4 6	4408·1836	·7519	6·411
5 5	81·3125	31·0539	9·224	5 5	55·9375	·9729	8·624
6 4	1712·3664	·3163	311·851	6 4	4503·9104	48·1933	480·831
7 3	43·6827	·5781	4·472	7 3	52·1037	·4131	3·032
8 2	75·2608	·8393	7·027	8 2	4600·5168	·6323	5·227
9 1	1807·1001	32·0999	9·696	9 1	49·1491	·8509	7·416
11·0 ·20	39·2	·3599	322·3	18·0 ·50	98·	49·0689	9·6
1 19	71·5599	·6193	4·896	1 ·49	4747·0689	·2863	491·776
2 8	1904·1792	·8781	7·487	2 8	96·3552	·5031	3·947
3 7	37·0573	33·1363	330·072	3 7	4845·8583	·7193	6·112
4 6	70·1936	·3939	2·051	4 6	95·5776	·9349	8·271
5 5	2003·5875	·6509	5·224	5 5	4945·5125	50·1499	500·424
6 4	37·2384	·9073	7·791	6 4	95·6624	·3643	2·571
7 3	71·1457	34·1631	340·352	7 3	5045·0267	·5781	4·712
8 2	2105·3088	·4183	2·907	8 2	095·6018	·7913	6·847
9 1	39·7271	·6729	5·456	9 1	146·3961	51·0039	8·976
12·0 ·10	74·4	·9269	8·	19·0 ·40	198·4	·2159	511·1
1 ·09	2209·3269	35·1803	350·536	1 ·39	249·6159	·4273	3·216
2 8	44·5072	·4331	3·067	2 8	301·0432	·6381	5·327
3 7	79·9403	·6853	5·592	3 7	352·6813	·8483	7·432
4 6	2315·6256	·9369	8·111	4 6	404·5296	52·0579	9·531
5 5	51·5625	36·1879	360·624	5 5	456·5875	·2669	521·624
6 4	87·7504	·4383	3·131	6 4	508·8544	·4753	3·711
7 3	2424·1887	·6881	5·632	7 3	561·3297	·6831	5·792
8 2	60·8768	·9373	8·127	8 2	614·0128	·8903	7·867
9 1	97·8141	37·1859	370·616	9 1	666·9031	53·0969	9·936
13·0 ·00	2535·	·4339	3·1	20·0 ·30	720·	·3029	532·
1² × 14·99	72·4339	·6813	5·576	1 ·29	773·3029	·5083	4·056
2 8	2610·1152	·9281	8·047	2 8	826·8112	·7131	6·107
3 7	48·0433	38·1743	380·512	3 7	880·5243	·9173	8·152
4 6	86·2176	·4199	2·971	4 6	934·4416	54·1209	540·191
5 5	2724·6375	·6649	5·424	5 5	988·5625	·3239	2·224
6 4	63·3024	·9093	7·871	6 4	6042·8864	·5263	4·251
7 3	2802·2177	39·1531	390·312	7 3	097·4127	·7281	6·272
8 2	41·3648	·3963	2·747	8 2	162·1408	·9293	8·287
9 1	80·7611	·6389	5·176	9 1	207·0701	55·1299	550·296
14·0 ·90	2920·4	·8809	7·6	21·0 ·20	262·2	·3399	2·3
1 ·89	60·2809	40·1223	400·016	1 ·19	317·5299	·5293	4·296
2 8	3000·4032	·3631	2·427	2 8	373·0592	·7281	6·287
3 7	40·7663	·6033	4·832	3 7	428·7873	·9263	8·272
4 6	81·3696	·7429	7·231	4 6	484·7136	56·1239	560·251
5 5	3122·2125	·9819	9·624	5 5	540·8375	·3209	2·224
6 4	63·2944	41·3203	412·011	6 4	597·1584	·5173	4·191
7 3	3204·6147	·5581	4·392	7 3	653·6757	·7131	6·152
8 2	46·1728	·7953	6·767	8 2	710·3888	·9083	8·107
9 1	87·9681	42·0319	9·136	9 1	767·2971	57·1029	570·056
15·0 ·80	3330·	·2679	421·5	22·0 ·10	824·4	·2969	2·
1 ·79	72·2679	·5003	3·856	1 ·09	881·6969	·4903	3·936
2 8	3414·7712	·7381	6·207	2 8	939·1872	·6831	5·867
3 7	57·5093	·9723	8·552	3 7	996·8703	·8753	7·792
4 6	3500·4816	43·2059	430·891	4 6	7054·7456	58·0669	9·711
5 5	43·6875	·4389	3·224	5 5	112·8125	·2579	581·624
6 4	87·1264	·6713	5·551	6 4	171·0704	·4483	3·531
7 3	3630·7977	·9031	7·872	7 3	229·5187	·6381	5·432
8 2	74·7008	44·1343	440·187	8 2	288·1568	·8273	7·327
9 1	3718·8351	·8649	2·496	9 1	346·9841	59·0159	9·216
16·0 ·70	63·2	·5949	4·8	23·0 ·00	406·	·2039	591·1

TABLE D—*continued.*

$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.	$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.
23·1² ×	13·99	7465·2039	59·3913	592·976	30·1² ×	13·29	12040·8729	71·0183	709·456
2	8	524·5952	·5781	4·847	2	8	111·8912	·1631	710·907
3	7	584·1733	·7643	6·712	3	7	183·0543	·3073	2·352
4	6	643·9376	·9499	8·571	4	6	254·3616	·4509	3·791
5	5	703·8875	60·1349	600·424	5	5	325·8125	·5939	5·224
6	4	764·0224	·3193	2·271	6	4	397·4064	·7363	6·651
7	3	824·3417	·5031	4·112	7	3	469·1427	·8781	8·072
8	2	884·8448	·6863	5·947	8	2	541·0208	72·0193	9·487
9	1	945·5311	·8689	7·776	9	1	613·0401	·1599	720·896
24·0	·90	8006·4	61·0509	9·6	31·0	·20	685·2	·2999	2·3
1	·89	067·4509	·2323	611·416	1	·19	757·4999	·4393	3·696
2	8	128·6832	·4131	3·227	2	8	829·9392	·5781	5·087
3	7	190·0963	·5933	5·032	3	7	902·5173	·7163	6·472
4	6	251·6896	·7729	6·831	4	6	975·2336	·8539	7·851
5	5	313·4625	·9519	8·624	5	5	13048·0875	·9909	9·224
6	4	375·4144	62·1302	620·411	6	4	121·0784	73·1273	730·591
7	3	437·5447	·3081	2·192	7	3	194·2057	·2631	1·952
8	2	499·8528	·4853	3·967	8	2	267·4688	·3983	3·307
9	1	562·3381	·6619	5·736	9	1	340·8671	·5329	4·656
25·0	·80	625·	·8379	7·5	32·0	·10	414·4	·6669	6·
1	·79	687·8379	63·0133	9·256	1	·09	488·0669	·8003	7·336
2	8	750·8512	·1881	631·007	2	8	561·8672	·9331	8·667
3	7	814·0393	·3623	2·752	3	7	635·8003	74·0653	9·992
4	6	877·4016	·5359	4·491	4	6	709·8656	·1969	741·311
5	5	940·9375	·7089	6·224	5	5	784·0625	·3279	2·624
6	4	9004·6464	·8813	7·951	6	4	858·3904	·4583	3·931
7	3	068·5277	64·0531	9·672	7	3	932·8487	·5881	5·232
8	2	132·5808	·2243	641·387	8	2	14007·4368	·7173	6·527
9	1	196·8061	·3949	3·096	9	1	082·1541	·8459	7·816
26·0	·70	261·2	·5649	4·8	33·0	·00	157·	·9739	9·1
1	·69	325·7649	·7343	6·496	1² ×	12·99	231·9739	75·1013	750·376
2	8	390·4992	·9031	8·187	2	8	307·0752	·2281	1·647
3	7	455·4023	65·0713	9·872	3	7	382·3033	·3543	2·912
4	6	520·4736	·2389	651·551	4	6	457·6576	·4799	4·171
5	5	585·7125	·4059	3·224	5	5	533·1375	·6049	5·424
6	4	651·1184	·5723	4·891	6	4	608·7424	·7293	6·671
7	3	716·6907	·7381	6·552	7	3	684·4717	·8581	7·912
8	2	782·4288	·9033	8·207	8	2	760·3248	·9763	9·147
9	1	848·3321	66·0679	9·856	9	1	836·3011	76·0989	760·376
27·0	·60	914·4	·2319	661·5	34·0	·90	912·4	·2209	1·6
1	·59	980·6319	·3953	3·136	1	·89	980·6209	·3423	2·816
2	8	10047·0272	·5581	4·767	2	8	15064·9632	·4631	4·027
3	7	113·5853	·7203	6·392	3	7	141·4263	·5833	5·232
4	6	180·3056	·8819	8·011	4	6	218·0096	·7029	6·431
5	5	247·1875	67·0429	9·624	5	5	294·7125	·8219	7·624
6	4	314·2304	·2033	671·231	6	4	371·5344	·9403	8·811
7	3	381·4337	·3631	2·832	7	3	448·4747	77·0581	9·992
8	2	448·7968	·5223	4·427	8	2	525·5328	·1753	771·167
9	1	516·3191	·6809	6·016	9	1	602·7081	·2919	2·336
28·0	·50	584·	·8389	7·6	35·0	·80	680·	·4079	3·5
1	·49	651·8389	·9963	9·176	1	·79	757·4079	·5233	4·656
2	8	719·8352	68·1531	680·747	2	8	834·9312	·6381	5·807
3	7	787·9883	·3093	2·312	3	7	912·5693	·7523	6·952
4	6	856·2976	·4649	3·871	4	6	990·3216	·8659	8·091
5	5	924·7625	·6199	5·424	5	5	16068·1875	·9789	9·224
6	4	993·3824	·7743	6·971	6	4	146·1664	78·0913	780·351
7	3	11062·1567	·9281	8·512	7	3	224·2577	·2031	1·472
8	2	131·0848	69·0813	690·047	8	2	302·4608	·3143	2·587
9	1	200·1661	·2339	1·576	9	1	380·7751	·4249	3·696
29·0	·40	269·4	·3859	3·1	36·0	·70	459·2	·5349	4·8
1	·39	338·7859	·5373	4·616	1	·69	537·7349	·6443	5·896
2	8	408·3232	·6881	6·127	2	8	616·3792	·7531	6·987
3	7	478·0113	·8383	7·632	3	7	695·1323	·8613	8·072
4	6	547·8496	·9879	9·131	4	6	773·9936	·9689	9·151
5	5	617·8375	70·1369	700·624	5	5	852·9625	79·0759	790·224
6	4	687·9744	·2853	2·111	6	4	932·0384	·1823	1·291
7	3	758·2597	·4331	3·592	7	3	17011·2207	·2881	2·352
8	2	828·6928	·5803	5·067	8	2	090·5088	·3933	3·407
9	1	899·2731	·7269	6·536	9	1	169·9021	·4979	4·456
30·0	·30	970·	·8729	8·	37·0	·60	249·4	·6019	5·5

TABLE D—*continued.*

$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.	$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.
37·1² ×	12·59	17329·0019	79·7053	796·536	44·1² ×	11·89	23123·7909	85·4523	854·216
2	8	408·7072	·8081	7·567	2	8	209·2432	·5131	4·827
3	7	488·5153	·9103	8·592	3	7	294·7563	·5733	5·432
4	6	568·4256	80·0119	9·611	4	6	380·3296	·6329	6·031
5	5	648·4735	·1129	800·624	5	5	465·9625	·6919	6·624
6	4	728·5504	·2133	1·631	6	4	551·6544	·7503	7·211
7	3	808·7637	·3131	2·632	7	3	637·4047	·8081	7·792
8	2	889·0768	·4123	3·626	8	2	723·2128	·8653	8·367
9	1	969·4891	·5109	4·616	9	1	809·0781	·9219	8·936
38·0	·50	18050·	·6089	5·6	45·0	·80	895·	·9779	9·5
1	·49	130·6089	·7063	6·576	1	·79	980·9779	86·0333	860·056
2	8	211·3152	·8031	7·547	2	8	24067·0112	·0881	0·607
3	7	292·1183	·8993	8·512	3	7	153·0993	·1423	1·152
4	6	373·0176	·9949	9·471	4	6	239·2416	·1959	1·691
5	5	454·0125	81·0899	810·424	5	5	325·4375	·2489	2·224
6	4	525·1024	·1843	1·371	6	4	411·6864	·3013	2·751
7	3	616·2867	·2781	2·312	7	3	497·9877	·3531	3·272
8	2	697·5648	·3713	3·247	8	2	584·3408	·4043	3·787
9	1	778·9361	·4639	4·176	9	1	670·7451	·4549	4·296
39·0	·40	860·4	·5559	5·1	46·0	·70	757·2	·5049	4·8
1	·39	941·9559	·6473	6·016	1	·69	843·7049	·5543	5·296
2	8	19023·6032	·7381	6·927	2	8	930·2592	·6031	5·787
3	7	105·3413	·8283	7·832	3	7	25016·8623	·6513	6·272
4	6	187·1696	·9179	8·731	4	6	103·5136	·6989	6·751
5	5	269·0875	82·0069	9·624	5	5	190·2125	·7459	7·224
6	4	351·0944	·0953	820·511	6	4	276·9584	·7923	7·691
7	3	433·1897	·1831	1·392	7	3	363·7507	·8381	8·152
8	2	515·3728	·2703	2·267	8	2	450·5888	·8833	8·607
9	1	597·6431	·3569	3·136	9	1	537·4721	·9279	9·056
40·0	·30	680·	·4429	4·	47·0	·60	624·4	·9719	9·5
1	·29	762·4429	·5283	4·856	1	·59	711·3719	87·0153	9·936
2	8	844·9712	·6131	5·707	2	8	798·3872	·0581	870·367
3	7	927·5843	·6973	6·552	3	7	885·4453	·1003	0·792
4	6	20010·2816	·7809	7·391	4	6	972·5456	·1419	1·211
5	5	093·0625	·8639	8·224	5	5	26059·6875	·1829	1·624
6	4	175·9264	·9463	9·051	6	4	146·8704	·2233	2·031
7	3	258·8727	83·0281	9·872	7	3	234·0937	·2631	2·432
8	2	341·9008	·1093	830·687	8	2	321·3568	·3023	2·827
9	1	425·8101	·1899	1·496	9	1	408·6591	·3409	3·216
41·0	·20	508·2	·2699	2·3	48·0	·50	496·	·3789	3·6
1	·19	591·4699	·3493	3·096	1	·49	583·3789	·4163	3·976
2	8	674·8192	·4281	3·887	2	8	670·7952	·4531	4·347
3	7	758·2473	·5063	4·672	3	7	758·2483	·4893	4·712
4	6	841·7536	·5839	5·451	4	6	845·7376	·5249	5·011
5	5	925·3375	·6609	6·224	5	5	933·2625	·5599	5·424
6	4	21008·9984	·7373	6·991	6	4	27020·8224	·5943	5·771
7	3	092·7357	·8131	7·752	7	3	108·4167	·6281	6·112
8	2	176·5488	·8883	8·507	8	2	196·0448	·6613	6·447
9	1	260·4371	·9629	9·256	9	1	283·7061	·6939	6·776
42·0	·10	344·4	84·0369	840·	49·0	·40	371·4	·7259	7·1
1	·09	428·4369	·1103	0·736	1	·39	459·1259	·7573	7·416
2	8	512·5472	·1831	1·467	2	8	546·8832	·7881	7·727
3	7	596·7303	·2553	2·192	3	7	634·6713	·8183	8·032
4	6	680·9856	·3629	2·911	4	6	722·4896	·8479	8·331
5	5	765·3125	·3979	3·624	5	5	810·3375	·8769	8·624
6	4	849·7104	·4683	4·331	6	4	898·2144	·9053	8·911
7	3	934·1789	·5381	5·032	7	3	986·1197	·9331	9·192
8	2	22018·7168	·6073	5·727	8	2	28074·0528	·9603	9·467
9	1	103·3241	·6759	6·416	9	1	162·0131	·9869	9·736
43·0	·00	188·		7·1	50·0	·30	250·	88·0129	880·
·1² ×	11·99	272·7439	·8113	7·776	1	·29	338·0129	·0383	0·256
2	8	357·5552	·8781	8·447	2	8	426·0512	·0631	0·507
3	7	442·4833	·9443	9·112	3	7	514·1143	·0873	0·752
4	6	527·3776	85·0099	9·771	4	6	602·2016	·1109	0·991
5	5	612·3875	·0749	850·424	5	5	690·3125	·1339	1·224
6	4	697·4624	·1393	1·011	6	4	778·4464	·1563	1·451
7	3	782·6017	·2031	1·712	7	3	866·6027	·1781	1·672
8	2	867·8048	·2663	2·347	8	2	954·7808	·1993	1·887
9	1	953·0711	·3289	2·976	9	1	29042·9801	·2199	2·096
44·0	·90	23038·4	·3909	3·6	51·0	·20	131·2	·2399	2·3

TABLE D—*continued.*

$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.	$T^2 \times g^1$.		Fall in the whole time.	Fall in each tenth of a second.	Velocity acquired.
51·1² ×	11·19	29219·4399	88·2593	882·496	52·8² ×	11·02	30721·9968	88·4973	884·927
2	8	307·6992	·2781	2·687	9	1	810·4941	·5059	5·016
3	7	395·9773	·2963	2·872	53·0	·00	899·	·5139	5·1
4	6	484·2736	·3139	3·051	1² ×	10·99	987·5139	·5213	5·176
5	5	572·5875	·3309	3·224	2	8	31076·0352	·5281	5·247
6	4	660·9184	·3473	3·391	3	7	164·5633	·5343	5·312
7	3	749·2657	·3631	3·552	4	6	253·0976	·5399	5·371
8	2	837·6288	·3783	3·707	5	5	341·6375	·5449	5·424
9	1	926·0071	·2929	3·856	6	4	430·1824	·5493	5·471
52·0	·10	30014·4	·4069	4·	7	3	518·7317	·5531	5·512
1	·09	102·8069	·4203	4·136	8	2	607·2848	·5563	5·547
2	8	191·2272	·4331	4·267	9	1	695·8411	·5589	5·576
3	7	279·6603	·4453	4·392	54·0	·90	784·4	·5609	5·6
4	6	368·1056	·4569	4·511	1	·89	872·9609	·5623	5·616
5	5	456·5625	·4679	4·624	2	8	961·5232	·5631	5·627
6	4	545·0304	·4783	4·731	3	7	32050·0863	·5633	5·632
7	3	633·5087	·4881	4·832					

Gunnery is the science which enables us to determine the path of a shot through the air; it is the application of the laws of motion to the flight of military projectiles of every description; but at present Gen. Anstruther confines his attention to the flight of spherical bodies projected from smooth-bored cylinders; when this is well understood, it will be time enough to inquire what would result from using elongated shot or rifled barrels.

"If a body be projected (vertically) upwards, with the velocity it acquired in any time by descending freely, it will lose all its (upward) velocity in an equal time, and will (if not acted upon by gravity) ascend just to the same height from whence it fell, and will describe equal spaces in equal times, both in rising and falling, but in an inverse order; and it will have equal velocities at any one and the same point of the line described, both in ascending and descending."—Rutherford's Mathematics, 1841, p. 842.

These words, excepting those in parentheses, are Hutton's; those in parentheses are absolutely required to make the passage truth. For we know, from the second law of motion, that the force of gravity, acting upon this body projected vertically upwards, produces the exact same vertical descent, in each and every second of the entire time of flight, as it had produced in equal time when the same body was suffered to fall freely from a state of rest.

Therefore, in the fall by gravity, as modified by the resistance of the atmosphere, we have at once the inverted reading of the ascent of any ball fired vertically upwards, and we must compound the two simultaneous motions, the ascent and the descent, to obtain the actual motion of the shot.

Most unfortunately, observes Anstruther, we possess no reliable record of a time of flight exceeding 30 seconds; this may justify our inferring the fall in 40 seconds, but no more. The fall in 40 seconds is ($40^2 \times 12 \cdot 3 =$) 19,680 ft., the velocity acquired by such fall is 824 ft. a second; we are to determine the path of a ball fired vertically upwards with initial velocity 824 ft. a second, which we show in the following tabulated form;—

Seconds of Time.	Simultaneous		Actual Height A – D.	Simultaneous		Seconds of Time.
	Ascent A.	Descent D.		Descent D.	Ascent A.	
20	13960	5720	8240	5720	13960	20
19	13417·8	5198·4	8219·4	6262·2	14481·6	21
18	12855·6	4698	8157·6	6824·4	14982	22
17	12274	4219·4	8054·6	7406	15460·6	23
16	11673·6	3763·2	7910·4	8006·4	15916·8	24
15	11055	3330	7725	8625	16350	25
14	10418·8	2920·4	7498·4	9261·2	16759·6	26
13	9765·6	2535	7230·6	9914·4	17145	27
12	9096	2174·4	6922·6	10584	17505·6	28
11	8410·6	1839·2	6571·4	11269·4	17840·8	29
10	7710	1530	6180	11970	18150	30
9	6994·8	1247·4	5747·4	12685·2	18432·6	31
8	6263·6	992	5273·6	13414·4	18688·1	32
7	5523	764·4	4758·6	14157	18915·6	33
6	4767·6	565·2	4202·4	14912·4	19114·8	34
5	4000	395	3605	15680	19285	35
4	3220·8	254·4	2966·4	16459·2	19425·6	36
3	2430·6	144	2286·6	17249·4	19536	37
2	1630	64·4	1565·6	18050	19615·6	38
1	819·6	16·2	803·4	18860·4	19663·8	39
0	0	0	0	19680	19680	40

If no force were acting upon the projectile, it would, by the first law of motion, move on for ever, in a straight line, with the uniform velocity 824 ft. a second, so that if fired vertically upwards, it would, in 40 seconds, ascend to a height of (824 × 40 =) 32,960 ft. This motion is, however, in fact, modified by the action of two forces—the resistance of the atmosphere and gravity; the former of these reduces the magnitude of the ascent from 32,960 ft. to 19,680 ft. in 40 seconds' time; while the other force, gravity, in the same 40 seconds of time, is counteracting this reduced ascent, so that it brings the ball back to the spot from whence it rose, after an actual ascent of 8240 ft., which is equal to the initial velocity multiplied by one-fourth of the time of flight. Fired vertically upwards, the ball will return to the spot from whence it rose, having no range at all; but at any elevation less than 90° there will be a range which is very easily found, the ascent and the descent being both known, the difference between their squares is the square of the range.

We give Anstruther's Table, E, showing forty ranges at varying elevations for the velocity 824 ft. a second; this will enable us at once to draw the trajectory for any angle of elevation whatever. In any right-angled triangle, lay off upon the hypothenuse as many of the forty-one ascents given in our Table as the case may require, and let fall perpendiculars to show by their length the simultaneous descent—a line joining the lower ends of all these is the trajectory.

TABLE E.—INITIAL VELOCITY = 824 FEET A SECOND.

Time.	Ascent.	Descent.	Range.	Yards.	Log. Sin.	Elevation.		
						°	′	″
1	819·6	16·2	819·44	273·15	8·2959131	1	08	57
2	1630	64·4	1628·72	542·91	8·5966982	2	16	
3	2430·6	144	2426·33	808·77	8·7726489	3	23	49
4	3220·8	254·4	3210·74	1070·25	8·8975533	4	31	49·2
5	4000	395	3980·9	1327	8·9945371	5	40	
6	4767·6	565·2	4733·98	1570	9·0739024	6	48	
7	5523	764·4	5469·84	1823·28	9·1411557	7	57	
8	6265·6	992	6186·15	2062·05	9·1995490	9	06	34·8
9	6994·8	1247·4	6882·67	2294·22	9·2512304	10	16	
10	7710	1530	7556·67	2518·56	9·2976371	11	26	45
11	8410·6	1839·2	8206·64	2735·55	9·3398020	12	37	52·6
12	9096	2174·4	8832·28	2944·09	9·3784889	13	49	49·7
13	9765·6	2535	9430·8	3143·61	9·4142791	15	2	43·5
14	10418·8	2920·4	10001·13	3333·71	9·4476246	16	16	41·4
15	11055	3330	10541·54	3512·85	9·4788855	17	31	51
16	11673·6	3763·2	11050·4	3686·8	9·5083525	18	48	22·4
17	12274	4219·4	11525·95	3841 95	9·5363675	20	06	41·5
18	12855·6	4698	11966·42	3988·81	9·5628206	21	26	05·6
19	13417·8	5198·4	12369·88	4123·29	9·5881916	22	47	40·5
20	13960	5720	12734·33	4244·78	9 6125106	24	11	19
21	14481·6	6262·2	13057·36	4352·45	9·6359104	25	37	17·5
22	14982	6824·4	13337·46	4445·82	9·6584947	27	05	51·1
23	15460·6	7406	13570·98	4523·66	9·6803578	28	37	18
24	15916·8	8006·4	13756·52	4585·51	9·7015844	30	12	
25	16350	8625	13890	4630	9·7222413	31	50	17·6
26	16759·6	9261·2	13968·33	4656·11	9·7424166	33	32	45·8
27	17145	9914·4	13987·7	4662·59	9·7621289	35	19	44·9
28	17505·6	10584	13943·63	4644·54	9·7814728	37	12	02
29	17840·8	11269·4	13830·93	4610·31	9·8004865	39	10	23
30	18150	11970	13648·37	4547·79	9·8192176	41	15	43
31	18432·6	12685·2	13373·35	4457·78	9·8377107	43	29	14·3
31·57	18599·23	13150·21	13150·21	4383·4	9·8494377	44	59	37·55
32	18688	13414·4	13011·34	4337·11	9·8560087	45	52	
33	18915·6	14157	12545·09	4181·7	9·8741511	48	27	16·4
34	19114·8	14912·4	11958·09	3986·03	9·8918864	51	13	34·9
35	19285	15680	11227·15	3742·72	9·9101265	54	23	48·2
36	19425·6	16459·2	10317·39	3439·13	9·9280243	57	55	06·9
37	19536	17249·4	9171·33	3057·11	9·9459384	62	—	03·1
38	19615·6	18050	7679·15	2559·72	9·9638778	66	57	14·3
39	19663·8	18860·4	5562·51	1887·5	9·9818834	73	33	55·3
40	19680	19680	0·0	0·0	10·0000000	90		

The following example illustrates the nature and use of Table D;—Suppose that at elevation 4° 31′ 49·2″ (log. sin. 8·8975533) we find that the range was 3210·74 ft., then the vertical descent must be 254·4 ft., which indicates a time of flight of 4 seconds exactly. The oblique ascent of this ball we find to be 3220·8 ft., this we divide by the time, 4 seconds, the quotient, 805·2 ft. a second, is the mean velocity of the ascent. A reference to our Table shows that a velocity of 805·6 ft. is generated by gravity in 38 seconds of time; this we accept as sufficiently near. Evidently, the mean velocity of the oblique ascent will be almost the exact measure of the velocity with which the ball will be moving at the expiration of half the time of flight, that is to say, two seconds of time after quitting the muzzle, therefore (38 + 2 =) 40 seconds is the time in which gravity would generate the velocity which at elevation 4° 31′ 49·2″ ranged 3210·74 ft.

Our Table gives the graduation of this oblique ascent in the forty lines preceding 4 seconds,

but this minuteness is not required; the following Table shows the trajectory for 4 seconds, calculated to half seconds of time—velocity 824 ft. a second.

Time.	Ascent.		Horizontal (Abscisses).	Vertical Descent.	Ordinates.
	Oblique.	Vertical.			
0·5	410·9125	32·456576	409·62868	4·0625	28·394076
1	819·6	64·737407	817·03931	16·2	48·537407
1·5	1225·7875	96·836569	1222·15713	36·3375	60·499069
2	1630	128·748148	1624·90737	64·4	64·346148
2·5	2031·5625	160·466210	2025·21526	100·3125	60·153710
3	2430·6	191·984831	2423·00604	144	47·984831
3·5	2827·0375	223·298087	2818·20494	195·3875	27·910587
4	3220·8	254·4	3210·7372	254·4	0·0

Table E shows that at elevation 15°, the range for initial velocity, 824 ft. a second, is 3143·6 yds., which is not very far short of the ranges obtained by the 32-pounder guns at the well-known practice at Deal in 1839. We deduce the following, among other, conclusions from our Table;—First, the angle of ascent and the angle of descent are almost exactly the same when a spherical ball is fired with the usual initial velocity at service elevation. Secondly, the height to which any ball, fired vertically upwards with velocity V, would ascend in any time T, is equal to ¼ T × V exactly. Third, the course of every round projectile is nearly level from the end of the first quarter of the time of flight to the end of the third quarter of it; and, lastly, that we can in all cases of real practice determine every point of any real importance from the facts patent to all.

To show this by an example, we must employ a French record of experiments; all English records give us the angle of inclination, to use Lefroy's phrase; what we want is the angle of departure, and the French give it. At Gavre, in 1830, some very carefully-conducted experiments were recorded; we take the largest gun, the "canon de 30 long"; the following are the results, in feet, with our own calculation of the time and the velocity; the charge is 4k·9 or 0·324 of the weight of the shot.

Real Elevation.			Feet Range.	Oblique Ascent.	Vertical Ascent.	Dip of the Ground.	Fall by Gravity.	Time of Flight.	Mean Velocity.
°	′	″							
0	11	13	1322·02	1322·3	4·314	13·1029	17·417	1·037	1275
1	33	22	2792·05	2793·1	75·849	17·3	93·15	2·4084	1159·7
5	4	28	5626·75	5648·8	499·644	19·886	519·53	5·75	980·66
10	27	37	8517·23	8661·2	1572·467	11·1077	1585·575	10·18	850·8

The French artillerists, reasoning on Hutton's principles, find that the initial velocity was 425 mètres, or 1394·38 ft. a second. Obviously it cannot be more than 1280 or 1290, the mean velocity in a time of only 1·037 second being 1275 ft. a second. The oblique ascent is as follows;—

Oblique Ascent.	Time.	Current Velocity.
1322·3	1·037	1275
1470·8	1·3714	1072·5
2855·7	3·3416	854·5
3012·4	4·43	850·8
8661·2	10·18	

Gen. Anstruther gives a Table, Table F, showing the fall by gravity in five seconds of time, calculated to the hundredth parts of seconds of time; this will enable us to find the time of flight for all ordinary musketry or field artillery. The range multiplied by the tangent of the elevation is the vertical ascent and descent; its length indicates the time.

TABLE F.

Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.
0·01	0·0016	0·09	0·1320	0·17	0·4706	0·25	1·0172	0·33	1·7715
2	·0065	0·1	·1629	8	·5275	6	·1001	4	·8803
3	·0147	1	·1971	9	·5877	7	·1863	5	·9925
4	·0261	2	·2345	0·2	·6512	8	·2757	6	2·1078
5	·0107	3	·2753	1	·7179	9	·3684	7	·2264
6	·0587	4	·3192	2	·7879	0·3	·4643	8	·3482
7	·0798	5	·3664	3	·8619	1	·5635	9	·4783
8	·1042	6	·4169	4	·9375	2	·6658	0·4	·6016

TABLE F—*continued.*

Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.
0·41	2·7331	1·11	19·9465	1·81	52·8075	2·51	101·1103	3·21	164·6492
2	·8679	2	20·3062	2	53·3893	2	1·9112	2	5·6663
3	3·0059	3	·6692	3	·9742	3	2·7152	3	6·6864
4	·1472	4	21·0353	4	54·5623	4	3·5224	4	7·7097
5	·2916	5	·4047	5	55·1536	5	4·3326	5	8·7359
6	·4393	6	·7772	6	·7480	6	5·1460	6	9·7653
7	·5903	7	22·1529	7	56·3455	7	5·9624	7	170·7977
8	·7445	8	·5318	8	·9463	8	6·7820	8	1·8332
9	·9019	9	·9139	9	57·5501	9	7·6046	9	2·8717
0·5	4·0625	1·2	23·2992	1·9	58·1571	2·6	8·4304	3·3	3·9133
1	·2264	1	·6877	1	·7672	1	9·2593	1	4·9580
2	·3935	2	24·0793	2	59·3805	2	110·0912	2	6·0057
3	·5638	3	·4742	3	·9970	3	0·9263	3	7·0565
4	·7373	4	·8722	4	60·6165	4	1·7645	4	8·1103
5	·9141	5	25·2734	5	61·2393	5	2·6058	5	9·1672
6	5·0941	6	·6778	6	·8651	6	3·4502	6	180·2272
7	·2774	7	26·0854	7	62·4941	7	4·2977	7	1·2902
8	·4638	8	·4962	8	63·1263	8	5·1482	8	2·3563
9	·6535	9	·9101	9	·7616	9	6·0019	9	3·4254
0·6	·8464	1·3	27·3273	2·	64·4	2·7	6·8587	3·4	4·4976
1	6·0425	1	·7476	1	65·0416	1	7·7186	1	5·5728
2	·2419	2	28·1711	2	·6863	2	8·5816	2	6·6512
3	·4445	3	·5978	3	66·3341	3	9·4476	3	7·7325
4	·6503	4	29·0277	4	·9851	4	120·3168	4	8·8169
5	·8593	5	·4607	5	67·6392	5	1·1891	5	9·9044
6	7·0715	6	·8969	6	68·2965	6	2·0644	6	190·9949
7	·2870	7	30·3363	7	·9569	7	2·9429	7	2·0885
8	·5057	8	·7789	8	69·6204	8	3·8244	8	3·1851
9	·7276	9	31·2247	9	70·2871	9	4·7091	9	4·2848
0·7	·9527	1·4	·6736	2·1	·9569	2·8	5·5968	3·5	5·3875
1	8·1810	1	32·1257	1	71·6298	1	6·4876	1	6·4933
2	·4126	2	·5810	2	72·3059	2	7·3815	2	7·6021
3	·6474	3	33·0394	3	·9851	3	8·2786	3	8·7140
4	·8854	4	·5011	4	73·6674	4	9·1786	4	9·8289
5	9·1866	5	·9659	5	74·2529	5	130·0818	5	200·9469
6	·3710	6	34·4339	6	75·0415	6	0·9881	6	2·0679
7	·6186	7	·9050	7	·7332	7	1·8975	7	3·1919
8	·8695	8	35·3793	8	76·4281	8	2·8099	8	4·3190
9	10·1235	9	·8568	9	77·1261	9	3·7251	9	5·4492
0·8	·3808	1·5	36·3375	2·2	·8272	2·9	4·6441	3·6	6·5824
1	·6413	1	·8213	1	78·5314	1	5·5658	1	7·7186
2	·9050	2	37·3083	2	79·2388	2	6·4906	2	8·8579
3	11·1719	3	·7985	3	·9493	3	7·4185	3	210·0003
4	·4420	4	38·2919	4	80·6629	4	8·3495	4	1·1456
5	·7153	5	·7884	5	81·3797	5	9·2825	5	2·2940
6	·9919	6	39·2880	6	82·0996	6	140·2206	6	3·4455
7	12·2716	7	·7909	7	·8226	7	1·1609	7	4·6000
8	·5546	8	40·2969	8	83·5487	8	2·1042	8	5·7575
9	·8407	9	·8061	9	84·2779	9	3·0505	9	6·9181
0·9	13·1301	1·6	41·3184	2·3	85·0103	3·	4·	3·7	8·0817
1	·4227	1	·8339	1	·7458	1	4·9525	1	9·2483
2	·7185	2	42·3526	2	86·4844	2	5·9082	2	220·4180
3	14·0174	3	·8744	3	87·2261	3	6·8669	3	1·5908
4	·3196	4	43·3994	4	·9710	4	7·8286	4	2·7665
5	·6250	5	·9275	5	88·7190	5	8·7935	5	3·9453
6	·9336	6	44·4588	6	89·4701	6	9·7614	6	5·1271
7	15·2454	7	·9933	7	90·2243	7	150·7234	7	6·3120
8	·5604	8	45·5310	8	·9816	8	1·7065	8	7·4999
9	·8786	9	46·0717	9	91·7420	9	2·6837	9	8·6908
1·	16·2	1·7	·6157	2·4	92·5056	3·1	3·6639	3·8	9·8848
1	·5246	1	47·1628	1	93·2723	1	4·6472	1	231·0818
2	·8524	2	·7131	2	94·0421	2	5·6336	2	2·2818
3	17·1834	3	48·2665	3	·8150	3	6·6230	3	3·4849
4	·5176	4	·8231	4	95·5910	4	7·6156	4	4·6910
5	·8550	5	49·3828	5	96·3701	5	8·6112	5	5·9001
6	18·1956	6	·9457	6	97·1524	6	9·6098	6	7·1122
7	·5394	7	50·5117	7	·9377	7	160·6116	7	8·3274
8	·8863	8	51·0809	8	98·7262	8	1·6161	8	9·5456
9	19·2365	9	·6533	9	99·5178	9	2·6243	9	240·7668
1·1	·5899	1·8	52·2288	2·5	100·3125	3·2	3·6352	3·9	1·9911

TABLE F—*continued.*

Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.	Time.	Fall.
3·91	243·2184	4·13	270·9830	4·35	300·2055	4·57	330·8795	4·79	362·9986
2	4·4487	4	2·2797	6	1·5683	8	2·3081	4·8	4·4928
3	5·6820	5	3·5794	7	2·9341	9	3·7398	1	5·9900
4	6·9184	6	4·8822	8	4·3080	4·6	5·1744	2	7·4901
5	8·1578	7	6·1879	9	5·6748	1	6·6120	3	8·9932
6	9·4002	8	7·4967	4·4	7·0496	2	8·0526	4	370·4993
7	250·6456	9	8·8084	1	8·4274	3	9·4962	5	2·0083
8	1·8940	4·2	280·1232	2	9·8082	4	340·9427	6	3·5204
9	3·1455	1	1·4410	3	311·1920	5	2·3923	7	5·0353
4·	4·4	2	2·7618	4	2·5788	6	3·8448	8	6·5533
1	5·6575	3	4·0856	5	3·9686	7	5·3003	9	8·0742
2	6·9180	4	5·4124	6	5·3614	8	6·7588	4·9	9·5981
3	8·1816	5	6·7422	7	6·7572	9	8·2203	1	381·1250
4	9·4482	6	8·0750	8	8·1560	4·7	9·6847	2	2·6548
5	260·7177	7	9·4108	9	9·5577	1	351·1521	3	4·1876
6	1·9903	8	290·7496	4·5	320·9625	2	2·6225	4	5·7233
7	3·2660	9	2·0915	1	2·3702	3	4·0959	5	7·2620
8	4·5446	4·3	3·4363	2	3·7810	4	5·5722	6	8·8037
9	5·8262	1	4·7841	3	5·1947	5	7·0516	7	390·3483
4·1	7·1109	2	6·1350	4	6·6114	6	8·5339	8	1·8959
1	8·3986	3	7·4888	5	8·0311	7	360·0191	9	3·4465
2	9·6893	4	8·8456	6	9·4538	8	1·5074	5·	5·

The quotation, p. 1732, showed one theory of Hutton's, which, with slight additions, is true; we shall now show another, which is totally wrong in every possible way.

Extract from Straith's Memoir of Artillery, pp. 81, 82.—"Dr. Gregory, in his lectures upon gunnery, observes on the difference between the times employed by a ball in ascending and descending vertically through the same space.

"If a 24-lb. iron ball were projected vertically upwards, with a velocity of 2000 ft. a second, it would ascend to the height of 6424 ft. before its upward motion was extinguished, and it would pass over that space in less than $9\frac{1}{4}$ seconds. (This is computed in Hutton's Mathematics, vol. iii.)

"It might, on a cursory view of the subject, be supposed that the circumstances of the descent would be analogous to those of the ascent, but in an inverted order; and so they would, in a non-resisting medium, but in the air the case is widely different.

"After the ball had descended 2700 ft., the resistance of the air would be equal to the weight of the ball, there would remain no further cause of acceleration, and the ball would descend uniformly with its *terminal* velocity (that is, the greatest velocity which a heavy body can acquire when falling in the air), which does not exceed 419 ft. a second.

"It would require, therefore, $\frac{6425 - 2700}{419}$ or 6 seconds, to descend the remaining 3724 ft., in addition to the time, about 10 seconds, which had been occupied in descending through the first 2700 ft.; so that, in this instance, the time of descent would be about double that of ascent. In all cases where the projectile velocity exceeds 300 or 400 ft., the time of descent will exceed that of ascent; and their difference is greater the more the initial velocity exceeds that limit."

Here we find, observes the General, an Addiscombe professor quoting from two Woolwich professors. We are therefore sure that, at one period of his life, each artillery officer in the service believed that the descent and ascent of a ball fired upwards were successive motions, not simultaneous.

But such belief could not last long; the first time the young officer attempted to draw the trajectory of any ball from given range and elevation, we know that he did assume the abscisses; he then multiplied each abscissa by the tangent of the elevation for the vertical ascent, from which he subtracted the vertical descent, or fall by gravity, and assigned the difference between the vertical ascent and vertical descent as the ordinates of the curve. He could not have drawn a trajectory at all on any other supposition.

In the Treatise on Artillery, by Colonel Boxer, we find, pp. 144, 145, sixteen descents subtracted from sixteen ascents, the sixteen differences being given in p. 146 as the ordinates of the curve. The trajectory which he gives would leave nothing to be desired, if he would adapt his mode of calculating the flight of a projectile to some actual fact, some range and elevation recorded as having actually been observed. Perhaps the following may serve as an example:—

At Shoeburyness, on the 24th of November, 1854, the 10-in. iron howitzer of 125 cwt. gave a range of 4440 yds., or 13,320 ft. at 42° elevation. Here we say 13,320 × secant 42° = 17923·827564 ft. the oblique ascent, 13,320 × tangent 42° = 11993·38128 ft. the vertical ascent, to which latter we add 15 ft., the height of the piece above the plane, and we have 12008·38128 ft. for the vertical descent, or fall by gravity. From this we determine the time of flight; we find it to be 30·0541 seconds for $30{\cdot}0541^2 \times 13{\cdot}29459 = 12008{\cdot}32415$ ft.

We divide the oblique ascent, 17923·827564 ft. by the time of flight, 30·0541 seconds, the

quotient is 596·38, the mean velocity of the oblique ascent. We look at our printed Table and find that a ball, falling freely from a state of rest, would acquire a velocity of 596,712 ft. in 23·3 seconds of time, therefore (23·3 + 15 =) 38·3 seconds is the approximation to the greater extreme of the series, or progression, which must form the graduation of the oblique ascent. But 38·3 seconds is not sufficient; we try 39 seconds, 39·5 seconds, and 39·55 seconds, which is sufficiently accurate.

For the fall in 39·55 seconds of time is

	$39{\cdot}55^2$	×	12·345	=	19310·0798625 ft.
and in 9·496 seconds	$9{\cdot}496^2$	×	15·3504	=	1384·2072152
the differences	30·054 seconds of time and				17925·8726473 ft.,

snow an excess of only 2·045 ft. over the oblique ascent. The velocity which the ball acquires by falling in 39·55 seconds, we find thus;—

	$39{\cdot}55^2$	×	12·345	=	19310·0798625 ft.
and in 39·54 seconds	$39{\cdot}54^2$	×	12·346	=	19301·8796136
the differences	0·01 second of time and				8·2002482 ft.,

show a velocity of 820 ft. per second.

The observed time of flight was 30 seconds; this is so nearly that which we assign, that we do not hesitate to say that it proves the truth of our theory, and utterly confutes that of Hutton.

We will now conclude this by showing the true initial velocity of an actual ascent of 6424 ft. at elevation 90°; it is very easily found by inverting the reasoning, p. 1732; we there found that an actual ascent of 8240 ft. resulted from velocity 824 ft. a second, and 40 seconds' time of flight, $\frac{VT}{4}$ = 8240 ft., we now say $\frac{VT}{4}$ = 6424 ft.; therefore V T = 25686 ft.

We try 33·8 seconds, 33·8 × 759·147 = 25659·1686, too little;
„ 33·9 „ 33·9 × 760·376 = 25776·7464, too much;
„ 33·83 „ 32·83 × 759·45581 = 25692·3900523 ft.,

with which we are satisfied. It cannot be necessary to repeat for 33·83 seconds the process shown in p. 1732 for 40 seconds, the initial velocity which Hutton gives as 2000 ft. a second should be 759·456 ft. a second.

General Anstruther invites the mathematicians of Cambridge, or of any other place, to confirm or negative the theory which assigns to a vertical flight of 10 seconds' duration an initial velocity of only 161 ft. a second, with an actual ascent of only 402·5 ft. *Vide* Cape's Mathematics, vol. ii., p. 216.

We say that the velocity of 10 seconds' time of flight is 296 ft. a second; one-fourth of this is 74 ft., then 74 × 10 = 740 ft. the actual ascent. We give the following Table showing the motion of a ball fired vertically upwards with initial velocity 296 ft. a second.

Time. Seconds.	Simultaneous		Actual Height.	Simultaneous		Time. Seconds.
	Ascent.	Descent.		Descent.	Ascent.	
5	1135	395	740	395	1135	5
4	964·8	254·4	710·4	565·2	1275·6	6
3	765·6	144	621·6	764·4	1386	7
2	538	64·4	473·6	992	1465·6	8
1	282·6	16·2	266·4	1247·4	1513·8	9
0	0	0	0	1530	1530	10

We say that at 45° elevation the range for 10 seconds' time of flight is 1530 ft., and that we can determine the initial velocity which would give that range by a very simple process; thus, we draw a square, each side representing 1530 ft., and we draw a diagonal measuring 1530 $\sqrt{2}$ ft., or 2163·75 ft. This diagonal will represent the oblique ascent in the time of flight, 10 seconds; we see at a glance that the mean velocity of the oblique ascent, 2163·75 ft. in 10 seconds, is 216·375 ft. a second, and our printed Table shows us that a fall of 7·1 seconds' duration would give a velocity of 216·336, almost exactly the mean velocity of the oblique ascent.

The time when the ball is moving with the mean velocity is sure to be very nearly the middle period of the time of flight, so that 5 seconds after, and 5 seconds before, 7·1 seconds, that is 12·1 and 2·1 seconds, will be the 10 seconds in our Table which give the graduations of the oblique ascent. More correctly it would be 12·2 seconds and 2·2 seconds, still more correctly 12·19 and 2·19 seconds. For the fall in 12·19 seconds of time is

	$12{\cdot}19^2 \times 15{\cdot}081$	=	2240·9777841 ft.
and in 2·19 seconds	$2{\cdot}19^2 \times 16{\cdot}081$	=	77·1260841
			2163·8517 ft., only 0·1 ft. too much.

The following are the abscisses and ordinates:—

Time.	Abscisses.	Differences. 1.	2.	3.	Ordinates.
1	240·467338	240·467338			240·267338
2	462·630509	222·203171	18·304167		398·231009
3	666·065249	203·434740	18·728431		522·065249
4	850·347294	184·282045	19·152695		595·947294
5	1015·052380	164·705086	19·576959		620·052380
6	1159·756243	144·703863	20·001223	0·424264	594·556243
7	1284·034619	124·278376	20·425487		519·634619
8	1387·463244	103·428625	20·849751		395·463244
9	1469·617850	82·154610	21·274015		222·217850
10	1530·074181	60·456331	21·698279		0·074181

We have given these abscisses and ordinates because that is the usual method of defining a curve; it is, however, a very bad method, not one figure being of any use at any other elevation. We give now the simultaneous ascent and descent for all elevations whatever.

Time.	Ascent.	Differences. 1.	2.	3.	Descent.
1	340·07217	340·07217			16·2
2	654·25834	314·18617	25·886		64·4
3	941·95851	287·70017	26·486		144
4	1202·57268	260·61417	27·086		254·4
5	1435·50085	232·92817	27·686		395
6	1640·14302	204·64217	28·286		565·2
7	1815·89919	175·75617	28·886	0·6	764·4
8	1962·16936	146·27017	29·486		992
9	2078·35353	116·18417	29·086		1247·6
10	2163·85170	85·49817	30·686		1530
11	2218·06387	54·21217	31·286		1839·2
12	2240·39004	22·32617	31·886		2174·4
12·19	2240·9777841				2240·9777841

The velocity acquired by falling in 12·19 seconds, we find thus;—

The fall in	12·2 seconds is	2244·5072 ft.
and in	12·19 „	2240·9777841
the differences being	0·01 second and	3·5124159 ft.,

show a velocity of 351·94, say 352 ft. a second.

That the range at 45° elevation is the exact measure of the fall by gravity in the time of flight is an acknowledged fact, yet we find the British gunner assigning different ranges to the same elevation and time of flight, 45° and 10 seconds.

We read that the range of the 10-in. land mortar was 534 yds. in 10 seconds, at 45° elevation; that of the 8-in., 560 yds.; so that the fall by gravity in 10 seconds was 1602 ft., or $t^2 \times 16·02$, as shown by the 10-in., or else 1680 ft., or $t^2 \times 16·8$, as shown by the 8-in. We say it is 1530 ft., or $t^2 \times 15·3$, as registered in the General's Table.

The President of the special committee on breech-loading rifles informed Anstruther that they do not consider the angles given in p. 23 of their report to be sufficiently accurate to furnish data for calculation, and that the initial velocity was really very much greater than he gave it.

It is to be regretted that the committee did not measure the true angle of departure in the manner adopted by the French at Gavre, in the years 1830 to 1840, who placed a screen 30 ft. in front of the muzzle, and measured the actual angle at which the shot did leave the gun.

The following were the oblique ascents and vertical descents of the largest gun, the "canon de 30 long," with the smallest charge, 2k·45.

Range 323 mètres, or 1059·73 ft., had

Oblique ascent	1059·8 ft.
Vertical descent	14·396 ft.
Therefore, a time of flight	0·94 second
And a mean velocity of ascent $\left(\frac{1059·8}{0·94} =\right)$ ..	1127·44 ft. a second.

Hence we know that at the expiration of 0·47 second, the velocity of this ball was 1127·44 ft. a second.

Again, the range being 712 mètres, or 2336 ft.,

The oblique ascent was	2337 ft.
The vertical descent	83·09 ft.
The time of flight	2·27 seconds
And a mean velocity of $\left(\frac{2337}{2\cdot 27}=\right)$	1029·5 ft. a second,

which is the velocity with which it was moving at 1·135 second after quitting the muzzle.

Comparing these two, at	0·47	second, the velocity was	1127·44 ft.
	1·135	„ „ „	1029·5
the differences are	0·665	„ and	97·94 ft.,

showing a rate of reduction of about 147 ft. in a second.

Supposing that the ball lost half of this velocity in the 0·47 second of its first flight, we add 74 ft. a second to the 1127·44 ft. of the first range, and give (1127·44 + 74 =) 1201 ft., say 1200 as the initial velocity of the ball. The French artillerists give it as 363 mètres, or 1190·966 ft., almost exactly what Anstruther's simple and easy calculation makes it.

Example, to show how Table E is applied.—On the supposition that the largest projectile will be most easily observed, and that the longest gun will give the most accurate results, we have selected our data from page 390 of the Ordnance Manual of the American Artillery. The bore of the piece is 15 in. in diameter, 165 in. in length.

The 15-in. columbiad, with 40 lbs. of powder, gave the following ranges, namely, at

9°	elevation,	2236 yds.,	or	6708 ft.	in	8·87	seconds' time.
10°	„	2425	„	7275	„	10	„
12°	„	2831	„	8493	„	12·07	„
15°	„	3078	„	9234	„	13·72	„
20°	„	3838	„	11514	„	17·82	„
25°	„	4528	„	13584	„	22·03	„
28°	„	4821	„	14463	„	24·18	„
30°	„	5018	„	15054	„	26·71	„

The angles here named are evidently the "angles of inclination," as shown by a spirit-level quadrant, and not the "angles of departure," those which the path of the shot makes with the horizon, as it clears the mouth of the piece; but the recorded time of flight enables us to find the vertical ascent, from which we can determine the angle of departure.

The vertical descent is the fall by gravity: our Table shows us that the fall in

8·87	seconds is	$8\cdot87^2 \times 15\cdot413$	= 1212·6470597 ft.
10	„	$10^2 \times 15\cdot3$	= 1530
12·07	„	$12\cdot07^2 \times 15\cdot093$	= 2198·8221857
13·72	„	$13\cdot72^2 \times 14\cdot928$	= 2810·0228352
17·82	„	$17\cdot82^2 \times 14\cdot518$	= 4610·2257432
22·03	„	$22\cdot03^2 \times 14\cdot297$	= 6841·5687273
24·18	„	$24\cdot18^2 \times 13\cdot882$	= 8116·4222568
26·71	„	$26\cdot71^2 \times 13\cdot629$	= 9723·2570589

There must have been a vertical ascent to enable the ball to have this vertical descent, we deduce the following eight triangles;—

Triangle.	Horizontal.	Vertical.	Oblique.	Triangle.	Horizontal.	Vertical.	Oblique.
I.	6708	1212·647	6818·7	V.	11514	4610·226	12402·6
II.	7275	1530	7434·15	VI.	13584	6841·569	15209·6
III.	8493	2198·822	8773	VII.	14463	8116·422	16584·7
IV.	9234	2810·023	9652·1	VIII.	15054	9723·257	17921·067

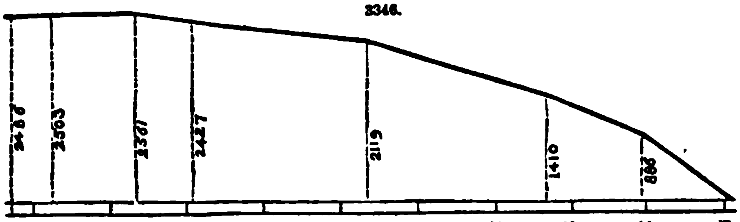
The last of these triangles we must examine carefully.

Knowing the three sides, we find the acute angles to be

32° 51′ 29·1″, log. sin. 9·7344479
and 57° 8′ 30·9″, „ 9·9242881;

we draw a triangle with these angles, and upon the hypothenuse we mark off all the eight oblique ascents given p. 1732, then from the points so found let fall successively the eight vertical

3346.



descents, join the lower ends of all the verticals; the curve so found is the trajectory required, as in Fig. 3346.

The calculated measurements of this curve are as follows;—

Abscissea.	Simultaneous		Ordinates.	Abscissea.	Simultaneous		Ordinates.
	Ascent.	Descent.			Ascent.	Descent.	
5726·145	3698·467	1212·647	2485·82	10416	6729·151	4610·226	2118·925
6224·812	4033·47	1530	2503·47	12777·322	8252·117	6841·569	1410·548
7369·47	4759·876	2193·842	2561·054	13934·637	9002·261	8116·422	885·839
8107·927	5236·841	2810·023	2426·818	15054	9723·257	9723·257	0·0

We must now determine the initial velocity of this ball, which we easily do, from the fact that it did ascend obliquely to a distance of 17921·067 ft. in 26·71 seconds, with a simultaneous vertical descent of 9723·257 ft. If this ball had been fired vertically upwards, then at 8197·81 ft. above the plane would have been its place at the expiration of 26·71 seconds of time.

We know that if a ball be projected upwards vertically, with the velocity which it acquired in any time by descending freely, it would, *if not acted upon by gravity*, lose all velocity in an equal time, and would, *if not acted upon by gravity*, ascend just to the same height from which it fell, and, in so doing, would describe equal spaces in equal times, in rising and falling, but in an inverse order; and it would have equal velocities at any one and the same point of the line described both in ascending and descending.

Our last paragraph, except the words in italics, is taken from Hutton, as quoted by Gregory and Rutherford. The words in italics added by the General are necessary.

We have therefore only to turn to our Table showing the fall of gravity as modified by the resistance of the atmosphere, to find the 267 lines which will give the graduation of the oblique ascent of this ball, and we have a very simple method of finding the place by finding the mean velocity; the space 17,921 ft. divided by the time 26·7 seconds, gives 671·2 ft. a second as the mean velocity of the ascent, which we see must be nearly the velocity with which the ball will be moving at the middle period of the time of flight, that is to say, at 13·35 seconds from clearing the mouth of the piece. Now velocity 671·2 is acquired by a fall of 27·6 seconds, we add 27·6 and 13·35, the sum 40·95 seconds is an approximation to the true time.

But 42·1 seconds is nearer the truth, as we shall show.

The fall in	42·1 seconds is $42{\cdot}1^2 \times 12{\cdot}09 =$	21428·4369 ft.
and in	15·4 „ $15{\cdot}4^2 \times 14{\cdot}76 =$	3500·4816
the differences being	26·7 seconds of time and	17927·9553 are only 7 ft. too much.

We shall, however, be satisfied with 42 seconds as near enough.

The fall in	42 seconds is $42^2 \times 12{\cdot}1 =$	21344·4
and in	15·3 „ $15{\cdot}3^2 \times 14{\cdot}77 =$	3457·5093
		17886·8907 ft.,
	which is too short by	34
	as it ought to be	17921·

Our Table shows that the velocity acquired by a fall of 42 seconds is 840 ft. a second; this is therefore the initial velocity, which at the elevation (true) 32° 51′ 29·1″ gave a range of 5018 yds.

Now let us suppose that the ball is fired vertically upwards with initial velocity 840 ft. a second, it is very evident that, if no force were acting upon the projectile, it would, by the first law of motion, move on for ever in the line of direction given to it, and would ascend to a height of (42 × 840 =) T V = 35280 ft. in 42 seconds; the resistance of the atmosphere reduces this to 21344·4 ft.; and the force of gravity causes a simultaneous descent of 21344·4 ft., so that the ball returns to the ground, having in exactly half the time of flight attained a vertical height of exactly one-fourth part of T V, that is to say, 8820 ft., the fourth part of 35,280 ft.

Why the actual ascent, at 90° elevation, should always be exactly the fourth part of T V, we need not inquire; such is the fact. We shall show another example of it. A ball fired vertically with initial velocity 636·224 ft. per second, will return to the ground in 25·5 seconds, having ascended in 12·75 seconds to a height equal to 25·5, multiplied by one-fourth part of 636·224, which is 159·056 ft., so that (25·5 × 159·056 =) 4055·928 ft. is the real height attained. This is a correction of the height assigned to this ball in p. 381 of vol. ix. of the Journal of the Royal United Service Institution, where the height is given as 4055·93275 ft., which is 0·00675 ft. too much; the velocity there assigned to this time of flight is 636·1385 ft., it should be 636·224. The cause of the error is to be found in page 375, where the velocity acquired by a fall of 25·5 seconds is found by taking the difference between 25·5 and 25·49 seconds; it should have been done by taking the difference between 25·5 and 25·49999999999999 seconds. The true ascent is one-fourth of T V in all possible cases, but this was not known in A.D. 1865.

Hutton supposed that an initial velocity of 2000 ft., at 90° elevation, would culminate in "less than 9½ seconds" at a height of 6424 ft., returning to the ground in "about" 16 seconds. He did not perceive that the ascent and the descent were simultaneous, and in this error he was followed by Gregory and Straith, and most probably by many others.

It is gratifying to be able to show that General Boxer, R.A., in his treatise on Artillery, gives the following;—

Abscissæ.	Simultaneous		Ordinates.	Abscissæ.	Simultaneous		Ordinates.
	Ascent.	Descent.			Ascent.	Descent.	
100	0·87269	0·05624	0·81645	900	7·85421	5·1328	2·72141
200	1·74538	0·2285	1·51688	1000	8·7269	6·4193	2·3076
300	2·61807	0·52211	2·09596	1100	9·59959	7·874	1·72559
400	3·49076	0·94233	2·54843	1200	10·57228	9·4982	0·97408
500	4·36345	1·4945	2·86895	1300	11·34497	11·296	0·04897
600	5·23614	2·1837	3·05244	1400	12·21766	13·274	— 1·05634
700	6·10883	3·0157	3·69313	1500	13·09035	15·436	— 2·34565
800	6·98152	3·9954	2·98612	1600	13·96304	17·79	— 3·82696

The following tabulated form shows the measurements of the curve described by the ball which, at elevation 32° 51′ 29·1″, obtained a range of 5018 yds. ;—

Abscissæ.	Simultaneous		Ordinates.	Abscissæ.	Simultaneous		Ordinates.
	Ascent.	Descent.			Ascent.	Descent.	
702·42	453·7	16·2	437·5	9601·4	6201·5	3330	2871·5
1298·12	903	64·4	838·6	10150·1	6555·9	3763·2	2792·7
2086·6	1347·7	144	1203·7	10684·5	6901·1	4219·4	2681·7
2767·35	1787·4	254·4	1533	11204·2	7236·7	4698	2538·7
3439·87	2221·8	395	1826·8	11708·5	7562·4	5198·4	2364
4103·65	2650·5	565·2	2085·3	12197·1	7877·9	5720	2157·9
4758·19	3073·3	764·4	2308·9	12669·3	8182·9	6262·2	1920·7
5403	3489·7	992	2497·7	13124·8	8477·1	6824·4	1652·6
6037·54	3899·6	1247·4	2652·2	13563	8760·2	7406	1354·2
6661·34	4302·5	1530	2772·5	13983·3	9031·6	8006·4	1025·2
7273·88	4698·1	1839·2	2858·9	14385·4	9291·3	8625	666·3
7874·66	5086·2	2174·4	2911·8	14768·6	9538·6	9261·2	277·4
8463·18	5466·3	2535	2931·3	15131·5	9773·6	9914·4	— 140·8
9038·93	5838·2	2920·4	2917·8				

If this ball were now fired vertically upwards, its time of flight would be 42 seconds, its motions as follows, namely ;—

Seconds of Time.	Simultaneous		Actual Height attained.	Simultaneous		Seconds of Time.
	Ascent.	Descent.		Descent.	Ascent.	
21	15082·2	6262·2	8820	6262·2	15082·2	21
20	14520	5720	8800	6824·4	15624·4	22
19	13938·4	5198·4	8740	7406	16146	23
18	13338	4698	8640	8006·4	16646·4	24
17	12719·4	4219·4	8500	8625	17125	25
16	12083·2	3763·2	8320	9261·2	17581·2	26
15	11430	3330	8100	9914·4	18014·4	27
14	10760·4	2920·4	7840	10584	18424	28
13	10075	2535	7540	11269·4	18809·4	29
12	9374·4	2174·4	7200	11970	19170	30
11	8659·2	1839·2	6820	12685·2	19505·2	31
10	7930	1530	6400	13414·4	19814·4	32
9	7187·4	1247·4	5940	14157	20097	33
8	6432	992	5440	14912·4	20352·4	34
7	5664·4	764·4	4900	15680	20580	35
6	4885·2	565·2	4320	16459·2	20779·2	36
5	4095	395	3700	17249·4	20949·4	37
4	3294·4	254·4	3040	18050	21090	38
3	2484	144	2340	18860·4	21200·4	39
2	1664·4	64·4	1600	19680	21280	40
1	836·2	16·2	820	20508·2	21328·2	41
			0	21344·4	21344·4	42

It is much to be regretted that we have not any record of practice at great elevations, which would give us the measure of the fall by gravity in 50, 60, or even 70 seconds of time. As there is no limit to the resistive strength that might be given to the 15-in. columbiad, by shrinking on steel tubes, there can be no reason why this piece, which, with a charge of 40 lbs. of powder, at

elevation 74° 3′ 38·7″, would get a range of 1950 yds., should not be fired at an elevation of 85° with a charge of 80 or 100 lbs. of powder; the observed time of flight and measured range would be very instructive. At present the 42 seconds' flight is the very outside of the limit we can calculate with certainty; our Table of the series or progression fails altogether at 54·4″.

TABLE G.

Time.	Abscissae.		Simultaneous Ascent.		Simultaneous Descent.	Ordinates.		
		Diff.		Diff.			Diff. 1.	Diff. 2.
1	702·4223	702·4223	453·6889	453·6889	16·2	437·4889	437·4889	
2	1298·1245	695·7022	903·0373	49·3484	64·4	838·6373	401·1484	36·066
3	2086·6026	88·4781	1347·7197	44·6824	144	1203·7197	365·0824	35·7915
4	2767·3526	80·75	1787·4106	39·6909	254·4	1533·0106	329·2909	35·617
5	3439·8705	72·5179	2225·7845	34·3739	395	1826·7845	293·6739	35·1425
6	4103·6523	63·7818	2650·5159	28·7314	565·2	2085·3159	258·5314	34·968
7	4758·1940	54·5417	3073·2793	22·7634	764·4	2308·8793	223·5634	34·6935
8	5402·9916	44·7976	3489·7492	16·4699	992	2497·7492	188·8699	34·419
9	6037·5411	34·5495	3899·6001	09·8509	1247·4	2652·2001	154·4509	34·1444
10	6661·3385	23·7974	4302·5065	02·9064	1530	2772·5065	120·3065	33·8701
11	7273·8798	12·5413	4698·1429	395·6364	1839·2	2858·9429	86·4364	33·5855
12	7874·6610	00·7812	5086·1838	88·0409	2174·4	2911·7838	52·8509	33·2472
13	8463·1781	588·5171	5466·3037	80·1199	2535	2931·3037	19·5199	33·3310
14	9038·9271	75·749	5838·1771	71·8734	2920·4	2917·7771	13·5266	33·0465
15	9601·4040	62·4769	6201·4785	63·3014	3330	2871·4785	46·2986	32·7720
16	10150·1048	48·7008	6555·8824	54·4039	3763·2	2792·6824	78·0961	31·7975
17	10684·5255	34·4207	6901·0633	45·1809	4219·4	2681·6633	111·0191	32·9230
18	11204·1621	19·6366	7236·6597	35·5964	4698	2538·6597	143·0036	32·9845
19	11708·5106	04·3485	7562·3821	25·7224	5198·4	2363·9821	174·6776	31·6740
20	12197·0750	488·5644	7877·9050	15·5229	5720	2157·9050	206·0771	31·3995
21	12669·3433	72·2683	8182·9029	04·9979	6262·2	1920·7029	237·2021	30·1250
22	13124·8115	55·4682	8477·0503	294·1474	6824·4	1652·6503	268·0526	30·8505
23	13562·9756	38·1641	8760·1762	82·9714	7406	1354·1762	298·4741	30·4215
24	13983·3316	20·356	9031·6461	71·4699	8006·4	1025·2461	328·9301	30·4560
25	14385·3755	02·0439	9291·2890	59·6429	8625	666·2890	358·9571	30·0270
26	14768·6083	383·2278	9538·6249	47·4904	9261·2	277·4249	388·8641	29·9070
27	15131·5110	63·9077	9773·6373	35·0124	9914·4	—140·7627	428·1876	29·3235

TABLE H.—ELEVATIONS.—INITIAL VELOCITY 840 FEET A SECOND.

Time.	Ascent.		Descent.	Logm. Sine	Elevation.			Ranges.	
					°	′	″	feet.	yards
1	836·2	836·2	16·2	8·2872048	1	6	36	836	279
2	1664·4	28·2	64·4	5876282	2	13		1663·1	554
3	2484	19·6	144	7632109	3	19		2479·8	826·6
4	3294·4	10·4	254·4	8872138	4	25		3288·5	1096
5	4095	00·6	395	9843432	5	32		4075·9	1359
6	4885·2	790·2	565·2	9·0633199	6	38		4852·4	1617·5
7	5664·4	79·2	764·4	·1301668	7	45		5612·6	1871
8	6432	67·6	992	881657	8	52		6355	2118
9	7187·4	55·4	1247·4	·2394339	9	59		7078·3	2359
10	7930	42·6	1530	854182	11	7		7845	2615
11	8659·2	29·2	1839·2	3271512	12	15		8461·6	2820·5
12	9374·4	15·2	2174·4	653959	13	24		9118·7	3039·6
13	10075	00·6	2535	·4007340	14	34		9750·8	3250
14	10760·4	685·4	2920·4	336139	15	44		10356·5	3452
15	11430	69·6	3330	643980	16	55		10938·3	3646
16	12083·2	54·2	3763·2	933753	18	8		11482·2	3827
17	12719·4	36·2	4219·4	·5207841	19	22		11999·3	3999·8
18	13338	18·6	4698	468223	20	37		12483·2	4161
19	13938·4	00·4	5198·4	716568	21	53		12932·7	4311
20	14520	581·6	5720	954294	23	11		13345·8	4449
21	15082·2	62·2	6262·2	·6182622	24	31		13720·7	4573·6
22	15624·4	42·2	6824·4	402612	25	53		14055·2	4685
23	16146	21·6	7406	615188	27	18		14343·8	4781
24	16646·4	00·4	8006·4	821270	28	44		14598·2	4866
25	17125	478·6	8625	·7021285	30	14	30·3	14794·44	4931·5
26	17581·2	56·2	9261·2	216188	31	47		14944·3	4981
27	18014·4	33·2	9914·4	406466	33	23		15040·2	5013

TABLE H—*continued.*

Time.	Ascent.		Descent.	Logm. Sine.	Elevation.			Ranges.	
					°	′	″	feet.	yards.
28	18424	409·6	10584	9·7592659	35	3		15080·5	5027
29	18809·4	385·4	11269·4	775259	36	48		15059·6	5020
30	19170	60·6	11970	954721	38	38		14973·6	4991
31	19505·2	35·2	12685·2	·8131469	40	34		14816·8	4939
32	19814·4	09·2	13414·4	305903	42	36		14583	4861
33	20097	282·6	14157	478400	44	47		14264·2	4754·7
34	20352·4	55·4	14912·4	649319	47	6		13850·2	4617
35	20580	27·6	15680	819007	49	38		12329·4	4443
36	20779·2	119·2	16450·2	987799	52	22		12687·3	4229
37	20949·4	170·2	17249·4	·9166024	55	37		11803·7	3934·6
38	21090	140·6	18050	324006	58	51		10908	3636
39	21200·4	110·4	18860·4	492068	62	49	34·02	9682	3227
40	21280	79·6	19680	660535	67	39		8095·4	2698·5
41	21328·2	48·2	20508·2	829733	74	3		5857·1	1952·4
42	21344·4	16·2	21344·4	10·0000000	90			0·0	0·0

TABLE I.—ELEVATION 45°.

Range.		Time of Flight.	Oblique Ascent.	Mean Velocity.	Initial Velocity.
yards.	feet.				
100	300	4·35	424·264	97·53	162·88
200	600	6·2	848·528	136·86	227·624
300	900	7·61	1272·792	167·25	275·88
400	1200	8·82	1697·056	192·31	315·7
500	1500	9·9	2121·32	214·27	349
600	1800	10·87	2545·584	234·07	380·5
700	2100	11·79	2969 848	251·9	407·2
800	2400	12·63	3394·112	268·73	433·2
900	2700	13·45	3818·376	283·88	454·4
1000	3000	14·2	4242·64	298·8	476·411
1100	3300	14·93	4666·904	312·58	496·112
1200	3600	15·63	5091·168	325·73	515·327
1300	3900	16·3	5515·432	338·37	533
1400	4200	16·96	5939·696	349·04	554
1500	4500	17·59	6363·96	361·19	565·1
2000	6000	20·52	8485·28	413·41	634·5
2500	7500	23·15	10606·6	458·17	690
3000	9000	25·6	12727·92	497·18	734·656
3500	10500	27·88	14849·24	532·54	772·336
4000	12000	30·05	16970·56	564·71	805·1
4500	13500	32·11	19091·88	594·46	830
5000	15000	34·12	21213·2	621·72	850

We will hereafter describe two useful instruments, adapted by Major-Gen. Anstruther, in his investigations respecting the motion of projectiles.

When the parabolic theory of the motion of projectiles in vacuo was first brought forward it was supposed by many clever men that the resistance of the fluid in which a projectile must necessarily move might be disregarded in calculation, or that at least a compensation might be found by experiment or otherwise which would render the parabolic theory capable of practical application. Persons who in those remote times were sceptical on this point would most likely have been treated with the same kind of good-natured contempt that anyone in the present day would meet with who dared to express a doubt as to the truth of the generally-received doctrine, that the application of the parabolic theory in practice is quite inadmissible on account of the resistance of the air to a projectile moving with the velocity which it is practically necessary to consider. It is the fashion to assert as an indisputable fact that nothing can be further from the true trajectory of a projectile subject to the resistance of the air, and moving with a considerable velocity, than a parabolic curve, and this we are told *everybody knows*; if everybody also knew what the true curve of the trajectory was, we should be in a very favourable position for solving all the useful practical problems appertaining to the science of gunnery; but here we are left completely in the dark, to arrive at the conclusion that what the real curve of a trajectory is, *nobody knows.*

The question therefore naturally arises, What are we to do? If theory, as it is called, unsupported by practical results, cannot be relied upon, and must consequently be altogether disregarded, isolated experiments, without the support of some theory enabling us to generalize, classify, and arrange the results of practice, so as to render them applicable to all circumstances, are little if at all better than the disregarded theory. That the range of a projectile moving through the air falls far short of the range calculated upon the supposition that this same projectile moves in vacuo, is

a well-known fact; but at the same time, we approach nearer and nearer to a coincidence with the calculated range as the projective force and velocity are diminished. It could hardly have been expected that calculations established upon the supposition that the projectile moved in vacuo should be found applicable in practice without certain modifications; but it was at first supposed that a system of compensation or rectification might be easily arrived at, which would bring the disturbing elements caused by the resistance of the atmosphere and other causes quite within manageable limits, and that the established theory would not only serve as a sure guide to experiment, but also as the means of generalizing and forming practical rules. But this idea seems to have been long abandoned. The force of resistance, at first almost ignored, seems to be now exaggerated to such a degree that all thoughts of attempting to establish a practical theory seem to be hopelessly relinquished. We seem to have got into a kind of chaos of ideas on the subject of projectiles, without anything to rest upon besides a blind reliance upon tables, which we worship and believe in, and which are framed according to a system which, of course, *everybody knows*, but which nobody could exactly describe to you if they were individually asked, and upon an implicit faith in some old traditional rules with reference to what are called the laws of gravity, which have been preserved for ages, and are reproduced as a matter of course in all works upon gunnery.

The laws of gravity seem to be like the laws of the Medes and Persians, which no one, except Anstruther, should gainsay or attempt to alter; and consequently they seem to have been taken as the starting point of all discussions respecting the theory of projectiles, and no one, except Anstruther, has been so wicked as for a moment to question the truth of these time-honoured institutions, consequently they have remained unaltered, as venerable monuments of the past.

That our theory of projectiles has proved a practical failure few will be disposed to deny. But the following question remains to be answered, Shall we relinquish all further theoretical inquiry, drop the matter in despair, and trust to experiment, tables, trial shots, and chance? or shall we do what a sensible workman generally does when his first effort at success is unsatisfactory, namely, look to his tools and machinery to see that there is no flaw of construction or no miscalculation as to the principles of action, and also look about him to see if there are no modern discoveries or new scientific inventions which may assist him in arriving at the required result? In the present article we propose to take the latter course, and in the first place to say a few words about the laws of gravity, which may be considered as the machinery and tools with which we have been working so long. That the resistance of the air must form an important element in all calculations with reference to the trajectory of projectiles there can be little doubt; but we think it is at least questionable whether we ought to attribute the whole of the glaring discrepancy which is found to exist between theory and practice to that cause alone. It will be at least worth while to examine the matter in detail, in order to ascertain if there are not other sources of error, and whether corrections cannot be applied so as to bring the total amount within manageable limits.

Suppose a curve either convex or concave to the axis of the abscisses to be of such a nature that while the abscisses denote the time, the corresponding ordinates shall denote the measure of the force during such time; then, because effects are proportionate to their causes, the instantaneous velocities produced or generated by the forces are proportionate to the forces which generate them. $d\,a = y\,.\,d\,x$ expresses the differential of the area included by a curve, and the ordinates of any points upon it, as the function of the co-ordinates; therefore, substituting v for a, f for y, and t for x, we have $d\,v = f\,.\,d\,t$. Taking as before the abscisses to express the time, but the corresponding ordinates to denote the velocities generated or produced, it will be evident that in this case in the expression $d\,a = y\,.\,d\,x$, a may be taken to represent the space passed over with the velocity indicated by the abscisses; we may therefore substitute s for a, v for y, and as before, t for x, which gives $d\,s = v\,.\,d\,t$. When we have to deal with uniform velocity we may take v as constant; therefore, integrating the last expression, $s = v\,t$, from which we deduce the following;—

In uniform velocity—

1. The spaces described in the same time are as the velocities.
2. The spaces described with the same velocity are as the times.
3. Spaces described with unequal velocities and in different times are as the velocities and times conjointly.

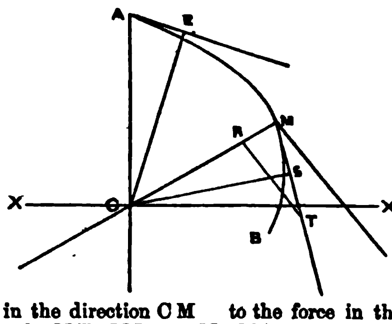
There can be no doubt whatever that the above expressions are correct exponents of the value of uniform forces or velocities under all the circumstances to which they may be applied. But with reference to the subject we are now dealing with, uniform force or the resulting uniform velocity can be only taken as a measure of, or as a means of indicating the comparative values of the varying forces, or of the accelerated or retarded velocities involved in the motion of bodies subjected to the influence of attraction combined with that of a projectile force, while passing through a resisting medium. We here enter upon a very intricate subject, so difficult that in order to facilitate calculation we seem to have condoned many errors in theory as being separately too minute and insignificant to have any important effect upon the practical result, the consequence of which appears to be that the aggregate has produced an amount of accumulated error which renders our calculations practically useless, and then we exonerate ourselves by laying the whole blame upon the atmosphere. We can only estimate or compare forces by noting their effects upon matter with which we are conversant, and then by a careful analysis and comparison endeavour to deduce as close an approximation as possible to the laws of action of the forces under investigation. Leaving out of consideration slight deviations and perturbations, we may assume as an established fact that the orbits of the heavenly bodies subjected to the influences of attraction and some original mysterious projectile force are elliptical. This, due to John Kepler, is at least the closest approximation to their line of motion which we have as yet arrived at.

Starting from this point, we shall now proceed to show that if the centre of force be situated in one of the foci of an ellipse, a body whose line of motion is on the periphery must be attracted by a force whose intensity is inversely proportional to the square of the distance from the centre of

attraction, and that therefore this is the closest approximation to the law of attraction which we have as yet attained to. In order to this we shall in the first place establish two equations with reference to centripetal forces in general.

We shall suppose that a body projected from a given point A, Fig. 3347, in a given direction A E, with a given velocity, is attracted by a force which acts according to any law whatever towards a fixed point C. Take the curve A M B, which we will express by z, to represent the locus of motion of a particle subject to the combined influences of the projectile and attractive forces; C A, C M radii vectors, which we will represent by y, indicating the lines of action of the attractive force at any points A and M in the curve A M B; A E, M S tangents at the points A and M, and C E, C S perpendiculars upon the tangents from the fixed point C.

3347..



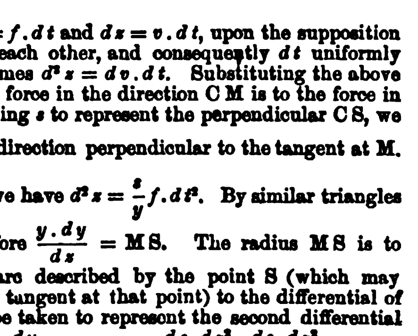
From any point T in the tangent at M let fall the perpendicular T R upon the line C M. Then T M, M R, T R, may be taken respectively to represent dz, dy and dx.

By the resolution of forces C M, M S as the force in the direction C M to the force in the direction of the tangent at the point M. By similar triangles M T : M R :: C M : M S; or if we take f to express the centripetal force at the point M, $dz : dy :: f : \frac{dy}{dz} f$, which evidently expresses the force in the direction of the tangent at M. Substituting this value for f in the expression $dv = f.dt$ (f is for force in general), we obtain $dv = \frac{dy}{dz} f.dt$; $dv.dz = f.dy.dt$; $\frac{dv.dz}{f.dy} = dt$. Eliminating dt by means of the equation $dz = v.dt$, which in this case is represented by $dz = v.dt$, we get $\frac{dv.dz}{f.dy} = dt = \frac{dz}{v}$; $v.dv = f.dy$, which, when y increases as the velocity v diminishes, becomes $v.dv = -f.dy$.

Taking the two equations already established, $dv = f.dt$ and $dz = v.dt$, upon the supposition that the evanescent intervals of time are equal to each other, and consequently dt uniformly constant, the differential of the latter expression becomes $d^2 z = dv.dt$. Substituting the above value for dv, we have $d^2 z = f.dt^2$. C M : C S as the force in the direction C M is to the force in the direction perpendicular to the tangent at M. Taking s to represent the perpendicular C S, we have $y : s :: f : \frac{s}{y} f$, which represents the force in the direction perpendicular to the tangent at M. Substituting this value for f in the above expression, we have $d^2 z = \frac{s}{y} f.dt^2$. By similar triangles T M : M R :: C M : M S, $dz : dy :: y : \frac{y.dy}{dz}$; therefore $\frac{y.dy}{dz}$ = M S. The radius M S is to the radius M T as the differential of the circular arc described by the point S (which may evidently be represented by ds, the differential of the tangent at that point) to the differential of the circular arc described by the point T, which may be taken to represent the second differential of z with respect to the space T M $= dz$. Therefore $\frac{y.dy}{dz} : dz :: ds : \frac{ds.dz^2}{y.dy}$, $\frac{ds.dz^2}{y.dy} = d^2 z$; therefore $\frac{s}{y} f dt^2 = d^2 z = \frac{ds.dz^2}{y.dy}$, $\frac{s}{y} f dt^2 = \frac{ds.dz^2}{y.dy}$; but $dz = v dt$, $\frac{dz^2}{v^2} = dt^2$. Substituting this value for dt^2, $\frac{s}{y} f \frac{dz^2}{v^2} = \frac{ds.dz^2}{y.dy}$, $f.dy.s = v^2.ds$.

Having now established the two equations $v.dv = -f.dy$ and $f.dy.s = v^2.ds$, we can eliminate f. $-\frac{v dv}{dy} = f = \frac{v^2 ds}{dy s}$, $-s.dv = v.ds$, $v.ds + s dv = 0$. Integrating, we get $v.s = C$. Taking d to represent the perpendicular C E to the tangent at the initial point A, and c the initial velocity, it will be evident that when v becomes c, s becomes d; therefore $cd = C$, and the corrected integral will be $v.s = cd$.

3348.



The areas of all parallelograms circumscribing an ellipse formed by drawing tangents at the extremities of two conjugate diameters are constant, each being equal to the rectangle under the axes.

Take a to represent the semi-transverse axis, and b the semi-conjugate axis; P C, C D, Fig. 3348, two semi-conjugate diameters, $b.a$ = P P′ C D, $\frac{b a}{\text{C D}}$ = P P′.

The angles made by the focal distances with the tangent are equal and the angle at P is equal to the angle at E on account of the tangent being parallel to the diameter E D; therefore, by similar triangles, $y : s ::$ P E : P P′, $\frac{s.\text{P E}}{y}$ = P P′. If straight lines be drawn from the foci to a vertex of any

diameter, the distance from the vertex to the intersection of the conjugate diameter with either focal distance is equal to the semi-transverse axis,

$$\mathrm{P\,E} = a, \frac{s\,.\,a}{y} = \mathrm{P\,P'}; \quad \frac{b\,.\,a}{\mathrm{C\,D}} = \mathrm{P\,P'} = \frac{s\,.\,a}{y}, \quad s = \frac{b\,.\,y}{\mathrm{C\,D}}.$$

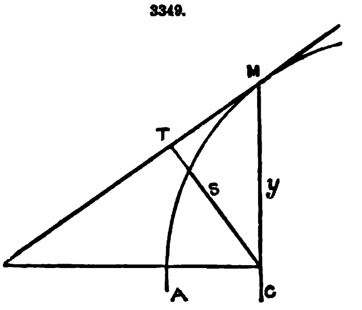
The rectangle under the focal distances of the vertex of any diameter is equal to the square of the semi-conjugate diameter, $\mathrm{F\,P}\,.\,y = \mathrm{C\,D}^2$; but the focal distances are equal to the transverse axis, $\mathrm{F\,P} + y = 2a$, $\mathrm{F\,P} = (2a - y)$. Substituting value for F P, $(2a - y)\,.\,y = \mathrm{C\,D}^2$, $\sqrt{(2a-y)y} = \mathrm{C\,D}$. Substituting this value in $s = \frac{b\,.\,y}{\mathrm{C\,D}}$, we have $s = \frac{b\,.\,y}{\sqrt{\{(2a-y)y\}}}$. Substituting this value for s in the equation $v\,.\,s = c\,.\,d$, $v\,.\,\frac{b\,.\,y}{\sqrt{\{(2a-y)y\}}} = c\,d$, $v\,.\,b\,.\,y = c\,.\,d\,.\,\sqrt{\{(2a-y)y\}}$, $v^2.\,b^2.\,y^2 = c^2.\,d^2.\,(2a-y)\,y$, $v^2.\,b^2.\,y = c^2.\,d^2.\,(2a-y)$, $v^2 = \frac{c^2.\,d^2}{b^2}\,\frac{2a-y}{y}$. Differentiating, we obtain $-v\,.\,dv = \frac{c^2.\,d^2}{b^2}\cdot\frac{a\,.\,dy}{y^2}$; but $-f\,dy = v\,.\,dv$; $\therefore f\,dy = -v\,.\,dv$, $f\,dy = -v\,.\,dv = \frac{c^2.\,d^2}{b^2}\cdot\frac{a\,.\,dy}{y^2}$, $f = \frac{c^2.\,d^2.\,a}{b^2}\cdot\frac{1}{y^2}$. Take $\frac{c^2.\,d^2.\,a}{b^2} = \mathrm{A}$, $f = \frac{\mathrm{A}}{y^2}$, which shows that the force of attraction is inversely proportional to the square of the variable distance y of a particle moving in an elliptical orbit, the centre of the attracting force being in one of the foci. This therefore is the closest approximation to the law of attraction which has as yet been attained. The same law will be found to exist with reference to a body moving on the arc of a parabola, the centre of attraction being situated in the focus, which can be shown as follows;—

Let C, Fig. 3349, be the focus of the parabola A M; A the vertex. The perpendicular s to the tangent at the point M is a mean proportional between the distance y from the focus to that point, and the distance C A from the focus to the vertex. Take $\mathrm{C\,A} = a$, $a\,.\,y = s^2$, $\sqrt{a\,.\,y} = s$. Substituting this value for s in the equation $v\,s = c\,d$, we have $v\,.\,\sqrt{(a\,y)} = c\,.\,a$. A C is perpendicular to the tangent at the vertex; therefore $a = d$, $v\,.\,\sqrt{(a\,y)} = c\,.\,a$, $v^2 = \frac{c^2.\,a}{y}$. Differentiating,

$$v\,.\,dv = -\frac{c^2 a\,.\,dy}{2\,y^2}, \quad -v\,.\,dv = \frac{c^2 a\,.\,dy}{2\,y^2};$$

but $f.\,dy = -v\,.\,dv$, $f.\,dy = -v\,.\,dv = \frac{c^2 a\,.\,dy}{2\,y^2}$, $f = \frac{c^2.\,a}{2}\cdot\frac{1}{y^2}$. Take $\frac{c^2 a}{2} = \mathrm{A}$, $f = \frac{\mathrm{A}}{y^2}$.

3349.



If the earth was perfectly spherical the directions of gravity would all concur at its centre. Therefore, considering the earth as a sphere and applying the laws of attraction founded upon astronomical observations, we might suppose that the force of gravity with reference to bodies near the surface of the earth acts upon lines tending towards such central point with an intensity varying inversely as the square of the distance from the centre of attraction. That this is the true law of attraction, so far as up to the present time human reason can recognize it, seems to be generally admitted by all writers upon the science of gunnery. But it seems to be thought that the working out of the problems involved in such a law in all their integrity, so as to bring them within the scope of practical utility, would require the command of a calculus more powerful than any we are at present in possession of. Therefore, in order to bring the subject within the grasp of comparatively easy calculation, we suppose;—

1st. That the lines of action of the force of gravity, instead of all tending to a common point at the centre of the earth, are all parallel to each other.

2nd. That within the space above the surface of the earth which we have to consider with reference to the motion of projectiles, we may dispense with a scrupulous adherence to the established laws of gravity.

In order to justify ourselves in these suppositions we endeavour to define the limits of error incurred as follows;—

And first as to the parallelism of the lines of action of the attractive force.

Taking the radius of the earth at 3965, nearly, miles, 6978400 yds.; the length of one minute of a degree to such a radius would be over 2000 yds. A mile is only 1760 yds.; therefore within a lateral range of one mile the limit of error involved in considering the lines of attraction parallel instead of tending to a common point would be within one minute of a degree. As the centre of attraction is more and more removed from any points under consideration, the more will the lines of action of the attracting force tend towards parallelism. Consequently, considering the lines parallel is tantamount to considering the centre of attraction removed to an infinite distance.

We have assumed the distance of one mile as the lateral space necessary to consider with reference to the motion of projectiles, in our first supposition, and in the second we shall consider the same vertical distance as being far beyond the greatest height which can be reached by the trajectory of projectiles propelled by any human contrivance as yet discovered.

Taking the radius of the earth at 3965 miles, and assuming that the power of attraction is

inversely proportional to the square of the distance from the centre, $(3966)^2 - (3965)^2$ will represent the difference in the intensity of the force at the surface of the earth and at a mile above it, $(3966)^2 - (3965)^2 = 7931$, $(3965)^2 = 15721000$, the thousandth part of which is 15721. The half of this is 7861. If therefore we take $f = (3965)^2$ as the representative of the attractive force at the surface of the earth, the difference in the intensity of the force at the surface and a point one mile above it may be expressed by the number 7931, which is very little in excess of $7861 = \frac{1}{2000} f$.

The supposition founded upon this reasoning is that we may assume the force of gravity (which is the name given to the attracting force) either as constant or as influenced by any law we please, or both, first one and then the other, for the vertical space of one mile above the surface of the earth; and that the limit of the error thus incurred will be the $\frac{1}{2000}$ part of the intensity of the force of attraction at the surface.

This mode of reasoning is very plausible, and would be conclusive if we had to deal with any of the palpable material subjects familiar to our daily experiences, for the $\frac{1}{2000}$ part of anything is in most cases a very small matter, and may be generally neglected without producing any sensible error in practice. But we have now to deal with a very subtle mysterious power, of the essence of which we know next to nothing; we approach those dark limits which circumscribe the action of the human intellect, and we ought therefore to feel our way very carefully as we proceed. So long as we have only to deal with ratios and comparisons, we are tolerably safe in drawing inferences, but we think it is very questionable whether we are justified in assuming that the neglect of the $\frac{1}{2000}$ part of the entire force of gravity at the surface of the earth will produce no sensible error in calculation, when at the same time we can form only a very vague conception of what the intensity of that force is, and if so, here at the very outset of our investigations is a most prolific source of error. Up to the present time, however, the subject has not been considered from this point of view; on the contrary, it has been taken as a fact, established by conclusive reasoning, that we may, within the limits assigned, assume almost any latitude in dealing with the force of gravity when applied to the theory of projectiles. And with this understanding we proceed to frame what are commonly called the laws of gravity.

Most people who have not closely considered the subject will be under the impression that there is no difficulty whatever in forming a perfectly clear and defined conception of the continued action of any given force. But when we come to analysis, reasoning, and calculation, we find that the only means we have of dealing with the matter is to consider that the force acts by successive impulses or solicitations, equal or otherwise, as the case may be, at the commencement or the end of very small equal intervals of time or of space measured on the line of action of the force.

If we take a to represent the small unit of time, and b the small unit of space, we must in the first place find an expression for the initial intensity of the force by supposing that it is such as to cause a particle of matter to move upon the line of action of the force through the space A b during the unit of time a, or else that it is such as to cause the particle to move on the same line over the unit of space b during the time A a, the coefficient A being a quantity determined by experiment or otherwise. For instance, assuming the law which seems to be received as the true law of attraction, namely, that the intensity of the force varies inversely as the square of the distance from the centre of force, and supposing that during the first unit of time, which is usually taken at one second, d represents the distance passed over by a particle of matter, in consequence of the initial solicitation of the force, d^2 will evidently express the intensity of the force at the termination of the first second. If the particle were subject to the influence of this force alone, it would evidently descend during the next second, with a uniform velocity, a distance $= d^2$; but during its descent it is subjected to d^2 solicitations, each successively equal to the square of that immediately preceding it. The distance will therefore be expressed by $d^2 + d^{2 \cdot 2} + d^{2 \cdot 2 \cdot 2} \ldots . d^{d^2 - 2} + d^{d^2}$, a series, the exponents of the consecutive terms of which form a geometrical series of which the first term and the common ratio are 2 and the number of terms of the series d^2.

The expression for the distance corresponding to each of the succeeding seconds will evidently be a series of the same form, the last term of each series being taken for the first term, as well as for the number of terms of the succeeding series. The sum of all the series will express the distance actually descended in any given time, the time t expressing the number of series. If we should suppose the intensity of the attractive force to be inversely as the distance from the centre of force, taking d as before to represent the distance passed over during the first second, the intensity of the force at the end of the first second will evidently be represented by $2 . d$; the particle of matter under the influence of this force alone would evidently descend a distance equal $2 . d$, but during the descent it will be subjected to two solicitations of gravity each separately equal to d. Therefore at the end of the second second the force will be represented by $4 . d$; at the end of the third second by $16 . d$, &c. Therefore $d + 2^2 . d + 2^{2 \cdot 2} . d \ldots . 2^{t-2} . d + 2^t . d$, a series of which the exponents of the coefficients of the terms form a geometrical series of which the first term and the common ratio are 2, and the number of terms t will express the distance descended by a particle of matter during the time t.

In framing the laws of gravity this latter supposition seems to have been to a certain extent adopted, for we are told that during the first second of descent d becomes $2 d$, and is then called g. This is so far intelligible, but what follows is, to say the least of it, rather startling, and a little difficult to be understood, for d expires in giving birth to g, and this posthumous offspring is not only twice as big as his progenitor, but he appears to be endowed with the most extraordinary and supernatural powers, for he has a capability of generating a constantly increasing velocity, which ought to be simply an exponent of his own increase, but wonderful to relate, g never alters. He goes down to infinity for the purpose of settling some small matters relative to terminal velocity (of which we shall speak presently), and comes up again quite unchanged; he is employed to solve complicated problems with reference to variably accelerated or retarded velocity; he is blown up by gunpowder, forced into steam boilers, up and down funnels, through fire and water, but is not

in the slightest degree affected by all these vicissitudes. Whenever we come across him we find him placidly and systematically doing his duty as the representative of the force of gravity—in all places and at all times the old familiar g, exactly as he appeared at his birth.

The fact of a heavy body being suspended in space without any support, and being in a state of rest, and then commencing suddenly to descend by the influence of an initial force which is equal to nothing, may be a conception easily formed in a well-trained mathematical mind, but to the uninitiated it is a little difficult of comprehension.

In order to illustrate as clearly as possible the method adopted of deducing the laws of gravity, we shall return to the language of the differential and integral calculus, taking again the expressions already established, $dv = f\,dt$, $ds = v\,.\,dt$, but supposing the generating power or force of gravity which is represented by f to be constant and invariable. Integrating $dv = f\,.\,dt$, we get $v = f\,.\,t$, $\frac{dv}{f} = dt = \frac{ds}{v}$, $v\,.\,dv = f\,.\,ds$; integrating $v^2 = 2\,.\,f\,.\,s$, $v = f\,.\,t$, $v^2 = f^2 t^2$, $v^2 = 2fs$, $f^2 t^2 = v^2 = 2fs$, $ft^2 = 2s$, $\frac{v}{t} = f = \frac{v^2}{2s}$, $2\,.\,s = vt$. From the equations thus established, we draw the following conclusions:—1st. From $v = f\,.\,t$ we assume that the velocity acquired by a body falling freely from a state of rest, and being acted upon by gravity alone, is proportional to the time elapsed. 2nd. From $v^2 = 2f\,.\,s$ we assume that the spaces described in the descent are proportional to the squares of the velocities; or by $ft^2 = 2\,.\,s$ to the squares of the times; or by $2\,.\,s = vt$ to the times and velocities conjointly.

Comparing the two equations, $s = t\,.\,v$ and $2\,.\,s = tv$, the first being referred to uniform velocity and the second being supposed to refer to accelerated velocity, the following inference is drawn. The space described uniformly with the velocity acquired by a body falling freely from a state of rest is double the space described by the body while generating such velocity. This is the foundation upon which a very complicated structure is raised, elaborate tables of reference are compiled, and the changes are rung upon all the various combinations of the quantities of space, time, and velocity, in all of which g plays his invariable part as the representative of gravity and unalterable sameness.

We have given the above few simple differential expressions as illustrating most easily the course followed in constructing the laws of gravity, but the matter is generally explained by a much more elaborate process. We shall now make a few remarks upon this mode of deducing the law of gravity.

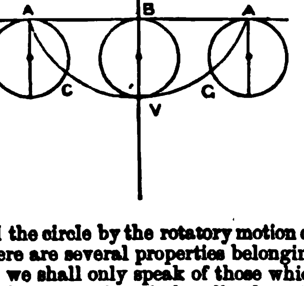
The effects of gravity are represented by supposing that the force acts by equal solicitations, at the end of equal intervals of time called seconds; it has been proved that the action of the force is inversely proportional to the square of the distance from the centre of attraction. It would therefore seem that the above supposition was as far from the truth as the parabolic curve is stated to be from the true trajectory of a projectile. The intensity of the initial solicitation of the force, and consequently its exponent velocity, are exhibited (upon the supposition that it may be considered constant within assigned limits) with reference to units of time and space as follows. The number of feet descended by a particle of matter during one second of time is in the first instance supposed to be ascertained by accurate experiments with the pendulum, &c. (and of the accuracy of these experiments we shall speak presently); this, which thus becomes a known quantity, is expressed by d. d is at first supposed to represent the value of each of the solicitations which are supposed to take place at the end of each succeeding second, for we find that at the end of the first second d becomes $2\,.\,d$, and is called g, which evidently represents the intensity of the force generated by or resulting from two successive solicitations. In accordance with this assumption, at the end of the second second, three equal solicitations would have taken place; and consequently the resulting force and its exponent velocity would be expressed by $3\,.\,d$. But it is not so; g is now taken to represent the value of the successive solicitations. Although we may have implicit faith in the accuracy of the experiment which introduced d to our notice, g evidently owes his existence to an arbitrary assumption which, although supposed to be admissible within assigned limits, is proved to be very far from the truth. I think, therefore, there is at least room for doubt whether g should be at once unhesitatingly recognized as the legitimate representative and successor of d. It would appear to be quite allowable to form a series according to a certain law, and having ascertained the value, reject the first term upon the supposition that the terms were so small individually that the neglect of one would in the aggregate produce no sensible error. But in the present instance we have not only done this, but we have altered the value of what ought to be the constant increment by substituting g for d. This little sin against fair logical reasoning is glossed over and concealed by ingenious illustrations and high-sounding terms specially framed and adapted for the confusion of useful knowledge; and it seems to be expected that the illegitimacy of g's birth will be condoned upon the ground of his being so very small; but we are afraid this sin against gravity, like all other sins, will be found to bear its fruits, and that something else besides the atmosphere is to be blamed, when we find our theory and practical experiences so very divergent.

The assumption that the spaces described by a falling body in its descent are proportional to the squares of the times is founded upon the supposition that we are justified in considering the generating power or force of gravity, which is represented by f in the expressions involving space, time, and velocity, as constant; but it seems absurd to suppose that we can separate the essence of anything from the exponent that marks its existence, and consider that one varies while the other does not. The laws of gravity now extant, whether fallible or infallible, are evidently dependent upon the accuracy of the experiment by means of which we are supposed to have ascertained the space described by a descending body during one second of time. We shall therefore now say a few words on this subject.

If the circumference of a circle be rolled on a right line, beginning at any point A, the move-

ment being continued till the same point A arrives at the line again, making just one revolution, and thereby measuring out a straight line A B A, Fig. 3350, equal to the circumference of the circle, while the point A in the circumference traces out a curve line A G V C A; the curve thus traced is called the common cycloid; the line A A is called the base; V the vertex; V B the axis;

3350.



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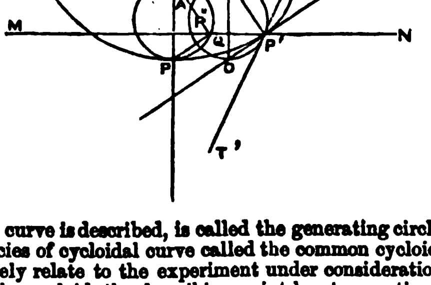
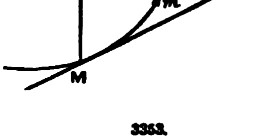


and the circle by the rotatory motion of which the curve is described, is called the generating circle. There are several properties belonging to the species of cycloidal curve called the common cycloid; but we shall only speak of those which immediately relate to the experiment under consideration. As the generating circle rolls along the base of the cycloid, the describing point has two motions; first a progressive motion in a direction parallel to the base B B', Fig. 3351, and secondly, a motion of rotation round the centre of the generating circle. These motions are equal, for, in the time of one revolution of the generating circle, the describing point moves by its progressive motion through the space B B', while by its motion of rotation it moves through a space equal to the circumference of the circle. Suppose the circle to roll from the position A in which the describing point P coincides with the vertex of the cycloid, to the position L in which the describing point has moved to P', when the point which was at A will be now at A'; the distance L A will then be equal to the arc L A' of the circle, since that arc has rolled over L A. The point P', in consequence of the two equable motions already explained, one in the horizontal direction P' N parallel to A B, and the other in the direction of the tangent to the generating circle P' T at the point P', will have an actual motion in a direction equally inclined to each of these lines. The direction of the curve at P', or, what is the same, the direction of a tangent to the curve at that point, will therefore be represented by a line bisecting the angle N P' T, and this line will be the continuation of the chord of the arc of the generating circle between P' and the highest point O; for if L P' be drawn the angle O P' M will be equal to the angle O L P', on account of the similarity of the triangles O Q P' and O P' L. The angle O P T' will also be equal to the angle O L P' in the opposite segment of the circle; therefore the angle O P' T will be equal to the angle O P' Q, or, what is the same, the angle N P' R will be equal to the angle T P' R. The line O P' R therefore bisects the angle T P' N, and is therefore a tangent to the cycloid at P'. Since the arcs A P'' and L P' are equal, and also the arcs P P'' and O P', the lines A P'' and L P' are equal and parallel, and the lines P P'' and O P' are likewise equal and parallel. The tangent at P' is therefore parallel to the corresponding chord P'' P of the generating circle on the axis. The direction of motion in any point of a curve is always in the tangent at that point; consequently, if the motion with which any point m, Fig. 3352, arrives at M was to become uniform, the point m would proceed in the direction of the tangent T M, therefore the directions of the motion in the abscissa A P, ordinate P M, and curve being in the sub-tangent T P, ordinate P M, and tangent T M. The differentials of the abscissa, ordinate, and curve may be represented by the three sides of the triangle T P M, or by the corresponding sides of any similar triangle.

3352.



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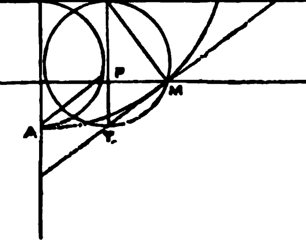


Draw M P, Fig. 3353, perpendicular to the diameter L T of the generating circle. Take L T $= a$, T P $= x$, the chord T M $= p$, and the arc A M $= s$.

$$\therefore \ \mathrm{TM : TL :: TP : TM}, \quad p : a :: dx : ds.$$

$$p \,.\, ds = a\, dx.$$

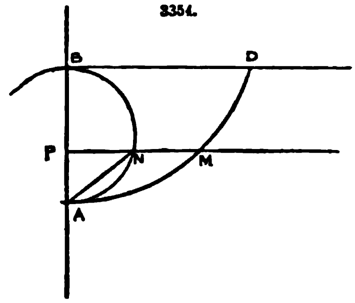
$$\mathrm{TL : TM :: TM : TP}, \quad a : p :: p : x.$$

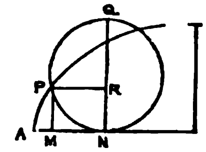
$$ax = p^2,\ a \,.\, dx = 2p \,.\, dp,\ p \,.\, ds = a \,.\, dx = 2p \,.\, dp,\ ds = 2\,dp,\ s = 2p.$$

Take a to represent the diameter B A, Fig. 3354, of the generating circle, and let A P $= y$. Upon the supposition that the lines of action of the gravitating force are parallel to the axis and perpendicular to the base of the cycloid, $ds : dy :: f : \frac{dy}{ds} f$. Substituting this value for f in the

equation $dv = f\,dt$, $dv = \frac{dy}{dz}.f.dt$, $\frac{dv.dz}{f.dy} = dt = \frac{dz}{v}$, $v.dv = f.dy$; and because in this case v increases as y diminishes, $v.dv = -f.dy$. If we take d to represent the height fallen through in a second, we must, according to the conventional rule, take $2d$ to express the uniform velocity equal to the velocity generated by falling through the height d; therefore in the equation $v.dv = -f.dy$ we are told that we must substitute $2d$ for f in order to obtain the actual value of d in known terms. The equation therefore becomes $v.dv = -2d.dy$. But this is in reality nothing more than our old friend g, who seems to be always turning up, like the clown in a pantomime, in some new disguise, and it is only by treating him differentially that we can detect him. However, he is here engaged in fulfilling a sacred duty, for he is on his way to search for the remains of his deceased parent d, who it will be recollected disappeared in such a mysterious manner at the commencement of g's career. Integrating $v.dv = -2d.dy$, we get $v^2 = C - 4.d.y$. When $y = a$, $v = 0$; therefore $C = 4.a.d$, and the corrected integral is $v^2 = 4.a.d - 4.d.y$, $v = 2\surd(a.d - d.y)$.

3354.





Take p to represent the chord AN,

$$AP : AN :: AN : AB, \quad y : p :: p : a.$$

$$a.y = p^2,\ \surd(a.y) = p,\ z = 2.p,\ z = 2\surd(a.y),\ dz = \frac{a.dy}{\surd(a.y)}.$$

Substituting the values just found for v and dz in the equation $dz = v.dt$, we get

$$\frac{a.dy}{\surd(a.y)} = 2\surd(a.d - dy)dt = 2\surd\left\{a.d\left(1 - \frac{y}{a}\right)\right\}dt.$$

Divide both sides of the equation by $2\surd\left(1 - \frac{y}{a}\right)$,

$$\frac{a.dy}{2\surd(ay - y^2)} = \surd(a.d)\,dt,\quad \int\frac{a.dy}{2\surd(ay - y^2)} = \surd(a.d)\,t = \text{versin.}^{-1}y$$

to diameter a. When $y = 0$, versin.^{-1}y expresses $\frac12$ the circumference of the circle, and t the time of descent through half the cycloid. Take c to represent versin.$^{-1}0$ to diameter a, $c = t(a.d)$. Take $t = \frac12''$, and consequently the time in the entire arc one second, $c = \frac12\surd(a.d)$, $c^2 = \frac14(a.d)$, $\frac{4c^2}{a} = d$; which may be put under the form $\frac{4c^2}{a^2}a = d$, $\frac{4c^2}{a^2} = \pi^2$, $\pi^2.a = d$.

Take AM $= x$, PM $= y$, Fig. 3354, QN $= 2a$, arc NP $= \theta$,

$$x = AM = AN - NM = \theta - \sin.\theta = \text{versin.}^{-1}y - \surd(2ay - y^2),$$

the general expression for the normal is $y\sqrt{1 + \left(\frac{dy}{dx}\right)}$

$$dx = \left\{\frac{a}{\surd 2ay - y^2} - \frac{a - y}{\surd 2ay - y^2}\right\}dy = \frac{y}{y^{\frac12}(2a - y)^{\frac12}}dy,$$

$$\frac{dx}{dy} = \frac{y^{\frac12}}{(2a - y)^{\frac12}} \quad \frac{dy}{dx} = \frac{(2a - y)^{\frac12}}{y^{\frac12}} \quad \left(\frac{dy}{dx}\right)^2 = \frac{2a - y}{y},\quad 1 + \left(\frac{dy}{dx}\right)^2 = \frac{2a}{y}$$

$$\surd\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = \frac{(2a)^{\frac12}}{y^{\frac12}},\quad y\surd\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = \surd\overline{2ay}.$$

An expression for the radius of curvature is $\frac{(1 + p^2)^{\frac32}}{q}$; when $p = \frac{dy}{dx}$ and $q = \frac{d^2y}{dx^2}$

$$1 + p^2 = \frac{2a}{y},\quad (1 + p^2)^{\frac32} = \frac{(2a)^{\frac32}}{y^{\frac32}},\quad \frac{dy}{dx} = \frac{(2a - y)^{\frac12}}{y^{\frac12}}.$$

Take $z = 2a - y$; $\frac{dy}{dx} = \frac{z^{\frac12}}{y^{\frac12}}$, $\frac{d^2y}{dx} = \frac{\frac12 y^{\frac12} z^{-\frac12} dz - \frac12 z^{\frac12} y^{-\frac12} dy}{y}$. But $dz = -dy$,

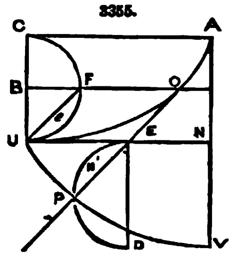
$$\frac{d^2y}{dx} = \frac{-\frac{1}{2}y^{\frac{1}{2}}z^{-\frac{1}{2}} - \frac{1}{2}z^{\frac{1}{2}}y^{-\frac{1}{2}}}{y}dy = \frac{-\frac{y^{\frac{1}{2}}}{2z^{\frac{1}{2}}} - \frac{z^{\frac{1}{2}}}{2y^{\frac{1}{2}}}}{y}dy = -\frac{2y+2z}{4yz^{\frac{1}{2}}y^{\frac{1}{2}}}dy.$$

Substituting value for z, $\frac{d^2y}{dx} = -\frac{2y+4a-2y}{4y(2a-y)^{\frac{1}{2}}y^{\frac{1}{2}}}dy = -\frac{4a}{4y^{\frac{3}{2}}(2a-y)^{\frac{1}{2}}}dy$; but

$dy = \frac{(2a-y)^{\frac{1}{2}}}{y^{\frac{1}{2}}}dx$. Substituting this value for dy, $\frac{d^2y}{dx} = -\frac{a}{y^2}dx, \frac{d^2y}{dx^2} = -\frac{a}{y^2} = q$. Substituting these values, $\frac{(1+p^2)^{\frac{3}{2}}}{q} = \frac{(2a)^{\frac{3}{2}}}{y^{\frac{3}{2}}} \cdot \frac{y^2}{a} = 2^{\frac{3}{2}}a^{\frac{1}{2}}y^{\frac{1}{2}} = 2\sqrt{2ay}$. Taking R to represent the radius of curvature and N the normal, we have $R = 2\sqrt{2ay}$, $N = \sqrt{2ay}$; therefore it is evident that in this curve the radius of curvature is equal to twice the normal; therefore at the vertex the radius of curvature is equal to twice the diameter of the generating circle.

The involute of a semi-cycloid A O U, Fig. 3355, is an equal semi-cycloid U P V in an opposite direction, the extremity of the base of the latter being in contact with the vertex of the former. From any point O draw O B parallel to A C, cutting the generating circle in F, and join F U; draw O P a tangent to the cycloid in O, and at E the point where it cuts the line U N, drawn from U, parallel to C A let fall E D perpendicular to U N and equal to C U. With E D as a diameter describe a circle intersecting the tangent O P in some point P. O E is equal and parallel to F U, p. 1749. O F is equal to the arc F e U. The circles C F U and D P E are equal by construction. The angles F U E, U E P are also equal, the chord being parallel to the tangent. The chords F U and E P are therefore equal, and as the angles they make with the common tangent to the circles at U and E are equal, the arcs F e U and P n E subtend equal angles, and are therefore equal. F O E U is a parallelogram, therefore U E is equal to F O. But F e U is equal to F O, therefore P n E is equal to U E. If the circle E P D had been placed on the line U N at U, and had rolled from U to E, the arc disengaged would have been equal to U E, and the point which was in contact with U would be at P in the periphery of a semi-cycloid U P V equal to A O U; the base line U N of the one being equal and parallel to the base line C A of the other; also the axis N V of the one equal and parallel to the axis C U of the other. And since the same may be shown to obtain with respect to any other point whatever in the arc A O U, the cycloid U P V is the involute of A O U.

3355.



To construct a pendulum which shall oscillate in any given cycloid whose base is parallel to the horizon.—Let V N represent the axis of the cycloid and diameter of the generating circle. Produce V N till V A equals 2 . V N. Through A draw a line A C parallel to N U, the semi-base of the given cycloid; then on A C as a semi-base with axis A N describe a semi-cycloid A O U, and in like manner describe another semi-cycloid turned the contrary way. Then if a pendulum be suspended by a flexible string to the point A, the length of the string being exactly equal to the line A V or arc A O U, which from the nature of the curve are equal to each other, the pendulum oscillating in the plane of the cycloids will in its motion come alternately into contact with the cycloidal cheek A O U and the one corresponding to it on the opposite side of the line A V, and will describe the cycloid of which N V is the axis and N U the semi-base.

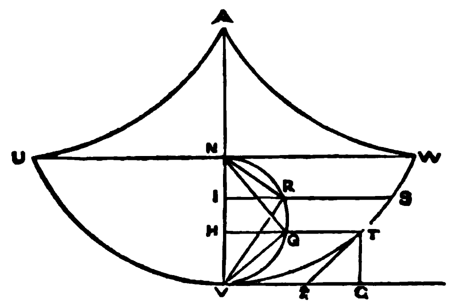
The length of a cycloidal pendulum vibrating seconds in any given latitude being ascertained, it will be evident that the diameter of the generating circle is also known, and therefore the quantity a in the equation $\pi^2 . a = d$ becomes a known quantity. The properties of the cycloid with reference to the vibrations of the pendulum are demonstrated upon the supposition that the whole mass of the pendulum is concentrated in a single point, but this cannot be assumed with reference to any vibrating body, for the centre of oscillation will not occupy the same place for any two points in the arc of vibration. This therefore in practice is a source of error.

Much time and trouble appear to have been expended in demonstrating the properties of the Isochrone. The name of this curve alone impresses one with a sort of reverence, and the investigations connected with the vibrations of pendulums and consequent determination of the force of gravity are highly interesting, and would no doubt have been found practically applicable, but for one slight drawback,—it was found that the exceedingly ingenious instrument by means of which the experiments were to be performed could not be made. The difficulties involved in the construction were so great that sufficient accuracy and durability could not be attained to render it practicably serviceable. All thoughts of making use of the cycloidal pendulum seem therefore to have been abandoned, and the theoretical demonstrations appear to have been placed on the same shelf with those well-known abstruse and ingenious investigations relative to the motion of projectiles in vacuo.

Having found out that the cycloidal pendulum would not do, we immediately turn to our old contrivance, based upon what ought now to be admitted as an axiom, namely, that all things which are very little are equal to each other, and that there is no difference to speak of between any of them. Upon the strength of this reasoning we invest the pendulum oscillating in a circular arc with all the properties of the cycloidal pendulum, upon the understanding that the arcs of

vibration are to be very small. That we are justified in doing this is supposed to be proved as follows;—Let U V W, Fig. 3356, be the cycloid in which a body is supposed to oscillate, V N = N A, A U = A W = A V = ½ U V W. The accelerating force in any point T of the curve has the same effect as if the body were placed upon the tangent T t; therefore the force of gravity is to the force in the direction T t as T t : T G, or by similar triangles as Q V : Q H, the tangent being parallel to the chord; or again, by similar triangles, as V N : V Q. In like manner, taking any other point S in the curve, the force of gravity is to the force in the curve as

3356.



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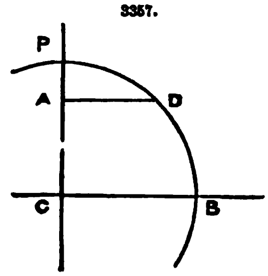


V R : V I or V N : V R. Therefore the accelerative force at different points T . S of the curve varies as the corresponding chords V Q . V R of the generating circle, or as the portions V T . V S of the curve measuring from the vertex, such portions of the curve being respectively double the corresponding chords. Therefore a body moving in a cycloidal arc is attracted by a force which varies directly as its distance from the lowest point V. We have therefore now to find expressions for the velocity and time under these circumstances. Let P, Fig. 3357, be the point from which a particle of matter commences its motion; take P C = a, its distance from the centre of force; let v = the velocity at any variable distance, A C = s, and take f to represent the force at some given distance l from C, compared with a unit of force which we will represent by m. As the intensity of the attractive force is directly proportional to the distance from the centre of force, $l : s :: f : \frac{fs}{l}$; therefore $\frac{fs}{l}$ is an expression for the force at the variable distance s compared with unity, and $\frac{m.f.s}{l}$ when measured by m. Substituting this value for f in the equation $v.dv = f.ds$, we have $v.dv = \frac{mf}{l}s.ds$; or, since v increases as s diminishes, $v.dv = -\frac{mf}{l}s.ds$. Integrating, $v^2 = -\frac{mf}{l}s^2 + C$. When $v = 0$, $s = a$; $0 = -\frac{mf}{l}s^2 + C$; $C = \frac{mf}{l}a^2$. The corrected integral is therefore $v^2 = \frac{m.f}{l}\{a^2 - s^2\}$, $v = \sqrt{\left\{\frac{mf}{l}(a^2 - s^2)\right\}}$. In the quadrant P D B described from centre C with radius C P, the ordinate A D = $\sqrt{(CP^2 - CA^2)} = \sqrt{(a^2 - s^2)}$; therefore $v = AD\sqrt{\frac{mf}{l}}$, from which we deduce the following;—The velocity corresponding to any space P A is proportional to the sine A D of the arc answering to the versed sine P A to radius C P.

To find an expression for the time.—Taking the expression $ds = v.dt$, which, as the velocity increases as the space diminishes, becomes $-ds = v.dt$, $-\frac{ds}{v} = dt = \sqrt{\frac{l}{m.f}}\,\frac{-ds}{\sqrt{(a^2 - s^2)}}$.

Take z to represent the arc P D,

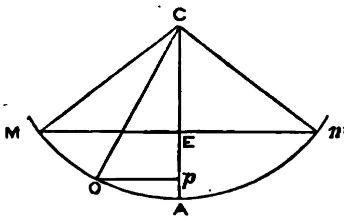
$$\text{sin. } z : R :: d.\text{ cos. } z : dz, \qquad \sqrt{(a^2 - s^2)} : a :: -ds : dz. \qquad \frac{-ds}{\sqrt{(a^2 - s^2)}} = \frac{dz}{a}$$

Therefore $dt = \frac{dz}{a}\cdot\sqrt{\frac{l}{m.f}}$. Integrating, $t = \frac{z}{a}\sqrt{\frac{l}{m.f}} = \frac{PD}{PC}\sqrt{\frac{l}{m.f}}$; for when $t = 0, z = 0$; when $AC = 0$, $t = \frac{PDB}{PC}\sqrt{\frac{l}{m.f}} = \frac{1}{2}\pi\sqrt{\frac{l}{m.f}}$. This is therefore an expression for the time in a semi-cycloidal arc, and $\pi\sqrt{\frac{l}{m.f}}$ for the full arc of vibration. l being the length of the semi-arc of vibration, and consequently the length of the pendulum; f being the representative of the force at a given distance l from the centre of force, and m being the representative of the unit of force, $\sqrt{\frac{l}{m.f}}$ is assumed as constant, the coefficient $\frac{1}{2}\pi$ is also constant; therefore t is constant for all the variable arcs represented by s. And this is supposed to prove the isochronism of vibration in the curve. mf expresses the intensity of force at the vertical point of the curve, which causes the pendulum to descend along the curve estimated at a distance from the centre of force = l, the length of the line of descent; but when referred to the descent through the length of the pendulum

from the point of vibration to the vertex, we must call it $2f$, according to the conventional rule, therefore the time of vibration $= \pi\frac{l}{2f}$.

We shall now find an expression for the time in the arc of vibration of the circular pendulum, in order to compare it with the expression first found, and thus ascertain the limit of the error involved in substituting the circular for the cycloidal pendulum. But we are sorry to say that $m.f$ will turn out to be our old friend g in a perfectly new disguise, for there is something mean in his insinuating himself into an experiment intended to test his own value.

3358.



Let M A m, Fig. 3358, be the circular arc in which a pendulum oscillates. Take the radius or length of the pendulum C M $= l$, A E $= b$, A $p = x$, $p.o = y$, and the variable arc $= s$. The velocity acquired by falling through the arc M o is equal to the velocity due to the vertical distance E p. Taking the equation $2fs = v^2$, we have $\sqrt{(2f.s)} = v$. Substituting E $p = (b-x)$ for s, $\sqrt{\{2f(b-x)\}} = v$; but $ds = v.dt$, $\frac{ds}{v} = dt$. Substituting value for v, $\frac{ds}{\sqrt{\{2f(b-x)\}}} = dt$.

The differential of an arc of a curve considered as a function of the ordinates of its extremities is expressed by $ds = \sqrt{(dy^2 + dx^2)}$; in the present case $y = (2lx - x^2)^{\frac12}$,

$$dy = \tfrac12(2lx - x^2)^{-\frac12}.d(2lx - x^2),\quad d(2lx - x^2) = 2l.dx - 2xdx = (l-x)2dx$$

$$dy = \tfrac12(2lx - x^2)^{-\frac12}(l-x)2dx = \frac{(l-x)dx}{\sqrt{(2lx - x^2)}},\quad dy^2 = \frac{(l-x)^2.dx^2}{(2lx - x^2)},$$

$$y^2 + dx^2 = \frac{(l-x)^2dx^2}{(2lx-x^2)} + dx^2 = \frac{(l-x)^2 + (2lx - x^2)}{(2lx - x^2)}dx^2 = \frac{l^2 - 2lx + x^2 + 2lx - x^2}{(2lx - x^2)}dx^2$$

$$= \frac{l^2.dx^2}{(2lx - x^2)},\quad \sqrt{(dy^2 + dx^2)} = \frac{l.dx}{\sqrt{(2lx - x^2)}},\quad ds = \frac{l.dx}{\sqrt{(2lx - x^2)}};$$

and because s diminishes as the time augments, $ds = \frac{-l\,dx}{\sqrt{(2lx - x^2)}}$. Substituting this value in the equation $\frac{ds}{\sqrt{2f(b-x)}} = dt$, $\frac{-l.dx}{\sqrt{\{(2lx - x^2).2f(b-x)\}}} = dt$. In order to obtain an expression for the time it only remains to integrate the first term of the above equation,

$$\frac{-l.dx}{\sqrt{2f(b-x).(2lx - x^2)}},\quad (b-x)x = b.x - x^2,\quad \frac{2lx - x^2}{x} = 2l - x.$$

Therefore the expression becomes

$$\frac{l}{\sqrt{2f}}\,\frac{-dx}{\sqrt{(bx - x^2)(2l - x)}} = \frac{l}{\sqrt{2f}}\,\frac{-dx}{\sqrt{(bx - x^2)}}(2l - x)^{-\frac12},$$

$$(2l - x)^{-\frac12} = 2l^{-\frac12}\left(1 - \frac{x}{2l}\right)^{-\frac12} = 2l^{-\frac12}\left\{1 + \frac12\,\frac{x}{2l} + \frac{1.3}{2.4}\,\frac{x^2}{4l^2} + \frac{1.3.5}{2.4.6}\,\frac{x^3}{8l^3}\cdots\right\},$$

$$\frac{-l.dx}{\sqrt{2f(b-x).(2lx - x^2)}} = \frac12\sqrt{\frac{l}{2f}}\cdot\frac{-dx}{\sqrt{(bx - x^2)}}\left\{1 + \frac12\,\frac{x}{2l} + \frac{1.3}{2.4}\cdot\frac{x^2}{4l^2}\cdots\right\}.$$

The question therefore resolves itself into the integration of a series of terms of the form $\frac{x^n.dx}{\sqrt{(bx - x^2)}}$, which, taking $b = 2a$, becomes $\frac{x^n.dx}{\sqrt{(2ax - x^2)}}$.

$$\int\frac{dx}{\sqrt{(2ax - x^2)}} = \frac1a.\text{versin.}^{-1}\,x \text{ to radius } a.$$

$$\int\frac{x.dx}{\sqrt{(2ax - x^2)}} = \int\frac{a.dx - xdx}{\sqrt{(2ax - x^2)}} + a\int\frac{dx}{\sqrt{(2ax - x^2)}} = \sqrt{(2ax - x^2)} + a\int\frac{dx}{\sqrt{(2ax - x^2)}},$$

$$\int\frac{x^2dx}{\sqrt{(2ax - x^2)}} = \int x^{\frac32}(2a - x)^{-\frac12}dx = 2x^{\frac32}.\sqrt{(2a - x)} + 3\int x^{\frac12}.\sqrt{(2a - x)}\,dx,$$

but $x^{\frac12}\sqrt{(2a - x)}\,dx$.

Multiply and divide by $x^{\frac12}(2a - x)^{\frac12} = \frac{2.a.x.dx}{\sqrt{(2ax - x^2)}} - \frac{x^2dx}{\sqrt{(2ax - x^2)}}$,

$$\int \frac{x^2 . d x}{\sqrt{(2 a x - x^2)}} = 2 x (2 a x - x^2) + 3 . 2 . a \int \frac{x d x}{\sqrt{(2 a x - x^2)}} - 3 \int \frac{x^2 . d x}{\sqrt{(2 a x - x^2)}},$$

$$4 \int \frac{x^2 d x}{\sqrt{(2 a x - x^2)}} = 2 x (2 a x - x^2) + 3 . 2 . a \int \frac{x . d x}{\sqrt{(2 a x - x^2)}},$$

$$\int \frac{x^2 d x}{\sqrt{(2 a x - x^2)}} = \tfrac{1}{2} x (2 a x - x^2) + \frac{3 . 2 . a}{4} \int \frac{x d x}{\sqrt{(2 a x - x^2)}},$$

$$\int \frac{x^3 . d x}{\sqrt{(2 a x - x^2)}} = \int x^{\frac{5}{2}} . (2 a - x)^{-\frac{1}{2}} . d x = 2 x^{\frac{5}{2}} \sqrt{(2 a - x)} + 5 \int x^{\frac{3}{2}} \sqrt{(2 a - x)} d x,$$

but $x^{\frac{3}{2}} \sqrt{(2 a - x)} d x.$

Multiply and divide by $x^{\frac{1}{2}} \sqrt{(2 a - x)} = \dfrac{2 a x^2 . d x}{\sqrt{(2 a x - x^2)}} - \dfrac{x^3 d x}{\sqrt{(2 a x - x^2)}},$

$$\int \frac{x^3 d x}{\sqrt{(2 a x - x^2)}} = 2 x^2 \sqrt{(2 a x - x^2)} + 5 . 2 . a \int \frac{x^2 . d x}{\sqrt{(2 a x - x^2)}} - 5 \int \frac{x^3 . d x}{\sqrt{(2 a x - x^2)}},$$

$$6 \int \frac{x^3 . d x}{\sqrt{(2 a x - x^2)}} = 2 x^2 \sqrt{(2 a x - x^2)} + 5 . 2 . a \int \frac{x^2 . d x}{\sqrt{(2 a x - x^2)}},$$

$$\int \frac{x^3 . d x}{\sqrt{(2 a x - x^2)}} = \tfrac{1}{3} x^2 \sqrt{(2 a x - x^2)} + \frac{5 . 2 . a}{6} \int \frac{x^2 . d x}{\sqrt{(2 a x - x^2)}}$$

We find therefore that

$$\int \frac{d x}{\sqrt{(2 a x - x^2)}} = \frac{1}{a} . \text{versin.}^{-1} x \text{ to radius } a, \qquad \int \frac{x d x}{\sqrt{(2 a x - x^2)}} = \sqrt{(2 a x - x^2)} + a \int \frac{d x}{\sqrt{(2 a x - x^2)}},$$

$$\int \frac{x^2 d x}{\sqrt{(2 a x - x^2)}} = \tfrac{1}{2} x (2 a x - x^2) + \frac{3 . 2 . a}{4} \int \frac{x . d x}{\sqrt{(2 a x - x^2)}},$$

$$\int \frac{x^3 . d x}{\sqrt{(2 a x - x^2)}} = \tfrac{1}{3} x^2 \sqrt{(2 a x - x^2)} + \frac{5 . 2 . a}{6} \int \frac{x^2 d x}{\sqrt{(2 a x - x^2)}}.$$

This will be sufficient to show the law of the series, and substituting b for $2 a$, we have

$$\int \frac{d x}{\sqrt{(b x - x^2)}} = \frac{2}{b} \text{ versin.}^{-1} x \text{ to radius } \frac{b}{2}, \qquad \int \frac{x d x}{\sqrt{(b x - x^2)}} = \sqrt{(b x - x^2)} + \frac{b}{2} \int \frac{d x}{\sqrt{(b x - x^2)}},$$

$$\int \frac{x^2 d x}{\sqrt{(b x - x^2)}} = \tfrac{1}{2} x (b x - x^2) + \frac{3 . b}{4} \int \frac{x d x}{\sqrt{(b x - x^2)}},$$

$$\int \frac{x^3 d x}{\sqrt{(b x - x^2)}} = \tfrac{1}{3} x^2 \sqrt{(b x - x^2)} + \frac{5 . b}{6} \int \frac{x^2 d x}{\sqrt{(b x - x^2)}}.$$

It will be evident that at the commencement of the arc of vibration the time must equal 0, therefore when $x = b$, $\displaystyle\int \frac{d x}{\sqrt{(b x - x^2)}} = \pi$, for R − cos. = versin., cos. 180° = − 1.

$$\begin{aligned} \text{R} &= 1 \\ \text{cos. } 180 &= -1 \\ \hline 2 &= \text{versin. } 180^\circ. \end{aligned}$$

$$\frac{2}{b} \text{ versin.}^{-1} b \text{ to radius } \frac{b}{2} = \pi,$$

$$\int \frac{x d x}{\sqrt{(b x - x^2)}} = \frac{1 . b}{2} \pi, \qquad \int \frac{x^2 d x}{\sqrt{(b x - x^2)}} = \frac{1 . 3 . b^2}{2 . 4} \pi, \qquad \int \frac{x^3 d x}{\sqrt{(b x - x^2)}} = \frac{1 . 3 . 5 b^3}{2 . 4 . 6} \pi.$$

It will be evident that when $x = b$ all terms containing $(b x - x^2)$ as a factor vanish.

It will therefore be evident that integrating between $x = b$ and $x = 0$, and substituting the values of the above integrals in the series, $\frac{1}{2}\sqrt{\frac{l}{2f}}\frac{-dx}{\sqrt{(bx - x^2)}}\left\{1 + \frac{1}{2}\frac{x}{2l} + \frac{1.3}{2.4}\cdot\frac{x^2}{4l^2}\ldots\right\}$, we get $t = \frac{1}{2}\pi\sqrt{\frac{l}{2f}}\left\{1 + \frac{1^2}{2^2}\cdot\frac{b}{2l} + \frac{1^2.3^2}{2^2.4^2}\cdot\frac{b^2}{4l^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2}\cdot\frac{b^3}{8l^3}\ldots\right\}$. This expresses the time of descent through half the arc of vibration, but with the velocity acquired during the descent the pendulum would proceed along an equal branch of the curve, its velocity being supposed to be extinguished after a lapse of time equal to the time of descent; therefore the time of a complete vibration will be expressed by $t = \pi\sqrt{\frac{l}{2f}}\left\{1 + \frac{1^2}{2^2}\cdot\frac{b}{2l} + \frac{1^2.3^2}{2^2.4^2}\frac{b^2}{4l^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2}\cdot\frac{b^3}{8l^3}\ldots\right\}$. Therefore, upon the supposition that the time of vibration in the cycloidal arc is expressed by $t = \pi.\sqrt{\frac{l}{2f}}$, it will appear that the error incurred by substituting the circular for the cycloidal pendulum is expressed by a series of the powers of the versin. of the same angular value as the arc of vibration, having unity for its radius. It is argued that as the versines of small angles are exceedingly minute, the series will converge rapidly, and the error may be neglected in practice; or at least all the terms but the first or second may be neglected, and corrections applied accordingly. Assuming therefore that the circular pendulum may be substituted for the cycloidal pendulum, without producing sensible error, and taking the equation $\pi^2.a = d$, $3{\cdot}14159^2 = \pi^2 = 9{\cdot}8696$, $9{\cdot}8696\,a = d$. It has been found by experiment that the length of a pendulum vibrating seconds in the latitude of London $= 39{\cdot}125$. The length of the cycloidal pendulum is equal to twice the diameter of the generating circle, therefore $39{\cdot}125 = 2.a$, $19{\cdot}562 = a$, $9{\cdot}8696 \times 19{\cdot}562 = 193{\cdot}0623$, $193{\cdot}0623$ in. $= 16{\cdot}088$ ft. It will therefore be evident that the value given to d, from which the value of g is supposed to be derived, depends upon the length of a pendulum vibrating seconds in a particular latitude, in a circular arc which is assumed to be a cycloidal arc, because it is very small, and this is to be ascertained by experiment. This is the experiment which is generally mysteriously alluded to in the following words;—

"It has been ascertained by accurate experiments with the pendulum and by other means, that in the latitude of London a heavy body falling freely from a state of rest will describe a space of $16\frac{1}{12}$ ft. during the first second of the descent, and will have generated a velocity of $32\frac{1}{6}$ ft. a second," &c., &c.

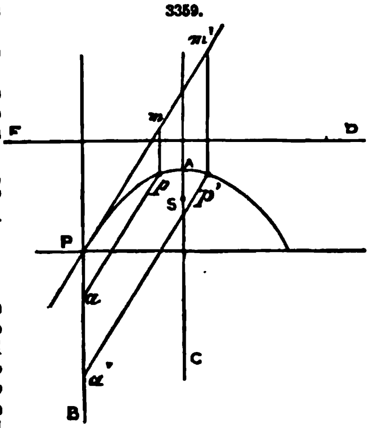
The method of determining the space descended during one second, said to have been suggested by Galileo by means of experiments with reference to the descent of bodies upon inclined planes, in consequence of the friction on the planes lead to no practically useful results.

The experiments made by means of Atwood's machine, a description of which will be found p. 7, are not sufficiently accurate for practical purposes.

We have gone into detail with reference to the apparent mixture of reasoning and arbitrary assumption upon which the laws of gravity seem to be founded, for it is upon the validity of these laws that the whole theory of gunnery as it now stands depends. If there is a flaw in this line of argument we should fail in arriving at practically useful results, even though we should be successful in determining accurately the law of resistance of the air to a body moving through it, to which point alone attention appears to be at present directed. We shall now exhibit as shortly as possible the most important of the principles which are supposed to constitute the present system of gunnery.

If a body be projected with a given velocity in a given direction from any given point of departure, it is considered that the body, if uninfluenced by any disturbing force, will proceed continually in the given direction with a uniform velocity equal to the initial velocity originally impressed upon it. The velocity being uniform, the spaces described on the line of direction will be proportional to the times. According to the conventional rule, the spaces described by a heavy body subject to the attractive force of gravity upon a line perpendicular to the horizon are proportional to the squares of the times. Admitting the above statements as true, and combining them together, we are in a position to define the curve of trajectory

3359.



Amongst the properties of the common parabola we find the following;—1. The ordinates to all diameters are parallel to the tangent at the vertex. 2. The abscisses are proportional to the squares of the semi-ordinates.

If therefore we take P B a diameter of any parabola, of which A, Fig. 3359, is the vertex, A C the axis, F D the directrix, s the focus, $ap^2 : a'p'^2 :: Pa : Pa'$; and completing the parallelograms we have

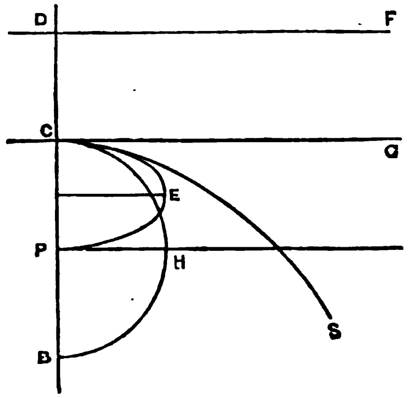
$$Pm^2 : Pm'^2 :: mp : mp.$$

If therefore we take Pm and Pm' to represent the spaces which would be described with a uniform velocity on the tangential line during any given times of the transit t and t', mp and $m'p'$ will evidently represent the spaces due to the action of gravity during the same time; therefore p and p' will represent the position of the projectile at the end of the times t and t', and as this will hold good for all distances which may be assumed on the tangential line, we conclude that the trajectory of the projectile, when subject only to the influence of the propelling force and the force of gravity, is the parabolic curve.

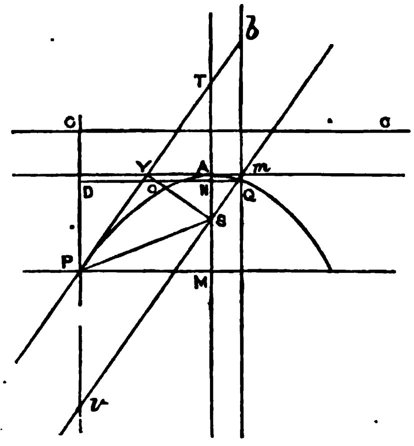
According to the laws of gravity, a body in its descent through any vertical distance will generate a velocity which, acting uniformly, would cause the same body to describe a distance equal to double the space descended in a time equal to the time of descent. The height due to any velocity is therefore taken as the exponent or measure of the initial velocity equal to the velocity generated.

Take the vertical line C P to represent the height due to any given initial velocity. The horizontal line C G, Fig. 3360, through the point C at right angles to the line C P will be the directrix; the semicircle C H B described with radius C P from centre P will be the locus of the foci; and the semi-ellipse C E P upon C P as conjugate axis, and of which the transverse is double the conjugate, the locus of the vertices of all the parabolæ representing the trajectories of a projectile impressed with the given initial velocity at the point P whatever may be the original direction given

3360.



3361.



to the projectile. And, lastly, the parabolic curve C S described with focus P, and directrix D F, vertex C, will be the locus of the extremities of the greatest ranges attained with the given initial velocity. These points are proved as follows;—

1. The horizontal line C G through the point C will be the directrix. Take P b, Fig. 3361, to represent the tangential line of direction P v equal to C P, and complete the parallelogram P b . Q v, C P = P v = b Q; therefore P b represents the space described in the tangential direction with a uniform velocity equal the initial velocity impressed in the time during which the initial velocity has been generated by the descent through C P = b Q. Therefore, according to the conventional rule, $Pb = 2\,CP$, $Qv = 2\,Pv$, $Qv^2 = 4\,Pv^2 = 4\,Pv . CP$, $\frac{Qv^2}{4\,Pv} = CP$. Take P A Q to represent the curve of trajectory corresponding to the tangential direction P b, S the focus, A the vertex. Let S Y be a perpendicular to the tangent P b, and join S P. It will be evident that if it can be shown that S P is equal to C P (for which we have already obtained an expression), the proposition will be proved, as the perpendicular from the point P in the curve upon the horizontal line C G will be equal to the distance of the same point P from the focus, which is a property of the parabolic curve.

The tangent at the vertex A is perpendicular to the axis A M, and the perpendicular from the focus S upon the tangent at the point P is a mean proportional between S P and S A; therefore the right-angled triangles S P Y and S A Y are similar.

The right-angled triangles S A Y . S m Y are similar, having a common angle at Y. The right-angled triangles S m Y . S Q O are similar, having a common angle at S. The right-angled triangles S Q O . Q D v are similar, having a common angle at Q; therefore the triangles S P Y and Q D v are similar, and $Qv^2 : QD^2 :: SP^2 : SY^2$; but $SY^2 = SP . SA$, $\therefore Qv^2 : QD^2 :: SP : SA$.

The right-angled triangles Q D v and P M T are similar, the opposite angles v and T in the parallelogram P v T S being equal, therefore

$$QD : Dv :: PM : MT$$
$$:: PM^2 : PM . MT.$$

Substituting $4\,AS . AM$ for PM^2, and $2\,AM$ for MT, we have

$$QD : Dv :: 4AS . AM : PM . 2AM$$
$$:: 4AS : 2PM;$$

therefore $4AS . Dv = 2PM . QD$, $PM^2 = 4AS . AM$, $QN^2 = 4AS . AN$.

$$PM^2 - QN^2 = 4AS . AM - 4AS . AN = 4AS(AM - AN) = 4AS . MN = 4AS . DP,$$

$$PM^2 - QN^2 = (PM + QN)(PM - QN), \text{ but } PM + QN = QD;$$

$$\therefore PM^2 - QN^2 = (PM - QN)\,QD.$$

$$4AS . DP = PM^2 - QN^2 = (PM - QN)\,QD, \quad 4AS . DP = (PM - QN)\,QD.$$

Subtracting this equation from the equation $4\,AS.Dv = 2\,PM.QD$,

$$\begin{aligned} 4\,AS.Dv &= 2\,PM.QD \\ 4\,AS.DP &= (PM - QN)\,QD \\ \hline 4\,AS.Pv &= (PM + QN)\,QD = QD^2 \\ 4\,AS.Pv &= QD^2. \end{aligned}$$

Substituting this value for QD^2 in the proportion $Qv^2 : QD^2 :: SP : SA$, we have

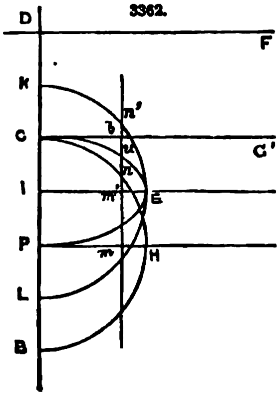
$$Qv^2 : 4\,AS.Pv :: SP : AS,\ Qv^2 = 4\,SP.Pv,\ \frac{Qv^2}{4\,Pv} = SP,\ CP = \frac{Qv^2}{4\,Pv} = PS;$$

therefore $CP = SP$, and consequently the line CG is the directrix of the parabola; and as this will hold good whatever direction the tangential line Pb may take, the proposition is proved.

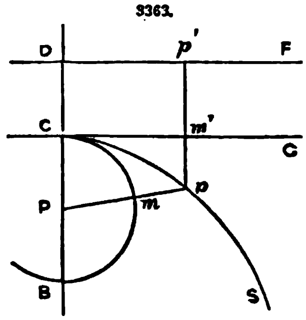
2. The semicircle C H B described with radius C B from centre P will be the locus of the foci. This follows as a corollary to the former proposition; for the foci being always at a distance from the point P in the curve equal to the perpendicular upon the directrix from the same point, the foci must necessarily lie in the periphery of the semicircle described with P as a centre and C P as radius.

3. The semi-ellipse C E P, Fig. 3362, upon C P as conjugate axis, and of which the transverse is double the conjugate, is the locus of the vertices; bisect C P in I, with I as centre and a distance equal C P as radius describe the semicircle K E L. $m'v : m'n' :: IC : IE :: 1 : 2$, $m'n' = 2\,m'v$, $\therefore m'v = vn'$, $mn = m'n'$, $mm' = m'b$, $\therefore m'n = bn'$, $m'v - m'n = vn' - bn'$, $nv = vb$; and as CG is the directrix and n the focus, v must evidently be the vertex; and as this will hold good for any other point in the periphery of the semicircle CHB, which may be assumed as the focus of the curve of trajectory, the proposition is proved.

3362.



3363.



4. The parabolic curve CS, Fig. 3363, described with focus P and directrix D F, vertex C, will be the locus of the extremities of the greatest ranges attained with the given initial velocity.

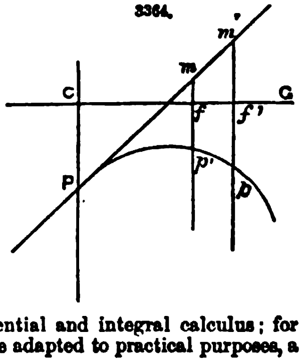
It has been already shown that the semicircle C m B described with CP as radius and P as centre is the locus of the foci of all the parabolæ representing the trajectory of a projectile discharged from the point P with the initial velocity represented by the descent through C P.

Let m be the focus of any one of these parabolæ; join Pm, and produce the connecting line to meet the parabola C S in p, and draw pp' perpendicular to D F; then, as D F is the directrix and P the focus of the parabola CS, $Pp = pp'$; but $Pm = PC = CD = p'm'$; therefore

$$Pp - pm = pp' - p'm', \quad mp = pm'.$$

Therefore, as it has been shown that CG is the directrix of the parabola of which m is the focus, p must be a point in the curve of that parabola; and since the tangent of such parabola as well as the tangent of the parabola C S at the common point p bisect the same angle Ppp', they must coincide. Consequently the two parabolæ having a common tangent at the point p touch each other at that point; and as this is true for every point in the semicircle CmB, it follows that the curves of all the trajectories of a projectile discharged with the given velocity from the point P will touch the concavity of the parabola C S, and lie wholly within it. No point without the parabola C S can be struck while the initial velocity remains unchanged; for if the elevation be increased, the focus of the parabola which the body would describe will be on the portion Cm of the circumference of the semicircle CmB, and the trajectory will touch the parabola CS in some point between C and p, and being wholly within the parabola C S, it must intersect the line Pp in some point nearer to the initial point P than p. If the elevation be diminished, the curve of trajectory will touch the parabola CS in some point below p, and will therefore intersect Pp in some point nearer to P than p.

3364.

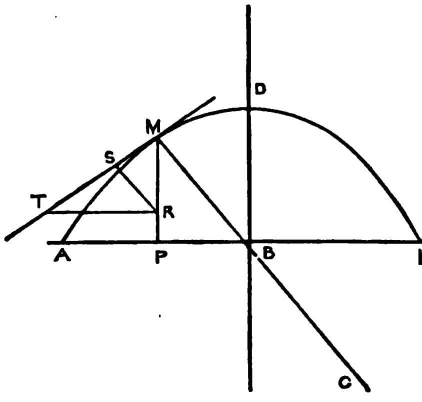


The times of describing any given portions $Pp.pp'$ of the curve are as the corresponding parts $Pm.mm'$, Fig. 3364, of the tangent or the intercepted parts $Cf.ff'$ of the directrix; for according to the original supposition $t.Pp = t.Pm$ and $tpp' = tmm'$; and because the directrix cuts the three parallels $PC.pm.p'm'$, $Pm : mm' :: Cf : ff'$. So far the subject may be most clearly illustrated under the form of geometrical reasoning; but in order to deduce practical formulæ adapted to actual calculation and comparison, we must again avail ourselves of the facilities afforded by the rudimental portion of the differential and integral calculus; for independently of the necessity of constructing algebraical formulæ adapted to practical purposes, a

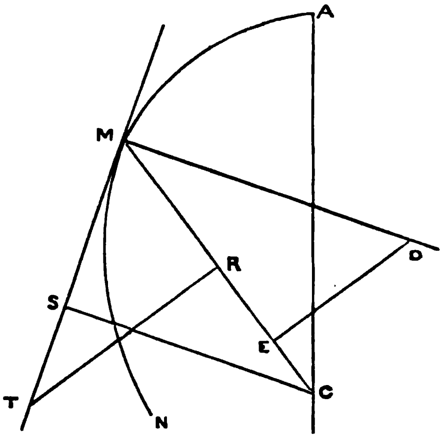
process of reasoning and a consequent result can be thus exhibited under the form of a few well-known symbols, which would require pages of geometrical reasoning to demonstrate.

Take the parabolic curve A D K, Fig. 3365, of which A K is the base and D B the axis, to represent the trajectory of a shot discharged from the point A with a given initial velocity. Let the ordinate M P be perpendicular to the horizontal base A K, M T a tangent to the curve at the point M. Take M P $= y$, A P $= x$, and A the origin of the co-ordinates. From any point R in the ordinate M P draw the perpendicular R S to the tangent M T; draw R T perpendicular to the ordinate to meet the tangent in T, the triangle M T R may be taken to represent the differential triangle. Then the force f in the direction M P is to the effect in the direction R S perpendicular to the tangent as M R : R S; or, by similar triangles, as M T : T R :: $dz : dx$; therefore $\frac{dx}{dz} f$ will express the force in a direction perpendicular to the tangent. Take C M to represent the radius of curvature at the point M, and let C M $= r$; then, as C M : M T as the differential of the circular arc described with C M as radius is to the differential of the circular arc described with M T as radius, we have $r : dz :: dz : \frac{dz^2}{r}$; but the differential of the arc described with M T as radius may be taken as representing the second differential of the arc described with C M as radius; we may therefore assume $d^2 z = \frac{dz^2}{r}$. Taking the original equations $dv = f\,dt$,

3365.



3366.



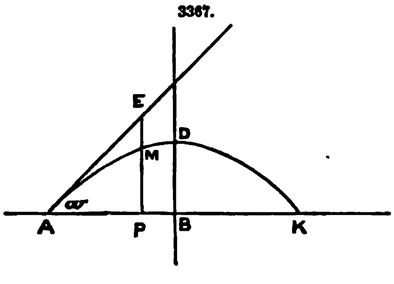
$dz = v\,.\,dt$, differentiating the last expression upon the supposition that dt is constant, we get $d^2 z = dv\,.\,dt$; substituting value for dv, $d^2 z = f\,.\,dt^2$, and substituting value for f, so as to render the expression applicable to the present case, $d^2 z = \frac{dx}{dz} f\,dt^2$, $\frac{dx}{dz} f\,dt^2 = d^2 z = \frac{dz^2}{r}$, $dx\,f\,dt^2 = \frac{dz}{r}$, $dz = v\,dt$, $dz^2 = v^2\,dt^2$; therefore $f\,dx\,r = v^2\,dz$. We shall now find a value for r applicable to the present case. Take D M $= r$ the radius of curvature at the point M of the curve A M N, Fig. 3366, M T the tangent at the same point, C M the radius vector $= y$, C S a perpendicular upon the tangent from the centre C = S; the lines s and r will evidently remain parallel for every point in the curve. Take dz to represent the differential of the circular arc described with radius r, and ds the differential of the circular arc described with radius M S; D M : M S :: $dz : ds$. Take M T R to represent the differential triangle, M S : M C :: M R : M T. Therefore we have r : M S :: $dz : ds$, M S : $y :: dy : dz$. Compounding, $r : y :: dy : ds$, $r\,.\,ds = y\,.\,dy$, $\frac{r}{dy} = \frac{y}{ds}$; T M : T R :: C M : C S, $dz : dx :: y : s$, $dz\,.\,s = dx\,.\,y$. Considering the lines of attraction parallel is equivalent to considering the centre of force C removed to an infinite distance, and consequently the radii vectors represented by y infinite, and consequently constant upon the supposition that the velocity in the direction of the curve is uniformly variable, and consequently dz proportional to the equal increments of time, dz may also be taken as constant. Differentiating under these conditions, we have $dz\,.\,ds = d^2 x\,y$, $\frac{dz}{d^2 x} = \frac{y}{ds}$, $\frac{r}{dy} = \frac{y}{ds} = \frac{dz}{d^2 x}$, $r = \frac{dy\,.\,dz}{d^2 x}$. Therefore the equation $f\,dx\,r = v^2.\,dz$ becomes $f\,dx\,\frac{dy\,.\,dz}{d^2 x} = v^2.\,dz$, $f.\,dx\,.\,dy = v^2\,d^2 x$. Comparing this with the equation $-f\,dy = v\,.\,dv$ already established, and eliminating f, we get $\frac{v^2.\,d^2 x}{dx\,.\,dy} = f = -\frac{v\,.\,dv}{dy}$, $v\,.\,d^2 x = -dv\,.\,dx$, $v\,.\,d^2 x + dv\,.\,dx = 0$. Integrating, $v\,.\,dx = C$, which we may put in the form $v\,.\,dx = A\,.\,dz$, dz having been treated as a constant during the investigation.

If we take w to represent the angle at the origin when the co-ordinates are rectangular, we evidently have the following proportion, $\cos.\,w : 1 :: dx : dz$, $\cos.\,w\,.\,dz = dx$. Taking c to represent the initial velocity, $c\,.\cos.\,w\,.\,dz = C$, $v\,.\,dx = c\,.\cos.\,w\,.\,dz$. Comparing this with

$dz = v.dt$, we get $dx = c.\cos.w.dt$; and integrating, $x = c.\cos.w.t$, $\frac{x}{c.\cos.w} = t$. Take $\sin.w = a$, $\cos.w = b$, $\frac{x}{c.b} = t$, $\frac{x^2}{c^2.b^2} = t^2$.

Produce the ordinate M P, Fig. 3367, to meet the tangent A E in E. E M will evidently represent the descent due to the attraction during the transit of a projectile discharged from the initial point A along the tangential line A E to the point E. In finding an expression for $MP = y$ in terms of $AP = x$ and known quantities, we might have some difficulty in reducing our formulæ to manageable dimensions; but our familiar spirit g comes to our assistance in his character of $2.d$, assuming for the time the title of his progenitor, and all the difficulties at once disappear, for according to the conventional rules $2fs = v^2$; and taking d to represent the descent during the first second, $2d$ not only represents the generated velocity at the end of the first second, but also upon all other occasions the quantity indicated by f. Substituting $2d$ for f, $2.2.d.s = v^2$, $4ds = v^2$. Taking l to represent the height due to the initial velocity c, we get $4.d.l = c^2$.

3367.



If we take t to represent the time of the transit from A to E and the contemporaneous descent through E M, and referring to the law that "the spaces are as the squares of the times,"

$$1 : t^2 :: d : EM.$$

Substituting value for t^2, $1 : \frac{x^2}{c^2.b^2} :: d : EM$, $\frac{d.x^2}{c^2.b^2} = EM$; $b : a :: x : EP$, $\frac{ax}{b} = EP$;

$$\{EP - EM\} = MP = y = \frac{ax}{b} - \frac{dx^2}{c^2.b^2}.$$

Substituting value for c^2, $y = \frac{a.x}{b} - \frac{x^2}{4l.b^2}$. It will be evident that when y becomes 0, x will represent the horizontal range, and $\frac{ax}{b} = \frac{x^2}{4l.b^2}$, $4abl = x$.

To find the value of x corresponding to the maximum value of y, or highest point of the trajectory, we must evidently make $dy = 0$, $0 = \frac{adx}{b} - \frac{2xdx}{4lb^2}$, $\frac{adx}{b} = \frac{2x.dx}{4lb^2}$, $2a.b.l = x$; x is therefore in this case equal to half the range. Substituting this value for x in the equation $y = \frac{ax}{b} - \frac{x^2}{4lb^2}$, we get $DB = \frac{2a^2bl}{b} - \frac{4a^2b^2l^2}{4l.b^2} = 2a^2l - a^2l = a^2l$. This is therefore the value for the ordinate of the highest point of the trajectory. As $\sin.2.w = 2.a.b$, and as $2l$ represents the initial velocity, and $4a.b.l = 2ab.2l$ represents the range, it will be evident that the horizontal ranges with the same projectile velocity are as the sines of an angle equal to twice the angle of elevation. The horizontal range will be greatest with a given projectile force when the angle of elevation is 45°; for in this case $a = b$, and their product in the equation $4a.b.l = x$ will be a maximum. Also all ranges obtained at elevations at equal angles above or below 45° are equal; for the sine in one case becomes the cosine in the other, and *vice versâ*. Oblique ranges (that is, when the object is above or below the level of the battery) may be obtained as follows:—

Take t to represent the tangent and s the secant of the angle of elevation, t' the tangent of the angle of elevation or depression of the object above or below the level of the battery,

$$1 : t' :: x : y = t'.x.$$

Giving this value to y in the equation $y = \frac{ax}{b} - \frac{x^2}{4l.b^2}$, we get

$$\pm t'x = \frac{ax}{b} - \frac{x^2}{4l.b^2}, \quad \pm t' = \frac{a}{b} - \frac{x}{4l.b^2} = \frac{a}{b} - \frac{1}{b^2}\,\frac{x}{4l},$$

$$\frac{a}{b} = t, \quad \frac{1}{b^2} = s^2, \quad \pm t' = t - s^2\frac{x}{4l}, \quad s^2\frac{x}{4l} = t \mp t', \quad x = \{t \mp t'\}\,\frac{4l}{s^2}.$$

This has all come out very smoothly and easily, thanks to the kind assistance of our unalterable friend g; and it would be quite satisfactory, but provokingly enough the shot and shells will not conform to the theory; on the contrary, when they ought to go about twenty miles they collapse, and come to a stop at about a tenth of the distance. This is, to say the least of it, very annoying, after all the trouble we have had in deducing the formulæ. We try to comfort ourselves by saying it is all in the air, and if there was no atmosphere it would be all right. But no one who has ever thrown a stone or shot with a bow and arrow can look at the last figure, or consider the calculation showing that the highest point of the trajectory is over the centre point of the range, without the conviction forcing itself upon him that whatever the curve of the trajectory may be, that is not it.

It is true that we do not as a rule throw stones or practise archery in vacuo; but there is an instinct more reliable than abstract reasoning which tells us that, even leaving out the consideration of the resistance of the air, the true curve should bear a greater analogy to the observable trajectory in nature than the one exhibited to us under the influence of g, who, like all familiar spirits, has the power of making all things easy and pleasant for the present, but can only bring us to infinite trouble and error at the end. This is the theory of the trajectory of projectiles in vacuo, founded upon the laws of gravity, which has been so long received with implicit faith, and only considered inapplicable in practice because the true correction for the resistance of the air has not as yet been discovered.

We shall now consider the theory proposed with reference to the resistance of the air, and in the first place we shall say a few words on the subject of what is commonly called terminal velocity. Upon the supposition that g (the force of gravity, as it is called) remains constant and never changes during the descent of a falling body, although the force of resistance is supposed to augment by successive increments at the end of equal intervals of time, it is assumed that if the descent is continued long enough there must arrive a period at which the aggregate of all the increments of resistance shall equal the force of gravity or g. But it is also supposed that the velocity has gone on constantly increasing during the descent; therefore it is again assumed that at the moment the force of gravity is neutralized by the generated resistance, the greatest velocity attainable by the falling body must have been reached. It is then generally stated that from that moment the body will continue to descend with a uniform velocity equal to the velocity attained. But this is difficult to understand, for the supposition seems to be that at some particular moment of the descent the constantly accumulating force of resistance has reached a degree of intensity which will compensate the impulse of attraction received by the falling body at that moment.

It would therefore seem that there was no further cause for the descent; for, according to the original supposition, in order to receive another impulse of attraction the body should descend during another second of time. But even supposing the possibility of the descent during the succeeding second, the body would then only descend with a constantly augmenting velocity till the impulsive force was again compensated by the resistance. But in order to support the supposition that the velocity is a constantly augmenting velocity, we must also suppose that the successive increments of velocity exceed the corresponding increments of resistance, otherwise it would be either a uniform velocity or else a constantly diminishing velocity. But, on the other hand, unless we suppose that the increments of resistance successively exceed the corresponding increments of velocity, we cannot establish our right to suppose that a period of the descent must arrive when the aggregate of the differences of the increments shall compensate the original balance to the credit of the velocity represented by g. But the whole matter has been rendered so intricate by the anomalous assumption that the velocity increases while the force of which it is the exponent remains constant, that it is impossible to deal with it according to the usual course of argument; and as our present object is simply to exhibit the theory with relation to the correction to be applied for the resistance of the air as it stands at present, we must only assume the generally-received supposition that when the resistance has become equal to the force of gravity (g) the falling body will have attained its greatest velocity, merely remarking, as we pass on, that we have arrived at this conclusion by supposing an absurdity

Take R to represent the resistance; then, according to the generally-received doctrine, when a falling body has attained its greatest velocity in a resisting medium $R = g$. Referring to the equations $dv = f.dt$; $ds = v.dt$; $\frac{dv}{f} = dt = \frac{ds}{v}$; $f.ds = v.dv$. Integrating upon the supposition that f is constant, $2.s.f = v^2$. f is in this case a constant quantity and is expressed by g. Taking r to represent $2s$ and substituting, we ha $rg = v^2$; $g = \frac{v^2}{r}$; $R = g = \frac{v^2}{r}$.

We have thus obtained an expression for the resistance in terms of the velocity and the space through which the body has descended in generating the greatest velocity attainable in the medium through which it moves. But it will be observed that this expression has been obtained by the adoption of the conventional rules, that in the first place f is invariable, and in the second that during the descent the spaces are proportional to the squares of the velocities. It is generally supposed that during the movement of a body through a resisting medium, the resistance acts only on the line of motion, any lateral pressure or action being compensated or neutralized by a corresponding action in the contrary direction. Assuming this supposition to be correct, we shall have only to consider the action of the resisting force on the line of direction of the moving body.

Take the ordinates of the curve A M, Fig. 3368, to represent the successive measures of the motive force, while the corresponding ordinates of another curve A N represent the successive measures of the force of resistance, the abscissa A P expressing the time from the commencement of the motion. It will be evident that the area A M P expresses the sum of all the motive forces during the time represented by the abscissa A P; also that the area A N P expresses the sum of all the forces of resistance from the beginning of the motion during the same time. It follows that the difference represented by the area A M N will express the intensity of the force which generates the actual velocity of a body moving in a resisting medium.

Take y to represent the variable ordinate of the curve A M and y' the ordinate of the curve A N, x the abscissa, a the area enclosed by the curve and ordinates $x.y$, and a' the area enclosed by the curve and ordinates $x'.y'$. Taking the common expression for the differential of an area bounded by a curve related to rectangular co-ordinates,

$$da = y.dx, \quad da' = y'.dx, \quad da - da' = \{y - y'\}.dx$$

Consequently the differential of the velocity of a body moving through a resisting medium is equal to the product of the difference of the measure of the motive force and the resistance, and the differential of the time elapsed, $dv = \{f - R\}.dt$.

Take C, Fig. 3369, the centre of force; $CM = y$, the radius vector; $CS = s$, a perpendicular from the centre upon the tangent at any point M of the curve A M; T M R, the differential triangle, whose sides may be considered as small as we please. The force in the direction C M is to the force in the direction of the tangent as C M : M S. But T M : M R :: C M : M S, $dz : dy :: f : \frac{dy}{dz} f$; therefore $\frac{dy}{dz} . f$ expresses the force in the direction of the tangent. Substituting this value for f in the equation

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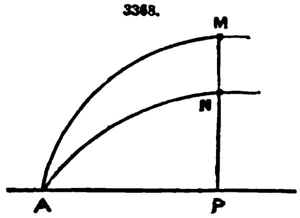

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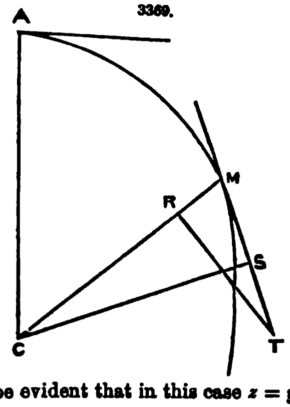

$$dv = \{f - \mathrm{R}\} . dt,$$

we get $dv = \left\{\frac{dy}{dz} f - \mathrm{R}\right\} . dt$; but $dz = v . dt$; $\frac{dz}{v} = dt$. Substituting this value for dt, $v . dv = dy . f - \mathrm{R} . dz$. We shall now apply this formula to the descent of a falling body. It will be evident that in this case $z = y$, and it has been shown that $\mathrm{R} = \frac{v^2}{r} = f$. We shall also, in order to simplify the calculation, take f equal to unity; g in this case representing f. Therefore $\frac{v^2}{r} = g = 1$; $r = v^2$. Making these substitutions, the expression $v . dv = dy . f - \mathrm{R} . dz$ becomes $v . dv = dy - \frac{v^2}{r} dy = \frac{r - v^2}{r} dy$; $\frac{r . v . dv}{r - v^2} = dy$. Therefore $y = \int \frac{r . v . dv}{r - v^2} = -\frac{1}{2} r \log . (r - v^2) + \mathrm{C}$. But when $y = 0$, $v = 0$; therefore $\mathrm{C} = \frac{1}{2} r \log . r$, and the corrected integral is

$$y = -\frac{1}{2} r \log . (r - v^2) + \frac{1}{2} r \log . r = \frac{1}{2} r \log . \frac{r}{r - v^2}\ ; \quad \frac{2y}{r} = \log . \frac{r}{r - v^2} .$$

Take q to represent the number whose log. is $\frac{2y}{r}$,

$$q = \frac{r}{r - v^2}\ ;\ q . r - q . v^2 = r\ ;\ r - v^2 = \frac{r}{q}\ ;\ \therefore r - \frac{r}{q} = v^2\ ;\ (q - 1)\frac{r}{q} = v^2\ ;\ \sqrt{\left\{(q - 1)\frac{r}{q}\right\}} = v.$$

We have thus obtained the value of v in terms of y and r.

Taking the equation $dz = v . dt$, and substituting dy for dz, we get $dy = v . dt$; but $dy = \frac{r . v . dv}{r - v^2}$; $v . dt = dy = \frac{r . v . dv}{r - v^2}$; $dt = \frac{r . dv}{r - v^2}$; $t = \int \frac{r . dv}{r - v^2}$. We have taken g equal to unity, and it has been already shown that under this supposition $r = v^2$. When v is the greatest velocity attainable, take a to represent this velocity, which must evidently be represented by a constant quantity $r = a^2$. Substituting this value, the equation $dt = \frac{r . dv}{r - v^2}$ becomes $dt = \frac{a^2 dv}{a^2 - v^2}$;

$$t = a^2 \int \frac{dv}{a^2 - v^2} = a^2 \frac{1}{2} a \log . \mathrm{C} \frac{a + v}{a - v} = \frac{1}{2} a \log . \mathrm{C} \frac{a + v}{a - v} = \frac{1}{2} a \left\{\log . \mathrm{C} + \log . \frac{a + v}{a - v}\right\}.$$

When $t = 0$, $v = 0$; therefore $\log . \frac{a + v}{a - v} = \log . 1 = 0$; therefore $\log . \mathrm{C} = 0$, and the corrected integral is $t = \frac{1}{2} a \log . \frac{a + v}{a - v}$.

We have thus obtained the time of descent of a falling body through a resisting medium in terms of a and v, which as $r = a^2$ amounts to the same as obtaining it in terms of r and v. v has been already obtained in terms of y and r. y is a known quantity, being the distance descended by the falling body. If therefore we can assign a value to r the problem will be solved and the time of descent known. But here again we are led into the region of conjecture and supposition, and find ourselves still under the influence of g. And therefore however sound or ingenious the reasoning may be, if our faith in g is shaken, or if we are not fully satisfied with the suppositions upon which the premises are founded, we must necessarily be sceptical as to the conclusions arrived at. Admitting the assumption that at the moment at which the greatest velocity is attained by a falling body in a resisting medium, the motive force which causes the body to descend must be equal to the force of resistance. Then if we can find expressions for both these forces and equate them together, we may arrive at the value of r, which expresses a uniform velocity, representing the greatest velocity, or, according to the conventional rule, twice the height due to the greatest velocity.

So far we have only considered abstractedly the attractive force which causes a particle of matter to descend; but now, when we are about to deduce a formula adapted to practical application, we must admit the considerations of form and relative density or specific gravity. The fluid with which we have to deal in matters relating to gunnery is the air, and therefore we may consider the specific gravity of the fluid as constant upon all occasions, and represent it by n. The

specific gravity of the falling body depends upon the material of which it is composed, but whatever that may be we shall represent it by N.

As we only propose to carry the present investigation so far as may enable us to test the truth of our theories by experiments upon falling bodies, we shall suppose the form of the falling body to be spherical. Take d to represent the diameter of the sphere, then the solid contents will be $\frac{1}{6}\pi d$ As we are about to institute a comparison between the motive force and the generated resistance, and as no resistance can be generated till motion takes place, it would be natural to suppose that $\frac{1}{6}\pi d^3 N$ would be taken to express the initial motive force; but on the contrary, $\frac{1}{6}\pi d^3 (N - n)$ appears to be generally taken to express the motive force, assuming an initial resistance which seems to be an absurdity.

The expression for the force of resistance is generally arrived at as follows;—The resistance is supposed to vary in proportion to the number of particles which strike a plane moving through the fluid in any given time multiplied into the force with which each particle respectively strikes the plane; but both the number of particles which strike the plane during any given time, as well as the force of each particle respectively, must be proportional to the velocity with which the plane is moving through the fluid; therefore it is concluded that the resistance is proportional to the square of the velocity. It is evidently considered that in arriving at this conclusion it is a sufficiently close approximation for practical purposes to suppose that the action of each particle ceases immediately upon impact with the plane, or, in other words, that the medium through which the plane passes is discontinued. It is supposed by Sir Isaac Newton that in continued media, as air, water, hot oil, quicksilver, &c., a body as it passes through them does not immediately strike against all the particles of the fluid that generate the resistance made to it, but presses only the particles that lie next to it, which press the particles beyond, which press other particles, and so on. It is assumed that the resistance to a plane moving perpendicularly through an infinite fluid at rest is equal to the force of the fluid on the plane at rest, upon the supposition that the fluid moves with the same velocity as the plane was supposed to move with in the first instance, but in a contrary direction. It is also supposed that the force of the fluid in motion is equal to the pressure which causes or generates the motion. If therefore we can find an expression for the pressure of an infinite fluid upon a plane at rest of given dimensions in terms of the velocity of the fluid and known quantities, we can find an expression for the resistance of the fluid at rest to a plane of the same dimensions moving with the same velocity. With reference to this matter there appear to be different theories; but assuming for the present that the pressure of a fluid upon a given plane is equal to the weight or pressure of a column of the fluid, the base of which is equal to the plane and the height equal to the altitude due to the velocity with which the fluid is moving, which according to the conventional rule $= \frac{v^2}{2g}$, $\frac{v^2 . n . a}{2g}$ will equal the weight or pressure of such a column, n representing the specific gravity of the fluid and a the area of the base, and will also express the resistance of the fluid to a body moving through it with the velocity v, the cross-section of the body perpendicular to the line of motion being expressed by a. If the moving body be a cylinder the diameter of the cross-section being represented by d, $a = \frac{\pi . d^2}{4}$.

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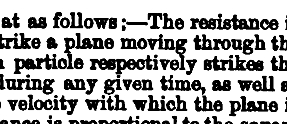

When the cross-section of the moving body is inclined to the direction of motion, let s express the sine of the angle of inclination, A B, Fig. 3370, being the direction of the plane and B D that of the motion. The number of particles or quantity of the fluid which strike the plane will be diminished in the ratio of $1 : s$, also the force of each particle will be diminished in the same ratio. Therefore on these accounts it is supposed the ratio will be diminished in the ratio of $1 : s^2$. But this is upon the supposition that the particles strike the plane perpendicularly, and as they strike obliquely it is considered that the effect will be diminished in the ratio of $1 : s^3$. Therefore upon this supposition $\frac{v^2 . n . s^3 . a}{2g}$ will express the resistance.

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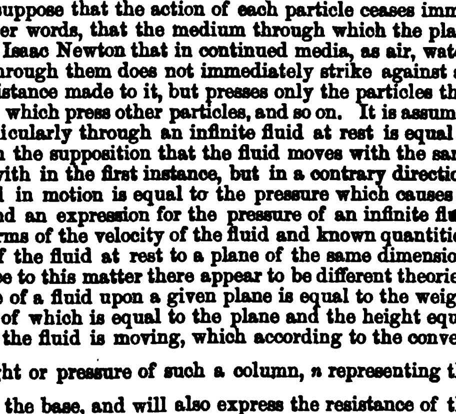


Let B E A D, Fig. 3371, be a section through the axis C A of the solid moving in the direction of that axis; let E G be a tangent to any point in the curve meeting the axis produced in G; let C F and F E represent the ordinates x and y to the point E, E o e the differential triangle; take z to express the arc B E, s the sine of the angle G; $2\pi y$ will equal the circumference described by the point E in revolving about the axis C A; and therefore $2\pi y . dz$ will express the differential of the area opposed to the motion, and $\frac{v^2 . n \; s^3}{2g} . 2\pi y . dz = \frac{v^2 . n . \pi s^3}{g} . y . dz$ will express the differential of the resistance. Let the solid be a sphere, and take r to express the radius C A;

$y = \sqrt{(r^2 - x^2)}$; $s = \frac{\text{E F}}{\text{E G}} = \frac{\text{C F}}{\text{C E}} = \frac{x}{r}$; $y.ds = (\text{E F}.\text{E}e) = (\text{C E}.oe) = r.dx$; therefore

$$\frac{v^2.n.\pi s^2}{g}.y.ds = \frac{v^2 n \pi}{g} \frac{x^2}{r^2} r dx = \frac{v^2 n \pi}{g r^2} x^3.dx; \quad \int \frac{v^2 n \pi}{g r^2} x^3.dx = \frac{v^2 n \pi}{4 g r^2} x^4;$$

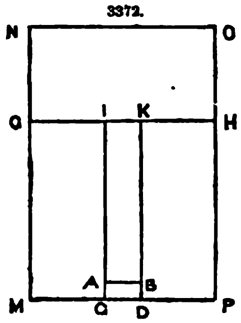
when $x = r$, $\frac{v^2.n.\pi}{4 g r^2} x^4 = \frac{v^2.n.\pi.r^2}{4 g}$. Taking d to represent the diameter of the sphere, $\frac{v^2 n \pi d^2}{16 g}$ will express the resistance. Equating this expression with $\frac{1}{6} \pi d^3 (\text{N} - n)$, the expression for the motive force, we have $\frac{v^2.n.\pi d^2}{16 g} = \frac{1}{6} \pi d^3 (\text{N} - n)$; $v^2 = 2g \frac{4}{3} d \frac{\text{N} - n}{n}$; $\frac{v^2}{2g} = \frac{4}{3} d \frac{\text{N} - n}{n}$. But as v has been taken to express the greatest velocity, $\frac{v^2}{2g}$ will express the height due to such velocity; but r = twice the height due to the velocity v, therefore $r = \frac{8}{3}.d.\frac{\text{N} - n}{n}$, or $r = \frac{8}{3} d \frac{\text{N}}{n}$, upon the supposition that at the commencement of the motion the resistance = 0.

Having thus obtained a value for r we can obtain a value for the time of descent of a spherical body of any given dimensions and specific gravity through any given space represented by y.

It has been assumed in the preceding investigation that the pressure of a fluid upon a given plane is equal to the weight or pressure of a column of the fluid, the base of which is equal to the plane and the height equal to the altitude due to the velocity of the fluid. This is generally proved as follows:—

Take M N O P, Fig. 3372, to represent a vertical section of a cylindrical vessel filled with a fluid up to the level G H, at which height it is supposed to be always retained; M P the diameter of the base, and C D the diameter of a circular orifice in the base, which is supposed to be very small compared with M P; C I K D a section of a column of the fluid standing directly above the orifice, and C A B D a section of a plate of the fluid immediately contiguous to the orifice. Take v to denote the velocity with which the plate C A B D would descend in vacuo through the space B D, subject only to the influence of gravity. Take V to denote the velocity with which the plate C A B D is discharged from the orifice when subjected to the pressure of the entire volume of the fluid, which a reference to any elementary work will show may, according to theoretical reasoning, be represented by the column C I K D. The velocities are as the moving forces, and the times in which they act directly and inversely as the quantity of matter moved; but it will be evident upon the supposition that the fluid is homogeneous that the moving forces will be as the heights B D and K D. The times in which they act are inversely as the velocities, the space being given, namely B D, and the quantities of matter moved equal, the quantity of matter in both cases being represented by the plate of fluid A C D B; therefore $v : \text{V} :: \frac{\text{B D}}{v} : \frac{\text{K D}}{\text{V}}$; $v^2 : \text{V}^2 :: \text{B D} : \text{K D}$; $v : \text{V} :: \sqrt{\text{B D}} : \sqrt{\text{K D}}$.

3372.

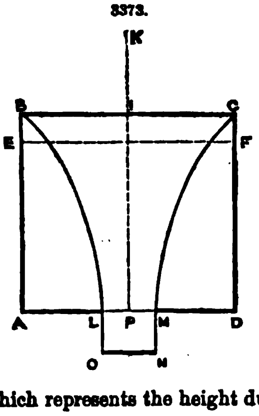


But B D is the height due to the velocity v, therefore K D is the height due to the velocity V, and K D is the height of the fluid. It will be observed that in this case V represents the velocity at the orifice, not the mean velocity of the descent of the fluid. The pressure on the orifice is equal to a column of the fluid of which the base is equal to the area of the orifice and the altitude equal to the height of the fluid. Therefore, admitting the usual suppositions, and also that the result in the case just investigated may be taken to represent the resistance in an unconfined fluid, the problem is solved.

The following may be taken as representing to a certain extent the line of reasoning employed in Prop. 36, Lib. II., of Newton's Principia in this matter, by means of which it will be seen a different result is arrived at.

Take A B C D, Fig. 3373, to represent the vertical section through a cylindric vessel which is supposed to remain constantly full of water. To illustrate this, Sir Isaac Newton supposes a block of ice on the top of the vessel, the lower surface of the ice being in contact with the upper surface of the water, so that as the water descends through an orifice in the bottom of the vessel, of which L M represents the diameter, the ice shall dissolve and constantly supply the deficit. If we take E F to represent the line of surface at any variable distance B E, which line of surface had been originally at B C, it will be evident that the quantity of water run out at the orifice during the descent from B C to E F will be represented by E B C F. Take U to represent the quantity of water contained in the vessel, m the area of the circular surface at B C, and m' the area of the circular orifice of which L M is the diameter; then as the velocities of equal quantities of water through different openings during the same time are inversely as the areas of the openings taking V to represent the velocity at the orifice and V′ the velocity at the surface B C, we have $\text{V} : \text{V}' :: m : m'$. Take x equal A B the height of the fluid, and $a + x$ the height due to the velocity at the orifice, $\text{V}^2 : \text{V}'^2 :: m^2 : m'^2 :: a + x : \frac{m'^2}{m^2}(a + x)$, which represents the height due

3373.



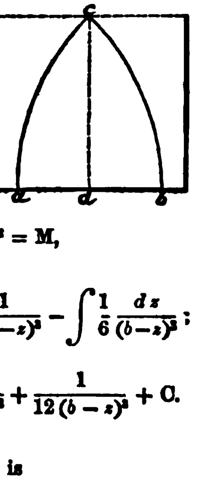
to the velocity at B C; therefore, while $a + x$ represents the height due to the velocity at the orifice, $\frac{m'^2}{m^2}(a + x)$ will represent the height due to the velocity at the surface. Differentiating both these expressions, we have $d\,x$ and $\frac{m'^2}{m^2}\,d\,x$. Assuming the differential of the height due to the mean velocity of the fluid as equal to the difference of the above differentials, we have $d\,x - \frac{m'^2}{m^2}\,d\,x$ to represent the differential of the height due to the mean velocity of the descending fluid; therefore $d\,.\,\mathrm{U} = \left\{d\,x - \frac{m'^2}{m^2}\,d\,x\right\} m = m\,d\,x - \frac{m'^2\,d\,x}{m}$, also $\mathrm{U} = m\,.\,x$. If v represents the mean velocity with which the water descends, the momentum of pressure upon the base may be expressed by $\mathrm{U}\,.\,v$ according to conventional rule. $m\,x$ evidently expresses the quantity of water contained in the vessel, which multiplied with the distance of the centre of gravity of the mass of water from the plane of the bottom of the vessel, will express the momentum also. But in this case the distance of the centre of gravity from the bottom of the vessel upon which the pressure is applied $= \frac{1}{2}\,x$; therefore taking M to represent the momentum, $\mathrm{U}\,.\,v = \mathrm{M} = \frac{1}{2}\,m\,x^2$. Differentiating, $v\,.\,d\,\mathrm{U} + \mathrm{U}\,d\,v = m\,.\,x\,.\,d\,x$. Substituting the values for $d\,\mathrm{U}$ and U already found, we have $v\,.\,m\,.\,d\,x - v\,\frac{m'^2}{m}\,d\,x + m\,x\,d\,v = m\,x\,.\,d\,x$; dividing by m, $v\,.\,d\,x - v\,\frac{m'^2}{m^2}\,d\,x + x\,.\,d\,v = x\,.\,d\,x$. Take $\left(1 - \frac{m'^2}{m^2}\right) = r$, $r\,.\,v\,.\,d\,x + x\,d\,v = x\,d\,x$. Multiplying both sides of this equation by x^{r-1}, $r\,.\,v\,.\,x^{r-1}\,.\,d\,x + x^r\,.\,d\,v = x^r\,.\,d\,x$; integrating, we obtain $v\,x^r = \frac{x^{r+1}}{1 + r}$; dividing by x^r, $v = \frac{x}{1 + r}$. We assumed $1 - \frac{m'^2}{m^2} = r$: therefore

$$2 - \frac{m'^2}{m^2} = \frac{2\,m^2 - m'^2}{m^2} = 1 + r; \quad v = \frac{x}{1 + r} = \frac{m^2\,x}{2\,m^2 - m'^2}.$$

Take K I to represent the height due to the velocity at the surface, and K P the height due to the velocity at the orifice, K I : K P :: $\mathrm{V}'^2 : \mathrm{V}^2$, $\mathrm{V}' : \mathrm{V}$ :: $\mathrm{PL}^2 : \mathrm{IB}^2$, $\mathrm{V}'^2 : \mathrm{V}^2$ :: $\mathrm{PL}^4 : \mathrm{IB}^4$, K I : K P :: $\mathrm{PL}^4 : \mathrm{IB}^4$; this determines the curve B L, and the cataract B L M C is formed by the revolution of this curve about the axis K P. It is supposed that the contents of this cataract expresses the quantity of water which presses upon the orifice in the same manner as if the rest was congealed into ice; therefore the portion of the water contained in the solid described by the same curve B L round the axis B A expresses the quantity which presses upon the ring described by A L in the rotation.

Let $a\,b$, Fig. 3374, represent the diameter of this ring, and $a\,c\,b$ a section of the solid described by the revolution. Take a to express K I, $x = $ I P, $b = $ B I, $y = $ P L, $z = $ A L, then $y = b - z$; therefore by the property of the curve, $a : a + x :: y^4 : b^4$; $\frac{a\,b^4}{y^4} = a + x$; $\frac{a\,b^4}{y^4} - a = x$. Substituting value for y^4, $\frac{a\,.\,b^4}{(b - z)^4} - a = x$. Differentiating, $\frac{4\,a\,b^4\,d\,z}{(b - z)^5} = d\,x$. But the differential of the solid described by the area A B L about the axis A B $= x$ is equal to the cylinder whose base is the circle described by the ordinate A L $= z$ and altitude equal to $d\,x$. This solid is therefore proportional to $z^2.\,d\,x$, while $z^2.\,x$ expresses the cylinder of the same base and altitude, $z^2\,d\,x = \frac{4\,a\,b^4\,z^2\,d\,z}{(b - z)^5} = \frac{a\,b^2}{3}\,\frac{12\,b^2.\,z^2\,d\,z}{(b - z)^5}$.

3374.



Therefore the solid $a\,c\,b$ will be proportional to $\frac{a\,b^2}{3}\int\frac{12\,b^3\,z^2\,d\,z}{(b - z)^5}$. Take $12\,b^3 = \mathrm{M}$,

$$\mathrm{M}\,.\int z^2\,\frac{d\,z}{(b-z)^5} = z^2\,\frac{1}{4\,(b-z)^4} - \int\frac{1}{2}\,z\,\frac{d\,z}{(b-z)^4}; \quad \int\frac{1}{2}\,z\,\frac{d\,z}{(b-z)^4} = \frac{1}{2}\,z\,\frac{1}{3\,(b-z)^3} - \int\frac{1}{6}\,\frac{d\,z}{(b-z)^3};$$

$$\int\frac{1}{6}\,\frac{d\,z}{(b-z)^3} = \frac{1}{6}\,\frac{1}{2\,(b-z)^2}. \text{ Therefore } \int z^2\,\frac{d\,z}{(b - z)^5} = \frac{z^2}{4\,(b - z)^4} - \frac{z}{6\,(b - z)^3} + \frac{1}{12\,(b - z)^2} + \mathrm{C}.$$

When $z = 0$, $0 = \frac{1}{12\,b^2} + \mathrm{C}$, $\mathrm{C} = -\frac{1}{12\,b^2}$; therefore the corrected integral is

$$\frac{z^2}{4\,(b - z)\,4} - \frac{z}{6\,(b - z)^3} + \frac{1}{12\,(b - z)^2} - \frac{1}{12\,b^2} = \frac{3\,z^2 - 2\,z\,(b - z) + (b - z)^2}{12\,(b - z)^4} - \frac{1}{12\,b^2}$$

$$= \frac{3\,b^2\,z^2 - 2\,b^2\,z\,(b - z) + b^2\,(b - z)^2 - (b - z)^4}{12\,b^2\,(b - z)^4}$$

$$\frac{3\,b^2\,z^2 - 2\,b^3\,z + 2\,b^2\,z^2 + b^4 - 2\,b^3\,z + b^2\,z^2 - b^4 + 4\,b^3\,z - 6\,b^2\,z^2 + 4\,b\,z^3 - z^4}{12\,b^2\,(b - z)^4} = \frac{4\,b\,z^3 - z^4}{12\,b^2\,(b - z)^4};$$

$$M\int z^2 \frac{dz}{(b-z)^5} = \frac{4bz^3 - z^4}{(b-z)^4}.$$

Therefore the solid $a\,o\,b$ is proportional to $\frac{a\,b^2}{3}\,\frac{4\,b\,z^3 - z^4}{(b-z)^4}$. Substituting value for x in $z^2\,x$, we have $\frac{a\,z^2\,b^4}{(b-z)^4} - a\,z^2 = a\,z^2\left\{\frac{b^4}{(b-z)^4} - 1\right\}$; therefore the solid is to the cylinder as

$$\frac{a\,b^2}{3}\,\frac{4\,b\,z^3 - z^4}{(b-z)^4} : a\,z^2\left\{\frac{b^4}{(b-z)^4} - 1\right\} = \frac{b^2}{3}\,\frac{4\,b\,z^3 - z^4}{(b-z^4)} : \frac{z^2\left\{b^4 - (b-z)^4\right\}}{(b-z)^4} = \frac{b^2}{3}(4\,b\,z^3 - z^4) : z^2\left\{b^4 - (b-z)^4\right\}.$$

But $z^2\left\{b^4 - (b-z)^4\right\} = 4\,b^3\,z^3 - 6\,b^2\,z^4 + 4\,b\,z^5 - z^6$; therefore the proportion becomes

$$\frac{4\,b^3\,z^3 - b^2\,z^4}{3} : \left\{4\,b^3\,z^3 - 6\,b^2\,z^4 + 4\,b\,z^5 - z^6\right\};$$

or dividing by z^3, $\left\{\frac{4}{3}\,b^3 - \frac{1}{3}\,b^2\,z\right\} : \left\{4\,b^3 - 6\,b^2\,z + 4\,b\,z^2 - z^3\right\}$. If we suppose z to become infinitely small, or in other words suppose the diameter at the orifice to approach without limit to an equality with the diameter at the surface, all the terms involving z may be neglected, and the proportion will become $\frac{4}{3}\,b^3 : 4\,b^3$, or $\frac{1}{3} : 1$; and also m', which has been taken to represent the area of the orifice, may be taken equal to m, the area at the surface, and $v = \frac{m^2\,x}{2\,m^2 - m'^2}$ becomes $v = x$. Therefore x equals twice the height due to the velocity v.

If we suppose the water to be at rest, and the small circle of which $a\,b$ represents the diameter to ascend with the velocity equal to v with which the water was supposed to descend, it will be evident that the same expression which represented the pressure when the fluid was in motion and the circle at rest will represent the resistance when the circle is in motion and the fluid at rest; but the solid representing the pressure in the first case is to the cylinder whose base is the small circle and altitude x, as $\frac{1}{3} : 1$, and consequently the resistance will bear the same proportion. It follows that in order to meet with a resistance equal to the pressure of the cylinder the small circle must move with a velocity equal $3\,.\,v$; but the heights due to any given velocities are as the squares of the velocities, therefore twice the height due to the velocity $3\,.\,v$ will equal $9\,.\,x$; but this is upon the supposition that the cylinder is of the same specific gravity as the fluid.

If we suppose the small circle which forms the base of the cylinder to remain constant, and also the velocity, then in order to express the pressure which denotes the resistance we must reduce the altitude x of the cylinder in proportion to the increase in the specific gravity. Take N to represent the specific gravity of the cylinder, and n the specific gravity of the fluid; then if we suppose the velocity to be the greatest velocity attainable in the medium in which the body is moving, and r to represent twice the height due to such velocity when the specific gravity of the moving body is represented by N and that of the fluid by n, we have $r : 9\,x :: \mathrm{N} : n$, the sphere is $\frac{2}{3}$ of the cylinder. If therefore we suppose the specific gravity of the sphere to be N, and the specific gravity of the circumscribing cylinder to be $\frac{2}{3}$ N, the pressure which is the exponent of the resistance will be the same if we substitute the cylinder for the sphere; therefore when the form of the moving body is spherical, the proportion becomes, taking d to represent the diameter of the sphere and consequently the height of the circumscribing cylinder, and substituting it for x, $r : 9\,d :: \frac{2}{3}\,\mathrm{N} : n$; $r = \frac{6\,\mathrm{N}\,.\,d}{n}$.

It will be observed that in arriving at this conclusion the resistance to solid bodies moving through a fluid is supposed to be the same, when the cross-sections of the solids at right angles to the line of motion are equal without reference to the form of the solid.

In Lemma IV., Lib. II., of the Principia, we find the following;—If a cylinder move forward uniformly in the direction of its length, the resistance made thereto is not at all changed by augmenting its length or diminishing that length, and is therefore the same with the resistance of a circle described with the same diameter, and moving forward with the same velocity in the direction of a right line perpendicular to its plane; for the sides are not at all opposed to the motion, and a cylinder becomes a circle when its length is diminished *ad infinitum*. The force of the last part of the reasoning is not immediately apparent, for if the cylinder is not diminished *ad infinitum* it does not become a circle.

In the report of a lecture on the flight of projectiles, delivered at the R. U. S. Institution in 1865 by General Anstruther, we find the following statement made by the Editor of the present work; —"It has been found by experiment on railways that the resistance of the atmosphere to the motion of a train depends chiefly upon the length of the train and not upon the frontage of the carriages; the resistance resembles more that of friction than the moving of a long parallelopiped of the fluid in which the body moves."

It is generally supposed that when the velocity of a body moving through a resisting medium exceeds a certain limit the resistance becomes increased, in consequence of a vacuum being formed in rear of the moving body, leaving the body to sustain the whole force of the resistance of the particles of the fluid opposed to the motion without any support from the particles moving in, in rear, upon the track of the moving body. This is supposed to take place when the velocity of the moving body exceeds the velocity with which the particles of the air subjected to the pressure of a

column equal in height to the height of the atmosphere will rush into the orifice supposed to be left by the advance of the moving body. Sound moves at the rate of 1142 ft. in a second, and as sound is propagated by the elastic force of the air, therefore the elastic force of the air is such as to produce an equivalent velocity; this is therefore sometimes taken to mark the limit from which the vacuum is formed.

Much has been written on these subjects by Newton, Bernoulli, D'Alambert, Bossut, Buat, and many other authors; and a good abridgment of all that can be said on the subject, assuming the established laws of gravity, will be found in the Treatise on Artillery for the practical class of the R. M. Academy, published by authority in 1866. But instead of entering further into detail in these matters, we should propose in the first place that we should test by actual experiment the accuracy of our present theory founded upon the laws of gravity generally received. If the experiment should prove the theory to be fallacious, the next step we should propose would be to endeavour to ascertain by experiment where the fallacy exists—whether in an erroneous estimate of the resistance of the air, or of the force of gravity, or both. We have facilities of experiment now which were not formerly attainable in the same degree. These may be furnished by the improvements in the construction and mode of working balloons, by electricity, photography, telegraphy, accuracy in measuring time, as well as in graduating and constructing optical instruments; and add to all these a calculus which will enable us to solve problems which were formerly beyond our reach

The first experiment we should propose is as follows;—That on a calm, favourable day an ascent should be made in a balloon carrying a spherical body of known density, as homogeneous as it is possible to make it; that an observer should be stationed at any convenient point for taking the elevation of the balloon at any given time; that the spherical body should be attached to the car of the balloon, so as to be detached suddenly, at a signal, by means of the suspension of an electric current—the weight of the shot, indicated by a spring balance, should be taken at the instant it is detached from the car; that the signal should be given from the point of observation at the moment that the elevation is taken by means of a theodolite or other suitable instrument; that the height of the balloon from which the body drops should be estimated by a single observation, the distance from the point of observation to the point where the spherical body drops being taken as radius, and the tangent of the observed angle to such radius, added to the height of the observer's eye, being taken as the height of the balloon. We have given a formula by means of which the time of descent may be calculated according to the theory now extant; the diameter and density of the falling body, the height descended as well as the moment of detachment being known; the time of descent being accurately noted and compared with the time by calculation will show the amount of discrepancy which may exist between them. If the discrepancy is great, we must conclude that our theory is fallacious; and, on the other hand, if the times nearly agree, we may conclude that our theoretical formulæ are sufficiently close approximations to the truth for practical purposes, and we may go further into detail to improve it. If we find a sensible discrepancy, which is most likely to be the case, we must then have recourse to further experiment, in order to discover where the fallacy exists.

We have assumed the descent of the spherical figure to be vertical; that is, that the falling body descends upon a vertical straight line in the direction of the plumb-line. It appears to us that the error incurred in consequence of this assumption will be less than the error arising from two observations which may not be simultaneous. If the single observation should be taken exactly at the moment that the spherical body is detached, and the time correctly noted, the only source of error will be the irregular motion during the descent of the spherical body, and the experiment will be much simplified, as it will be only necessary to measure the distance from the point of observation to the point where the spherical body reaches the ground. Even upon the supposition that the exact vertical height of the balloon was ascertained by a double observation, the error arising from irregularity of the motion in the descent would remain.

We shall now give a short extract from a small pamphlet lately published, entitled Theory of Gunnery, offered to the Institution of Civil Engineers by Gen. Anstruther;—

"When a ball has been projected obliquely upwards it is acted upon by two forces—the resistance of the atmosphere and gravity; the former of these two can only act in reducing the magnitude of the ascent, the latter of the two deflects the ascent vertically, so as to bring the ball to the ground at the expiration of a certain time of flight, at a distance from the gun called the range.

"If the true angle of departure is given to us as the elevation, and the horizontal space passed over as the range, we can determine the trajectory to an inch.

"We multiply the given range in feet by the tangent of the elevation; the product is the measure of the vertical descent, the fall by gravity in the time of flight."

This offers a useful suggestion for further experiment in order to test the validity of our present theory of gravity. The angle made by the axis of the gun, or the tangent to the curve of trajectory at the initial point and the direction of the object, is called the angle of elevation. The angle of departure required by the General is the true angle of elevation with the horizon, or complement of the zenith distance, which latter is the angle made by the vertical passing through the point of projection and the direction of the piece. This angle being known, the General considers that the vertical deflection during the time of flight will be equal to the tangent of the given angle to a radius equal to the measured range. If this be the case, it evidently suggests another means of ascertaining the time of descent through a given distance under the influence of gravity and the resistance of the air; for the height descended will be known by calculation, and the time by observation (being the time of flight).

The results of these two experiments being compared together, and with the results by calculation according to the existing theory, might lead to some useful conclusions. The resistance of the air is evidently a retardative force, with the law of which we are at present unacquainted. It is generally supposed that it acts only on the line of motion of the projectile, but as the initial impulse becomes weaker in consequence of the augmenting force of resistance, the ratio of the

impulsive force to the force of gravity is constantly changed, and the deflecting power of the force of gravity becomes comparatively stronger as the projectile proceeds; from this cause alone the force of gravity also varies by some law, as the moving body approaches nearer and nearer to the centre of force. These variations would render the determination of the curve rather an intricate matter, even if we were acquainted with the laws of variation, which at present we are not. But if a tolerably close approximation to the laws of variation could be arrived at by experiment, the means of calculation for the formation of reliable tables could be soon found.

Admitting the statement that a body impelled by a given force in a given direction will proceed with a *uniform velocity* in that direction *ad infinitum*, unless influenced by some counter-acting force; admitting also that the force of gravity acts on parallel vertical lines, and the force of resistance of the air only on the line of trajectory of the projectile, it would follow that the time of flight should be a very close approximation to the time of descent through a distance equal to the vertical deflection, calculated according to General Anstruther's theory; and the matter might be tested by dropping shot from a balloon at different heights, and also observing the time of flight corresponding to different ranges, interpolating in both cases, and comparing the results.

We shall now pass on to experiments with a view to ascertain as nearly as possible by a practical test the respective laws of variation, of gravity, and the resistance of the air. We shall here extract some remarks, by the Editor of the present work, on this subject, to be found in the report of General Anstruther's lecture, delivered at the R. U. S. Institution in 1866; also the calculation of certain formulæ, which we shall here modify so far as to put them in a more explanatory form than that in which they appear in the report;—

"To find whether the resistance of the air or any other fluid medium is proportional to the square of the velocity (V) or not, and also to find whether the value usually given to (g) the force of gravity near the surface of the earth is under or over estimated, generally g is put $= 31\frac{1}{6}, 32\frac{1}{6}$, 32, &c. Let v^2 multiplied by some constant coefficient express the retarding force, and to simplify the investigation put this coefficient under the form $n^2 g$; then the motive force will be expressed by $g - g n^2 v^2$. Taking the equation $dv = f.dt$, and substituting $g(1 - n^2 v^2)$ for f, we have $dv = g(1 - n^2 v^2)\,dt$; $\dfrac{dv}{1 - n^2 v^2} = g.dt$; $\dfrac{1}{g n^2} \dfrac{dv}{\frac{1}{n^2} - v^2} = dt$; $\dfrac{1}{g n^2} \displaystyle\int \dfrac{dv}{\frac{1}{n^2} - v^2} = t$. Assume $\dfrac{1}{n} = a$, the expression becomes $\dfrac{a^2}{g} \displaystyle\int \dfrac{dv}{a^2 - v^2} = t$, which reduces it to a well-known form, and we have $\dfrac{a}{2g}\left\{\log. \dfrac{a+v}{a-v} + \log. \mathrm{C}\right\} = t$. When $v = 0$, $t = 0$; therefore $\dfrac{a}{2g} \log. \mathrm{C} = 0$, and the corrected integral is $\dfrac{a}{2g} \log. \dfrac{a+v}{a-v} = t$; $\log. \dfrac{a+v}{a-v} = \dfrac{2gt}{a}$.

Take $2gt = m$, and ϵ to represent the base of the hyp. logs.,

$$\frac{a+v}{a-v} = \epsilon^{\frac{m}{a}}; \quad a + v = a.\epsilon^{\frac{m}{a}} - v\,\epsilon^{\frac{m}{a}}; \quad v\left\{\epsilon^{\frac{m}{a}} + 1\right\} = a\left\{\epsilon^{\frac{m}{a}} - 1\right\}; \quad v = a\,\frac{\epsilon^{\frac{m}{a}} - 1}{\epsilon^{\frac{m}{a}} + 1}.$$

Substituting value for a, $v = \dfrac{1}{n}\dfrac{\epsilon^{m.n} - 1}{\epsilon^{m.n} + 1}$.

Taking the equation $v.dv = f.ds$; substituting as before $g(1 - n^2 v^2)$ for f, we have $v.dv = g(1 - n^2 v^2)\,ds$; $\dfrac{v.dv}{1 - n^2 v^2} = g.ds$; $\dfrac{v.dv}{g(1 - n^2 v^2)} = ds$; which may be put under the form $-\dfrac{1}{2n^2 g}\dfrac{-2n^2 v.dv}{(1 - n^2 v^2)} = ds$; therefore $-\dfrac{1}{2n^2 g} \log. \dfrac{1}{1 - n^2 v^2} + \log. \mathrm{C} = s$. When $v = 0$, $s = 0$; therefore $\log. \mathrm{C} = \log. 1 = 0$, and $-\dfrac{1}{2n^2 g} \log. \dfrac{1}{1 - n^2 v^2} = s$; but

$$v = \frac{1}{n}\frac{\epsilon^{m.n} - 1}{\epsilon^{mn} + 1}; \qquad v^2 = \frac{1}{n^2}\frac{(\epsilon^{2mn} - 1)^2}{(\epsilon^{2mn} + 1)^2};$$

therefore
$$n^2 v^2 = \frac{(\epsilon^{2mn} - 1)^2}{(\epsilon^{2mn} + 1)^2}; \quad 1 - n^2 v^2 = \frac{(\epsilon^{2mn} + 1)^2 - (\epsilon^{2mn} - 1)^2}{(\epsilon^{2mn} + 1)^2};$$

$$(\epsilon^{2mn} + 1) + (\epsilon^{2mn} - 1) = 2\,\epsilon^{2mn}; \quad (\epsilon^{2mn} + 1) - (\epsilon^{2mn} - 1) = 2.$$

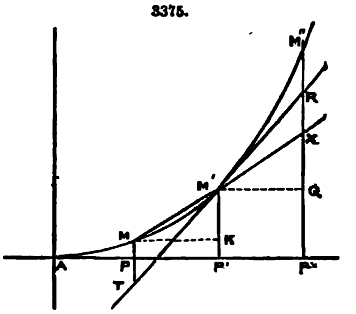
Therefore $1 - n^2 v^2 = \dfrac{4\,\epsilon^{2mn}}{(\epsilon^{2mn} + 1)^2}$; $\dfrac{1}{1 - n^2 v^2} = \dfrac{(\epsilon^{2mn} + 1)^2}{4\,\epsilon^{2mn}} = \left\{\dfrac{\epsilon^{2mn} + 1}{2\,\epsilon^{mn}}\right\}^2 = \left\{\dfrac{1}{2}\left(\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}}\right)\right\}^2$. Substituting this value for $\dfrac{1}{1 - n^2 v^2}$ in the expression $-\dfrac{1}{2n^2 g} \log. \dfrac{1}{1 - n^2 v^2} = s$, we have $\dfrac{1}{n^2 g} \cdot \dfrac{1}{2} \log. \left\{\dfrac{1}{2}\left(\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}}\right)\right\}^2 = s$; or $\dfrac{1}{n^2 g} \log. \dfrac{1}{2}\left(\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}}\right) = s$; $\log. \dfrac{1}{2}\left(\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}}\right) = s\,n^2 g$. Therefore $\dfrac{1}{2}\left(\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}}\right) = \epsilon^{sn^2 g}$; $\epsilon^{mn} + \dfrac{1}{\epsilon^{mn}} = 2\,\epsilon^{sn^2 g}$; we assumed $m = 2gt$. Substituting this value, we have $\epsilon^{2gtn} + \dfrac{1}{\epsilon^{2gtn}} = 2\,\epsilon^{sn^2 g}$, which is under the form $y + \dfrac{1}{y} =$ a function of n, g, and may be solved by dual arithmetic." See DAMMING.

Then if we drop a spherical body, as homogeneous as it can be made, from a balloon upon a calm day and can mark exactly the time of descent, s and t in the equation become known quantities, ϵ being the base of the hyperbolic system of logarithms, dual log. of $\epsilon = 100000000, = 10^8$; there evidently remain only n and g unknown quantities. Repeating the experiment and ascertaining two new values for s and t, we obtain a second equation, in which n and g are the only unknown quantities. Therefore, having two equations and only two unknown quantities, we can ascertain the values of n and g. Taking the equation $v = \frac{1}{n} \frac{\epsilon^{mn} - 1}{\epsilon^{mn} + 1}$, then since g and t are known quantities m becomes a known quantity, and consequently v. The corresponding resistance also becomes known, having been originally assumed as $g\, n^2 . v^2$; we can therefore compare the velocities with the resistance in the two experiments, and thus ascertain the ratio which subsists between them.

We commenced this article by assuming as a fact that bodies under the influence of the force of gravity and uninfluenced by any sensibly counteracting force move in elliptical orbits round the centre of force. This being all that we may be said to know of the action of the law of gravity by actual observation, the rest being more or less founded upon ingenious suppositions of clever men, which suppositions have become so familiar to us, and such an imposing structure of mathematical reasoning has been raised upon them, that we forget the original instability of the foundation. We should therefore propose that the ratio of the times of descent with reference to the space descended through found by experiment should be compared with the ratio of the times calculated with reference to the law of gravity found by observation, namely, that the force or velocity is inversely proportional to the square of the distance from the centre of force. And then if the differences of the times found by experiment and calculation should bear a constant ratio to each other, we may assume that this difference represents a close approximation to the correction to be applied for the resistance of the atmosphere. A formula for this purpose might have been deduced by considering the minor axis in the ellipse to be indefinitely diminished, in which case the expression for the time in the elliptical orbit would become an expression for the time during the descent along the line of the transverse axis. But in the present case we think we shall arrive at a more practically useful result, as follows;—

We can only represent a force or its exponent velocity by a reference to space and time; when the force is singly impulsive and the velocity uniform we can express the velocity by saying that it is such as to cause a motion of so many feet or yards in one given unit of time; if the force is constant and the velocity uniformly accelerated or retarded, we can express it by stating the space generated or due to the acceleration or retardation during a given unit of time. But if the force is variably accelerated or retarded we can only express it by a complicated and elaborate formula, or else approximate to it as closely as may be desirable in a more simple form. If a force should act according to any law of uniformly accelerated or retardative motion with reference to space or distance from the centre of force, it will be evident that the same force must be considered variably accelerated or retarded when referred to equal intervals of time. For instance, if a force be inversely proportional to the square of the distance from the centre of force, and consequently with relation to a falling body directly proportional to the square of the distance from the point of departure, it will be evident that the motion, although uniformly accelerated with reference to the space or distance descended, will be variably accelerated when referred to equal portions of the time of descent, that is, the space which expresses the distance due to the acceleration during any one interval of time will not be equal to the space which expresses the distance due to the acceleration during any other equal interval of time.

3375.



Take the abscisses A P, A P′, A P″, &c., Fig. 3375, to represent the times, and the corresponding ordinates P M, P′ M′, P″ M″, &c., to represent the spaces described. We shall suppose the ordinates P M, P′ M′, P″ M″, &c., to be indefinitely near to each other; also the distances P P′, P′ P″, &c., to be equal to each other, or in other words $d\,t$; which they represent, to be constant. Draw M K parallel to the axis of the abscisses and produce the chord M M′ to X. Let T R be a tangent to the curve at the point M′; then $\mathrm{K\,M} = \mathrm{Q\,X} = d\,s$ will represent the space due to the acceleration during the interval of time P P′. If the motion was to become uniform at the point M′ the line of motion would be represented by the tangent M′ R. It will be evident therefore that M″ R will represent the space due to the acceleration during the interval of time P′ P″; $\mathrm{T\,M'} = \mathrm{M'\,R}$; T M and M″ R are parallel; therefore $\mathrm{T\,M} = \mathrm{M''\,R}$ upon the supposition that the curvature is the same throughout. In the triangles T M M′, R M′ X, $\mathrm{T\,M'} = \mathrm{M'\,R}$, and the angles are equal; therefore

$$\mathrm{T\,M} = \mathrm{R\,X},\ \mathrm{M''\,R} = \mathrm{T\,M} = \mathrm{R\,X},\ \mathrm{X\,M''} = 2\,\mathrm{M''\,R}:\ \mathrm{K\,M} = d\,s,$$
$$\mathrm{K\,M'} - \mathrm{Q\,M''} = \mathrm{Q\,X} - \mathrm{Q\,M''} = -\,\mathrm{X\,M''} = d^2\,s;\ -\,2\,\mathrm{M''\,R} = d^2\,s.$$

Here the motion has been considered with reference to the point of departure of a body falling towards the centre of force, and consequently the velocity is accelerative and the curve convex towards the axis of the abscisses. But if we consider it with reference to the point at which the falling body reaches the ground, estimating from that point towards the point of departure, the velocity must be taken as retardative; the curve will be concave towards the axis of the abscisses, and the expression becomes $2\,\mathrm{M''\,R} = d^2\,s$. The equation $2\,s = f\,d\,t^2$ was arrived at upon the supposition that f was constant. Although this supposition appears to be inadmissible with reference

to the entire time of descent of a falling body, and only productive of error and confusion, still the application may be considered admissible without sensible error with reference to the movement through the first of the indefinitely short equal intervals of time into which the line of descent is supposed to be divided. In this case s in the formula expresses the space due to the acceleration during the short interval of time $d\,t$, s will therefore be represented by M″ R,

$$f\,d\,t^2 = 2\,\text{M}''\,\text{R} = d^2 s\,;\quad f = \frac{d^2 s}{d\,t^2}.$$

The intensity of the force of attraction may be supposed to become infinitely great at the centre of force, or whether infinite or not it is constant and may be represented by m; but we can always suppose the representation of the intensity of the force at some constant distance from the centre nearer to it than any distance which we shall have to consider, and therefore in our calculations the quantity expressing this force may be considered constant; let it be represented by m. The fact of its being a constant quantity whatever its actual value may be will be sufficient to enable us to institute a comparison of ratios with reference to other forces at a greater distance from the centre. The distance of the point of departure of a falling body from the centre of force is known, being equal to the radius of the earth added to the line of descent, and may therefore be also represented by a constant quantity. Let this latter be represented by a. Then when the intensity of the attractive force is inversely proportional to the square of the distance from the centre of force, taking s to represent the line of descent, we have $\frac{m}{(a-s)^2} = f$; therefore $\frac{d^2 s}{d\,t^2} = f = \frac{m}{(a-s)^2}$; $\frac{d\,s}{d\,t} = v$. Upon the supposition that $d\,t$ is constant,

$$\frac{d\,s}{d\,t}\,.\,d\left(\frac{d\,s}{d\,t}\right) = v\,.\,d\,v\,;\quad \frac{d^2 s}{d\,t^2}\,.\,d\,s = v\,.\,d\,v = \frac{m\,d\,s}{(a-s)^2}.$$

Integrating, $v^2 = \frac{2\,m}{a-s} + \text{C}$. Upon the usual supposition that when $v = 0$, $s = 0$, $\text{C} = -\frac{2\,m}{a}$: and the corrected integral is

$$v^2 = \frac{2\,m}{a-s} - \frac{2\,m}{a} = \frac{2\,m\,a - 2\,m\,a + 2\,m\,s}{a\,(a-s)} = \frac{2\,.\,m\,.\,s}{a\,(a-s)}\,;\quad v = \sqrt{\frac{2\,m}{a}}\cdot\sqrt{\frac{s}{(a-s)}}\,;$$

therefore when $s = a$, v is infinite. In $v = \sqrt{\frac{2\,m}{a}}\cdot\sqrt{\frac{s}{a-s}}$, substitute $\frac{d\,s}{d\,t}$ for v;

$$\frac{d\,s}{d\,t} = \sqrt{\frac{2\,m}{a}}\;\sqrt{\frac{s}{a-s}}.$$

Taking the reciprocals,

$$\frac{d\,t}{d\,s} = \sqrt{\frac{a}{2\,m}}\cdot\sqrt{\frac{a-s}{s}}\,;\quad d\,t = \sqrt{\frac{a}{2\,m}}\,\sqrt{\frac{a-s}{s}}\,d\,s\,;\quad t = \sqrt{\frac{a}{2\,m}}\int\sqrt{\frac{a-s}{s}}\,d\,s.$$

Multiply both terms of the fraction by $\sqrt{(a-s)}$, we get

$$\frac{a-s}{\sqrt{(a\,s-s^2)}}\,d\,s = \frac{\frac{1}{2}a-s}{\sqrt{(a\,s-s^2)}}\,d\,s + \tfrac{1}{2}\,a\,\frac{d\,s}{(a\,s-s^2)}\,;$$

$$t = \sqrt{\frac{a}{2\,m}}\left\{\int\frac{\frac{1}{2}a-s}{\sqrt{(a\,s-s^2)}}\,d\,s + \frac{a}{2}\int\frac{d\,s}{(a\,s-s^2)}\right\};\quad \int\frac{\frac{1}{2}a-s}{\sqrt{(a\,s-s^2)}}\,d\,s = \sqrt{(a\,s-s^2)}.$$

To find the integral of $\frac{d\,s}{\sqrt{(a\,s-s^2)}}$, take $s = \frac{1}{2}\,a - z$; $d\,s = -\,d\,z$;

$$a\,s - s^2 = \tfrac{1}{2}\,a^2 - a\,z - (\tfrac{1}{4}\,a^2 - a\,z + z^2) = \tfrac{1}{2}\,a^2 - a\,z - \tfrac{1}{4}\,a^2 + a\,z - z^2 = \tfrac{1}{4}\,a^2 - z^2 = \frac{a^2 - 4\,z^2}{4}\,;$$

$$\sqrt{(a\,s-s^2)} = \frac{\sqrt{(a^2-4\,z^2)}}{2}\,;\quad \frac{d\,s}{\sqrt{(a\,s-s^2)}} = \frac{-\,2\,d\,z}{\sqrt{(a^2-4z^2)}}\,;\quad \int\frac{-\,2\,d\,z}{\sqrt{(a^2-4\,z^2)}} = \cos.^{-1}\frac{2\,z}{a}.$$

We have taken $s = \frac{1}{2}\,a - z$, therefore

$$z = \tfrac{1}{2}\,a - s\,;\quad 2\,z = a - 2\,s\,;\quad \frac{2\,z}{a} = \frac{a-2\,s}{a}\,;\quad \int\frac{d\,s}{\sqrt{(a\,s-s^2)}} = \cos.^{-1}\frac{a-2\,s}{a}\,;$$

$$t = \sqrt{\frac{a}{2\,m}}\left\{(a\,s-s^2) + \tfrac{1}{2}\,a\cos.^{-1}\frac{a-2\,s}{a}\right\}.$$

When $t = 0$, $s = 0$; therefore the above is the correct integral.

If $s = a$, $\sqrt{\frac{a}{2\,m}}\,(a\,s-s^2) = 0$, and $\frac{a-2\,s}{a} = -\,1 = \cos.\,180^\circ$; therefore in this case

$$\cos.^{-1}\frac{a-2\,s}{a} = \pi\,;\quad t = \sqrt{\frac{a}{2\,m}}\cdot\frac{1}{2}\,a\,\pi = \frac{1}{\sqrt{8\,m}}\,a^{\frac{3}{2}}\,;$$

therefore the times of the descent of falling bodies from different heights would be as the square roots of the cubes of the lines of descent, upon the supposition that the force of attraction is inversely proportional to the square of the distance, and that there is no resistance; for the radius of the earth being constant the lines of descent will be as the distance from the point of departure of the falling body to the centre of force represented by a.

A careful comparison of the results of these experiments might lead to the discovery of the two great secrets, the law of gravity near the surface of the earth, and the correction to be applied for the resistance of the air. We believe that it would be also possible to obtain the trace of the trajectory of a shell if not of a round shot at short range and slow velocity by means of photography. The instrument should be placed at such an angle as to bring a sufficient portion of the range within the field of the lens to admit of our determining the general law of the curve. We should then have an oblique view of the range, the abscissæ being foreshortened, while the ordinates would be in their due proportion; but the angle at which the instrument had been placed with reference to the line of direction being known, the abscissæ could be calculated in their true proportions, and thus the law of the curve might be obtained.

There are three forces to which a projectile during its trajectory is subjected—the attractive force of gravity, the resistance of the air, and the initial propelling force. We have spoken of the first two in this article, and the third and last will be found treated of under the heads GUNPOWDER.

We have little doubt that a series of experiments carefully conducted, utilizing the improved means and appliances now at our disposal, would lead to a much closer approximation of calculated formulæ to practical results than has been as yet attained, enabling us to generalize, interpolate, and form corrected and at the same time practically useful tables. But to the prosecution of such experiments an insuperable objection appears to arise; they require appliances and means not usually at the disposal of private individuals, and the same stimulus to enterprise and risk of loss which exists in all matters relating to improvements in mechanical action when profit follows immediately upon successful results, does not exist in this case. Probably some would be inclined, in the interests of science alone, to carry out experiments of this kind if they were in possession of the means; but an assortment of large guns, balloons, voltaic batteries, and photographic apparatus, does not, as a general rule, form a portion of the properties of a gentleman of the period. Those therefore who are specially interested in facilitating the destruction of their fellow-creatures must only wait patiently till the matter is taken up by some of our foreign neighbours, and then, if they should be successful in discovering the true laws of gravity and resistance, we may be able to purchase the secret. But in the meantime we may console ourselves with the reflection that there is no absolute necessity for further discovery or improvement, for we find it stated by authority with reference to the effect of the resistance of the atmosphere to the motion of a projectile, that sufficient is known to guide the practical artillerist, and that it is only as a scientific question that any further prosecution of the subject is of interest, and this more on account of the difficulty of solution than for its practical importance. This fact being established, there remains little inducement to proceed further with an inquiry into the laws of gravity and the resistance of the air, for it is a difficult and intricate subject to deal with; and if no practically useful results are to be expected, and if it is merely an abstract inquiry into the laws of nature which is aimed at, the world is going much too fast for anything of that sort now. But even if we have arrived at the summit of perfection in the matter of artillery practice, it seems to be generally admitted, since long ranges have become the fashion, that some mode of ascertaining the distance in action more accurate than the unassisted estimate by the human eye is desirable. We shall therefore say a few words on this subject presently, p. 1772.

Fig. 3376 is of an instrument employed by General Anstruther to illustrate his system of gunnery.

1. Two bars, A B and B C, are joined by a hinge at B; they are to represent the ascent and descent of a projectile fired at any elevation with any velocity. A third bar A′ C′ represents the horizontal range; it is graduated by being divided into equal spaces of 100 yds. each, from A′ towards C′, 100, 200, 300, 400, &c., &c., yards, for the greatest range attainable. This bar A′ C′ has two collars, one fixed at A′, representing the place of the gun, the other movable to represent the point where the ball falls. Each of these collars has a ring through which the bars A B and B C will pass, and a milled-headed screw fixes them in their desired place. The horizontal bar is mounted on trestles to admit of the unemployed portion of A B and B C passing under the horizontal range.

2. Then A′ B C′ will always form a right-angled triangle, and we graduate A B and B C by dividing them so as to show the fall by gravity in the successive seconds, 1, 2, 3, 4, &c., &c., as far as we wish to do it, or can calculate the fall; then on B C we write the spaces described in the successive seconds, and on B A the velocity acquired; the two bars will therefore be graduated thus;—

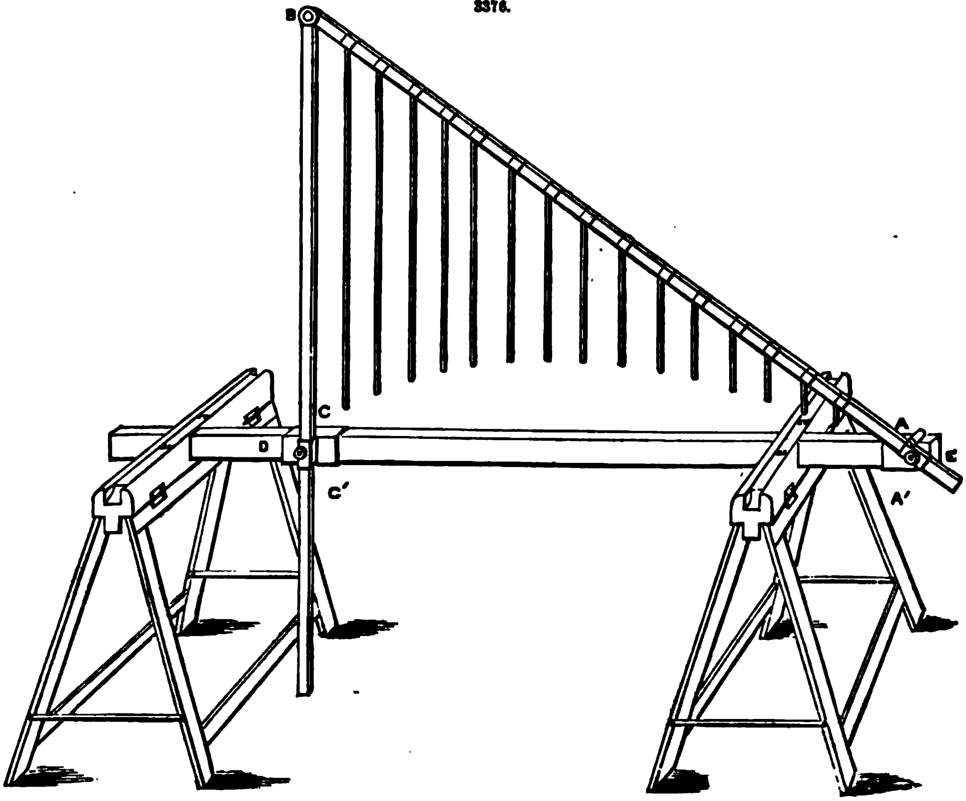
Time	1	2	3	4	5	6	7	8	9	10, &c.
B C	16·2	64·4	144	254·4	395	565·2	764·4	992	1247·4	1530, &c.
A B	31·8	64·1	95·1	125·6	155·5	184·8	213·5	241·6	269·1	296, &c.

3. These three bars thus graduated enable us to show the right-angled triangle formed by the simultaneous ascent and descent of any ball fired obliquely upwards; and we proceed to show how the trajectory is drawn.

4. For this purpose a set of metal rods, tinned or plated iron wire, are prepared, one for each second of time, as shown in our paragraph 2, where we show the space described by a falling body.

These are hung on the points of graduation of the oblique ascent, in an inverted order of succession, that is to say, the shortest rod lowest down; then a line drawn through the lower extremities of all these rods is the trajectory.

3376.



Example.—To draw the trajectory of a ball fired at an elevation of 11° 58′ 56″·25, with initial velocity 736 ft. a second, we set the bar A B in the ring of the collar at A, and clamp it to show velocity 736 ft., and we beard A B down to the desired angle, clamping its vertical bar B C at a distance of 5877·5 ft. from A′, and proceed to hang the metal rods for 1, 2, 3, 4, 5, 6, 7, 8, and 9 seconds on the oblique ascent; the last of these meets the range exactly, and we see the trajectory marked by ten given points; our Table showing the fall by gravity will enable us to give this curve for the tenth parts of seconds if required.

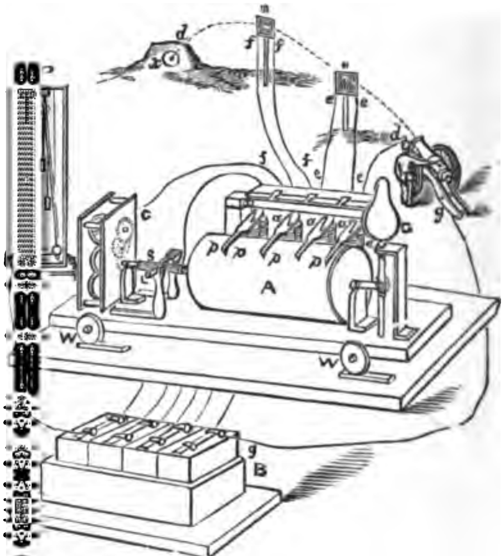
This instrument was offered to the service in 1864, when the model of which the above is a drawing was lodged in the United Service Institution.

Fig. 3377 is of an Electro-Chronoscope, invented by Major-General Anstruther, C.B. The object of the electro-chronoscope is to measure exactly the time of flight of a projectile between two given points. Various modes of taking the time of flight have been in use for years. The simplest is a sort of clock, which is divided into 600 parts, and which is traversed by a hand once a minute, and which may be set going and stopped by touching a lever, and from which a second hand may be detached in its course. It indicates to tenths of seconds; but, as the accuracy of this instrument depends upon the manipulation of the observer, it is subject to considerable error. The self-registering principle is therefore the only one which will give reliable results.

General Anstruther had an apparatus designed by Mr. Holmes, which was entrusted to Messrs. Elliott for execution. The principle was in the main the same as represented in the woodcut. A cylinder, covered with paper soaked in a solution of ferrocyanide of potassium, had to revolve driven by a weight. Small iron wheels attached to slight springs had to trace, by decomposition, blue lines on the paper, on the principle of Bain's electric printing telegraph, as long as an electric current passes; but when put into practice it was found that, if we may call it so, a sort of ink was formed which continued to mark after the current was broken, and consequently the object aimed at, extreme accuracy, was lost. In conjunction with Mr. Bashley Britten, Messrs. Elliott altered the plan. Instead of making use of chemical decomposition by an electric current, they substituted metallic paper for the cylinder A, and a clockwork C for the weight. At *a a a a* of sketch are four electro-magnets made of the same material, in exactly the same manner, a matter of some importance, as we shall presently see. The keepers are attached to springs which carry metallic points *p p p p*. When the electric current makes the iron magnetic, the keeper is attracted, and the metallic point presses gently on the paper; one of the electro-magnets is in connection with an accurately-timed seconds pendulum, which at every beat makes connection for a

… magnet attract the keeper every second, and dots … registered independently of the velocity with which …

3377.



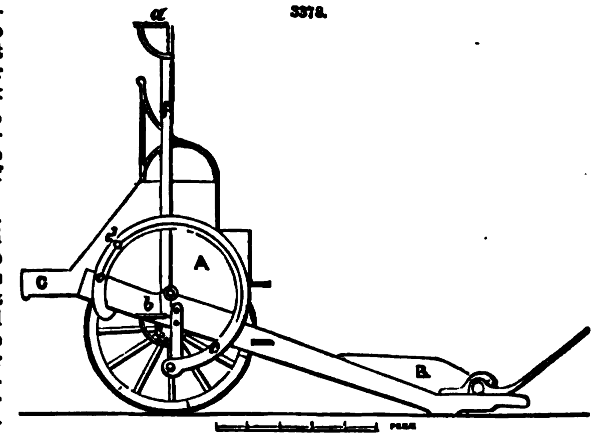
… equal velocity of rotation. This governor should … the governor of House's printing telegraph. A … ment complete.

…ner;—A galvanic battery is connected with each of … circuit the pendulum, the second the target at the … yards distance, the fourth the target at 200 yards; … the cylinder, which is, however, not yet in motion. … points draw lines. After one or two revolutions the … ving the gun, breaks the wire of No. 1 target, and … target is struck, point No. 2 ceases to draw; and … ses to draw. The clockwork is now stopped. To … paper is taken off; where point No. 1 has ceased … of the second and third lines will give seconds and … distance of the two dots made by the pendulum, for … ous contrivance of Mr. Holmes's, a compass to one … a nut turns in the second leg and subdivides one … e trammels either way the points of the compass … present the second, and each turn of the screw will … ce great or small. A second cylinder is provided, … orehand so as to save time. Various objections may … we will briefly allude. Electricians will point out … tism in the soft iron will retard the release of the … firstly, by making the electro-magnets exactly alike, … retardation in the release of the three different …; secondly, by not bringing the keeper into actual … brass pin. A second objection might be that the … t or unequal effects on the magnets; if this should … in the currents to make them all alike. A third … ient one, is, that when the three points draw, there … ely released; but as the amount of friction can be …

… *nge-finders*, each of which, with the exception of the … little or no use in action. The instrument, Fig. … ut 9 ft. (3 yds.) long, mounted upon a carriage in … re. Two quadrants $a\,b$ are attached, one at each … of rotation of the movable limbs of the quadrants … f the instruments and the centres of rotation of the … ine. With the aid of a telescope, movable in the … ter screw, tangents of angles of elevation or depres- … degree of accuracy. The axletree-boxes are raised so … he higher level to place his feet upon, and there is … him to rest his hand upon while observing. The … the level of the ground, so that an observation can

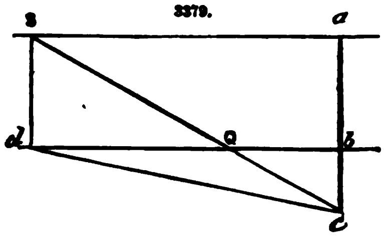
be taken with what we term the quadrant on the lower level in a kneeling position. The bar and boxes rotate upon an axle placed a little above the lower quadrant, so as to admit of the bar being depressed when the observation is completed, the quadrant a falling into a small case fitted to receive it at B, and the quadrant at b fitting into a similar case near the point C. A handle and crank d, moving upon a circular bar $c\,c$, is attached, and there is a counterweight at C in order to facilitate the elevation and depression of the bar.

3378.



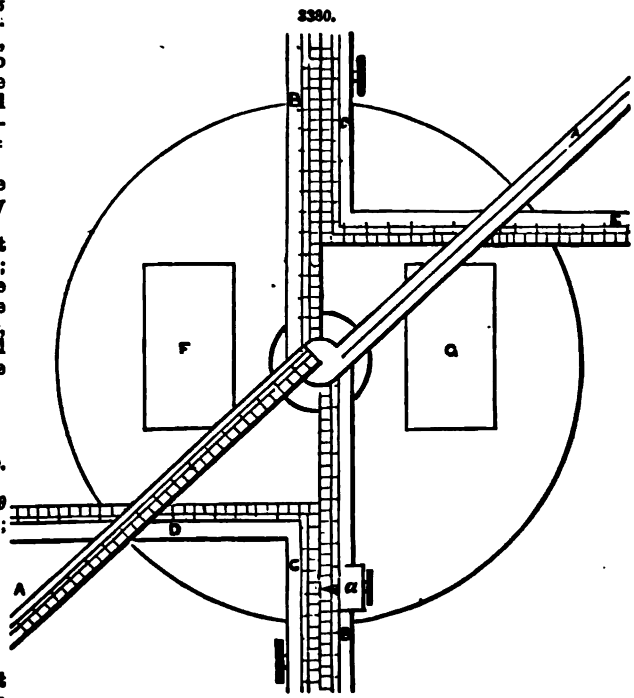
Use of the Instrument.—In Fig. 3378 the near wheel is removed as well as the axletree-box on the near side. The tangents of angles of depression of a distant point are taken at both levels and read off by the observers, the distance or range being thus calculated or worked out by means of a calculating instrument, as hereafter described, by a third man appointed for that purpose. The calculating instrument is shown in Fig. 3380, a small platform at B being constructed to work the calculating instrument upon.

3379.



If the radius or range of the quadrants $a\,b$ equal 10 in., divided into tenths, and if the micrometer screw divides $\frac{1}{10}$ of an inch into 100 equal parts, then the 10-in. radius = 10000; such equal parts and the difference of the tangents of the two observed angles at a and b may be found to be $\frac{1}{10000}$ of an inch = ·0001 = 20″·6 nearly. If greater accuracy be required, and a $\frac{1}{10}$ vernier be applied to the micrometer screw, then the radius is taken = 100000 equal parts, and the tangents are measured to $\frac{1}{100000}$ of an inch = ·00001 = tangent of 2″ nearly.

3380.



The principle upon which the instrument, Fig. 3378, works may be shown as follows;—

Let $\theta = \angle$ of observation at the higher level, $a\,s\,c$, Fig. 3379; $\theta' = \angle$ observed angle at the lower elevation, $b\,d\,c$; $r = s\,d$, the length of the vertical bar $a\,b$, Fig. 3378; R = $d\,b$, the horizontal range; c the position of the enemy's battery; then

$$s\,d : b\,c :: d\,o : o\,b;$$

$$\therefore\ s\,d : (s\,d + b\,c) :: d\,o : (d\,o + o\,b).$$

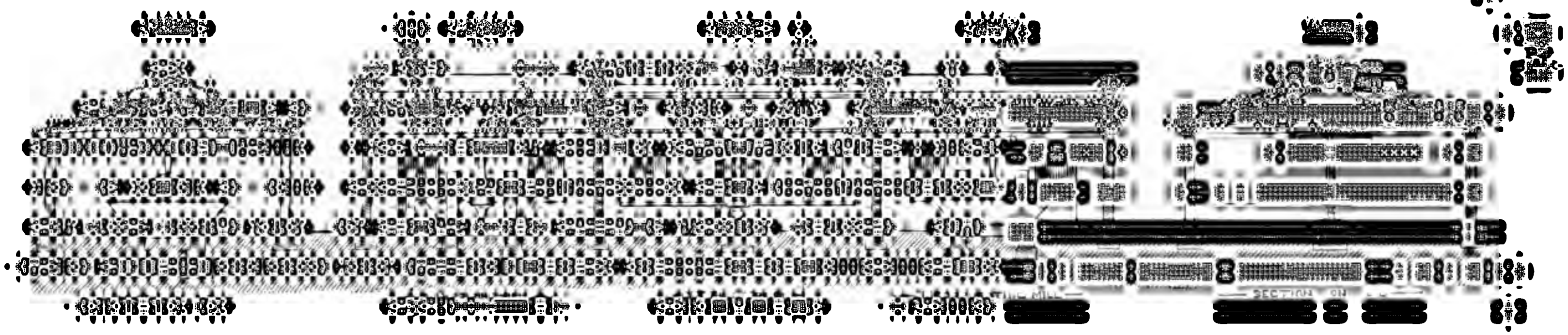
Since $s\,d = a\,b$, we have $(\tan.\theta - \tan.\theta') : \tan.\theta :: r \operatorname{cotan}.\theta : \mathrm{R}$;

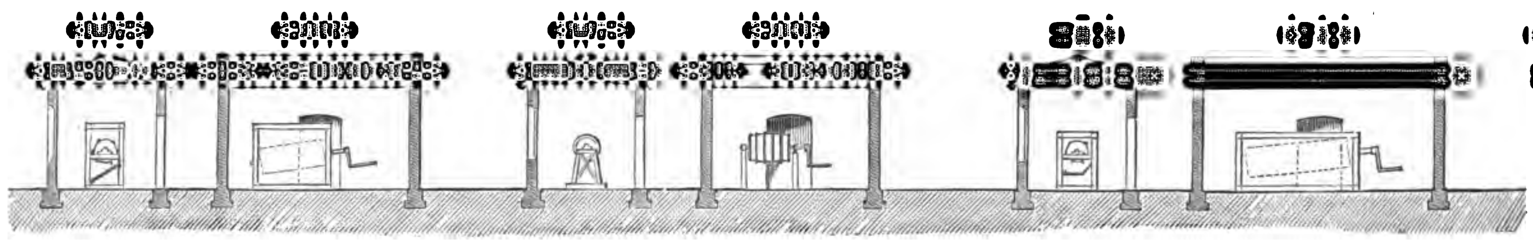
$$r\frac{\tan.\theta \operatorname{cotan}.\theta}{\tan.\theta - \tan.\theta'} = \mathrm{R};$$

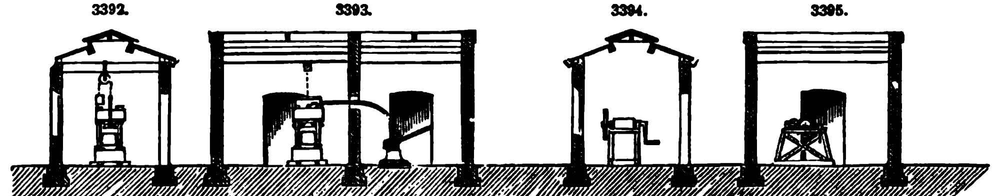
$$\therefore\ r\frac{1}{\tan.\theta - \tan.\theta'} = \mathrm{R}.$$

In this demonstration we put the radius = 1; but suppose the radius = 10000 equal parts, which we take as a whole number, then the horizontal range $\mathrm{R} = \frac{r \times 10000}{\tan.\theta - \tan.\theta'}$. Now if tan. θ measures 2 in. 4-10ths and 69 from the micrometer circle, and

OWDER.




CHARCOAL SIFTER
MIXING BARREL
GLAGING CYLINDER


3392.
3393.
3394.
3395.
HYDRAULIC PRESS AND PUMP
BREAKING DOWN CYLINDER

tan. $\theta' = 2$ in. 4-10ths and 48 from the micrometer circle, then tan. θ − tan. $\theta' = 2469 - 2448 = 21$, and as $r = 3$ yds., we have $B = \frac{30000}{\cdot 21} = 1429$ yds., the horizontal range very nearly. The calculating instrument before alluded to, Fig. 3380, forms a vast variety of similar triangles, which instantly give a fourth proportion to tan. θ − tan. θ', r, and 10. The divisions on the sides B, B; D, E, are all equal and not in logarithmic order. The calculating instrument consists of a circular disc F G attached to a vertical limb B B, upon which two horizontal limbs D and E, with clamping screws, slide vertically; upon the lower branch of the limb B B a small marker a, with clamping screw, slides vertically; a diagonal bar A A revolves upon the centre point of the circular disc; two small tables of tangents F and G are engraved upon the disc. The instrument is worked as follows;—While the first observation is being taken on the higher level, move the horizontal limb E to 10 on the upper branch of the vertical limb B B and clamp it; upon the observation at the higher level being taken and read off, move the marker a on the lower branch of the vertical limb B B to the number answering to tan. θ; upon the observation at the lower level being read off, move the horizontal limb D till the number answering to tan. θ' coincides with the number on the vertical limb B B indicated by the marker a, and clamp it. Bring the revolving diagonal bar to the number on the horizontal limb D corresponding to r. The upper arm of the diagonal bar will indicate a number which multiplied mentally by 10, 100, or 1000, according to the graduation of the instrument, will give the horizontal range. Nolan makes $r = \frac{b}{\sin. (\beta + \gamma)}$, and tries to obtain results by divisions put in logarithmic order on circular rings. Nolan's reasoning looks extremely scientific, but in practice all such performances are extremely ridiculous. See ANEMOMETER. ANGULAR MOTION. ARTILLERY. DAMMING. DYNAMOMETER. GUNPOWDER. GYRATION. ORDNANCE. OSCILLATION. PERCUSSION.

GUNPOWDER. FR., *Poudre à canon;* GER., *Schiesspulver;* ITAL., *Polvere da cannone;* SPAN., *Pólvora.*

The mechanical part of the manufacture of gunpowder is essentially the same in principle, and differs only to an immaterial extent in detail, whether in the various gunpowder mills in Great Britain or in those of the Continent.

The annexed engravings show a complete set of gunpowder machinery, manufactured for the Japanese Government by J. and H. Gwynne, of the Hammersmith Iron Works, London. In consequence of the objection to steam-power as increasing the danger of explosions, and the absence of sufficient water, it was found necessary to drive the mills by cattle-power.

Fig. 3381 shows a side view of the pulverizing mill, Fig. 3382 a front view of the same; Fig. 3383 shows the arrangement of the cattle track; Fig. 3384 the front view of the incorporating mill; Fig. 3385 is a side elevation of the cattle track; Figs. 3386, 3387, different views of the charcoal sifter; Figs. 3388, 3389, different views of the mixing barrel; Figs. 3390, 3391, different views of the glazing cylinder; Figs. 3392, 3393, different views of the hydraulic press and pump; Figs. 3394, 3395, different views of the breaking-down cylinder.

Figs. 3396 to 3402 exhibit another arrangement of gunpowder machinery, made for the Italian Government by J. and H. Gwynne. In this case water-power was available, and was thus made use of to drive the machinery by means of two turbine water-wheels, as shown. Figs. 3396, 3397, show a plan and section of the house containing the mills; Figs. 3398, 3399, show a section and plan of the house containing mixing barrels, glazing cylinders, machines for breaking down and sifting the gunpowder; Fig. 3400 is the packing house; Fig. 3401 the house containing hydraulic presses; Fig. 3402 the house for sifting charcoal. The houses are all made of as slight material as possible, so that in case of an explosion the powder has as little surface to act upon as possible, and are placed at such a distance apart so that the one may not be endangered by the other. The machinery in Figs. 3396 to 3402 is exactly similar to that in Figs. 3381 to 3395, but on a somewhat larger scale.

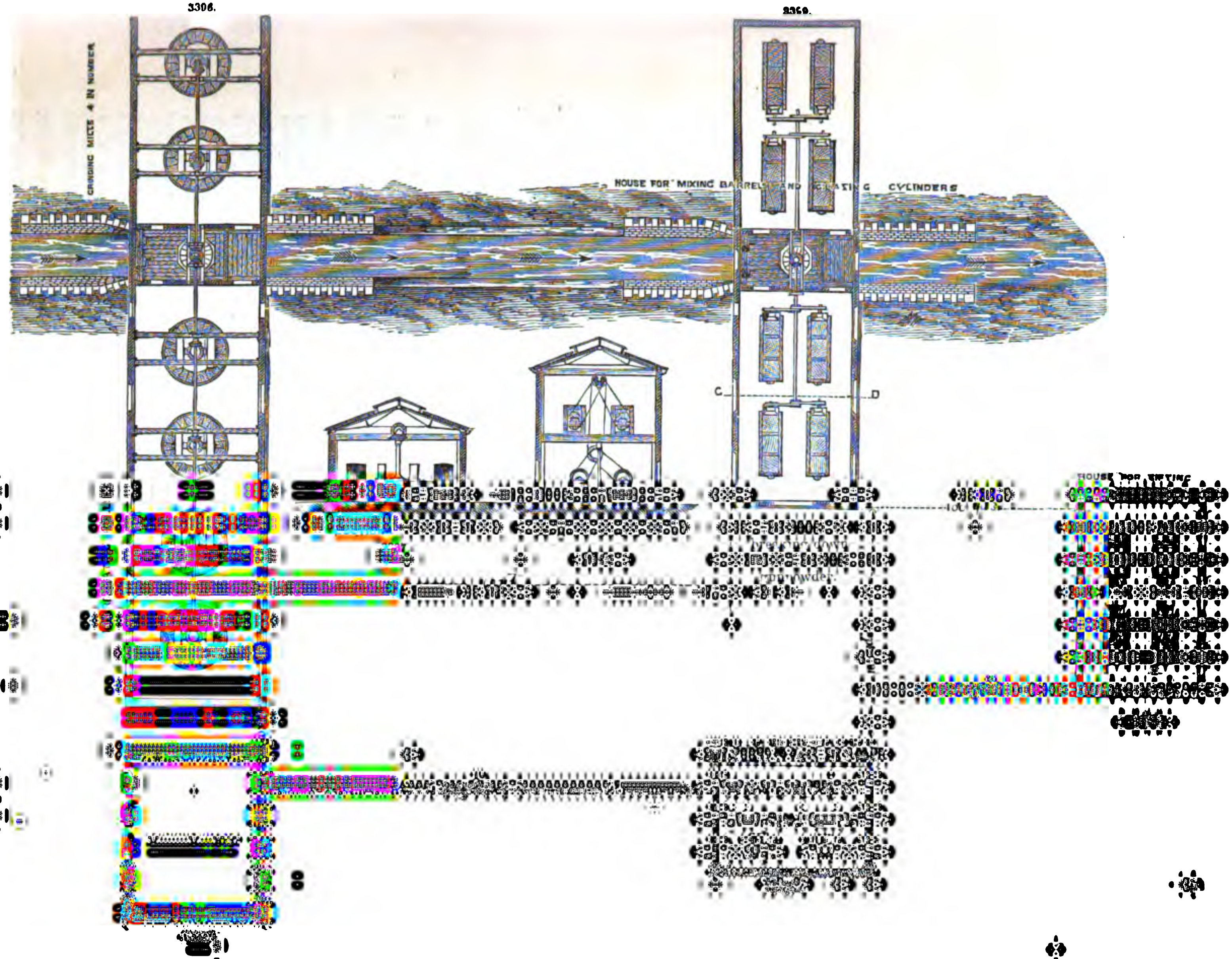
Figs. 3403 to 3406 give sectional views of the pulverizing and incorporating mill to a larger scale. This machine is made upon the latest and most improved construction, the rolls and pan being made of cast iron, which after many experiments has been found to be the most suitable material for the purpose; the upright and cross spindles are both made of wrought iron, working in gun-metal bearings at each end; the outside frame of the pan is made of wood, and the whole is worked from the main driving shaft by a pair of bevel-wheels, as shown in Fig. 3403. These details are necessary in order to explain the full working of the machinery.

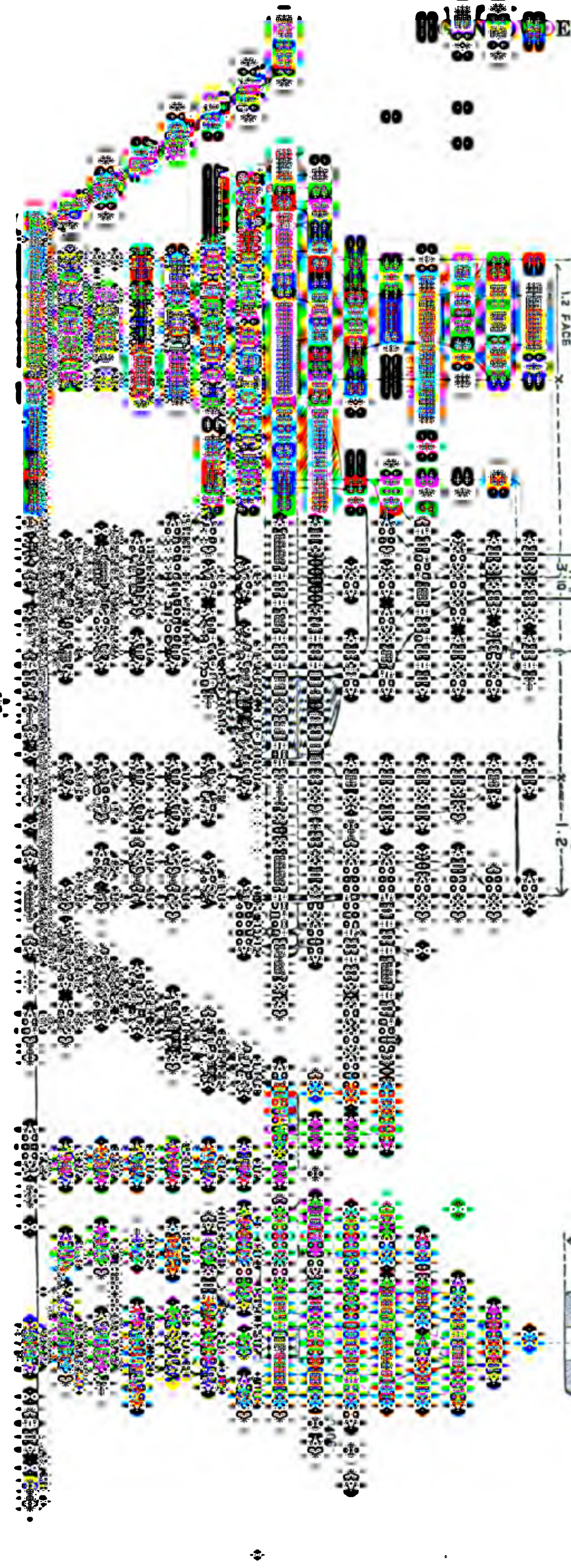
The materials having been pulverized in the mill, Fig. 3381, are apportioned out, the following being the proportions used at the Government mills, Waltham Abbey, and also by other makers generally;—

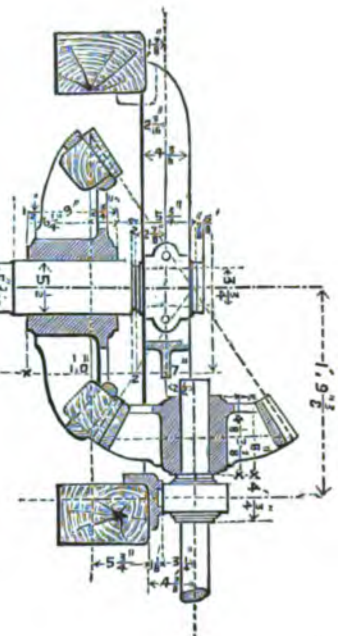
	lbs.	oz.	drachms.
Saltpetre	31	8	0
Charcoal	6	4	13
Sulphur	4	3	3
	42	0	0

This quantity of the ingredients, termed a charge, is placed in the mixing apparatus, Figs. 3388, 3389, which consists of a wooden cylinder, traversed in its centre by an octagonal shaft provided with several fan-like arms. Both the shaft and cylinder are kept in motion, but in different directions. This latter arrangement so facilitates the commingling that the homogeneous powder is ready for removal and further manipulation in from five to ten minutes, when it is transferred to bags which are pressed, and the mouths of which are firmly secured, in order to prevent the disunion or separation of the ingredients in the order of their density during the transport to the incorporating mill. It is evident if the charges were too lightly packed the ingredients would be liable to be separated, the saltpetre finding its way to the bottom, while the

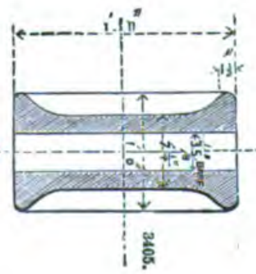
3306.


GRINDING MILLS 4 IN NUMBER
HOUSE FOR MIXING BARRELS AND ... ATING CYLINDERS
C
D

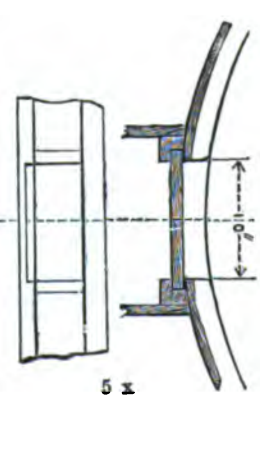




3403.



3405.



3404.

sulphur and charcoal would form layers above, the latter, from its light pulverulent state, escaping in the form of dust between the fibres of the cloth of which the bag is formed.

The next process is the incorporation. The mixture is spread out upon the bed of the mill, Fig. 3384, and distilled water is added to ensure the intimate cohesion of the particles, and the whole is again submitted to the action of the mill to ensure the uniform pressure and bruising of the ingredients; but in this case the motion is less rapid than during the grinding of the ingredients. A great degree of caution must be exercised to prevent the intervention of any hard or silicious matters. The requisite amount of moisture must be maintained during the operation, and for this purpose water is slowly added at suitable intervals till the incorporation is complete, which is generally the case at the expiration of from three to four hours.

The compound has now the properties of gunpowder; it indurates in a very short period, forming hard cakes, termed mill-cake. When it has lost some of its moisture, but before it becomes completely dry, it is passed between the breaking-down cylinder, Figs. 3394, 3395, and is then submitted, between copper plates placed in strong boxes, to hydraulic pressure of about 150 tons to the sq. ft., in the press house, Figs. 3392, 3393.

The powder has now to be granulated, and for this purpose it is placed in the breaking cylinder, Figs. 3394, 3395, and afterwards sifted by hand to the various degrees of fineness. The finest powder is then placed in the glazing cylinder, Figs. 3390, 3391, in a slightly moist state, and revolved very slowly, so that the grains become polished by attrition. The gunpowder is finally dried, which is now generally done by means of steam heat, and sometimes by a current of air, previously heated in another chamber from 130° to 150°, passing over the powder placed on canvas shelves.

On Fig. 3403 the footstep B will have to be covered with wood, in order to prevent the grains of powder being exploded from getting between the shaft and brass.

By mixing intimately saltpetre with charcoal or with sulphur, we obtain substances which, when subjected to a high temperature, deflagrate and suddenly develop a large volume of gas. When the combustion takes place in a contracted space considerable pressure is exerted on the surrounding walls of this space, and if one of these be movable, it may be projected with more or less force.

If, for example, 1 equivalent of nitre KO,NO_5 is mixed with 1 equivalent of carbon, there are produced, by detonation, 1 equivalent of carbonate of potassa, 1 equivalent of nitrogen, and 3 equivalents of oxygen; $KO,NO_5+C=KO,CO_2+N+3O$; 2 volumes of nitrogen and 3 of oxygen will therefore be disengaged.

We may calculate by approximation the volume of gas developed by one volume of the detonating mixture. 1 equivalent of nitrate of potassa weighing 1264·3, and 1 equivalent of carbon weighing 25·0, the weight of the mixture will therefore be 1339·3. Assuming that this pulverized mixture occupies the same volume as an equal weight of water, we can admit that a weight 1339·3 grammes of the mixture will occupy a volume of 1·339 lit. Now, this weight of the mixture develops 1 equivalent = 175 of nitrogen, and 3 equivalents = 300 of oxygen.

1 lit. of nitrogen, at 32°, under a pressure of 0·760 m., weighs 1·257 gramme.
1 „ oxygen „ „ „ 1·429 „

The volume occupied by the nitrogen at 32°, and under a pressure of 0·760 m., will be given by the proportion $1{\cdot}257 : 1{\cdot}000 :: 175 : x$, whence $x = 139{\cdot}2$ lit. The volume occupied by the disengaged oxygen under the same circumstances will be deduced from the proportion

$$1{\cdot}429 : 1{\cdot}000 :: 300 : y, \text{ whence } y = 209{\cdot}9 \text{ lit.}$$

Thus a volume of detonating mixture represented by 1·339 lit. yields 349·1 lit. of gas at 32°, and under a pressure of 0·760 m.—a volume 253 times greater than that of the explosive substance. The volume of gas, at the moment of development, is really much larger than we have just found, being strongly dilated by the high temperature produced by the combustion; and we may safely admit that the expansion is at least three times greater than that given by calculation, when the gas was supposed to have a temperature of 32°.

If 1 equivalent of nitrate of potassa is mixed with 2 equivalents of carbon, then 1 equivalent of carbonate of potassa, 1 equivalent of nitrogen, 1 of carbonic acid, and 1 of oxygen are formed; $KO,NO_5+2C=KO,CO_2+N+CO_2+O$. The equivalent of carbonic acid being represented by 2 volumes, it will be seen that 5 volumes of gas are still disengaged; that is, that the expansion is the same as in the preceding case. The projectile force may, however, be greater if a high temperature be developed during the combustion.

Lastly, if 4 equivalents of carbon are added to 1 equivalent of nitre, then 1 equivalent of nitrogen and 3 equivalents of oxide of carbon are disengaged; $KO,NO_5+4C=KO,CO_2+N+3CO$. 1 volume of oxide of carbon containing only a ½ volume of oxygen, it is evident that 6 volumes of oxide of carbon will be developed; the gaseous volume will therefore be equal to 8. Thus there will be a greater production of gas than in the two preceding cases. The projectile force might, however, be less, if the heat developed be not so great. Moreover, in the mixture we have just supposed, a great portion of the carbon does not ignite.

Mixtures of nitre and sulphur also produce, by detonation, considerable volumes of gas. Thus a mixture of 1 equivalent of nitre and 1 equivalent of sulphur yields 1 equivalent of sulphate of potassa, 1 equivalent of nitrogen, and 2 equivalents of oxygen; $KO,NO_5+S=KO,SO_3+N+2O$; 4 volumes of gas will therefore be formed.

With 1 equivalent of nitre and 2 equivalents of sulphur we have

$$KO,NO_5+2S=KO,SO_3+N+SO_2;$$

that is, again 4 volumes of gas; for the equivalent of sulphurous acid is represented by 2 volumes.

A mixture of 1 equivalent of nitre with 4 equivalents of sulphur gives

$$KO,NO_5+4S=KS+N+3SO_2;$$

2 volumes of nitrogen and 6 volumes of sulphurous acid will therefore be disengaged; in all, 8 volumes of gas. In fact, however, the gaseous volume is less considerable, owing to the incomplete combustion of the sulphur.

Mixtures of nitre and carbon generally produce a greater volume of gas than mixtures of nitre and sulphur; but the latter have the advantage of being more combustible.

Experiments have proved that the mixtures possessing the greatest projectile force consist of nitre, carbon, and sulphur. A mixture of

1 equivalent of nitre	1264	66·0
1 equivalent of sulphur	200	10·5
6 equivalents of carbon	450	23·5
	1914	100·0

gives $KO,NO_5+S+6C=KS+N+6CO$; that is, 14 volumes of gas. But in reality the gaseous volume is less considerable, because a large portion of the carbon escapes combustion, and the temperature does not rise very high.

The following mixture possesses a greater projectile force;—

1 equivalent of nitre	1264	74·8
1 equivalent of sulphur	200	11·9
3 equivalents of carbon	225	13·3
	1689	100·0

We then have $KO,NO_5+S+3C=KS+N+3CO_2$, with the disengagement of 8 volumes of gas.

We may calculate by approximation the volume of gas produced by a volume (1) of this mixture. Let us again admit that the mixture occupies the same volume as an equal weight of water. We shall say that 1689 grammes of the mixture, or a volume of 1·689 lit., disengages 175 grammes of nitrogen = 139·2 lit., and 825 grammes of carbonic acid = 417·3 lit.; total gaseous volume = 556·5 lit. A volume (1) of the detonating mixture will therefore produce 329 times its volume of gas at 32° and under a pressure of 0·760 m. of mercury.

The numerous experiments made in all countries to discover empirically the best composition for powder show that it should be as approximate as possible to that just now theoretically developed.

In France three different compositions are in use.

For war powder—

Saltpetre	75·0
Sulphur	12·5
Charcoal	12·5
	100·0

For sporting powder—

Saltpetre	76·9
Sulphur	9·6
Charcoal	13·5
	100·0

For blasting powder—

Saltpetre	62·0
Sulphur	20·0
Charcoal	18·0
	100·0

Prussian war powder shows the following composition—

Saltpetre	75·0
Sulphur	11·5
Charcoal	13·5
	100·0

English and Austrian war powder—

Saltpetre	75·0
Sulphur	10·0
Charcoal	15·0
	100·0

Swedish war powder—

Saltpetre	75·0
Sulphur	16·0
Charcoal	9·0
	100·0

Chinese powder—

Saltpetre	75·7
Sulphur	14·4
Charcoal	9·9
	100·0

French blasting powder is the only one which differs remarkably from the theoretical composition just indicated; this is because a great projectile force is not required, and the Government, which imposes a considerable tax on sporting powder, endeavours to manufacture a blasting powder such that it cannot be substituted for the former. This powder has, indeed, less strength, and fouls the gun very rapidly.

Powder should satisfy several conditions, which vary according to the weapon in which it is to be used. When it is very explosive, and the explosion of the charge is instantaneous, the reaction on the walls of the weapon is sudden and violent, frequently causing the weapon to burst; the powder is then said to be too *explosive*. If the powder is not sufficiently explosive, the projectile is thrown from the weapon before all the charge is burned; a portion of the latter, therefore, is uselessly inserted and wasted. The powder most suitable for any given weapon is that which, burning perfectly whilst the projectile passes through the chamber of the piece, communicates to it, gradually, and not instantaneously, the whole projectile force of which it is capable. Hence

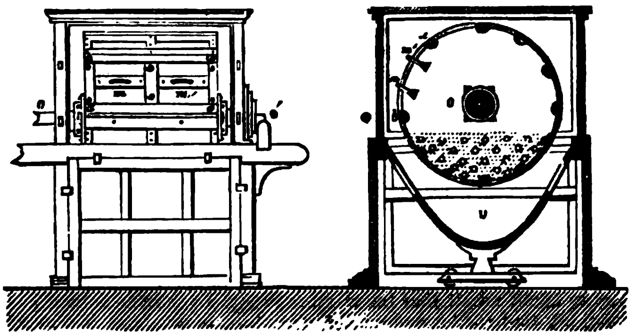
the quality of the powder must vary according to the nature of the piece in which it is used. With equal quantities of the ingredients the quality of the powder can still be altered, by using charcoal more or less carbonized, by giving the substance a greater or less degree of compactness, or by varying the size of the grain.

Before proceeding to study the manufacture of the various kinds of powder, we shall investigate the preparation of its primary components.

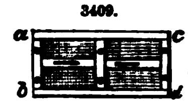
Saltpetre.—The saltpetre used in the manufacture of powder is the refined nitre. This nitre is remarkably pure, and rarely contains more than two or three thousandths of sea-salt. It comes from the refinery in very small crystalline grains, and in this state is used in the manufacture of powder.

Sulphur.—Powder mills purchase the refined sulphur in rolls. It must be reduced to an impalpable powder, which is effected in wooden drums, Figs. 3407, 3408, having on the inside wooden brackets *a*, *b*, arranged along the edges of the cylinder. These drums are cylindrical, and about $1^{m}\cdot 10$ long, with a diameter of about $1^{m}\cdot 15$: they revolve on a horizontal iron axis O O′. Through a door *a b c d*, which is furnished with iron handles *m′ m*, the material is introduced. Pulverization is effected by means of small brass balls, of about 5 or 8 millimètres in diameter, of which each drum contains 150 kilogrammes: 30 or 40 kilogrammes of sulphur are added, and the drum is made to revolve for six hours, during which time the balls, rolling with the sulphur, crush it and reduce it to extreme fineness. In order to withdraw the sulphur, the door of the drum is removed, and replaced by a similar door *a b c d*, the panels of which are of wire gauze, Fig. 3409; by causing the drum to revolve five or six times, the sulphur escapes through this door, leaving the balls in the drum.

3407. 3408.



3409.



The powdered sulphur is sifted in a bolting machine, similar to that used for bolting flour; the particles which have not been sufficiently pulverized are thus separated, as well as any small grains of sand, which might occasion accidents in the manufacture of the powder.

The charcoal destined for the fabrication of powder must be most carefully selected. All kinds of wood are not suitable for the preparation of this charcoal: the tender and light woods, which yield a friable, porous charcoal, leaving very little ash, are preferred.

The woods most esteemed are the black alder and spindle-tree: poplar and chestnut may also be used. Hemp stalks, likewise, yield a very good charcoal.

The wood of the black alder is exclusively used in France. The branches of about 15 or 20 millimètres in diameter are preferred; and if larger branches are used, they are first split. The bark is always removed, as it gives too much ash. These branches are cut into lengths of from 1·5 to 2 mètres, and tied in bundles weighing from 12 to 15 kilogrammes.

The carbonization is never effected in kilns, as common charcoal is made, but in pits or in cylinders.

Carbonization in Pits.—Cylindrical pits, about $1^{m}\cdot 5$ in diameter and $1^{m}\cdot 2$ in depth, are excavated in the earth and lined with bricks, and filled with the wood, cut into pieces of $0^{m}\cdot 30$ in length, until the heap rises to the height of a few decimètres above the mouth of the pit. Fire is communicated through a hole at the bottom; and as the combustion advances, the branches are raised with a fork, so as to allow the fire to be regularly distributed. The pile gradually sinks, and fresh wood must be added to keep the pit full. When a flame is no longer seen, the mouth of the pit is hermetically closed by a sheet-iron lid, and the carbonization is then finished without access of air. The pit remains closed for three or four days, in order entirely to extinguish and cool the charcoal. It is then opened, the charcoal removed, and conveyed to the sorting room, where it is most carefully sorted by hand; such branches as have not been sufficiently carbonized and the half-burnt pieces are rejected, as also those which are too much carbonized, and therefore would make bad powder. The good charcoal should be used immediately, as it sensibly deteriorates by exposure to the moist air.

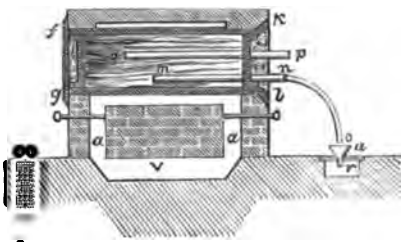
By carbonization in pits, about 18 to 20 per cent. of charcoal is obtained.

Carbonization in Cylinders.—This process yields a much larger proportion of charcoal; its quality is also more constant and uniform, because the fire can be regulated at will, and the carbonization can be arrested at the proper moment.

The cylinders C, C, Figs. 3410, 3411, are arranged in pairs in the same furnace: they are made of cast iron, having 2 mètres in length and about $0^{m}\cdot 70$ in diameter. One end of the cylinder is closed by a cast-iron lid, having four circular openings, through which pass four sheet-iron tubes, as *p q*, *m n*. Three of these tubes, which serve for the introduction of sticks of wood, are closed externally with wooden plugs, which can be withdrawn from time to time, so as to observe the progress of the carbonization. The fourth is open, and gives exit to the gases which are evolved during the process. A curved copper tube *n o* is fitted to one end of it, opening above a funnel *v*,

…ging along the furnace and opening into the …nged in the same mason-work.
…ne and smoke ascend between the two cylin… u and u′ into a horizontal canal V V′, which …imney built in the middle of the room. The … r and r′, in the vertical pipes u and u′. The …y exposed to the action of the fire is covered …um of temperature is thus found at the top of …peration.

3411.



… $1^m\cdot5$ in length: when the cylinders are filled …is end is made of two sheets of iron, the space …e then introduced into the tubes *p q*, *m n*.
… on the grate: turf is the fuel generally used. …under four or five hours. The progress of the …of the smoke which escapes from the pipe *n o*. …cing, the assay sticks are withdrawn, and an …ss of decomposition in the various parts of the …an in others, the combustible is pushed to the …s is also regulated by the registers *r* and *r′*. In …er from the pipe *n o*; the operation is then ter…ation is completed without further aid. On the …d in sheet-iron extinguishers (étouffoirs).
… per cent. of charcoal, which is sorted by hand,

… charcoal is intended for sporting powder: it is …bon roux); its colour then is brown. For war …e state of *black charcoal* (charbon noir), called …arcoal would be too explosive for muskets or

…*cess.*—Blasting powder is made in France by a …s obtained the name of *Bernese process.* This …d and musket powder.
…arcoal, which is unfit for other powder, is used: …jurious to the quality of the powder, as blasting …ity.
… in the manufacture—pulverization, mixing,

… in iron drums, exactly as has been previously … time balls of $4^m\cdot5$ in diameter, and some vary…g more difficult to grind. The drum contains …es of sulphur and 27 of charcoal, which is the …or is closed, and the drum made to revolve for …enty-eight revolutions a minute: the binary … from the drum.
…14·25 kilogrammes of the substance taken from …, and 23·25 kilogrammes of saltpetre added. … compound, namely;—

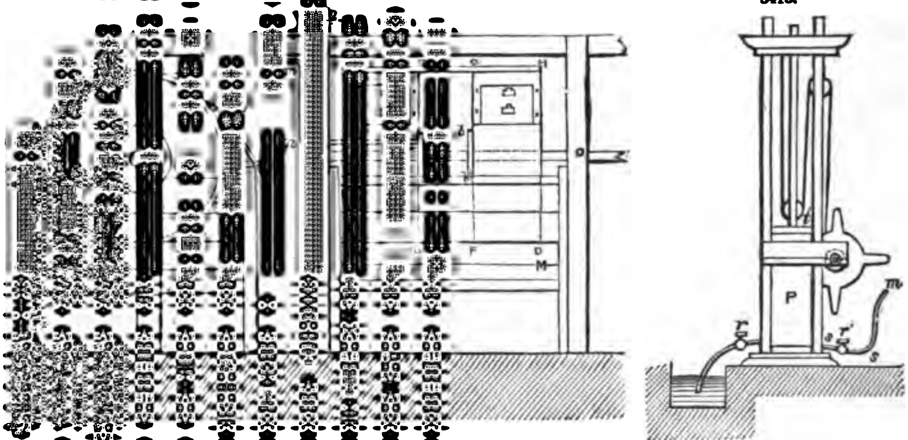
…	23·25	62·0
…	7·50	20·0
…	6·75	18·0
	37·50	100·0

… which are leather drums, containing 60 kilo…he 37·50 kilogrammes of it are introduced, and …five or thirty revolutions a minute. After four …aterial is then conveyed into a maie, and placed

…ture of round grains consists of two large oak …$0^m\cdot63$ in height. Each of them has only one … being furnished with a circular opening U of

[illegible] are traversed by the same iron axes I O, supported [illegible] r chains. Two copper discs $a\,a'$, $b\,b'$, fixed on the [illegible] th the ends E B and C F of the drum, while four [illegible] y. Each drum has a door M, of $0^m\cdot35$ by $0^m\cdot60$, [illegible] roducing and withdrawing the material. All the [illegible] large trough N, furnished with inclined copper [illegible] thdrawn by the doors, and conduct it into barrels [illegible] the other O H F D for glazing the powder.

3413.



[illegible] G B is furnished with twelve small cleats x, x, x, [illegible] and cause a small wooden hammer p, fastened by [illegible] stantly on its surface, detaching, by its blows, any [illegible] the drum. A copper watering tube $n\,u$, 2 centim. [illegible] side pierced with very minute holes, enters the [illegible] nd communicates, by a curved copper tube $n\,m\,s$, [illegible] is composed of a copper pump-tree P, in which a [illegible] $t\,t'$, fastened to the upper part of the piston, works [illegible] t in motion by means of a winch and a rope which [illegible] lower part of the pump-tree communicates, on the [illegible] other, with the injecting tube $s\,m\,n\,u$; two stop- [illegible] bes. When the stopcock r is opened and the piston [illegible] r: if this stopcock be closed and that at r' opened, [illegible] g the water to escape through the watering tube

[illegible] the workman removes the door M of the granulating [illegible] der already grained, called the *nucleus* (noyau), the [illegible] replaces the door, and sets the machine in motion [illegible] g this motion, the first sprinkling of 5 per cent. [illegible] ucleus which occupies the lower part of the granu- [illegible] he rotary motion of the drum constantly renewing [illegible] ed.

[illegible] ces through the opening U 50 kilogrammes of the [illegible] inserting 1 kilogramme at a time with a wooden [illegible] drum. The movement of the machine rolling the [illegible] owder causes the latter to adhere to their surface, [illegible] yers.

[illegible] e, and then 50 kilogrammes of the ternary mixture [illegible] hine to revolve for a quarter of an hour, the work- [illegible] sorbed; he then empties the machine, by dropping [illegible] These operations last from thirty-five to forty

[illegible] ine, is composed of variously-sized grains, which [illegible] one by shaking the grains over two leather sieves; [illegible] ates those grains which are too large, while the [illegible] oo fine to pass through. The holes in the equalizer [illegible] r pieces which do not pass through are set aside; [illegible] equalizer, the holes of which are $1^{mm}\cdot2$ in diameter. [illegible] he diameter of which is comprised between $1^{mm}\cdot2$ [illegible] they are deposited in a barrel to undergo a subse- [illegible] sub-equalizer is composed of grains smaller than [illegible] is grain need only be increased in the granulating [illegible] operation yields the quantity of nucleus necessary

for a succeeding operation, it is sufficient to obtain some for the first operation, for which the angular powder, of the size of musket powder prepared in the stamping machine, is employed. The grains which are too large, and the irregular pieces which remained on the equalizer, are broken by means of the cake, and used as a nucleus for the succeeding operation.

Blasting powder is glazed as well as sporting powder, in order to increase its density. This operation is effected in the second drum C H F D. 200 kilogrammes of equalized grains are introduced, and it is turned for four hours; by direct experiment it is ascertained when the grain has acquired sufficient density. For this purpose 60 grammes of the glazed grains are poured into a graduated test-glass; the grain is considered as sufficiently glazed when the level of the material rises to a certain division in the instrument. The glazed grain is dried in the ordinary way.

Round war powder is manufactured by the same method, the usual proportion of the ingredients for war powder, 25 of saltpetre, 12·5 of sulphur, and 12·5 of charcoal being employed. Two kinds of equalized grains are separated; those of which the diameter is between $1^{mm}\cdot 2$ and $2^{mm}\cdot 1$ constituting cannon powder; and musket powder, the diameter of the grains of which varies from $1^{mm}\cdot 0$ to $1^{mm}\cdot 20$.

Analysis of Powder.—The analysis of powder is a tedious and delicate operation, when the proportions and nature of its components are to be ascertained very exactly. The first operation is to determine the proportion of hygrometric water the powder contains, for which purpose a known weight of powder is exposed for several days in a dry vacuum, and the loss it experiences ascertained; or else the substance is placed in a U-shaped tube, kept at a temperature of 60° or 70°, and traversed by a current of dry air.

Ten grammes of dry powder are then treated with hot water, which dissolves the nitrate of potassa. The insoluble residue, composed of sulphur and charcoal, is collected on a small filter, which has been previously dried and weighed. When this residue has been properly washed, it is dried with the filter at a moderate temperature, and weighed: by subtracting from this weight that of the filter above, the weight of the sulphur and charcoal is obtained. After separating, as carefully and completely as possible, the substance from the filter, it is again weighed in a small bottle and treated with sulphuret of carbon, which may be mixed with an equal volume of ether, without too much impairing its solvent power. The charcoal which remains isolated is collected on a small filter previously dried, and weighed after a second desiccation, after having been well washed in a mixture of sulphuret of carbon and ether. The weight of the sulphur is thus obtained by the difference. It may, however, also be weighed directly after evaporating, at a low temperature, the solvent which contains it. The charcoal of the powder is not pure carbon; it contains, as the carbonization is always imperfect, a considerable proportion of oxygen and hydrogen; but, as the chemical nature of the charcoal exerts great influence on the quality of the powder, it is important, in an accurate analysis, to ascertain the amount of carbon exactly. An idea may be formed of the composition of charcoal by the following numbers, obtained from the analysis of the red charcoal used in the manufacture of sporting powder;—

Carbon	71·42
Hydrogen	4·85
Oxygen and nitrogen	22·91
Ashes	0·82
	100·00

The quantity of sulphur contained in powder may also be determined by operating directly on the powder itself. To effect this 10 grammes of dry powder are dissolved in a small quantity of hot water, nitric acid is added, and, after allowing the fluid to boil, small quantities of chlorate of potassa are gradually introduced. Under the influence of these oxidizing agents the sulphur is dissolved in the state of sulphuric acid, which is precipitated, after the liquid is filtered, by chloride of barium. The precipitate is allowed to settle, the clear liquid poured on a filter, and the precipitate, after being boiled for a few moments with chlorohydric acid, to dissolve the nitrates it might contain, is collected on the same filter and weighed after calcination.

Ten grammes of dry powder may also be mixed with an equal weight of nitrate of potassa and four or five times its weight of chloride of sodium; the mixture being thrown, in small quantities at a time, into a platinum crucible, deflagrates slowly, without any loss of the material. It is subsequently treated with water, and the sulphuric acid is precipitated by chloride of barium, after supersaturating the liquid with chlorohydric acid.

It has also been proposed to dissolve the sulphur of the mixture of sulphur and charcoal, by a solution of monosulphide of sodium, or of hyposulphite of soda; but this process is useless, because the charcoal, being always considerably attacked by these alkaline liquids, gives off a peculiar acid, called *ulmic acid.*

Very frequently only the quantity of saltpetre contained in powder is to be ascertained. This is easily done by treating 50 grammes of powder with 200 grammes of hot water, and filtering the liquid into a test-glass having a mark at the level corresponding to 500 cubic centimètres. The material is washed with water on a filter until the filtrate reaches the level. The liquid is then cooled to 60°, the contraction it undergoes by cooling being compensated by the addition of a small quantity of water; it is then well shaken to render it homogeneous, and a peculiar areometer, graduated so that its level will mark immediately the hundredths of nitrate of potassa contained in the 50 grammes of powder, is dipped in. In this manner the proportion of nitrate of potassa may easily be determined, to very nearly a half-hundredth.

Gun-cotton.—The chemical constitution of gun-cotton has been conclusively established by the researches of Hadow. In the formation of substitution-products by the action of nitric acid upon cotton or cellulose, three atoms of the latter appear to enter together into the chemical change, and

the number of atoms of hydrogen replaced by peroxide of nitrogen in the treble atom of cellulose $C_{18}H_{30}O_{15}=3(C_6H_{10}O_5)$ may be nine, eight, seven, or six, according to the degree of concentration of the nitric acid employed.

The highest of these substitution-products is trinitro-cellulose, pyroxilin, or gun-cotton—$C_{18}\left\{\begin{matrix}H_{21}\\9NO_2\end{matrix}\right\}O_{15}=3\,C_6\left\{\begin{matrix}H_7\\3NO\end{matrix}\right\}O_5$; this being the substance first produced by Pelouze in an impure condition, in 1838, by the action of very concentrated nitric acid upon paper, or fabrics of cotton or linen, and afterwards obtained in a purer form by Schönbein, who employed a mixture of concentrated nitric and sulphuric acids for the treatment of cotton-wool; the object of the sulphuric acid being to abstract water of hydration from the nitric acid, and also to prevent the action of the nitric acid from being interfered with by the water which is produced as the chemical transformation of the cotton into gun-cotton proceeds. The formation of trinitro-cellulose is represented by the following equation;—

$$\underset{\text{Cotton.}}{C_6H_{10}O_5}+\underset{\text{Nitric Acid.}}{3\left\{\begin{matrix}H\\NO_2\end{matrix}\right\}O}=\underset{\text{Gun-cotton.}}{C_6\left\{\begin{matrix}H_7\\3NO_2\end{matrix}\right\}O_5}+\underset{\text{Water.}}{3\left\{\begin{matrix}H\\H\end{matrix}\right\}O}.$$

The lowest substitution-product for cotton, of those named above, appears to have the same composition as the substance which Braconnet first obtained in 1832, by dissolving starch in cold concentrated nitric acid, and adding water to the solution, when a white, highly-combustible substance is precipitated, to which the name of *Xyloïdin* was given. The substitution-products from cotton, intermediate between the lowest and highest, are soluble in mixtures of ether and alcohol, and furnish by their solution the important material *collodion*, so invaluable in connection with photography, surgery, experimental electricity, and so on.

According to Schönbein's original prescription, the cotton was to be saturated with a mixture of one part of nitric (sp. gr. 1·5) and three parts of sulphuric acid (sp. gr. 1·85), and allowed to stand for one hour. In operating upon a small scale, the treatment of cotton with the acid for that period is quite sufficient to effect its complete conversion into the most explosive product *pyroxilin*, or *trinitro-cellulose;* but when the quantity of cotton treated at one time is considerable, especially if it is not very loose and open, its complete conversion into pyroxilin is not effected with certainty unless it be allowed to remain in the acid for several hours. This accounts in great measure for the want of uniformity observed in the composition of gun-cotton and its effects as an explosive in the earlier experiments instituted; and it is moreover very possible that the want of stability and consequently even some of the accidents which it was considered could only be ascribed to the spontaneous ignition of the material, might have been due to the comparatively unstable character of the lower products of substitution, some of which existed in the imperfectly-prepared gun-cotton.

The system of manufacture of gun-cotton elaborated by General von Lenk is founded upon that described by Schönbein; the improvements which the former has adopted all contribute importantly to the production of a thoroughly uniform and pure gun-cotton; there is only one step in his process which is certainly not essential, and about the possible utility of which chemical authorities are decidedly at variance with General von Lenk.

The following is an outline of the process of manufacture of gun-cotton as practised by Lenk. See Atomic Weights. The cotton, in the form of loose yarn of different sizes, made up into hanks, is purified from certain foreign vegetable substances by treatment for a brief period with a weak solution of potashes, and subsequent washing. It is then suspended in a well-ventilated hot-air chamber until all moisture has been expelled, when it is transferred to air-tight boxes or jars, and at once removed to the dipping tank, or vessel where its saturation with the mixed acid is effected. The acids of the specific gravity prescribed by Schönbein are very intimately mixed in a suitable apparatus in the proportions originally indicated by that chemist; that is, three parts by weight of sulphuric acid to one of nitric acid. The mixture is always prepared some time before it is required, in order that it may become perfectly cool. The cotton is immersed in a bath of the mixed acids, one skein at a time, and stirred about for a few minutes, until it has become thoroughly saturated with the acids; it is then transferred to a shelf in this dipping trough, where it is allowed to drain, slightly pressed to remove any large excess of acid, and afterwards placed in an earthenware jar, provided with a tightly-fitting lid, which receives six or eight skeins, weighing from 2 to 4 oz. each. The cotton is tightly pressed down in the jar, and if there be not sufficient acid present just to cover the mass, a little more is added: the proportion of acid to be left in contact with the cotton being about 10½ lbs. to 1 lb. of the latter. The charged jars are set aside for forty-eight hours in a cool place, where they are kept surrounded by water to prevent any elevation of temperature and consequent destructive action of the acids upon the gun-cotton. The same precaution is also taken with the dipping trough, as considerable heat is generated during the first saturation of the cotton with the acids. At the expiration of forty-eight hours the gun-cotton is transferred from the jars to a centrifugal machine, by the aid of which the excess of acid is removed as perfectly as is possible by mechanical means, the gun-cotton being afterwards only slightly moist to the touch. The skeins are then immersed singly in water, and moved about briskly, so as to become completely saturated with it as quickly as possible. This result is best accomplished by plunging the skeins under a fall of water, so that they become at once thoroughly drenched. If they are simply thrown into the water and allowed to remain at rest, the heat produced by the union of a portion of the free acids with a little water would be so great as to establish at once a destructive action upon the gun-cotton by the acid present. The washing of the separate skeins is continued until no acidity can be detected in them by the taste; they are then arranged in frames or crates and immersed in a rapid stream of water, where they remain undisturbed for two or three weeks. They are afterwards washed by hand to free them from mechanical impurities derived from the stream, and are immersed for a short time in a dilute

boiling solution of potashes. After this treatment, they are returned to the stream, where they again remain for several days. Upon their removal they are once more washed by hand, with soap if necessary; the pure gun-cotton then only requires drying by sufficient exposure to air at a temperature of about 27° C. to render it ready for use. A supplementary process is, however, adopted by General von Lenk, about the possible advantage or use of which his opinion is not shared by others. This treatment consists in immersing the air-dried gun-cotton in a moderately strong hot solution of soluble glass (silicate of potassa or soda) for a sufficient period to allow it to become completely impregnated, removing the excess of liquid by means of the centrifugal machine, thoroughly drying the gun-cotton thus silicated, and finally washing it once more for some time until all alkali is abstracted. Lenk considers that by this treatment some silica becomes deposited within the fibres of the gun-cotton, which, on the one hand, assists in moderating the rapidity with which the material burns; and, on the other hand, exercises (in some not very evident manner) a preservative effect upon the gun-cotton, rendering it less prone to undergo even slight changes by keeping. The mineral matter contained in pure gun-cotton which has not been submitted to this particular treatment amounts to about 1 per cent. The proportions found in specimens which have been silicated in Austria and in this country, according to Lenk's directions, vary between 1·5 and 2 per cent. It is difficult to understand how the addition of 1 per cent. to the mineral matter, in the form chiefly of silicate of lime and magnesia, the bases being derived from the water used in the final washing, which are deposited upon and between the fibres in a pulverulent form, can influence to any material extent either the rate of combustion or the keeping qualities of the product obtained by Lenk's system of manufacture.

Gun-cotton prepared according to the system just described is exceedingly uniform in composition. The analyses prepared both in Austria and at Waltham Abbey have furnished results corresponding accurately to those required by the formula $C_6\left\{\begin{array}{l}H\\3\,NO_2\end{array}\right\}O_5$. In its ordinary air-dry condition it contains, very uniformly, about 2 per cent. of moisture—an amount which it absorbs rapidly from the air when it has been dried. The proportion of water existing in the purified air-dried cotton, before conversion, is generally about 6 per cent. When pure gun-cotton is exposed to a very moist atmosphere or kept in a damp locality, it will absorb as much as from 6 to 7 per cent.; but if it be then exposed to air of average dryness, it very speedily parts with all but the 2 per cent. of moisture which it contains in its normal condition. It may be preserved in a damp or wet state apparently for an indefinite period without injury; for if afterwards dried by exposure to air, it exhibits no signs of change.

The general properties of gun-cotton as an explosive agent have long been popularly known to be as follows;—When inflamed or raised to a temperature ranging between 137° and 150° C., it burns with a bright flash and large body of flame, unaccompanied by smoke, and leaves no appreciable residue. It is far more readily influenced by powerful percussion than gunpowder; the compression of any particular portion of a mass of loose gun-cotton between rigid surfaces will prevent that part from burning when heat is applied. The products of combustion of gun-cotton in air redden litmus paper powerfully; they contain a considerable proportion of nitric oxide, and act rapidly and corrosively upon iron and gun-metal. The explosion of gun-cotton when in the loose, carded condition—the form in which it was always prepared in the early days of its discovery—resembles that of the fulminates in its violence and instantaneous character. In the open air it may be inflamed when in actual contact with gunpowder without igniting the latter; in a confined space, as in a shell or in the barrel of a gun, the almost instantaneous rapidity of its explosion produces effects which are highly destructive as compared with those of gunpowder, while the projectile force exerted by it is comparatively small.

In 1864 the members of a committee appointed by Government examined a number of miners on the question of the relative advantages of gunpowder and gun-cotton when the latter was used in the form of hollow rope. The conclusion arrived at was, that gun-cotton was superior to gunpowder, especially when used in solid rocks; but that, taking into consideration the respective prices of the two materials, the extra care required with the cotton, and the injurious effects from inhaling its vapours, it was not the more useful of the two.

Experiments were, however, made by Thomas Sopwith and F. A. Abel, in 1865, with gun-cotton made from *pulp*.

The gun-cotton used in the experiments was in two forms, granulated and compressed.

The granulated gun-cotton was prepared from gun-cotton pulp, by mixing it, when dry, with 10 per cent. of its weight of gum arabic, dissolved in sufficient water to render the gun-cotton operated upon at one time just wet to the touch; after which the material was shaken for some time in a drum. By this treatment the gun-cotton assumes a granulated form: the globular grains produced vary in size between that of coarse small-arm powder and the coarsest form of blasting powder.

The compressed gun-cotton consisted of square pieces measuring from ½ to ¾ in. across, and ¼ to ½ in. in thickness. These were obtained by cutting up slabs of gun-cotton, prepared from pulp compressed to a density of about 50 lbs. the cubic foot.

The gun-cotton was sent to Allenheads, for the experiments, in the form of 1 oz. and ½ oz. charges, contained in cylindrical pasteboard cases of 1 in. diameter and upwards.

The mode of charging the holes was varied somewhat with the direction in which these were bored. If they were vertical, or bored at an angle inclining downwards, the paper case containing the charge was inserted into the opening—a size being selected which fitted loosely into the latter—and the gun-cotton was allowed to fall into the hole, the case being gently shaken and squeezed to aid the exit of the charge. If the inclination of the hole was inconsiderable, it was necessary to push down the gun-cotton occasionally by means of a wooden rod, especially when the granulated form was employed. Holes which were horizontal, or had an inclination upwards, were more difficult to charge. A contrivance prepared for this purpose, but which was not sufficiently long

for the horizontal holes in these particular experiments, consisted of a slightly conical tin tube about 12 in. long, the external diameter of one end being 1¼ in., and of the other ⅝ in. The narrow opening of this tube was plugged with a small piece of cotton wool, and the charge of gun-cotton was poured in at the other end. This wide end was afterwards inserted into the hole, the tube being passed into the latter as far as possible, and the charge was then pushed out of the tube by means of a wooden rod which fitted tightly into the narrow or outer end of the tin tube. This arrangement is similar in its nature to the measure used by the miners for charging the holes with powder, and, if made of sufficient length, would probably remove the difficulty which was experienced in charging the horizontal holes.

All the holes were fired with the ordinary miner's safety fuze, which was inserted before the entire charge was introduced, as is the practice in blasting with gunpowder.

After the charge was inserted into the holes and pressed down with the wooden rod, a plug of cotton waste was pushed down, for the purpose of carrying to the bottom any particles of the gun-cotton clinging to the sides of the hole. The latter was afterwards tamped, as usual, first with clay, and afterwards with débris of limestone in a nearly powdered state.

The experiments in the mines were tried in the upper part of the Great Limestone in East Cross Vein. The stone is very hard.

All the holes were nominally 1¼ in. in diameter. Some of them, in the hard limestone, tapered considerably, and would hardly have received 1-in. gun-cotton rope throughout their entire length.

Experiments at Thorn Green Quarry, Allenheads.—Eight holes were experimented upon in this quarry. The first three were vertical holes bored in ledges left by previous operations, about 2 ft. from the face of the ledge. They were numbered 1, 2, and 3, but were fired in reverse order, so that the work might be equalized, No. 3 being nearest the side face of the ledge, and No. 1 farthest from it, but about equally near to a flaw in the limestone.

No. 3 was 14 in. deep. It was charged with 6 in. of 1¼-in. gun-cotton rope (= 1·56 oz.), which left 8 in. for tamping. The proportion of gun-cotton used was that which the quarrymen had been in the habit of employing for similar holes. The face was well opened up by the explosion, and the work done pronounced highly satisfactory.

No. 2, 14¼ in. deep, which now had a side face similar to that of No. 1, was charged with 1·5 oz. of the compressed gun-cotton, which left 8¼ in. for tamping. The report of this explosion was more violent than in the case of No. 3; pieces of the stone were thrown into the air to a great height, and the rock was very much broken up all round the hole. There was decidedly more work done than by the former explosion, and fissures were produced running back in the ledge to a considerable distance. The charge of the gun-cotton used was evidently in excess of the work to be accomplished.

No. 1 hole, 14 in. deep, was charged with 1 oz. of the granulated gun-cotton, leaving 7 in. for tamping. In this instance the rock was well broken in all directions, and beyond the depth of the hole; large blocks were perfectly separated, and though not thrown off to any important extent, were readily removed by the men. The work to be performed by this explosion was perfectly accomplished, and apparently with a well-proportioned amount of powder.

No. 4 hole was vertical, 17 in. deep, and 2 ft. 4 in. from the face. It was a difficult hole, being in the midst of very solid rock. The men proposed to employ 8 in. of 1¼-in. rope for the hole (= 2·08 oz.), and they would have used 8 oz. of gunpowder. It was charged with 1·5 oz. of the granulated gun-cotton, leaving 6 in. for tamping. The work done by the explosion was pronounced by the quarrymen to be not only complete, but in excess of what they would have expected, from their usual experience: a mass of rock, 4 ft. by 2 ft. 4 in., and 14 in. thick, was blown off the front of the face; the latter was fissured in several places to a depth of about 4 ft. 6 in., and one fissure extended backwards, at right angles to the face, about 4 ft. in length.

No. 5 hole, vertical, was 18¼ in. deep, and 2 ft. 2 in. from the face. It was very similar in position to No. 4 hole, and considered to present about the same work to be accomplished. It was charged with 1 oz. of the compressed gun-cotton, which left 14 in. for tamping. The work done was very similar, and pronounced fully equal to that performed by the 1·5 oz. of granulated gun-cotton in the preceding experiment.

No. 6 hole, vertical, 14 in. deep and 2 ft. from the face, appeared somewhat weaker than the two preceding ones, as the face of the rock exhibited two flaws, one on either side of the hole, about 1 ft. and 18 in. to the right and left. It was charged with ¾ oz. of compressed gun-cotton; the tamping amounted to 12 in. Two wide fissures were produced in the face, by the explosion, to a depth of about 5 ft., and some fissures (one about 4 ft. long) were produced towards the back of the ledge. The masses of rock detached by this operation, which admitted of removal by the workmen, appeared nearly equal in quantity to the work done in the two preceding operations. The quarrymen were greatly astonished at the work done with the ¾ oz. of material, in a position where they would have employed at least 6 oz. of gunpowder.

No. 7. This was a horizontal hole, 30 in. deep, in the face of a ledge of the limestone. The distance of the hole from the upper surface or ledge was 3 ft. The total width of solid rock to the right and left of the hole was 5 ft., there being a vertical fissure on the one side about 2 ft. 5 in. from the hole. There was also a horizontal flaw, near the base, in the block of rock to be operated upon. The hole was considered a very strong one by the men. They proposed to employ 10 oz. of powder, or 12 in. of 1¼-in. gun-cotton rope (= 3·12 oz.). The hole was charged with 1 oz. of granulated gun-cotton, leaving 23 in. for tamping. By the explosion a large block of the rock, constituting the principal mass of the ledge, situated above the hole, was detached and moved forward some distance, so that it was easily thrown off by the workmen; in addition, a smaller block, considerably to the left of the hole, was detached, and the rock was broken up beneath the hole, so that a considerable quantity could be easily removed. The amount of stone detached by this operation, with 1 oz. of the granulated gun-cotton, was about 53 cub. ft.

No. 8. This was a hole, 26 in. deep, driven at a slight angle into a face of the rock which had not yet been operated upon. The height of the face above the hole was about 6 ft., and the undisturbed surface soil was above it. It was charged, in the first instance, with 1 oz. of compressed gun-cotton, the tamping occupying 23½ in. of the hole. The explosion blew only about 2 in. off the mouth of the hole, but produced vertical fissures, extending to some distance above and below the hole. This hole was afterwards re-charged with 2 oz. of compressed gun-cotton, the explosion of which considerably increased the fissures, but the mass of rock was so closed in upon all sides, excepting the face, that a more considerable charge than 2 oz. would evidently have been required in the first instance (when the rock was quite sound) to effect any important dislodgment. After the second charge, the hole was too unsound for further experiments.

Experiments in the Mines at Allenheads.—There is some difficulty in forming an accurate estimate of the comparative work done, by different charges of gun-cotton or gunpowder, in the confined space of a drift, or other similar locality; and the débris of previous blasting operations sometimes render it scarcely possible to estimate correctly the work accomplished.

The following is a brief statement of the most definite observations made;—

Ten holes had been previously prepared and were operated upon. In three cases of what was pronounced to be difficult work, the holes had been driven either in a horizontal or slightly sloping direction into the perfectly sound rock, in positions where no weakness had been induced by previous blasts. The work accomplished by the compressed gun-cotton was pronounced excellent; the rock being removed up to the extremity of the holes. In one of these experiments, in a hole 13 in. deep, 1½ oz. of compressed gun-cotton were used, leaving 7 in. for the tamping. In another hole, 11 in. in depth, ¾ oz. of compressed gun-cotton, with 8 in. of tamping, performed the work allotted to it perfectly. In a third hole, 16 in. deep, at the base of the rock, quite apart from previous blasting operations, 2 oz. of the compressed gun-cotton were used, leaving 12 in. for tamping. The rock was cleanly detached to the base of the hole. The miners stated that the usual charge of gunpowder for a hole of this kind (about 8 oz.) would not have been likely to do any useful work, the position of the hole being a very difficult one.

Another hole of the same depth as the last, and similarly situated, was charged with only 1 oz. of the compressed material. In this instance a length of 10 in. of the hole was blown away with the surrounding rock, leaving 6 in. undestroyed.

A hole, 15 in. deep, driven almost horizontally into the rock, was also charged with 1 oz. of compressed gun-cotton. In this instance, again, a portion of the hole was not blown away, but this result may have been, to some extent, due to an unsoundness of the hole at the base, which was discovered after the explosion.

Four holes were operated upon, which were known to be unsound (that is, to pass into cavities or fissures of more or less considerable size). Two of these, each 12 in. deep, which were pronounced very unsound, were charged with 1 oz. of the granulated gun-cotton. In both instances, the gases resulting from the ignition of the gun-cotton escaped through the fissures in the rear, and there was no destruction. (The same kind of failure took place on several occasions, with the employment of both gun-cotton rope and gunpowder, when the committee operated upon unsound holes at this place in October, 1864.)

A third hole, 14 in. deep, and unsound at the base, was charged with 1½ oz. of compressed gun-cotton, and 9 in. of tamping. On this occasion, the work done was pronounced good, the rock being blown away to within 4 in. of the bottom of the hole.

The fourth hole, 12 in. deep, in which the unsoundness was less obvious than in the preceding three, was charged with 1 oz. of granulated gun-cotton, leaving 7 in. of the hole for tamping. The work allotted to this hole was perfectly accomplished, the rock being cleared to the base of the hole.

These trials of gun-cotton charges prepared from the pulp, though insufficient to furnish conclusive results as regards the merits of the explosion, when employed in a granulated form, nevertheless afford sufficient experimental evidence to warrant the following statements;—

1. The *granulated* gun-cotton pulp (not compressed) appears decidedly more effective as a blasting material than an equal weight of gun-cotton in the form of rope. This result is to be ascribed mainly to the greater rapidity of explosion of the gun-cotton when in the form of light granules; it may also, in some measure, be due to the circumstance that, in a hole charged with gun-cotton in the form of grains, or small fragments, the charge is in close contact on all sides with the rock, which is not the case when gun-cotton rope is employed, unless the hole is perfectly cylindrical, which it very seldom is. This remark as to the irregular shape of the holes does not apply to those made by the boring apparatus of Percy Westmacott, which has been partially in use at Allenheads Mines, and by which a perfectly cylindrical hole is effected. The operation of this machine was witnessed by the committee in 1864, but it has not yet been brought into continuous use.

This form of the material occupies a little more space in a hole than an equivalent weight of gun-cotton rope; this circumstance may perhaps somewhat lessen the full destructive power of the granulated gun-cotton; but a careful comparison of results is yet needed before a definite opinion can be expressed on this point.

2. The *compressed* gun-cotton, in the form of small lozenge-shaped fragments, exhibited a superiority in destructive power over the granulated gun-cotton, which may be ascribed to the fact of its occupying less space, and therefore affording room in a hole for a proportionately large amount of tamping, and also bringing the destructive agent well in rear of the work to be performed. The superiority of the gun-cotton in this form over charges of gun-cotton rope, as regards the comparative amount of work done by equal weights of the two, appears unquestionable.

3. No decided evidence was obtained of a superiority of the granulated and the compressed gun-cotton over gun-cotton rope, when employed in *unsound* holes.

4. The charging of holes, the position of which was horizontal, or nearly so, with gun-cotton in

the form of granules or small fragments, presented difficulties, to overcome which, special mechanical appliances would have to be used if the gun-cotton be introduced into the hole without an envelope. There is also considerable risk, under these circumstances, of small fragments of the gun-cotton becoming lodged on the sides of the hole, unless their removal can be ensured by carefully cleaning the holes with plugs of cotton waste, hemp, or soft paper, before the tamping is proceeded with. The charges might, however, be inserted into holes in thin paper cases, which would obviate the difficulties and possible risk attending the charging of the holes, though it would probably do away, to some extent, with the advantage resulting from the tendency of the granulated gun-cotton, when employed without a case, to accommodate itself to any irregularity in the shape of the hole.

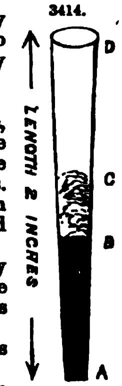
It has been found that the explosive force of gun-cotton may, like that of nitro-glycerine, be developed by the exposure of the substance to the sudden concussion produced by a detonation: and that if exploded by that agency, the suddenness and consequent violence of its action greatly exceed that of its explosion by means of a highly-heated body or flame. This is a most important discovery, and one which invests gun-cotton with totally new and valuable characteristics; for it follows, as recent experiments have fully demonstrated, that gun-cotton, even when freely exposed to air, may be made to explode with destructive violence, apparently not inferior to that of nitro-glycerine, simply by emoloying for its explosion a fuze to which is attached a small detonating charge.

The mode of operation is as follows:—

The detonating substance is placed in a tin tube of the dimensions shown, Fig. 3414, and it occupies in the inside of the tube the space from A to B.

On this at C is placed a small plug of gun-cotton, and the rest of the tin tube from C to the open end at D is empty.

3414.



Before leaving the manufactory a small piece of paper is pasted on the end merely to prevent anything falling into it, and this paper, so long as it remains, serves to distinguish the charged or useful primers, as the tin tubes are called, from empty tubes.

It is in this form that the detonating primers are supplied from the manufactory.

These primers are, in fact, large percussion caps, and are to be handled with care, as also to be protected from fire, and from all violent concussion. They explode with some violence when ignited, or if struck a violent blow, but with reasonable care are quite harmless, as much so as ordinary percussion caps for fowling-pieces. They may not only be safely handled, but may be thrown about with any freedom short of actual and intentional violence. Even when thrown on the ground or allowed to fall from a height of 20 or 30 ft., they are in no way affected by such usage.

When the primer is to be used, the paper cover at D is removed, and an ordinary fuze is then inserted, so as to be in contact with the gun-cotton at C. The tube is made large enough to receive an ordinary fuze, and as soon as the insertion has been made, the tube is pressed close to the fuze by a pair of common pliers.

This preparation is most conveniently done before entering a mine, but there is nothing to prevent its being done in a mine or quarry at any time.

The charges of compressed gun-cotton are made with a circular hole to receive the fuze. Into this hole the small end of the tin primer is inserted, instead of the fuze, and this is the only difference in the mode of firing as compared with the usual mode of exploding gun-cotton with a fuze.

It is important to observe that when the primer is thus used for blasting, it is not necessary to fill up the hole by any stemming or tamping; the hole may be left perfectly open, and those who know how often accidents occur in the process of stemming or tamping will at once appreciate the saving of time and the amount of personal safety thus obtained.

In the case of failure of explosion, accidents often occur from the miners attempting to remove the stemming material. All this is avoided, as after a proper interval of time the fuze with the primer attached can be safely and easily withdrawn from the open bore-hole.

Some remarkable results have been already obtained with this new mode of exploding gun-cotton. Large blocks of granite and other very hard rock, and iron plates of some thickness, have been shattered by exploding small charges of gun-cotton, which simply rested upon their upper surfaces—an effect which will be sufficiently surprising to those who have hitherto believed, as everyone has believed, that unconfined gun-cotton was scarcely to be considered as explosive at all, that it puffed harmlessly away into the air, not exerting sufficient force upon the body on which it might be resting to depress a nicely-balanced pair of scales, supposing the charge to be fired upon one plate of the scale. Further, long charges or trains of gun-cotton, simply placed upon the ground against stockades of great strength, and wholly unconfined, have been exploded by means of detonating fuzes placed in the centre or at one end of the train, and produced uniformly destructive effects throughout their entire length, the results corresponding to those produced by eight or ten times the amount of gunpowder when applied under the most favourable conditions. Mining and quarrying operations with gun-cotton applied in the new manner have furnished results quite equal to those obtained with nitro-glycerine, and have proved conclusively that if gun-cotton is exploded by detonation it is unnecessary to confine the charge in the blast-hole by the process of hard-tamping, as the explosion of the entire charge takes place too suddenly for its effects to be appreciably diminished by the line of escape presented by the blast-hole, some loose sand or broken rubbish to hold the fuze in position being all that is required. Thus the most dangerous of all operations connected with mining may be dispensed with when gun-cotton fired by the new system is employed.

Abel and Sopwith, in their report upon the use of gun-cotton with the detonating primer, give the following as an illustration of the different results obtained by the use of the primer as compared

with the explosion of gun-cotton by an ordinary fuze;—A disc of gun-cotton, weighing 1 oz., was laid upon a large slab of sandstone, fired by means of the ordinary fuze; it merely ignited with a sudden burst of flame, without much noise, entirely without violence, and quietly burnt away in about thirty seconds, doing no injury whatever to surrounding substances; but when the same quantity of gun-cotton of the same quality was laid on the same stone, and fired by means of a detonating primer, the whole mass instantaneously exploded with a report as loud as a cannon, and with an amount of destructive energy which could with difficulty be understood by any who had not quietly seen and carefully examined the result; not only was the stone shattered and broken into many pieces, but those portions of it which were immediately under the charge were literally ground and crushed into sand.

It will readily be observed that this discovery, which we believe is due to Mr. Brown, of the War Office Chemical Establishment, is likely to be attended with the most important results. Not merely is the strength of gun-cotton exploded in this way much greater than that of the same substance fired by simple ignition, but it now operates under conditions which were sufficient under the old system practically to deprive gun-cotton of its power.

See Boring and Blasting. Quarrying.

GUN MACHINERY, Rifled. Fr., *Machine à rayer les canons de fusil, ou les bouches à feu;* Ger., *Ziehbank für Geschütze;* Ital., *Macchina da rigare;* Span., *Máquina de estriar les cañones.*

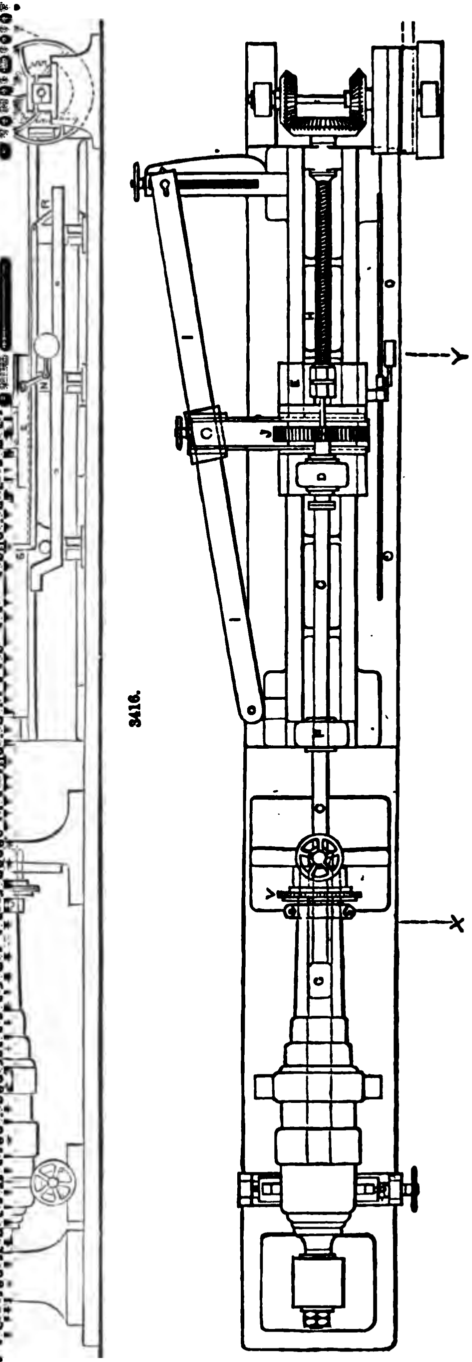
John Anderson, writing in P. I. M. E., 1862, observes that in the manufacture of guns, more especially of rifled cannon, one great object is to have the bore of definite dimensions, perfectly straight and parallel. The difficulty of accomplishing this depends entirely on what is considered straightness or parallelism, and on the closeness of measurement which may be adopted. With reference to dimensions: if the bore were completed in its boring up to the exact size previous to rifling, it would, from the rubbing of the rifling block and the rusting and cleaning after proof, be considerably over the size when actually finished. Hence it is found necessary to bore only up to within $\frac{3}{1000}$ of an inch of the proper dimensions, and two plug-gauges are employed for the purpose, one $\frac{3}{1000}$ of an inch under the proper size and the other exactly the proper size; the first is 12 in. long and must pass through the bore like the plug in the Whitworth gauge, while the other should not enter. In working so near there is much liability of exceeding the dimensions; hence the entrance for the final boring tool is made from the muzzle end, where an enlargement is of the least consequence. In the preparation of instruments for such precise boring it is found in practice that adjustable cutters are the most economical and convenient, with packings of the finest paper, which may now be obtained less than one thousandth of an inch in thickness. But in every instance these tools wear to some extent before reaching the other end, even if there is nothing left for the last cutter in the series to cut away. The farther end of the bore is therefore smaller than the other to an extent which is never less than one thousandth of an inch; but this difference is not considered sufficient to warrant the risk that would be incurred in proceeding from the other end a second time with a newly-adjusted instrument still untried. In dealing with muzzle-loading guns the difficulty is much increased in comparison with breech-loading, as the latter afford great facility of arrangement; and it is to breech-loading guns that the present paper chiefly refers.

In order to prepare for the last boring but one, the original bore of the innermost tube becomes the basis to work from, on the same plan as already described with reference to the previous preparation of this tube for building up the gun. It has lost its truth to some extent by the shrinking on of the exterior tubes, but that is recovered by future steps. A true bearing is then turned upon the exterior of the gun at both ends, and it is placed in bearings on a long saddle in a vertical machine. A boring bar with several sets of cutters is used, which works in bearings at both ends of the gun, and has upon it a block that follows the last set of cutting instruments. The bar revolves in fixed bearings, the gun having a slow motion upwards. There is usually about $\frac{1}{16}$ of an inch in the diameter to be cut out by this preliminary operation, and the aim is to continue the bore up to the required size, namely, $\frac{3}{1000}$ of an inch below the finished dimension, but this is seldom done; care is taken, however, that the bore is not above the size. It might be supposed that the turned bar and bored bearings would give a round hole, but this is not the case unless they are perfectly round themselves; hence these portions of the machine are looked upon as a foundation of truth, and are prepared as carefully as if intended for gauges. The boring bars, although made of steel like the gauges, are constantly wearing, and require vigilant attention to keep them up to truth. The hole from this boring is generally nearly straight, but never parallel; hence it is difficult to examine it with gauges, although no other mode of measurement is of any value in giving precise information on so delicate a point.

The next and last boring is done with the intention of making the hole parallel, but with no effort at straightness except what is derived from the bore itself as already made. The tool employed is a long broaching bar, shown in Fig. 3421, with six cutters A A arranged in two sets of three each, as shown enlarged in Figs. 3422, 3423. The first three cutters have all the work to do, the second set on entering being adjusted to the same diameter and intended only to scrape any of the surface that may be left from the first, which is not much, as there is seldom more than one thousandth of an inch altogether to be cut away. Both sets of cutters cut on the side rather than the front. The value of three cutters for steady cutting is well known; but it is also found that such an instrument is very apt to make a bad polygonal bore unless it copies a true circular form from something else. This true circular form, in addition to straightness of bore, is taken from the bore itself as already made. The transfer is effected by means of the bearing surfaces B B on the broaching block, Fig. 3422, which are long spiral surfaces made of gun-metal and filling the bore. In the earlier instruments it was found that straight bearing surfaces on the broaching block were liable to allow the roundness of the bore to wander into a polygonal shape; but by twisting the bearing surfaces into a spiral form round the block, as shown at B B in Fig. 3422, this liability has been prevented. An ordinary horizontal lathe is the most convenient for this operation, but it

... r from the cuttings; hence the lathes are placed ... soapy water to flow through.

... inch of being parallel, but is never positively ... present stage of the manufacture All the tool

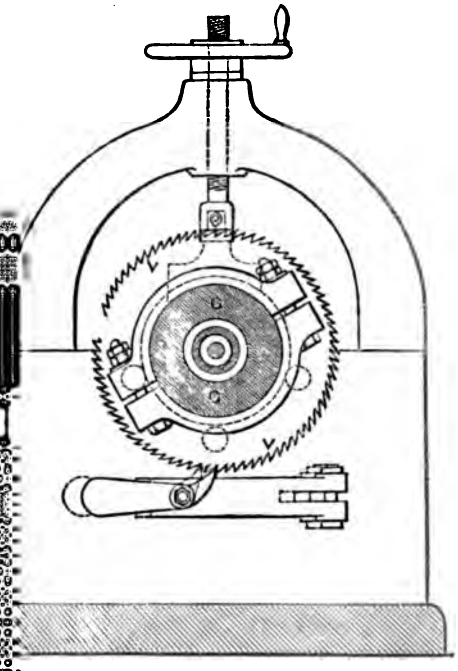


3416.

... red spiral was cut on the rifling bar, as shown in ... in a bearing. The nut for the rifling bar to work ... and the machine being set in motion, its recipro- ... de groove, and an ordinary dividing plate gave the

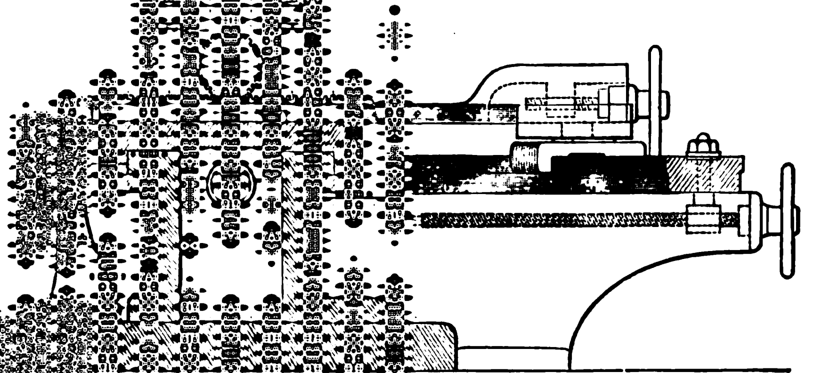
[illegible] possessed all the elements for rifling guns with a [illegible] bottom; but in practice guns have to be rifled [illegible] of groove, with sudden turns, with the shape

3417.



[illegible]

[illegible] 18.



[illegible] where the grooves shunt, or indeed for any [illegible]

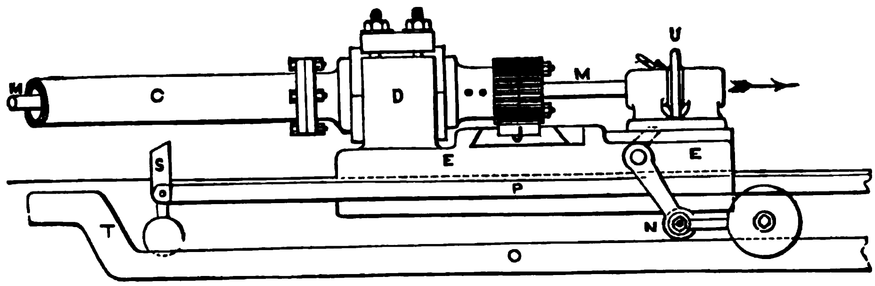
[illegible] for irregular grooving it is necessary that there [illegible] groove, first as regards the sides of the spiral, [illegible] and the two requirements must be so combined [illegible]

[illegible] which represent the rifling machine employed [illegible] side elevation of the machine, and Fig. 3416 [illegible] sections to a larger scale, and Fig. 3419 an en- [illegible] carries the rifling bar. Fig. 3420 is a com- [illegible] tangent bar I which gives the twisting motion [illegible] vertical plane, in order that it may [illegible] een in [illegible] feed motion to the cutter in the rifling head: [illegible] ensions for convenience of illustration, but the [illegible] correct dimensions and relative positions of [illegible]

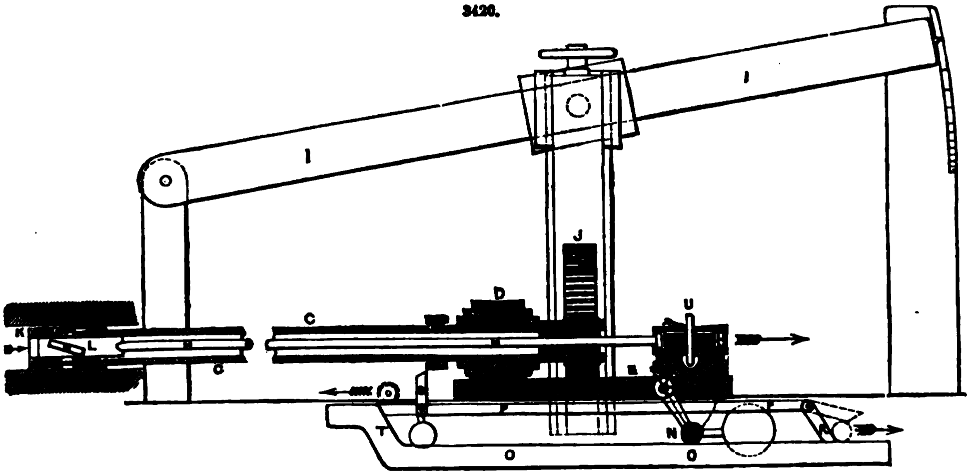
[illegible] and parallel, one end being held firmly in a [illegible] number of collars to take the pull of the cutter;

while the other end is free to turn and slide in a stationary bearing F near the muzzle of the gun G. The longitudinal motion of the rifling bar may be given by any of the planing-machine motions; that by the screw H, Fig. 3416, is preferred on account of the smooth action which it

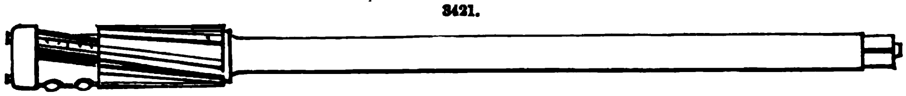
3419.



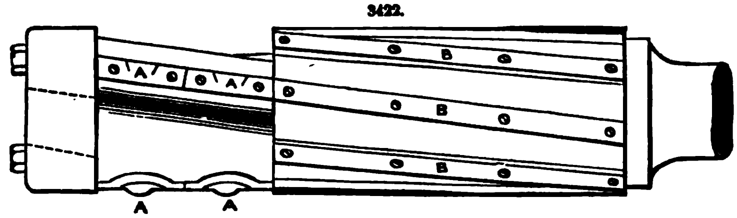
3420.



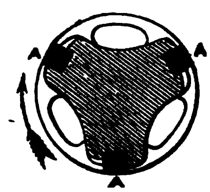
3421.



3422.

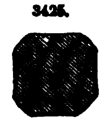


3423.



3424.

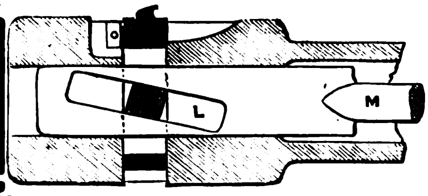
3425.



affords. The twisting motion of the rifling bar is derived from the tangent bar I by means of the rack J sliding transversely on the traversing saddle E, and gearing into a pinion on the end of the rifling bar C, Figs, 3418, 3419. The tangent bar I can be set at any angle by means of the adjusting screw and graduated arc, or can be made of any shape within the limits that the machine is capable of following the quirks of the rifling. Hence to produce any description of twisting in the grooves of the gun it is only necessary to employ a tangent bar of suitable pattern for the purpose, which will be faithfully copied on the interior of the bore by means of the rack J tracing the pattern. In guns where there are several twists or alterations of form in a single groove it is sometimes necessary to have several differently-shaped tangent bars piled one on the top of the

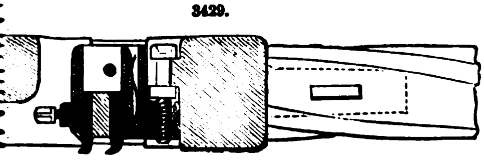
...he tracing rack J to the bar to be copied; and
...omplished as easily as a regular spiral.
... of the grooves is uniform, but in others it is a
...ence it is necessary to have the cutting instru-
...along the gun. It is also of importance that
...since the rubbing affects the maintenance of a
...tting is an essential condition. It is therefore
...est or holder in the head of the rifling bar, and
...quired. For this purpose the rifling bar C is
...d K, Fig. 3420, is actuated by an inclined slot
...arged sections of the rifling head, Figs. 3426,
...t or in, a radial motion is given to the cutting

3427.



... the groove in the gun. To prevent the cutting
...er rail P is provided, having a trap R and S to
...acing roller N to pass. The drawings represent
...cutting a groove in the gun, the arrows showing
...he roller N is tracing the copy bar O; but on
... the trap R, as shown dotted in Fig. 3420; and
... falls and forms an incline for the roller to run
...the upper rail P, thus pushing the feed-rod M
...wing the cutting tool, which remains withdrawn
...hine. When the roller N reaches the other end,
...eight; but the roller folds the trap downwards,
...dge to enable it to pass over. The trap is then
...ng again in the forward motion the roller drops
...opy bar O, thus drawing out the feed-rod M and
...osition. The incline T gives the form to the
...ly of very definite shape. It will thus be seen
...to the cutting tool; and hence by means of the
...ing can be accomplished without difficulty. To
...imen rifled tube has been made (shown in the
...different ways, one of which is spiral and wavy,
...the groove formed with a progressive irregularity.
...ch traverse so as to obtain the additional depth
...rod M has a screw and hand-wheel U upon it,
...per in each successive traverse, until the groove
... also affords the means of taking up the wear of
... exactly the same depth. When one groove is
...c required arc by means of the ratchet-wheel V
...ding plate, being made with the same number of
...gun.
...other kind of cutting instrument, by which the
...eans of a circular rifling head carrying as many
...s of these rifling heads are used in succession,
... one cutting the groove a little deeper than the
...ve of them the grooving is effected. This kind
...but so far as economy is concerned it is the most
... tools made on the former plan of withdrawing
...used; but it is doubtful whether they are more
... in obtaining perfect adjustment with so many
...Where no variation is required in the depth of
... be used, as shown in Figs. 3428, 3429. The
... centre pin in the rifling head, to allow them to

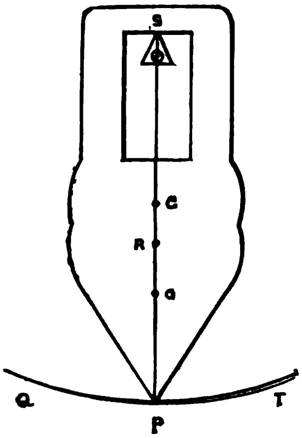
3429.



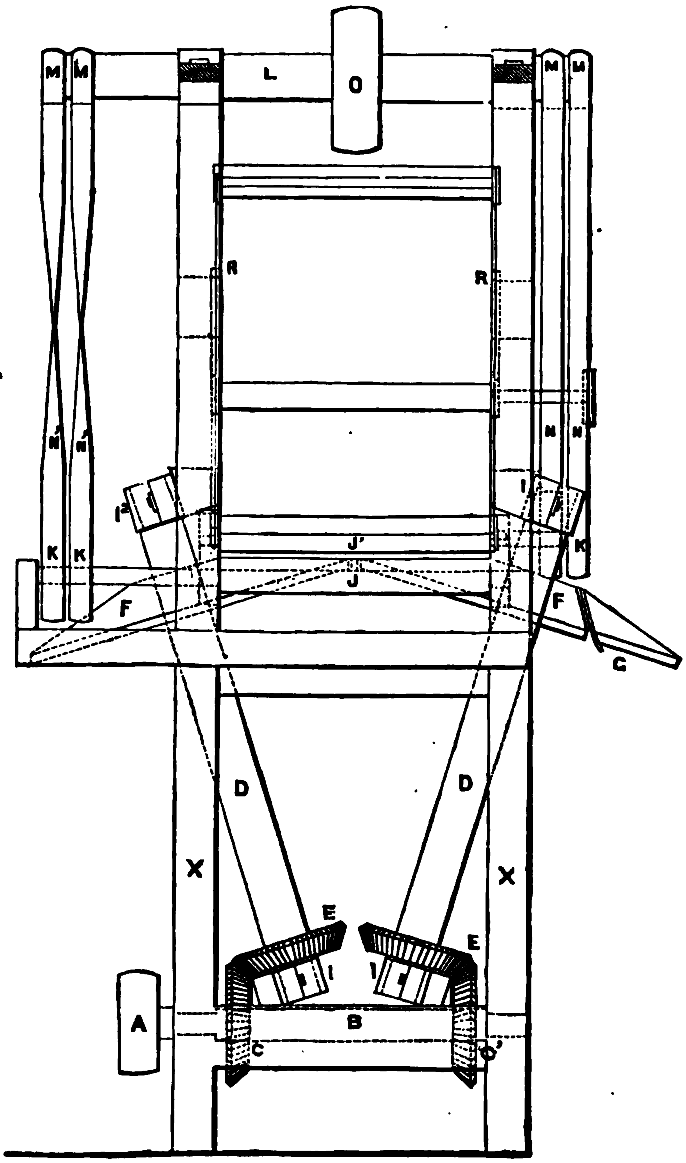
...ine; they are set up after each traverse by an
...e block in which the cutters are fixed. This
...ding guns, the cutters being set to cut inwards

from the muzzle towards the breech as the rifling head is pushed down the gun, instead of in the contrary direction as in the rifling head previously described and shown in Figs. 3426, 3427.

The copying principle is also used in drilling the various holes for the sights and other parts upon the outside of the guns. In a gun which is intended to hit a target at 2000 or 3000 yds. distance, the value of the thickness of a line in half the length of the gun is important; and as all the Armstrong guns are made so that the several parts interchange, absolute precision in the positions of the several holes is essential. Most of the holes have to be drilled on the side of the gun, where the difficulty of entering correctly is greatly increased on account of the surface being oblique to the direction of the holes; so that the drill requires to be guided very steadily, and the ordinary plan of dividing off the holes and the use of a centre punch are altogether inadmissible. A cast-iron saddle is therefore made to fit upon the gun and also upon the trunnions, being cast in halves, so that the whole of that part of the gun in which the holes have to be made is enveloped in it. The saddle is correctly made with copy holes lined with steel, the several holes being of the required dimensions of the holes to be made in the gun. Cylindrical drills are employed, which, fitting the holes in the copy, give the utmost accuracy to the sight-holes without any effort.



3431.



GYRATION. Fr., *Mouvement giratoire;* Ger., *Kreisförmige Bewegung;* Ital., *Girazione;* Span., *Rotacion.*

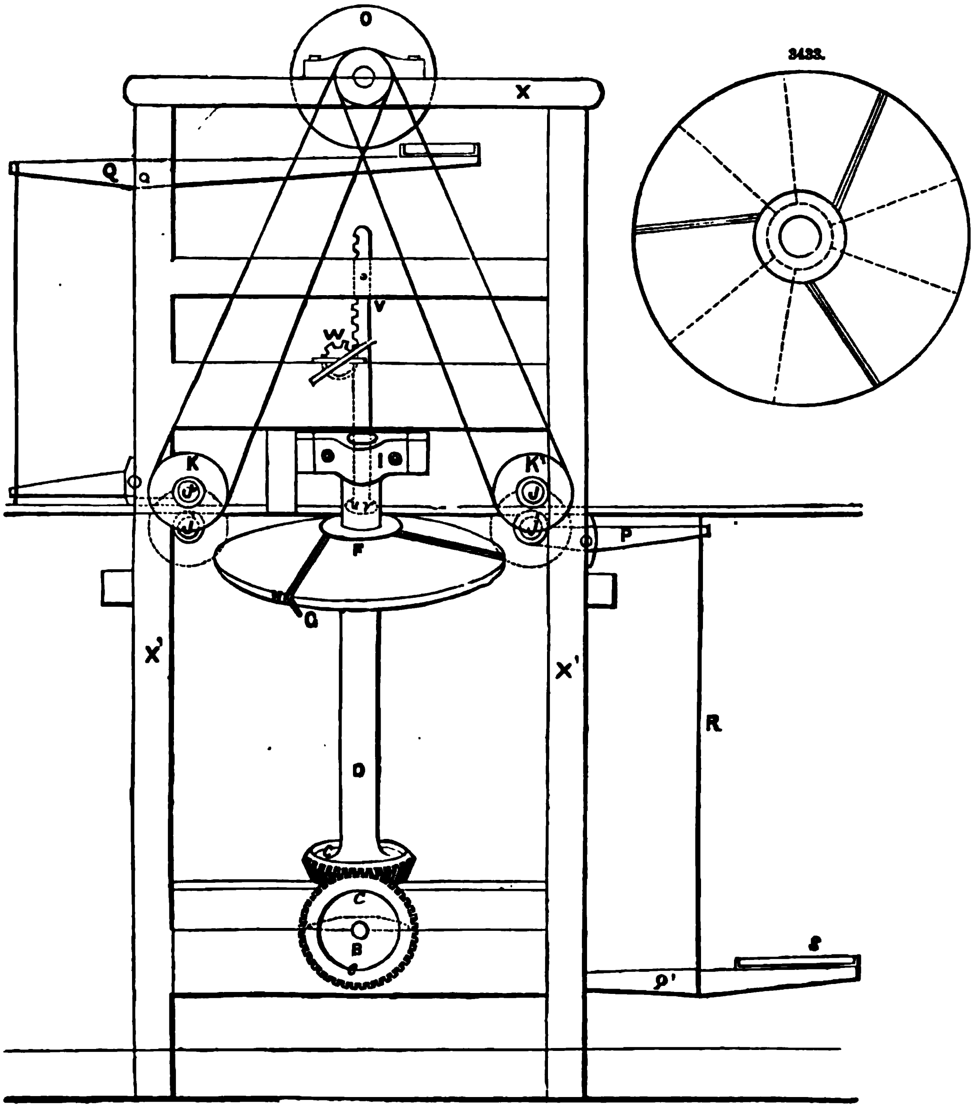
The centre of gyration is that point in which, if all the matter contained in a revolving system were collected, the same *angular velocity* will be generated in the same time by a given force acting at any place as would be generated by the same force acting similarly in the body or system itself.

The distance of the centre of gyration from the point of suspension, or axis of motion, is a mean proportional between the distances of the centres of oscillation and gravity, from the same point or axis. If S, Fig. 3430, be the point of suspension of any regular or irregular body P S; G the place of the *centre of gravity;* O that of the centre of oscillation; and R that of the centre of gyration; then $RS = \sqrt{SO \times SG}$; hence, $SO \times SG$ = a constant quantity for the same body and the same plane of vibration S Q T. If $SG = 25$, and $SO = 36$ units of length, then $SR = \sqrt{25 \times 36} = 30$.

Ira Gay's Conical-Plate Planing Machine.—In this machine the centre of the cutting edge coincides with the centre of gyration of the revolving cutter-holder. Ira Gay was an experienced

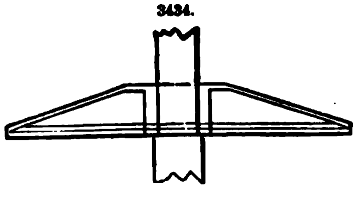
mechanical engineer; he was born in Dunstable, in the county of Hillsborough, New Hampshire, U.S. His machine, Figs. 3131 to 3435, for planing boards, plank, and other articles, is not of a complicated nature. Fig. 3431 is a front view of the machine; Fig. 3432, a side view; Fig. 3433, the plane-stock; Fig. 3434, a section through the centre of the plane-stock; and

3432.



3433.

3434.



3435.

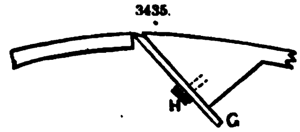
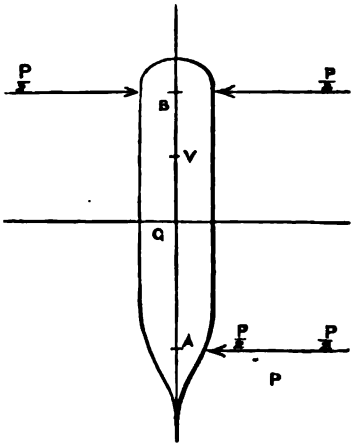


Fig. 3435, a section of the plane-stock cut across so as to show the plane-iron and the manner of fixing it to the stock. A is the main pulley to which the power for driving the planes is applied by means of a belt. B, the main shaft on which the driving wheels C C are fastened. D D, upright shafts inclining a little from a perpendicular position. E E, wheels attached to the upright shaft D, and driven by the wheels C C. F F the plane-stocks attached to the shaft D D.

The face of the plane-stock may be levelled to suit the views of the operator, but must be levelled enough to throw off the shavings freely; more than this tends to render the surface planed uneven. An angle of about 5 degrees from a right angle to the shaft is a very good inclination. The plane-iron is shown at G, Fig. 3435, which also shows the manner of fastening it to the stock by means of the screw H. J J are the receiving and delivering rolls driven by the pulley K. T T, boxes for the upright shafts to revolve in. At the top of the machine is the shaft L for the purpose of driving the receiving and delivering rolls, upon the ends of which are pulleys M M, corresponding with those on the rolls over which pass the belts N N. O, the pulley for communicating motion to the shaft L. P P are levers attached to the rolls and made to pass them together while the boards are passing through the rolls; these levers are assisted by other levers Q Q, connected by the rod R R, and weighted as seen at S S. T is a board passing through the machine and confined down to the plane by means of two small rolls U V, which are adjusted to their proper place by means of the rack V and pinion W. X is the frame of the machine. The mode of using the machine is simply to insert the end of the board between the receiving rolls, after which the board is carried through the machine by means of the rollers, and planed in its passage.

Centre of Spontaneous Rotation.—No matter what the nature of an impulsive force may be, or how that force is applied to a body at liberty to move in any direction, the whole force, when not applied to the centre of gravity, is employed to turn the body, and at the same time the whole force is employed to move the centre of gravity in the direction of such force. To illustrate this important proposition, let A B, Fig. 3436, be a body placed anyhow, at liberty to move in any way. The force P, or two forces, each equal to ½ P, is exerted at the point A; P has the power to turn that body, and to drive on the centre of gravity. We have then to consider the centre of oscillation, the centre of percussion, the *centre of spontaneous rotation*; we have nothing to do with the centre of gravity once the force acts upon a body and attempts to move it. If we apply two new opposing forces each equal to ½ P at the same distance as P from the centre of gravity G, they would balance each other, and would not interfere with either the progressive or the turning motion. But we shall now consider another arrangement of these forces; two ½ P's will drive on the centre of gravity; the other two ½ P's will turn the body round with a force equal to P. ½ P at A and ½ P at B turn the body round, and ½ P at B and ½ P at A have a tendency to give a progressive motion to G, consequently the force P acting at A loses none of its effect to turn the body, nor any of its driving effect. What will happen;—The point A will advance by two forces, the force of turning and the force of percussion, while the point B will advance by one force $= \frac{P}{2}$, and will turn in an opposite direction; the result will be the difference, so that the point B will recede while the point A will advance. Therefore this body will turn round a point V between G and B, which is called the centre of *spontaneous rotation*. See ANGULAR MOTION. GRAVITY. OSCILLATION. PERCUSSION.

3436.



HACKLE OR HECKLE. FR., *Séran, sérin, séranzoir*; GER., *Hechel*; ITAL., *Pettine*; SPAN., *Rastrillo*.

See FLAX MACHINERY, p. 1498.

HAMMER. FR., *Marteau*; GER., *Hammer*; ITAL., *Martello*; SPAN., *Martillo*.

See HAND-TOOLS.

HAND-GEAR. FR., *Levier à main de distribution*; GER., *Handsteuerungshebel*; ITAL., *Messa in moto*; SPAN., *Palanca de trasmision de movimiento*.

The contrivances in a steam-engine for working the valves by hand; the starting gear.

HAND-PUMP. FR., *Pomp à main*; GER., *Handpumpe*; ITAL., *Tromba a mano*; SPAN., *Bomba de mano*.

A pump situated at the side of the fire-box in a locomotive, and worked by means of a lever when the engine is standing with steam up.

HAND-SAW. FR., *Scie à main*; GER., *Handsäge*; ITAL., *Segone*; SPAN., *Serrucho*.

See HAND-TOOLS.

HAND-TOOLS. FR., *Outils à main*; GER., *Handwerkzeuge*; ITAL., *Strumenti da mano*; SPAN., *Herramientas de mano*.

Hand and Foot Lathes.—John Anderson, the Superintendent of the Woolwich Machine Shops, in the P. I. M. E., 1862, observes, "In looking back to the early days of the turning lathe, before the introduction of the transfer principle in the sliding rest, it is interesting to observe that even then the lathe was a perfect instrument so far as it was a copying machine; those common lathes that were made with a perfectly round spindle-neck, if any such existed, would yield a round figure in the article under operation, providing that the cutting instrument was held steadily. And even in a still higher degree was correct workmanship attained in the old-fashioned dead-centre lathes; if the centre holes in the article to be turned were formed with moderate care, and the article held steadily between the centres, then the surface developed by the cutting instrument when firmly held would be as perfect a circle as one described by a pair of compasses."

With such apparatus, however, the chances of error were numerous, arising principally from the spindle-necks not being perfectly round; for even in the case of modern lathes a perfect spindle-neck is more rarely obtained than is generally supposed, as a close examination will show, the polygonal form being much more predominant than the true circle. There are lathes, even among

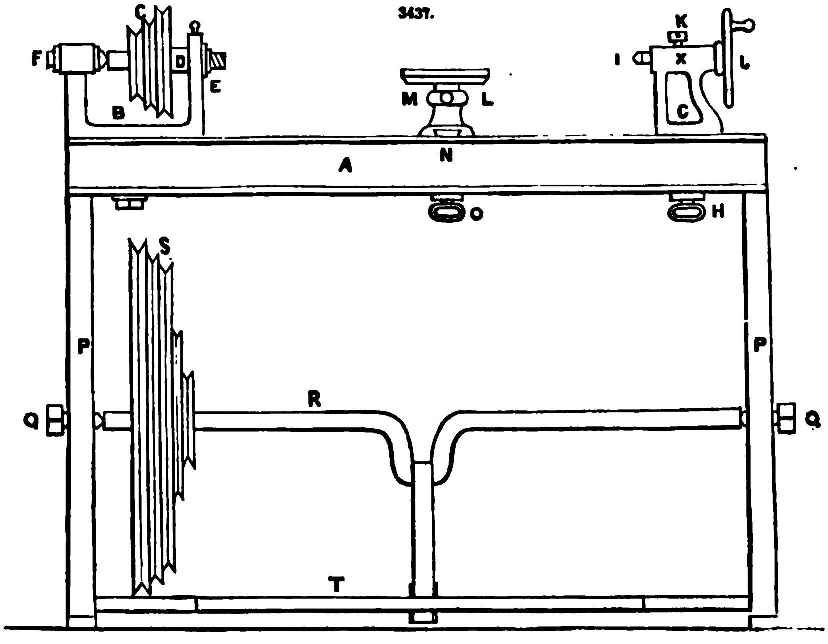
those of the most recent make, which have only to be handled gently to show their condition in this respect. Until recently such approximations to roundness were sufficient; but the extensive introduction of *accurate gauges* into workshops has, besides teaching the importance of precise dimensions, made engineers familiar with true circles. Hence there is now a much greater appreciation of positive truth of workmanship, and positive truths are always important; and in well-conducted workshops there is a constant striving after that condition and a gradual closing up of every avenue whereby error can creep in.

Such extreme accuracy is sometimes thought to be more costly than a less careful system; but practical men, like Anderson, have arrived at a contrary opinion, and are convinced that while extreme accuracy may be more expensive at the outset, especially from the want of workmen competent to carry it out, yet with a little perseverance the advantage arising from it will be clearly perceived, and the apparently inordinate cost will shortly be brought below that of less perfect arrangements. Many articles after being carefully turned and planed have to undergo a long course of filing and scraping before they are brought to the required quality of surface; whereas, if a small fraction of this outlay were spent in making the copy in the lathe spindle or the copy in the plane perfect as patterns, the great expense of subsequent fitting would be avoided. Many examples bearing on this point could be given were it required; an illustration may be named that came under Anderson's experience in the manufacture of guns at Woolwich Gun Factory. Certain rings about 1 ft. in diameter had to be fitted on corresponding cylinders, and were required to be perfectly easy to move, yet without shake, as any looseness in the fit rendered them useless; they had therefore to fit approximately like the Whitworth gauges. Several good new lathes were tried in vain; endless scraping and grinding had to be resorted to. Anderson was convinced that if the source of roundness were positively round, the result ought not to be out of truth. Measures were accordingly adopted to obtain perfection of roundness and steadiness in the lathe, at little more than the cost of fitting one of the rings; and the subsequent cost of the rings was thereby reduced from the value of nearly three days' work to less than an hour's. The lathe spindle became a true copy, the sliding rest a correct medium of transfer, and the combination of the two yielded the required truth and roundness. A similar case occurred in the manufacture of a number of large fire-cocks: the sockets and plugs were carefully turned, but they would not resist the water pressure without a great deal of scraping and grinding, until the lathe spindle was positively brought to perfect roundness, when the turning alone made them fit with scarcely any grinding. The lathe is a copying machine, and just as its bearing surfaces are so is the work produced.

The apparatus generally employed by wood and ivory turners is termed a foot-lathe, on account of its being driven by the foot in the same manner as the common grinder's wheel; some are constructed partly in metal and partly in wood, but those made entirely of metal are far superior to these, and are of the following construction;—

A, Fig. 3437, is the bed of the lathe, upon which two supports, called poppet-heads, rest; the surfaces of contact vary in form, in some beds both are flat, in others both angular, and in others one angular and the other flat.

3437.



By many the angular or V beds are preferred, from the idea that the heads are more likely to retain their proper position than when resting on plane surfaces; but the latter, when accurately

planed and fitted, are quite as worthy of reliance, and far more convenient than the angular-bedded lathes.

B represents the head to which the chucks are attached, and by means of which the power requisite for rotating the work is applied. This poppet-head consists of a strong frame of cast iron F B E; in the standard E is fixed a hard conical bearing, in which one end of the mandrel D revolves, and by which it is supported, the other end resting against the hard conical point of a screw placed in a nut at F; by means of this screw the mandrel is kept tight up to its bearings, any tendency of the screw to shift being prevented by one or two nuts upon it, which are screwed up tight against the standard F.

At the bottom of the head is a solid projection, which is made to fit the opening between the sides of the lathe-bed, and by which the parallelism of the lathe-bed and mandrel is maintained. The head is firmly fixed in its position by a bolt, which draws a strip of metal up tight against the bottom of the lathe-bed. A number of groove pulleys G are attached to the mandrel, one of which is connected with the pulleys S on the driving shaft R by means of a cord of catgut or gutta-percha, although in a case of necessity a sash-line may be made to answer the purpose. The catgut is, however, the most satisfactory, on account of its great durability. The plan usually adopted for joining the ends is to screw on hooks and eyes; the end of the gut is slightly tapered and damped, so that the hooks and eyes may squeeze the gut into a screw rather than cutting it, by which latter the band would be much weakened.

It must not be used until the gut is dry and hard. Gutta-percha bands are united by heat, the ends being cut off obliquely, thus, , and gently heated by means of a hot piece of smooth clean iron, until soft, when they are firmly pressed together, and kept in that position until cold. This, of course, necessitates the stoppage of the lathe for some time, besides shortening the band every time it is united.

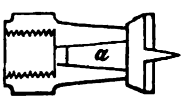
When the work is too long to be supported entirely by one end, a second poppet-head is required, which is of the form shown at C; this head is accurately fitted to the lathe-bed, and can slide upon it to allow of adjustment to the length of the work; it is fitted with a clamping screw H to fix it when in position, also a conical point I, called a centre, which is movable through a small space by the handle J, to allow the removal of the work from the lathe without shifting the poppet-head. The mandrel carrying the centre is fixed after adjustment by the capstan-headed screw K.

The next part of the apparatus to which our attention is called is the rest, upon which the operator supports the turning tool. There are two kinds, the common rest and the slide-rest; the former is that represented in our figure. M L is a short hollow column, provided with a foot sufficiently long to reach across the lathe-bed; in the bottom of the foot is planed a dovetailed groove N, which retains the head of a clamping screw O, but at the same time allows of a sliding motion when not clamped. From this it is evident that the rest can be placed and fixed in any position.

Within the hollow column is a cylindrical rod, which carries a straight strip of metal, the whole being raised or lowered by sliding the rod vertically in the column; when the proper elevation has been attained, the rest is fixed by a screw working in a thread cut in the thickness of the column.

The lathe-bed is supported on standards or frames P P, which also serve to carry the crank-shaft R by means of two conical-pointed screws Q Q, which enter countersunk recesses in the ends of the shaft. The shaft is made with one or two cranks, or throws, according to its length. This shaft is also fitted with grooved driving pulleys S, made of various diameters, in order to obtain any speed which may be required. The pressure imparted to the treadle T is communicated to the crank by a link with a hook at each end, or by a chain; some turners preferring the former, and others the latter.

3438.



We now proceed to consider the means by which the work is held in the lathe and caused to rotate with the mandrel.

Fig. 3438 represents the fork, prong, or strut-chuck, so called from the steel fork or prong a, which is fitted into the square socket of the chuck; this chuck is used for long pieces, the point supporting one end of the work, the other being supported by the back centre. The chisel edges on each side of the point take hold of the work and ensure its rotation. The fork being fitted into a square recess in the chuck may be replaced by drills, &c., or small pieces of wood or ivory to be turned. It is usually made of metal, and attached to the mandrel by an internal screw corresponding to that on the nose of the mandrel.

3439.

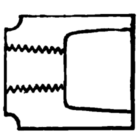


Fig. 3439 represents the hollow or cup-chuck; it is used for holding short pieces, or pieces that are to be turned out hollow. Its inside is turned slightly conical, so that the work may be driven tightly into it.

This chuck is usually made of boxwood, sometimes strengthened by a metal ring round the mouth of it; but this is scarcely necessary, as a very slight blow is sufficient to fix the work if it has previously been reduced to a form nearly approaching the circular by the chisel, paring knife, or other hand-tools.

3440.

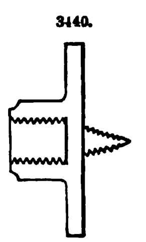


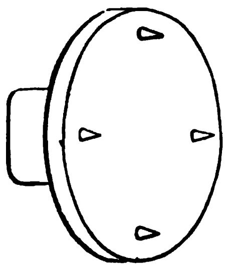
Fig. 3440 represents the face-plate or facing chuck; it may be made of iron or other suitable material.

This chuck is turned flat and perfectly true, and is fitted at its centre with a conical screw to hold objects to be turned on the face. This chuck can only be used when the hole made in the work is not objectionable, or can be plugged up. The screw should only be very slightly taper, otherwise the work will not hold when reversed.

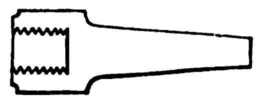
Fig. 3441, a chuck for flat work, where a hole in the centre would be detrimental. It is a face-plate with three or more small spikes projecting from its surface to penetrate the material to be wrought, which is held against it by the back centre.

A plane face-plate is used where the work cannot be conveniently fixed to either of the two foregoing, as in the case of thin pieces of horn, tortoiseshell, and so on. The work is attached by means of glue, or of jewellers' or turners' cement.

3441.



3442.



3443.

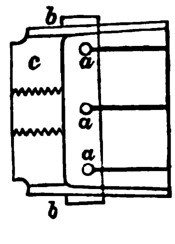


Fig. 3442 represents the arbor-chuck, usually made of brass. It is used for holding small hollow works or rings.

For very small work, Fig. 3438 is useful for holding the arbors in the place of a strut *a*.

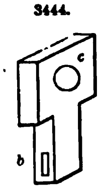
Fig. 3443 represents a spring-chuck which is used for holding very slight work that requires to be hollowed out.

It is turned conical externally, the apex of the cone being to the left. A few holes *a a* are drilled through the chuck near its base and at equal distances from each other. From these holes saw kerfs or slits are cut longitudinally to the front of the chuck, which allow the chuck to expand slightly to take a firm hold of the work, and when the work has been forced into the chuck, the grip is rendered still more firm by drawing a strong ring towards the front of the chuck.

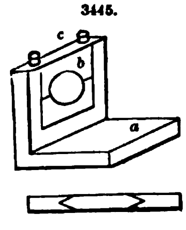
These chucks are sometimes made of wood, but those of metal are much neater and more convenient; they may be made of a piece of brass tube firmly driven on a wooden block.

A similar chuck is used for holding hollow work, but instead of being provided with an external ring, it is fitted with a short solid plug, which is forced forwards after the chuck has been inserted into the work. When long and slender pieces have to be turned, an extra poppet or a support is required to keep the work from shaking, or chattering, as it is termed. It is generally made of wood, and is formed similar to Fig. 3444. It consists of a head, in which is bored a hole *c* of the proper diameter, and a tail-piece fitted to the lathe-bed and sufficiently long to receive an aperture *b*, through which a wedge may be passed to hold it down firmly upon the lathe-bed.

3444.

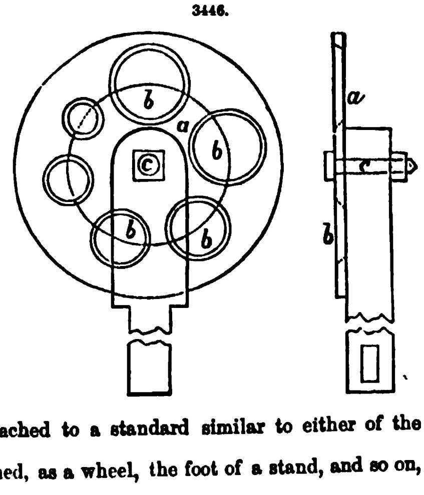


3445.



Another and more convenient form of support is shown at Fig. 3445; *a* is a cast-iron frame, having a foot fitted to the lathe-bed and furnished with a bolt and nut by which it is firmly bolted down to the lathe-bed; *b* is a block of wood fitted into the frame, where it is secured by the cross-bar *c*. An aperture of the required diameter is now bored in the block; it is then taken out of the frame and sawed in half, so as to form a top and bottom bearing; *d* shows a section of the frame; any other form of groove may be used, but we have selected the V on account of the ease with which the blocks may be fitted to them. One great advantage of the latter apparatus is, that the two bearings may be brought together when the hole is worn. When a slide-rest is used, this additional support should be attached to it; it will then keep close to that part of the work on which the tool is acting, by which a more satisfactory piece of work is turned out, and the trouble of shifting the poppet avoided. The application of a little grease to these bearings will sometimes be found beneficial.

3446.

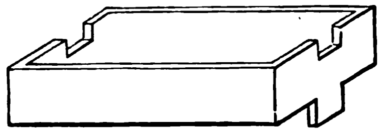


An apparatus called a boring collar, somewhat similar to that just described, is used for supporting the ends of pieces of which the ends are to be bored, and which are too long to be held by the cup-chuck alone. It consists of a plate similar to a face-chuck, Fig. 3446, through which a number of conical holes are bored, whose centres are equidistant from the centre of the plate, so that when the latter is turned on its axis any hole can be brought exactly in a line with the two centres. The plate may be attached to a standard similar to either of the foregoing.

It may sometimes occur that the work to be turned, as a wheel, the foot of a stand, and so on,

... it is convenient to have frames truly planed and ... is made of cast iron, the top being fitted to the ... ted to ... rel is ... ay be ... rest ... itable

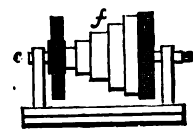
3447.



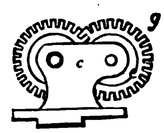
... self- ... irals, ... 3449.

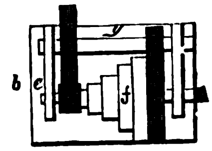
... lan. a is a slide which fits the lathe-bed very ... in a direction exactly parallel to the axis of the ... he lower one and sliding upon it in a direction at ... a screw attached to the lower slide, which gears ... on the slide b is fitted a small slide c, upon which ... This slide is moved in a direction parallel to the ... ide b, gearing in a similar manner to that in the ... the bed either by hand or by means of a screw ... into a nut made in two halves, so that it may be ... e nut. The use of this screw, which is called the ... ppet-head, and constitutes what is called a screw- ... process.

3450.



3451.





3452.

... ing lathes is represented in Figs. 3450 to 3452. ... vation. This head is fitted with speed pulleys f, ... drive it direct or loosened, and geared by a tooth- ... e mandrel, which is supported in bearings at each ... of gear with those on the mandrel by sliding the ... n or out of gear by a pin passing into the bearing, ... g. On the end e of the mandrel a toothed wheel ... may act directly upon another placed on the end ... it by means of one or two intermediate wheels, ... of the intended screw.

... ratio between the speeds of the mandrel and lead- ... turning or screw cutting.

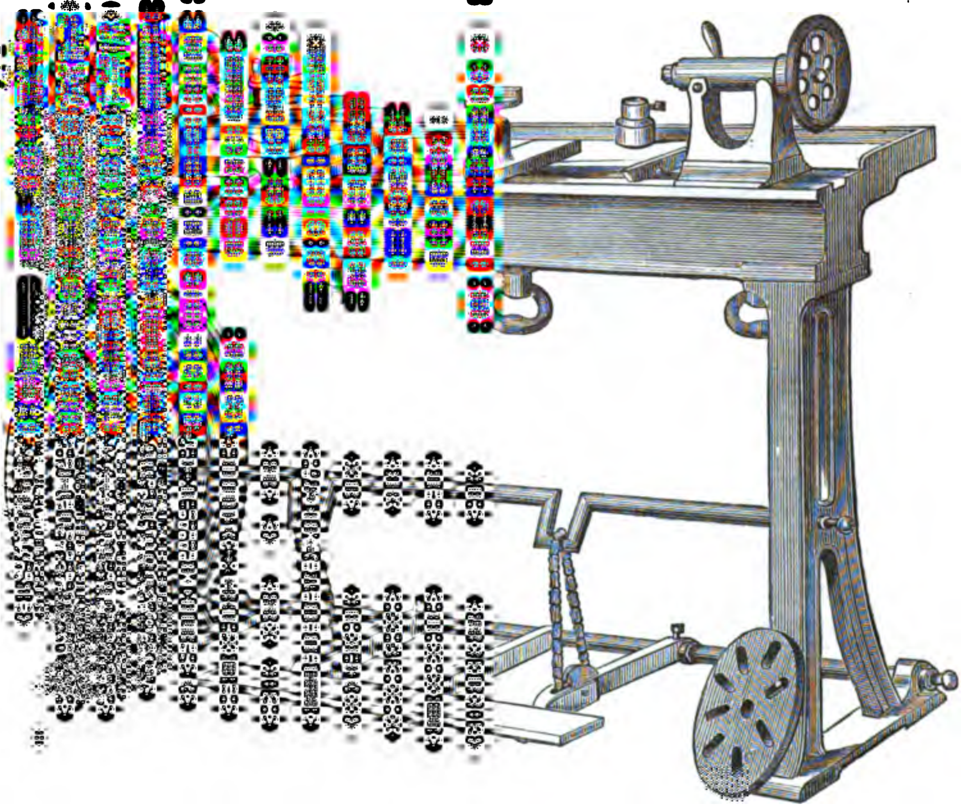
... foot-lathe, with planed bed, standards, anti-fric- ... wheel, hand-rest, face-plate, drill-chuck, and two

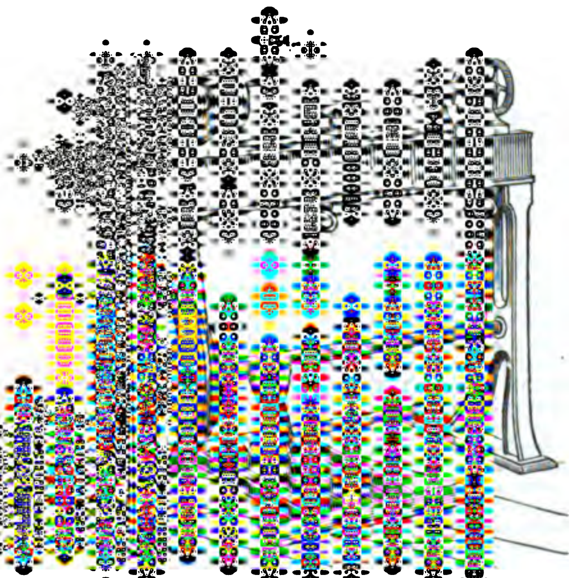
... planed bed, standards, anti-friction treadle, with ... plate, drill-chuck, and two centres.

... fast and loose headstock, hand-rest, and metal

... er arrangement of compound slide-rest is shown in ... illustrate principles; Figs. 3453 to 3457 are not ... the highest style by J. and H. Gwynne, of Ham- ... chester.

... Merritt's machine for boring angular holes. B is ... and E E bevel-gear for communicating motion to ... boring machine consists in attaching around the ... a cam H, on the under-side of which is a path of ... runs a truck attached to a box having a rack and ... aft as well as to an inside shaft. To the bottom of

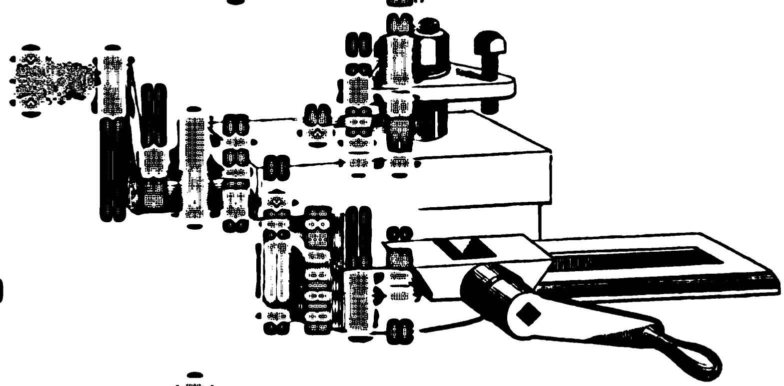




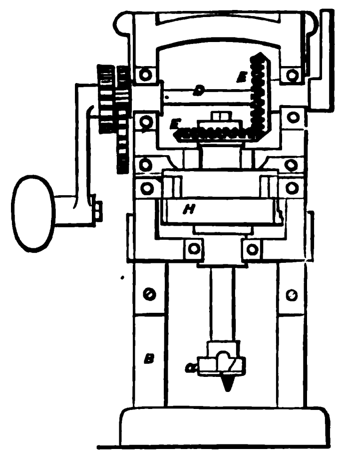
3455.

... a tug screw to draw the auger into the work. ...de shaft is a pinion which gears into two racks,

...7.



3458.



...ged that when the main shaft rotates, the inner ...the cam, turns one-half way round, causing the ...circular hole to the angle, as at J J; the opera- ...des are thus cut to form the angular hole.

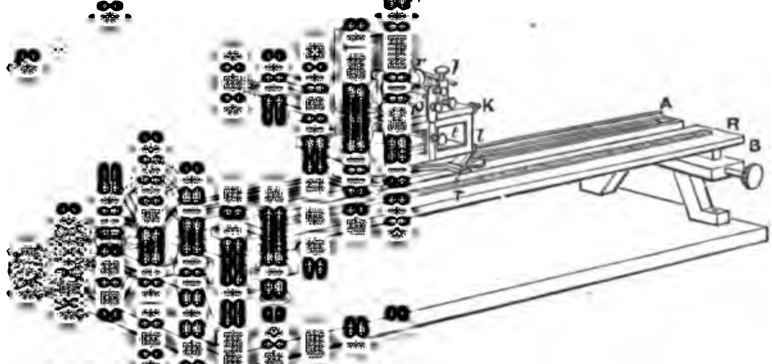
...ons upon mathematical and other instruments, ...re of two kinds, designed respectively to divide

...incipal part of these machines consists of a screw ...s some exact measure of an inch, or some other ...lways equal to 1 millimètre. It is obvious that ...the machine.

...ed shoulder, and works into a female screw, to ...frame C, Fig. 3460, bearing these instruments, ... It will be seen that when the screw is turned ... female screw advances a linear quantity pro- ...rew. The part C is furnished with a *tracer* to ...e marked upon metal, the tracer or graver is of ... required. The piece to be divided is placed ...onvenient distance. The graver may be raised ...during the time the latter is in motion; when ...ulled down upon the piece to be divided, and a ...imparted to it by pressing it gently upon the ...ch the graver is affixed being arranged to move

...ibed above are those which are adopted for all ...t these kinds differ widely from each other with ...g the distance and the length of the division-

[illegible] of engine, we will select one which seems to be [illegible] Balleron, whose instruments are justly esteemed [illegible] metal slab upon which the part C slides. The [illegible]



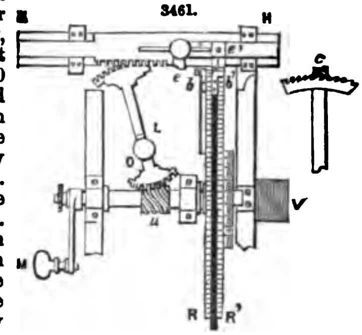
[illegible] turned by the crank M. B B is the plane table, [illegible] divided is placed. This table turns about the [illegible] piece may be brought exactly parallel to the [illegible] raised by turning about the axis $l\,m$; at the same [illegible]. In this way the graver may be brought into [illegible] point its construction will allow; then pressing [illegible] returns of itself to its original position, and the [illegible]

[illegible] following arrangements:—To the part $i\,k\,l\,m$ are [illegible] piece o is another vertical piece turning about [illegible], but the wheel b which it bears on its upper end [illegible] with which it is provided laterally. This third [illegible] horizontal piece t. It follows that when the graver is [illegible] $i\,k$, the third piece turns about its lower end, [illegible] the plane d meeting the end of a fixed screw v, or [illegible] a screw o working in the piece fixed to the frame. [illegible] towards its position when at rest, the third piece [illegible] contrary direction about its lower end, and its [illegible] plane d meeting the end of a fixed screw v' or by the [illegible] screw o'. Let us suppose, in the first place, that we [illegible]; the distance of the screws v and v' is regulated [illegible] required length, and the screws o and o' are with- [illegible] contact with them. The plane d strikes, in this case, [illegible] excursions of the frame $i\,k\,l\,m$ are thus regulated in a [illegible] of equal length. Suppose now it is required to [illegible] every fifth, for example. The excursions of the [illegible] plane d, because the screws v and v' will have been [illegible] regulated by the wheel b. This wheel is provided [illegible] difference between two notches comes in contact with [illegible] small and it marks a small division; but if the [illegible] graver is greater and it marks a longer division. [illegible] there is upon the axis of the wheel b a small ratchet- [illegible] the end of the piece h works into the teeth of this [illegible] the end h is brought back, the click presses upon [illegible] graver returns to its place, the click escapes from the [illegible] evident that at each division traced, the wheel b [illegible] regulated in such a way that at each fifth division, [illegible] notches to the screws o and o', which causes a longer [illegible] enables the operator to see whether or not the divisions [illegible] provided to ensure to the motion of the graver a direc- [illegible] motive screw.

[illegible] regulating the interval of the divisions. The wheel E [illegible] parts; hence when it turns by n divisions, the screw [illegible] means of a system of gearing we may turn the [illegible] number of divisions of this wheel which pass a fixed [illegible] independent of the attention of the operator, an arrange- [illegible] somewhat automatically, thus avoiding a chance [illegible] plane by Fig. 3461. The axle of the crank M is [illegible] is fixed a ratchet-wheel R having 100 teeth. [illegible] a similar wheel R'. The first is provided with a [illegible] teeth of the second. If the crank is turned in the

...h we will call direct, the click of the wheel R ...rank thus turns the screw; but if the crank is ...arrow, a ...des over ...e screw, ...he direct ...eeth, 100 ...s is fixed ...nd of an ...is of the ...y easily ...wo stops. ...on a rule ...length. ...motion ...placed in ...inverse ...eets the ...e screw ... to the ...nating motion, limited by the projecting pieces ...y rigorously equal quantities regulated before... screw forward half a millimètre at a time, we ...ace which separates the stops shall be exactly ... projections being supposed to coincide exactly ...to be obliged to consider the thickness of the

3461.



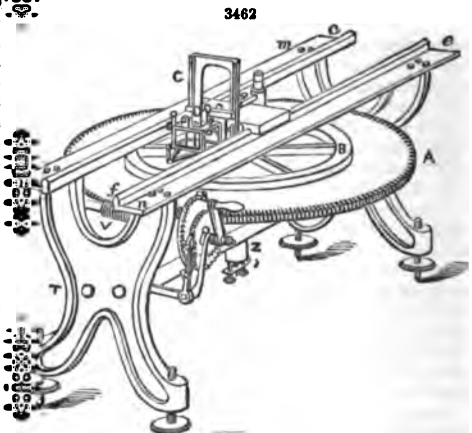
...uality of the spaces which separate the divisions ... But if the interval is to exceed 1 millimètre a ... is the following;—

...u, working into the end of a lever L, which turns ...s into a straight rack upon the rule K H. Con... backwards or forwards in the direction of its ...ate the interval of the projections so that the ...mber of turns and a fraction of a turn correspond...s, the stop b' being in contact with the projec...

... mark divisions, the interval of which may vary

...s the circle to be divided is placed concentrically ...ng about an axis passing vertically through its ... means of a crank. A graver moving in the ... reaching the limb, whatever the radius of the ...e centre. Fig. 3462 shows this arrangement. ...ower end of which may be seen in z; B is the

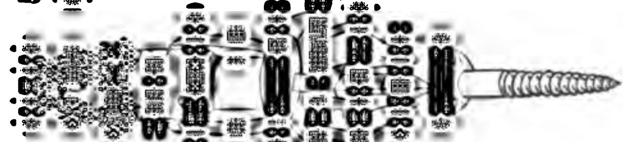
3462



...le carefully measured. When this distance is ...mixture of wax and resin, and the operation of ... of centring is a delicate one, but it is indis... limb of an instrument used for astronomical

...head of the common wood screw is frequently ...move it, and every mechanic has been annoyed

the head of the screw. To provide a remedy for
inventor of the screw and driver shown in Figs.
notches cut equidistant in the edge, instead of
perspective in Fig. 3463, has three correspond-
atically open and close upon the screw-head,

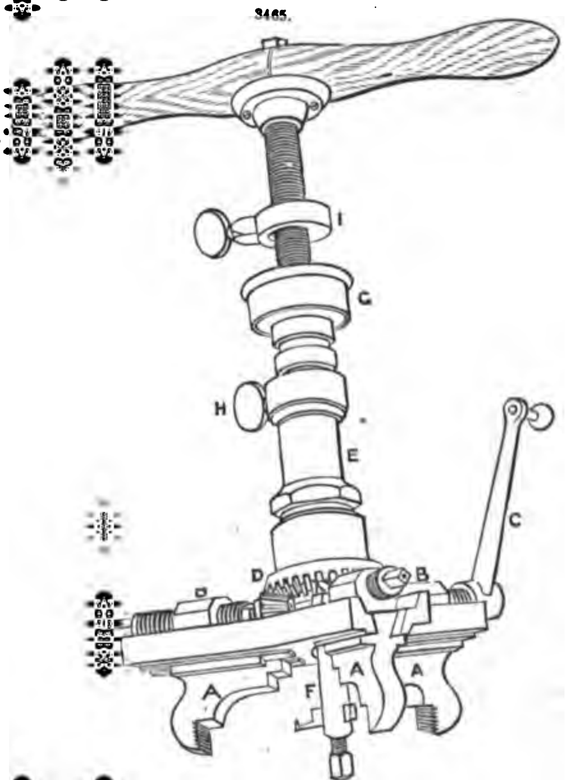


3464.

and is hollow for the larger part of its length,
o jaws B, all moved simultaneously by a sliding
ed and closed by means of the incline of their
rtures in the collar end of the implement, desig-

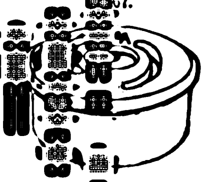
wn, the combined weight of jaws and ring causes
iently to receive the head of an ordinary screw.
wn and the jaws compressed, griping the screw-
xerted; the harder the pressure the greater the
when they are seated on the screw-head, project
preventing the necessity of using a separate tool
, there will be no necessity for previously boring
nearly home, the driver may be raised and the
s driver is equally effective. One advantage of
hat is, the absolute connection between the screw
ive the screw into wood at any angle, perfectly
of the screw-head from this style of construction,
re control over the course of the screw, are the

3465.

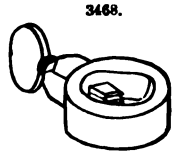


sufficient for ordinary feed for wood cutting, and
d-nut G with which the mandrel thread engages

... Fig. 3466. The nut is in two halves, A, which ... piece B. The whole is covered by the cap, Fig. ... screw by a pin or screw in each projecting into ... the lower portion of this cap are seated into an ... that the cap may be turned without lifting from ... in the stock E, when the machine is in use, by ... groove on the shank of the circular bed-piece ...



3468.

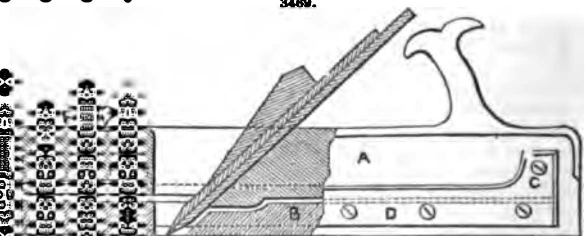


... th of the hole to be bored; seen also at I, Fig. ... rior being threaded to fit the screw of the man- ... led on its end, fitting in a chamber, and moved ... ded portions prevent injury to the screw of the ...

... ed on the mandrel at a proper height above the ... of hole in the hub. The hub being held in the ... the feed-nut at the top of the stock E, until the ... The set screw H is then slightly loosened, which ... nd a few turns of the handle forms a perfectly ... withdraw the mandrel from the bored hub, it is ... slight turn to the left, separating the two halves ...

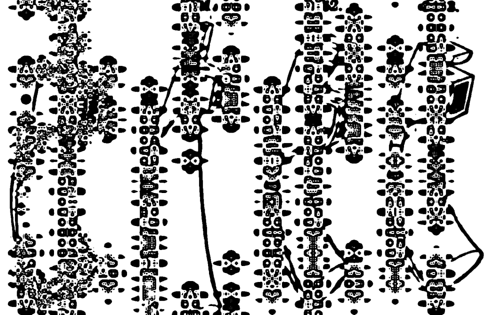
... vention of G. Buckel, shown in Fig. 3469, are to ... nd depth of the cut by the pressure of the hand,

3469.

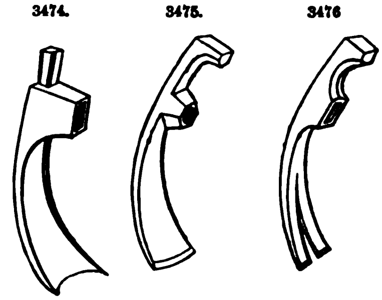


... ifts the upper portion together with the bit or ... erned by the thumb-screw F.

... nd-tool used by carpenters for chipping. It is ... edge at right angles to the handle. The edge ... easily removed when the tool is to be ground. ... hip-carpenter's adze; Fig. 3472 a cooper's adze;

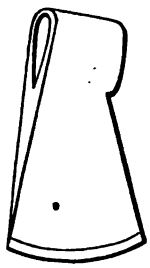


3474. 3475. 3476



... a spout adze; Fig. 3475 a cooper's adze, with a ... This hand-tool, to be perfect, must have the ... ng edge.

An axe is a hand-tool usually of iron, with a steel edge or blade, for hewing timber, chopping wood, and so on. It consists of a head with an arching edge, and a wooden helve or handle. Fig. 3477 is of a colonial felling axe; Fig. 3478 an Australian felling axe; Fig. 3479 a wheeler's axe; Fig. 3480 a north country ship-axe; Fig. 3481 a Dutch side-axe; Fig. 3482 a Brazil axe; Fig. 3483 a broad axe; Fig. 3484 a Kent axe; Fig. 3485 a Scotch axe; Fig. 3486 a blocking

3482.

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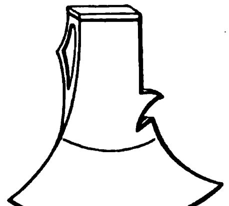
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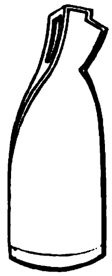
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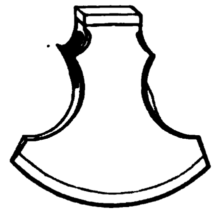
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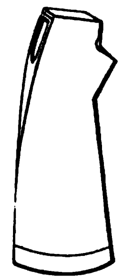
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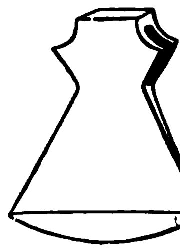
3483.

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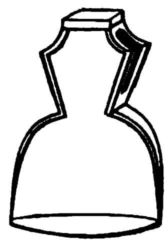
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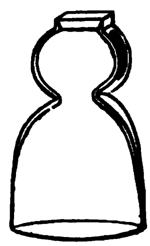
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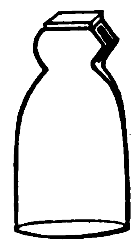
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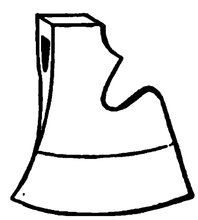
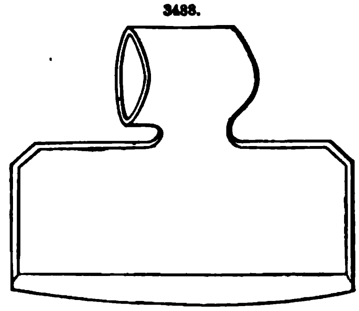


3489.

3488.

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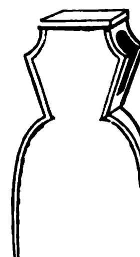
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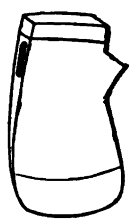


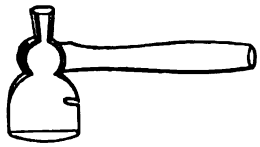
3492.

3493.


axe; Fig. 3487 a coachmaker's axe; Fig. 3488 a cooper's axe; Fig. 3489 a long felling axe; Fig. 3490 a common ship axe; and Fig. 3491 a Kentucky wedge-axe. This important tool must have either the centre of percussion or centre of gravity of the moving mass directly over and in the plane of the cutting edge. Fig. 3492 is of a Canada hatchet, handled; Fig. 3493 an American shingling hatchet, with claw; Fig. 3494 a shingling hatchet, with hammer head; Fig. 3495 an iron-handled butcher's cleaver; Fig. 3496 a bright meat chopper; Fig. 3497 a Norfolk and Suffolk single-edge bill, tanged and handled; Fig. 3498 a Yorkshire socket bill; Fig. 3499 a socket lopping bill; Fig. 3500 a Nottingham tanged bill, handled; and Fig. 3501 a strapped switch hook, single or double hand. When the cutting edge is required to throw chips, the plane passing

through the centre of percussion must also pass through the bevel, and not through the cutting edge of the blade.

3494.



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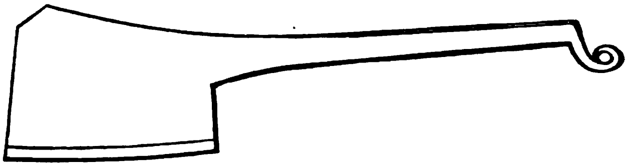

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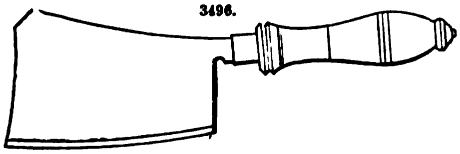

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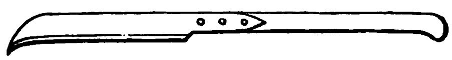

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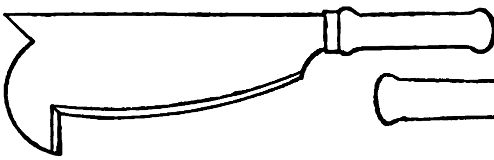

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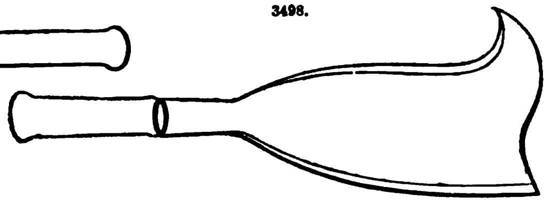

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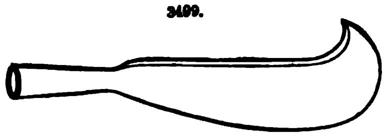

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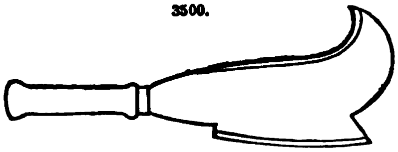
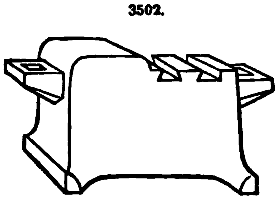


Anvils and Swage-blocks.—See ANVILS. Fig. 3502 is an anvil for light edge-tools; Fig. 3503 an anvil for heavy edge-tools; Fig. 3504 a peculiar-shaped anvil, used in the forging of shears; Fig. 3505 a saw anvil; Fig. 3506 a sickle anvil; Fig. 3507 a pocket-knife blade anvil; Fig. 3508 a single hand file anvil; Fig. 3509 a table-knife blade anvil; and Figs. 3510, 3511, swage-blocks.

3502.



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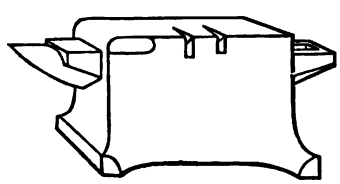

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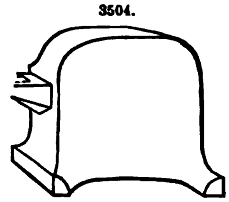

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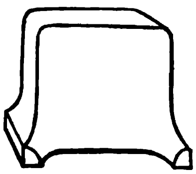

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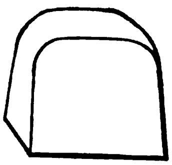

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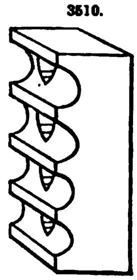

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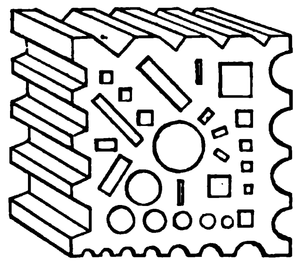

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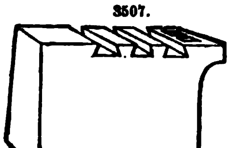

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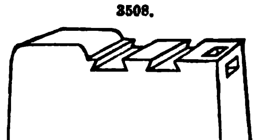

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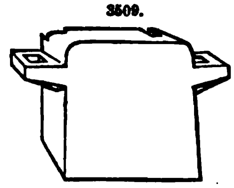
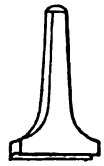


Tin and Coppersmiths' Tools.—Fig. 3512 is a block hammer; Fig. 3513 a concave hammer; Fig. 3514 a rivet set; Fig. 3515 a groove punch; Fig. 3516 a hollow punch; Fig. 3517 a teapot neck tool; Fig. 3518 a tea-kettle bottom stake; Fig. 3519 a kettle lid swage; Fig. 3520 a funnel

3512.

3513.

3514.

3515.

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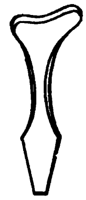
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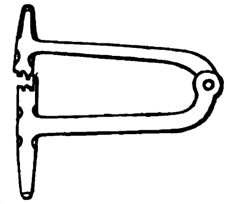

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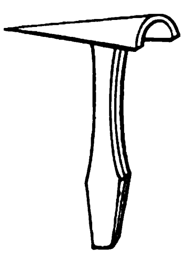
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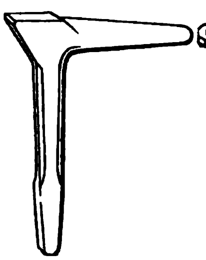
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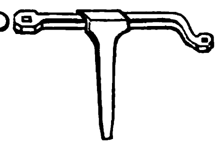
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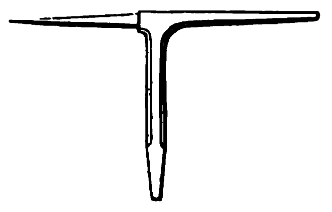

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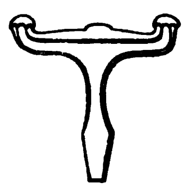
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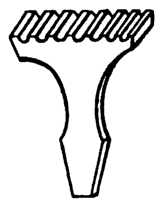
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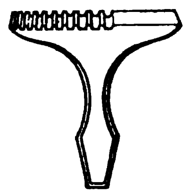
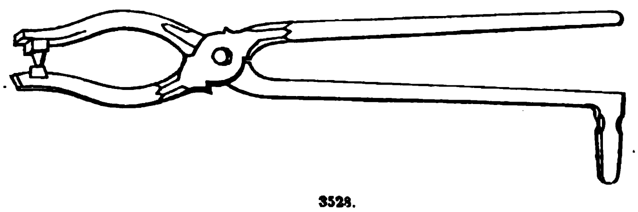


stake, Fig. 3521 a side stake; Fig. 3522 a tinman and brazier's horse; Fig. 3523 a beck iron; Fig. 3524 a saucepan bellie stake; Fig. 3525 a grooving stake; Fig. 3526 a creasing iron.

Fig. 3527 follies; Fig. 3528 stock shears; Fig. 3529 block shears; Fig. 3530 a bottom closing

3527.


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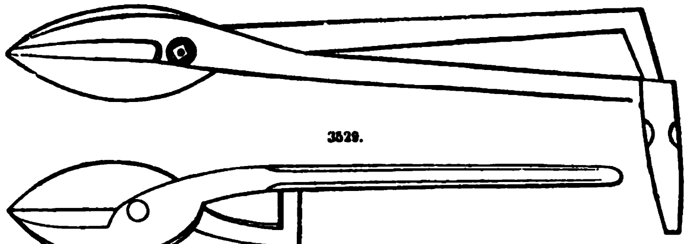

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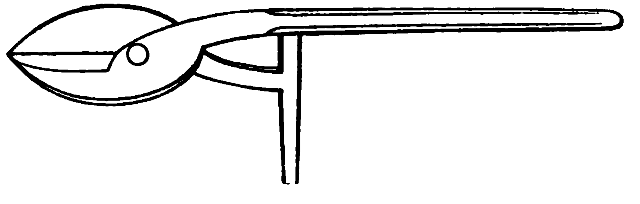
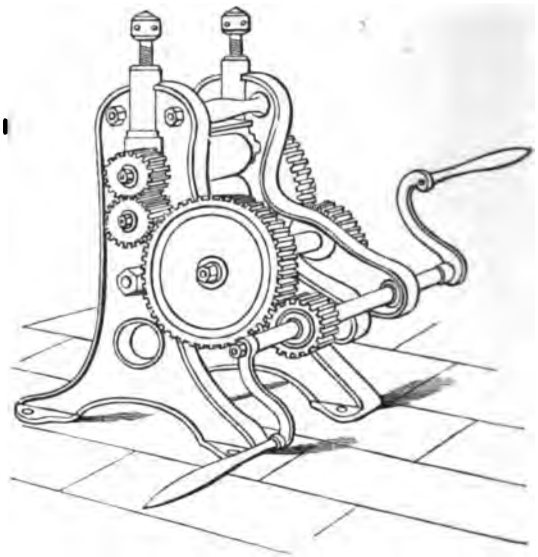
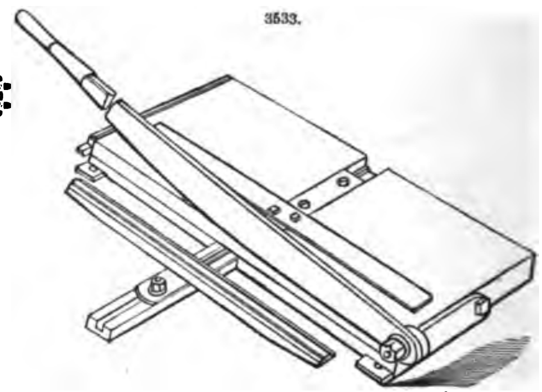


[illegible] rolls; Fig. 3532 a jeweller's mill; Fig. 3533

3531.

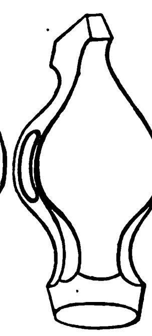


3533.



[illegible] driving nails, beating metals, and the like, usually [illegible] to a handle. See STEAM-HAMMER. Figs. 3534 to [illegible] hammer-heads; Figs. 3538 to 3543 boiler-makers'

3537.



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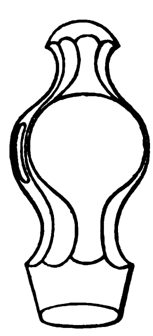

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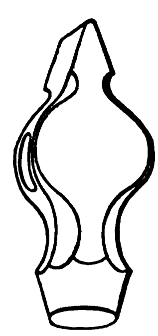
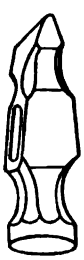


hammer-heads; Fig. 3544 a sledge hammer-head; Fig. 3545 a contractor's hammer-head for stonework; Fig. 3546 a hammer-head for riveting; Fig. 3547 a mason's hammer-head; Fig. 3584 a shingling hammer-head; Fig. 3549 a ship-carpenter's hammer-head; Fig. 3550 a coach-trimmer's

3540.



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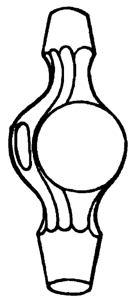

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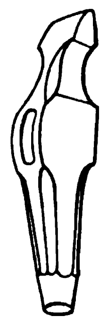

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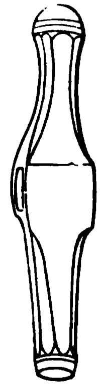

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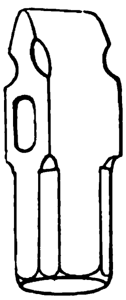

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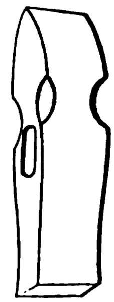

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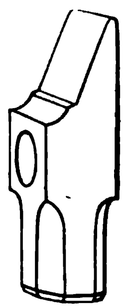

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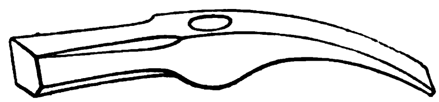
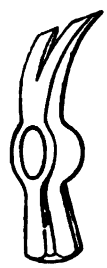


3549.



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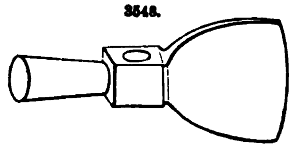

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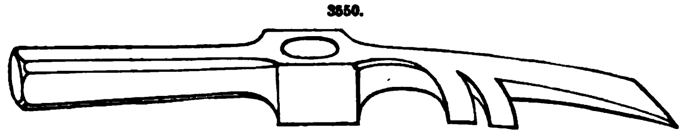

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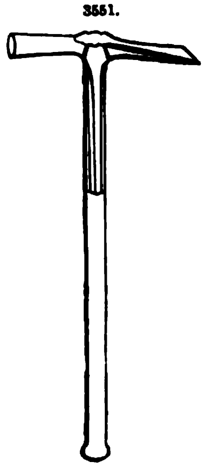

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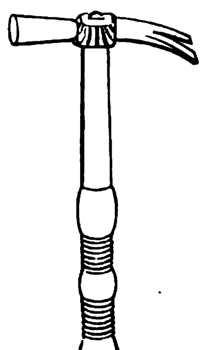

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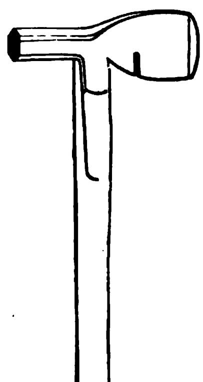

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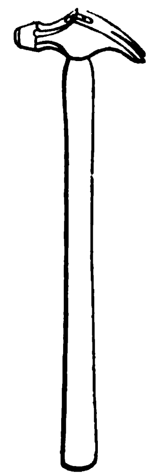

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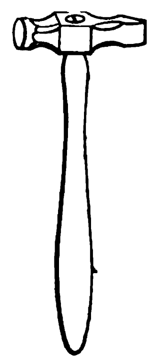
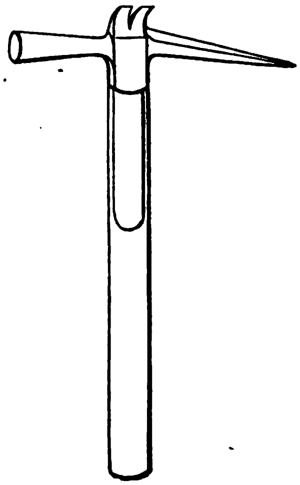


hammer-head; Fig. 3551 a saddler's hammer; Fig. 3552 a London glazier's hammer, Fig. 3553 a lathing hammer; Fig. 3554 a farrier's shoeing hammer; Fig. 3555 a plumber's hammer; Fig. 3556

a slater's hammer, with pick and claw; Fig. 3557 a brick hammer; and Fig. 3558 a fireman's hatchet or tomahawk.

3556.



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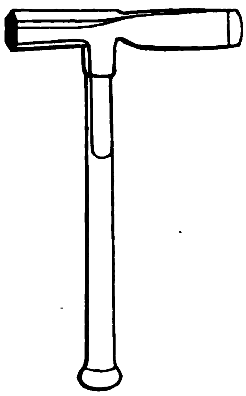

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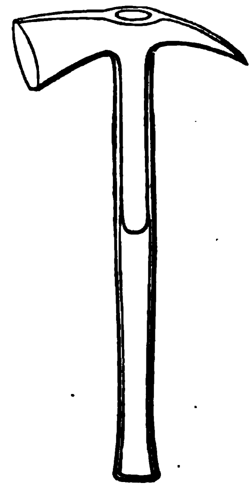
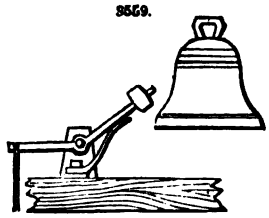


The handle of a hammer must be so formed and fixed that an operator may deliver blows without shock to his hand and arm; in this case the centre of percussion of both head and handle and the point struck must be in the line in which the centre of percussion is forced to move. It often happens that the centre of gravity or the centre of gyration of a hammer has to be directed on a given point; this is effected by giving to the head and handle peculiar shapes.

Fig. 3559 is an arrangement of a hammer for striking bells. The spring below the hammer raises it out of contact with the bell after striking, and so prevents it from interfering with the vibration of the metal in the bell.

Fig. 3560 is of a tilt or trip hammer. In this the hammer helve is a lever of the first order.

3559.



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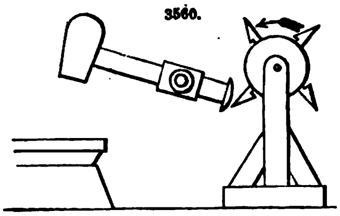

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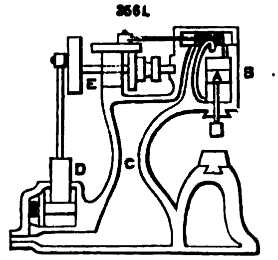

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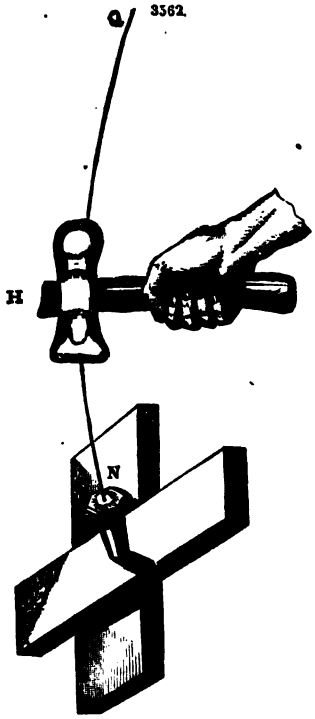


Fig. 3561 exhibits the mechanical combinations of Grimshaw's compressed air hammer. The head of this hammer is attached to a piston A, which works in a cylinder B, into which air is

admitted—like steam to a steam-engine—above and below the piston by a slide-valve on top. The air is received from a reservoir C, in the framing, supplied by an air-pump D, driven by a crank on the rotary driving shaft E.

The succeeding examples indicate how the power of hammers may be calculated.

Example.—Suppose a hammer H, Fig. 3562, strikes a nail N, and drives it $\frac{1}{4}$ of an inch, the hammer weighs 11·58 lbs., and in delivering the blow it passes over the space Q N = 10 ft. in a second; required the force in pounds delivered by the hammer upon the head of the nail. 10 ft. : 1″ :: $\frac{1}{48}$ ft. : $\frac{1}{480}$″. Hence the time occupied in driving the nail $\frac{1}{4}$ of an inch cannot be less than $\frac{1}{480}$ of a second = t.

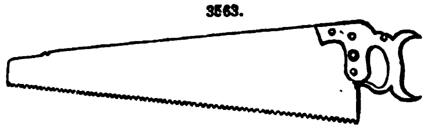
$$m = \frac{11 \cdot 58}{32\frac{1}{6}} = \cdot 36 ; \qquad \therefore \quad F = \frac{m}{t} \times v = \cdot 36 \times 480 \times 10 = 1728 \text{ lbs.}$$

$\therefore$ The force in pounds delivered upon the head of the nail is nearly = a ton.

Let us take another example, and suppose a large hammer, weighing 1930 lbs., moving with a velocity of 40 ft. a second, and to strike a mass of iron which it indents; the indentation is $\frac{1}{2}$ in. deep with a surface area of 16 sq. in.; after the blow is struck the hammer rebounds 2·5 ft. What is the force in pounds delivered on the sq. in. by a blow of this hammer? $\frac{1}{2}$ in. = $\frac{1}{24}$ of a foot; then 40 ft. : 1″ :: $\frac{1}{24}$ ft. : $\frac{1}{960}$″; hence the time occupied in making the indentation cannot be less than $\frac{1}{960}$ of a second. But $F = \frac{m}{t} \times v$. (See Essential Elements of Practical Mechanics, by the Editor of the present work.) The mass of this hammer = $\frac{1930}{32\frac{1}{6}} = 60$; then in this case $F = \frac{m}{t} \times v = 60 \times 960 \times 40 = 2304000$ lbs., which is the force in pounds delivered upon 16 sq. in. $\therefore \frac{2304000}{16} = 144000$ lbs. on each square inch. $\frac{40^2 \times 1930}{32\frac{1}{6}} = 96000$ units of work in the hammer (Essential Elements of Mechanics, p. 97). $1930 \times 2 \cdot 5 = 4825$ units of work in the rebound of the hammer; hence $96000 - 4825 = 91175$. Then if x be the area $\frac{1}{24}$ ft. the depth of the iron displaced by the blow, we have $x \times 144000 \times \frac{1}{24} = 91175$ units of work. $\therefore x = 15 \cdot 196$ sq. in. the area for a uniform depth of $\frac{1}{2}$ in., as if the iron was displaced by a punch.

Saw.—A saw is an instrument for cutting and dividing substances, as wood, iron, and so on, consisting of a thin plate or blade of steel, with a series of sharp teeth on one edge which remove successive portions of the material by cutting or tearing.

3563.



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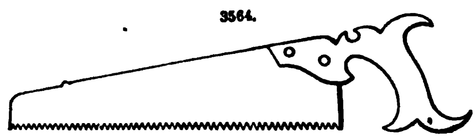

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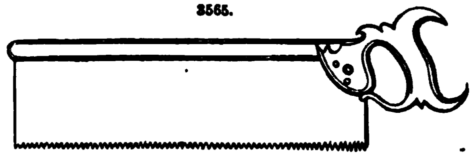

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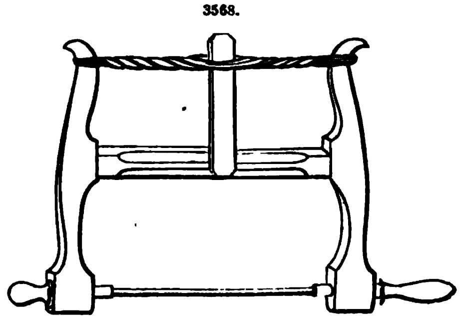

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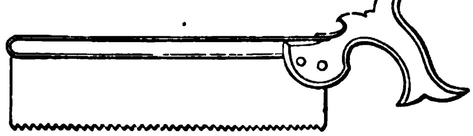

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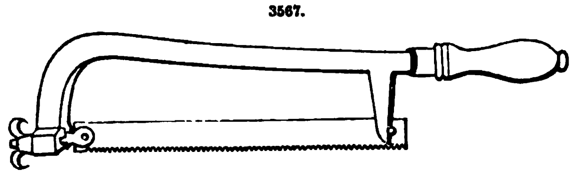

Fig. 3563 is a hand, panel, and ripping saw; Fig. 3564 is a grafter saw; Fig. 3565 a tenon saw; Fig. 3566 a dovetail saw; Fig. 3567 an iron bow-saw; and Fig. 3568 a turning saw in its frame.

...-TOOLS.

... man has perfected is the barrel, hogshead, or cask; ... nce to place with greater ease than any other form ... 3569 to 3571 are of coopers' drivers for tightening

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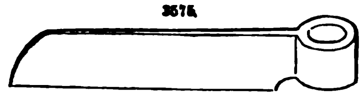
3572.

3573.

3575.



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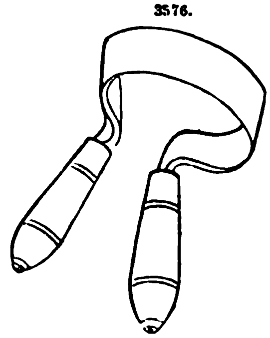

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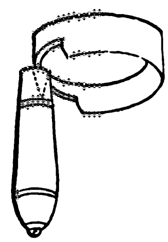

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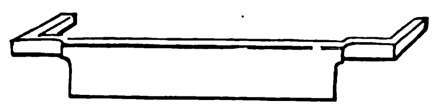

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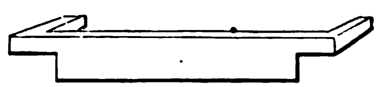

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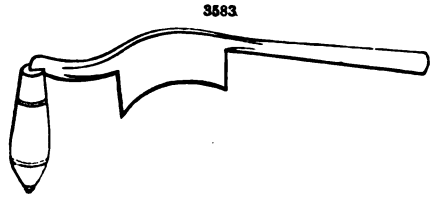
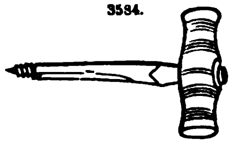


... ch; Fig. 3573 a cooper's chisel; Fig. 3574 a bung ... is a cooper's two-hand round shave; Fig. 3577 a ... ve, and Fig. 3579 a cooper's shave-iron; Fig. 3580 ... on shave-iron; Fig. 3582 a cooper's jigger knife;

Fig. 3583 a London jigger knife; Fig. 3584 a brewer's gimlet; Fig. 3585 a cooper's vice; Fig. 3586 a cooper's bick-iron, used principally for punching holes in hoop iron; and Fig. 3587 a cooper's flagging iron.

3584.



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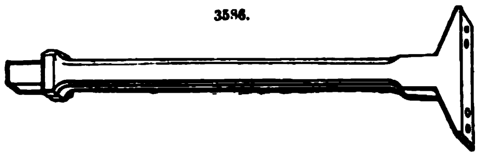

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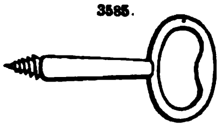

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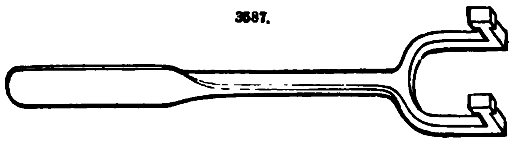

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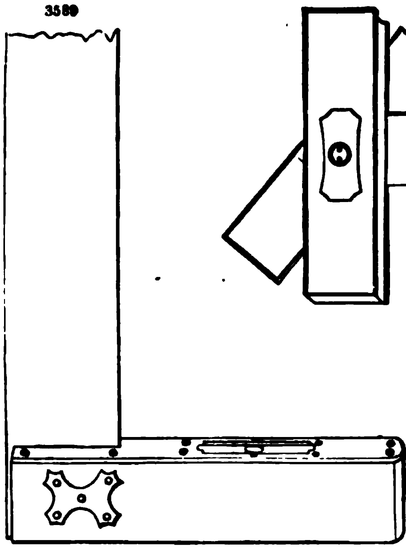

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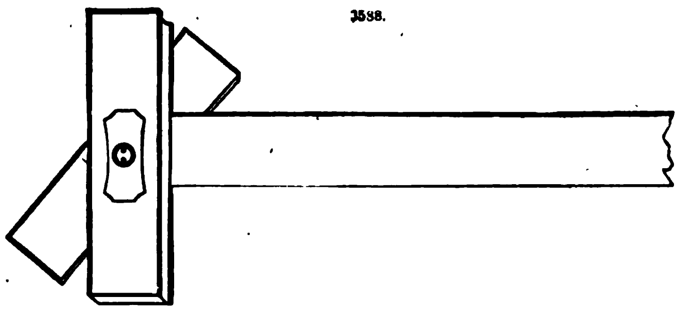

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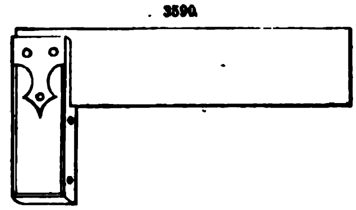

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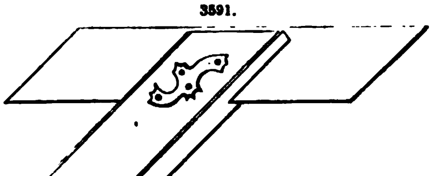

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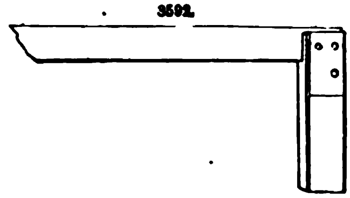

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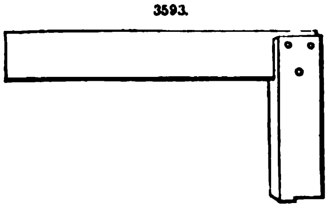

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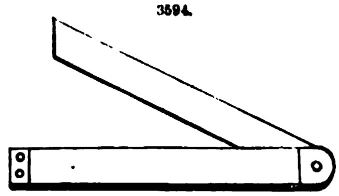
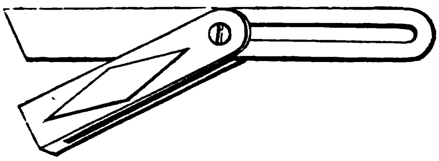


Squares and Bevels.—Fig. 3588 is a T drawing square; Fig. 3589 an ordinary square, with a level attached; Fig. 3590 a common brass-mounted square; Fig. 3591 a mitre square; Fig. 3592 a bricklayer's square, London pattern, Fig. 3593 a brass-stocked sash square; Fig. 3594 an angle

bevel; Fig. 3595 an improved metallic frame sliding bevel; and Fig. 3596 a boat-builder's bevel with two brass blades.

3595.



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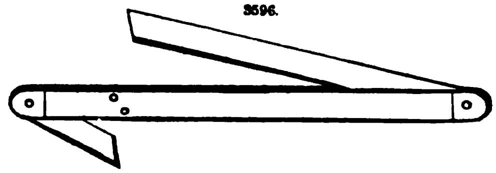
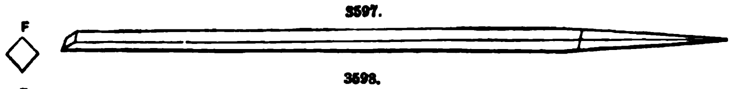


Engravers' Tools.—Fig. 3597 is of a square graver; Fig. 3598 a lozenge graver; Fig. 3599 a flat-edge graver; Fig. 3600 a bent square graver; Fig. 3601 a bent lozenge graver; Fig. 3602 a round

3597.



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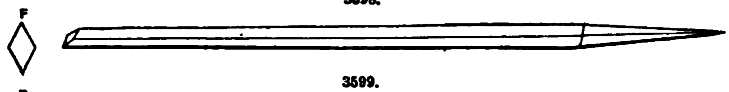

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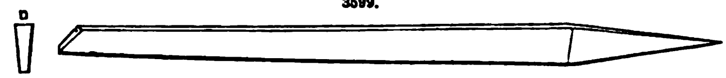

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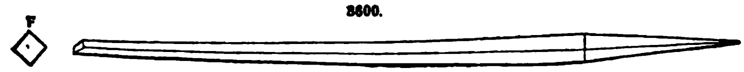

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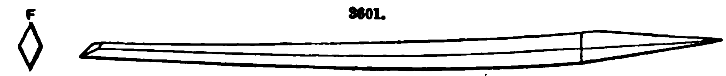

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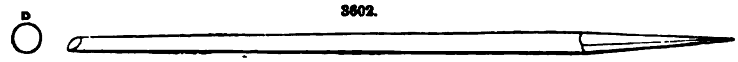

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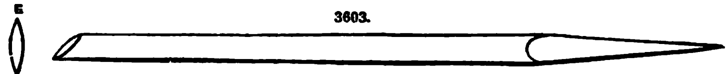

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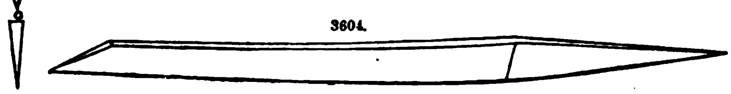

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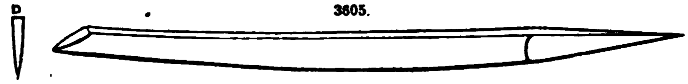

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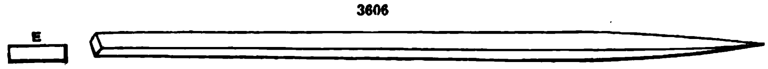

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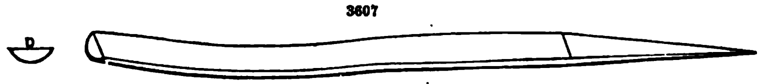

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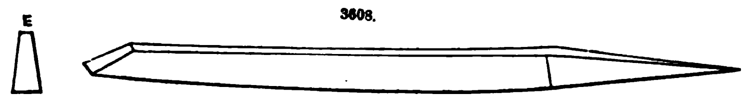

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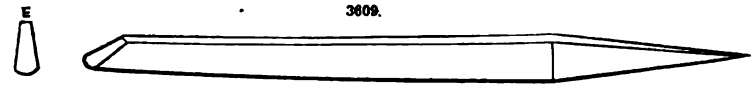
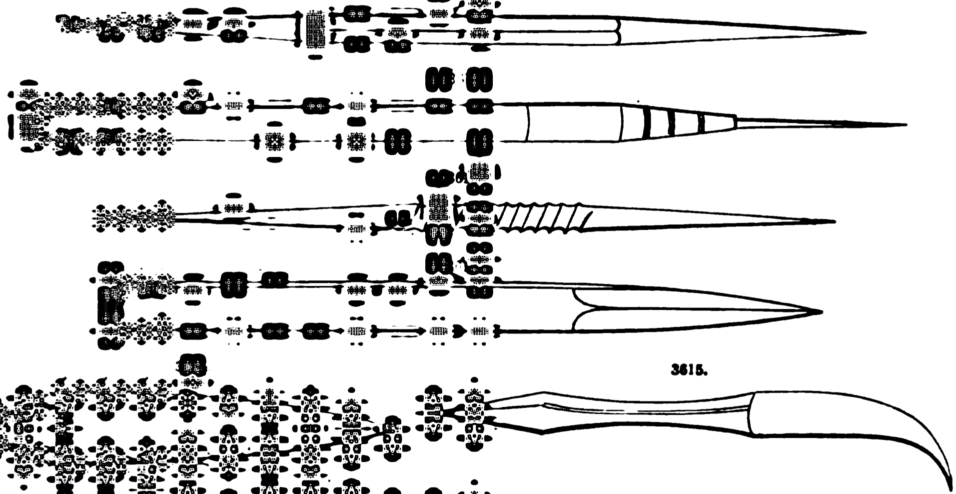


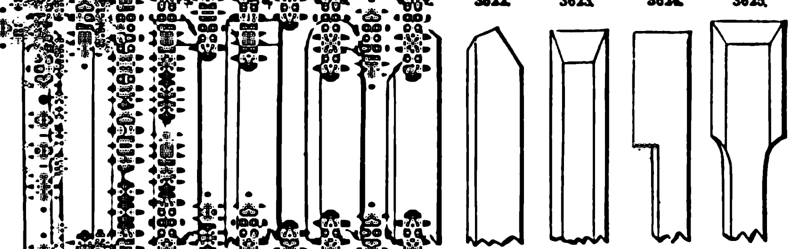
graver; Fig. 3603 an oval graver, Fig. 3604 an engraver's knife; Fig. 3605 a flat oval graver; Fig. 3606 a flat engraver's chisel; Fig. 3607 a half-round bent engraver's chisel; Fig. 3608 a flat

double needle; Fig. 3611 an etching point; scraper; Fig. 3614 an oval burnisher; and

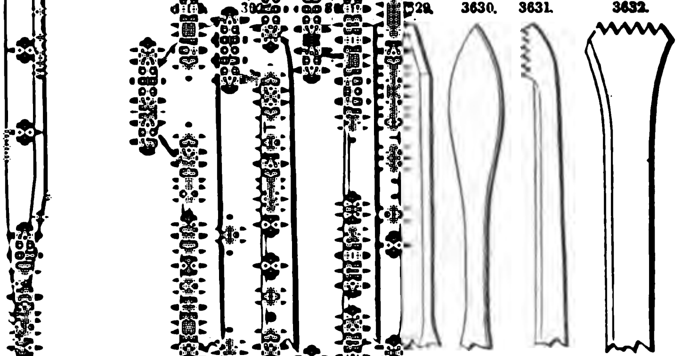


3615.

effected by a slow motion, comparatively
, or bone, which require in most cases a rapid
angle to form the cutting edge than the tools



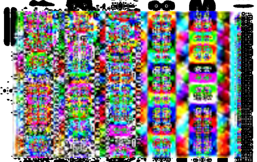
3622. 3623. 3624. 3625.



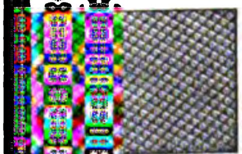
29. 3630. 3631. 3632.

forming the cutting edge of metal-turning tools
Figs. 3616 to 3632 are of a set of turning tools
cutting.
having the surface covered with sharp-edged
other substances, as metals, wood, and so on.
de by straight cuts of a chisel, either single or
raised by the pyramidal end of a triangular

… e coarsest and the second cut. Figs. 3633 to 3650 … ng; the cuts of longer and shorter files are larger … MACHINERY.

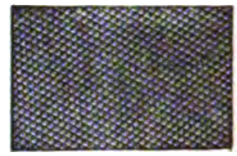


…634.

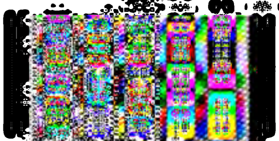


…ddle.

3635.



Bastard.



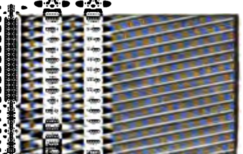
…637.

…mooth.

3638.

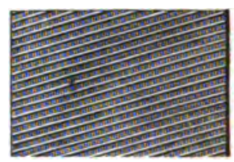
Dead Smooth.

3640.



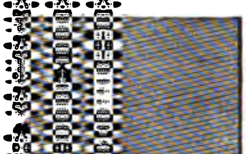
…w Cut.

3641.



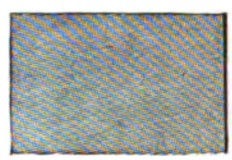
Middle.

3643.

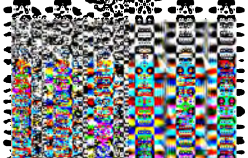


…nd Cut.

3644.



Smooth.

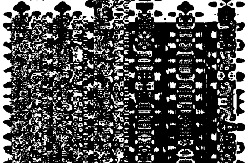


3646.

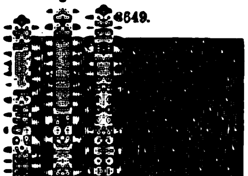
…ugh.

3647.

Middle.



3649.



…nd Cut.

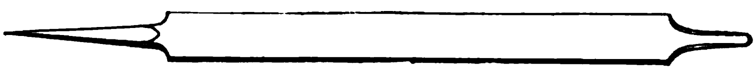
3650.

Smooth.

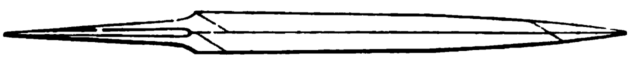
Fig. 3651 is of a smooth needle file; Fig. 3652 a round-off file; Fig. 3653 a three-square taper file; Fig. 3654 a bastard knife file; Fig. 3655 a bastard riffler; Fig. 3656 a saddle-tree rasp; Fig. 3657 a round rasp; Fig. 3658 an improved shoe rasp; Fig. 3659 a horse mouth rasp.

3651.

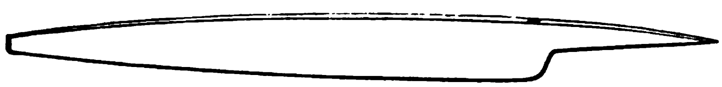
3652.



3653.



3654.



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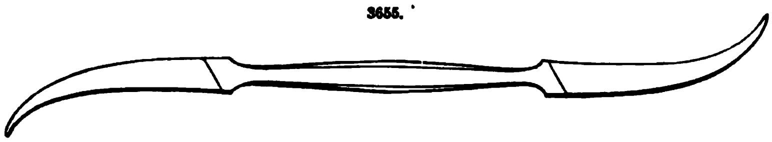

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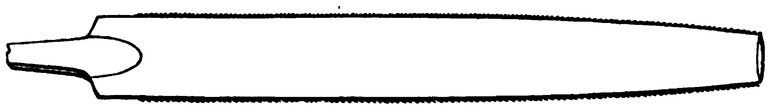
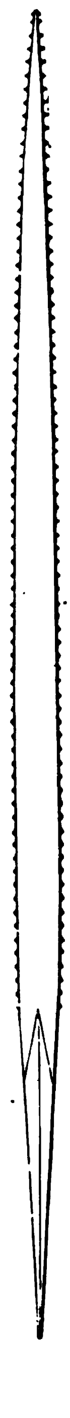
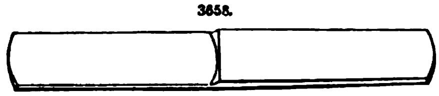


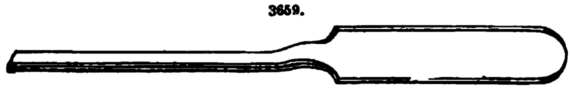
3657.



3658.



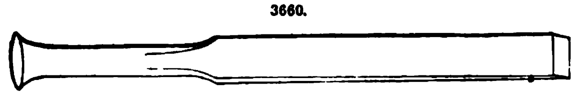
3659.



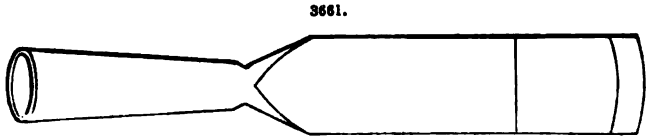
The effective power of the file resembles that of the saw, which has the power of a wedge not encumbered by the friction of one of the faces. The angle of the faces of the wedge is formed by the direction of the applied power and a tangent to the teeth. The diagonal position of the furrows of the file gives an additional shearing wedge power.

Chisels, Gouges, and Planes.—Fig. 3660 is of a shipwright's sharp iron chisel; Fig. 3661

3660.



3661.

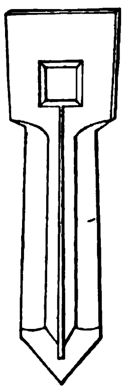


a ship slice; Fig. 3662 a turning chisel; Fig. 3663 a turning gouge; Fig. 3664 a bookbinder's plough knife; Fig. 3665 a common plane-iron; Fig. 3666 a round nose plane-iron; Fig. 3667 a cut plane-iron; Fig. 3668 a round nose double plane-iron; Fig. 3669 an ordinary double plane-iron;

3662.

3663.

3664.



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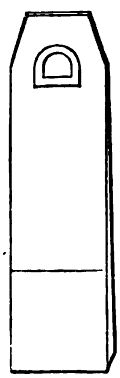

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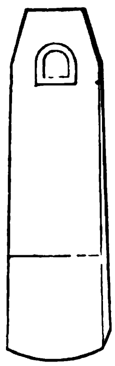

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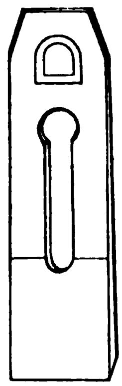
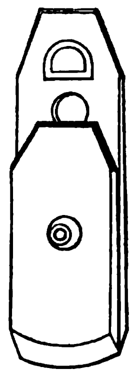
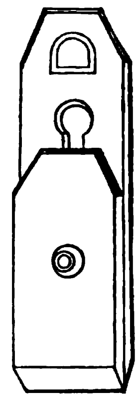


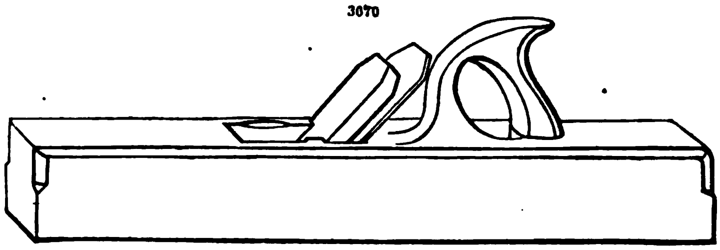
3668.



3669.



3670



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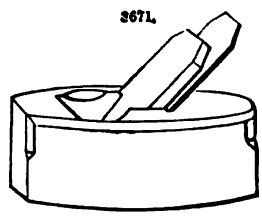
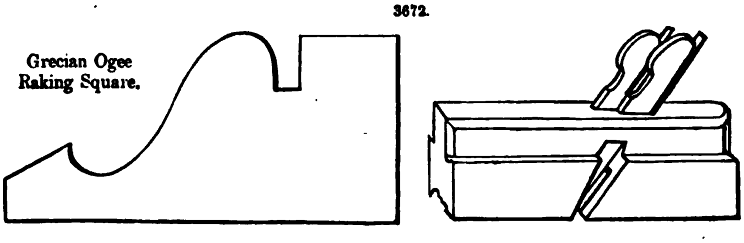


3672.



3673.

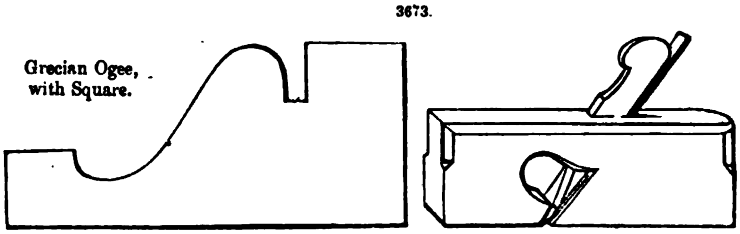
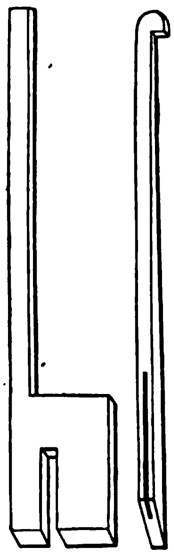


Fig. 3670 a trying plane; Fig. 3671 a smoothing plane, Fig. 3672 an ovolo sash plane, to stick and rebate; Fig. 3673 a rabbet or square plane with skew eye; Fig. 3674 grooving irons; Fig. 3675 a cooper's jointer iron; Fig. 3676 a coachmaker's T iron; Fig. 3677 a skew rabbet iron;

Fig. 3678 hollow and round rabbet irons; Fig. 3679 a striking knife; Fig. 3680 a carpenter's drawing knife; Fig 3681 a cooper's staff knife. Fig. 3682 a mast shave; Fig. 3683 a London

3674.



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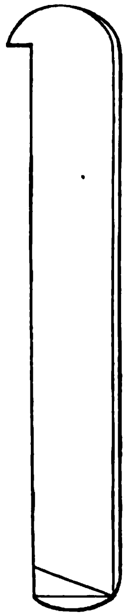

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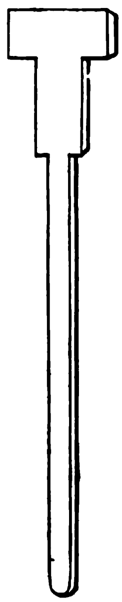

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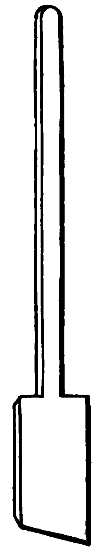

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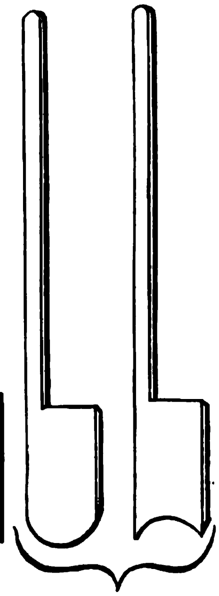

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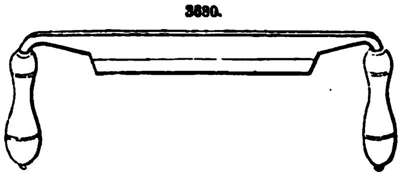
3680.



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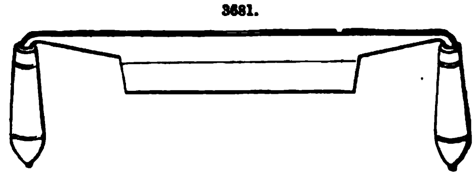

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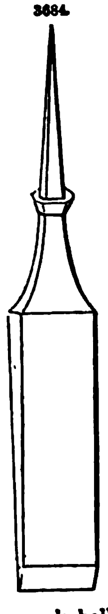

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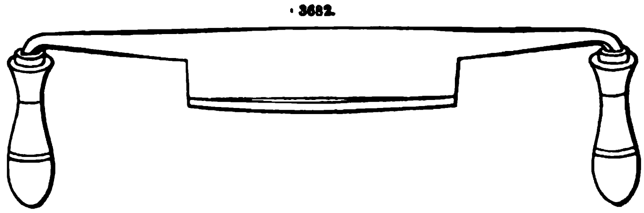

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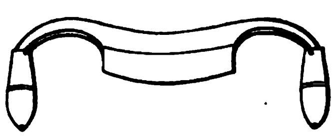

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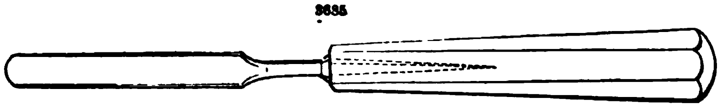
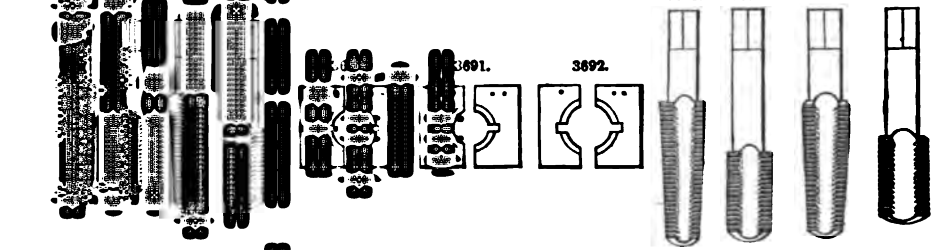


cooper's bollowing knife; Fig. 3684 a common chisel; and Fig. 3685 a common gouge, fixed to its handle.

-TOOLS.

3697 show a double-handed screw stock, with four
taps; Fig. 3698 a clock screw plate; Fig. 3699 a
00 Whitworth's screw stock; Fig. 3701 an ordinary
3703 a plated brace; Fig. 3704 a plug centre bit;

3691. 3692. 3693. 3694. 3695. 3696.



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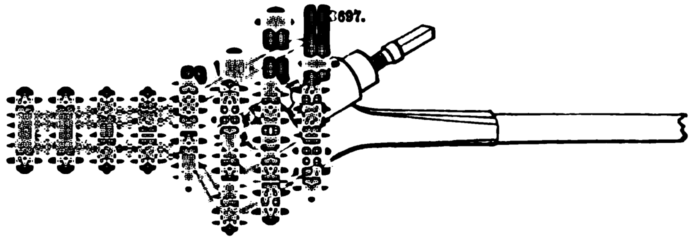

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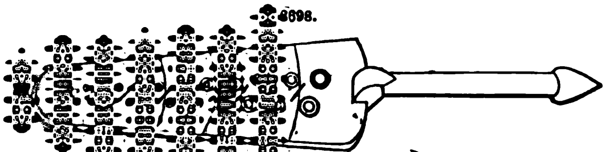
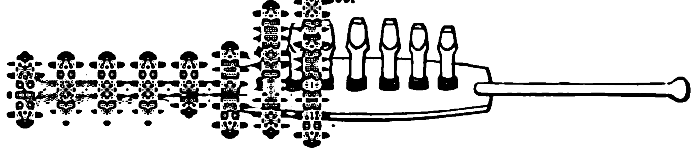
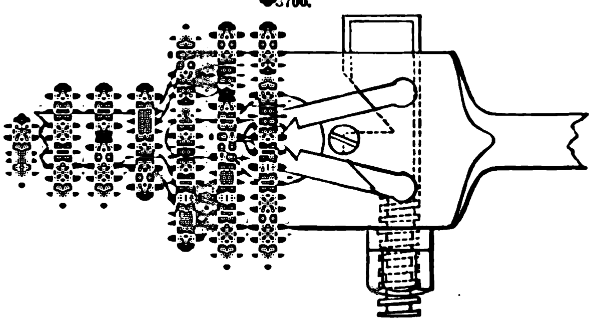


3699.

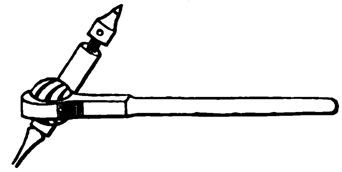


3700.

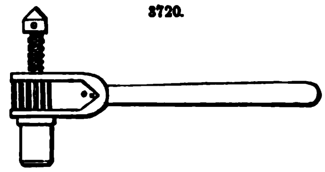


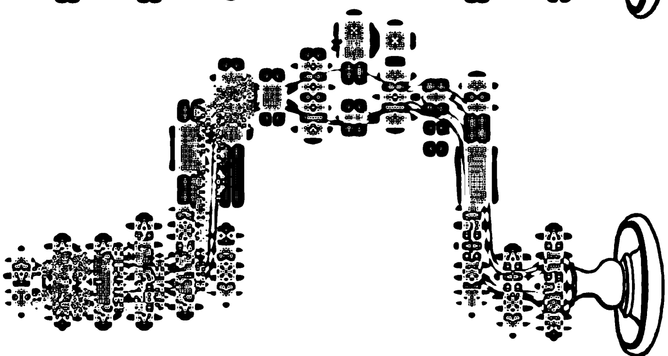
06 a rose-head countersink; Fig. 3707 a flat-head
Fig. 3709 a bobbin bit; Fig. 3710 a taper bit;
; Fig. 3713 a nose bit; Fig. 3714 a spoon bit;
round rinder; Fig. 3717 a gimlet bit; Fig. 3718
sal ball ratchet brace; Fig. 3720 a self-feeding
tchet brace; Fig. 3722 Calvert's ratchet brace;

3719.

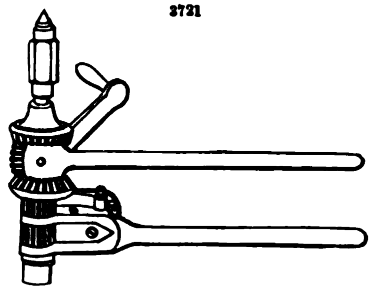


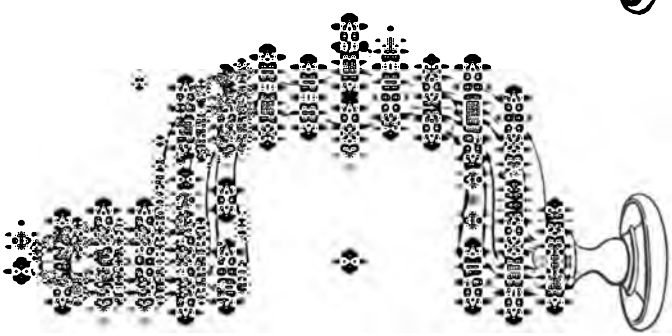
3720.





3721





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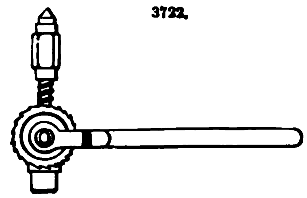
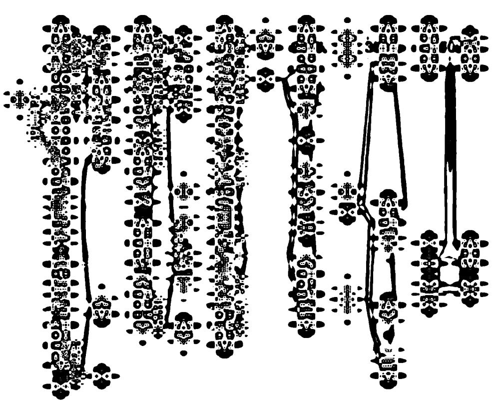
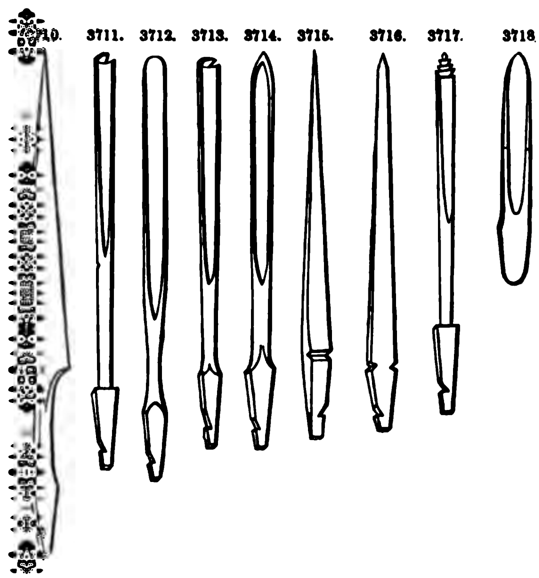
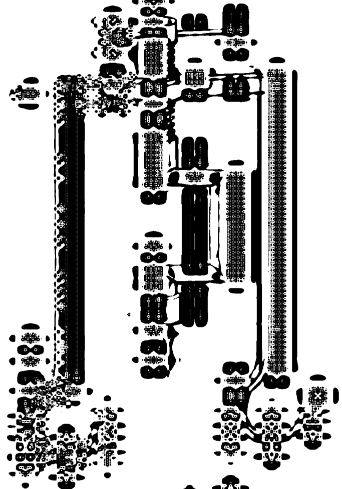




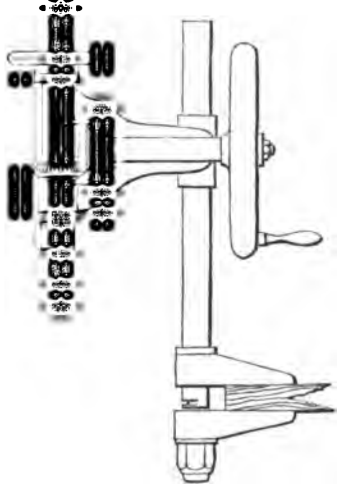
3710. 3711. 3712. 3713. 3714. 3715. 3716. 3717. 3718.



...inds; Fig. 3726 a gas-pipe wrench for iron pipe, ... -pipe tongs for iron pipe.



3724.



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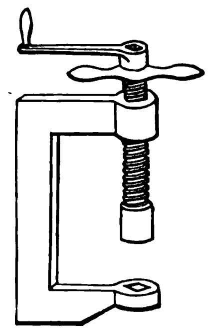
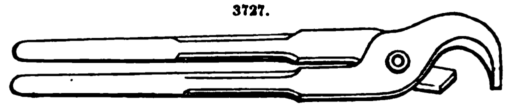
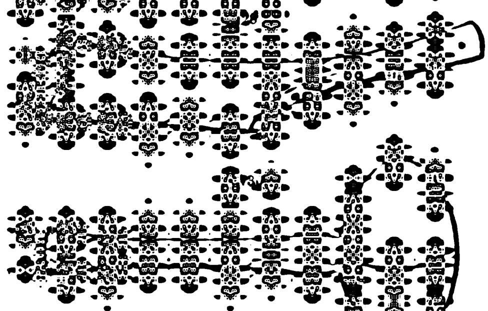


3727.



... dresser; Figs. 3729, 3730, bossing mallets; Figs. ... soldering and bossing irons; Fig. 3737 a ladle;



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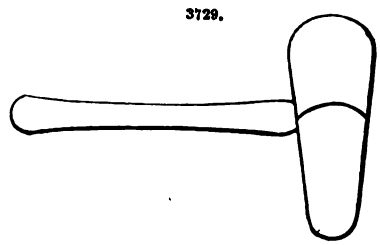
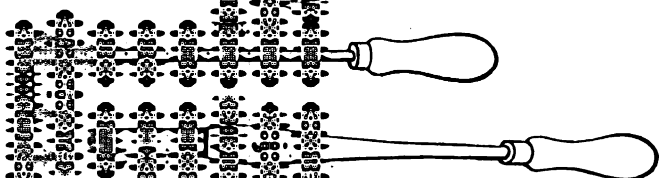
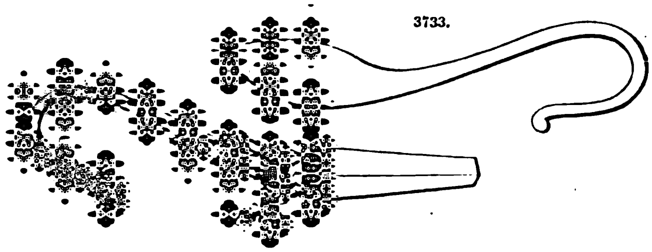


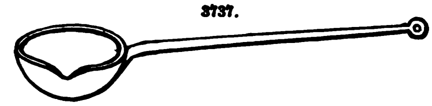


...732.

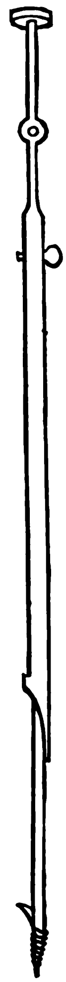
3733.



3737.

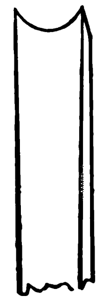


3738.



...k carving chisel; Fig. 3740 a skew ... Fig. 3742 a medium black carving ...; Fig. 3744 a deep black carving ...g. 3746 a black bent fluting gouge; ...

3743.



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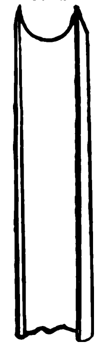
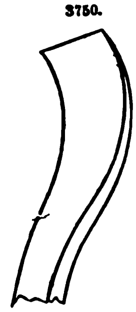


3749.

3750.



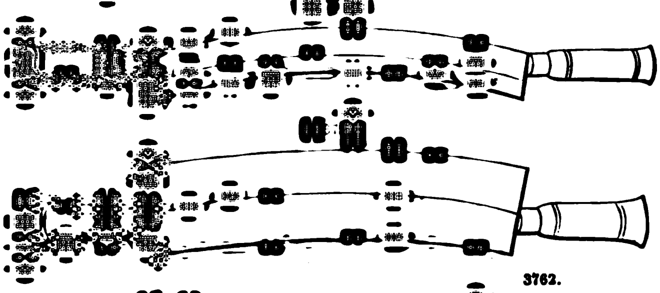
3755.

3756.

3757.

...arting tool; Fig. 3749 a spoonbit parting tool; ... skew spoonbit or entering chisel; Fig. 3752 ...bit or entering gouge for scribing; Fig. 3754 a ... bent spoonbit or entering gouge; Fig. 3756 ... cutter's chisel.

3759, tanners' knives; Fig. 3760 a currier's knife; ws; Fig. 3762 a skinner's round moon knife; Fig.



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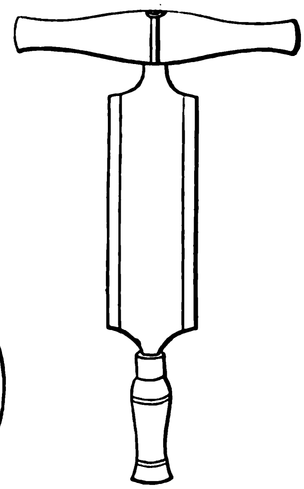

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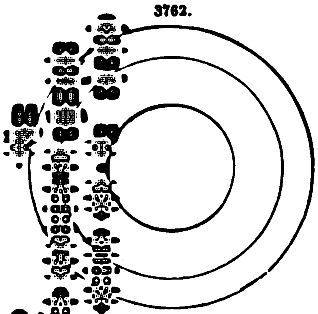
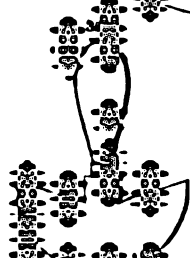
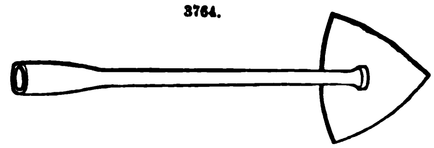


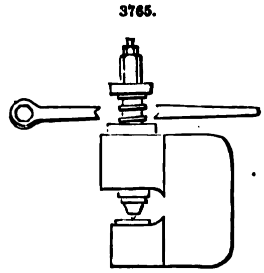


3764.

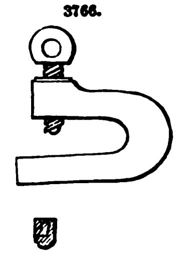


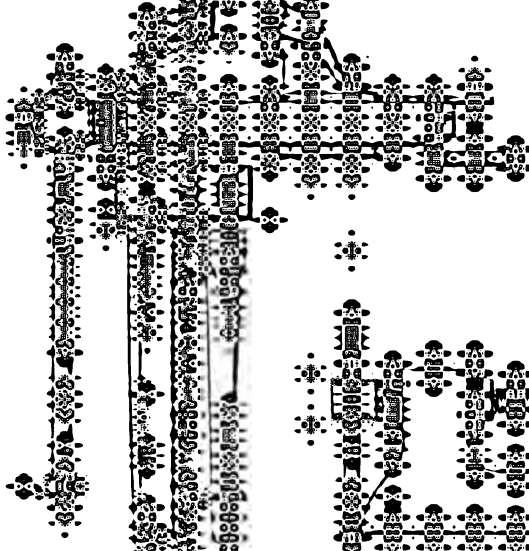
bear; Fig. 3766 a boiler cramp; Fig. 3767 an elwrights, or railway carriage builders; Fig. 3768

3765.

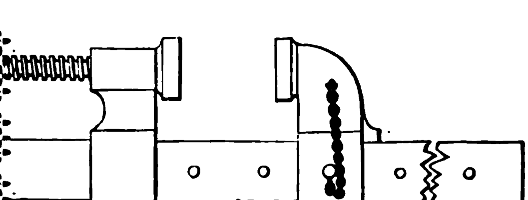


3766.



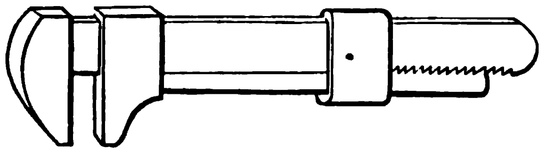


3767.

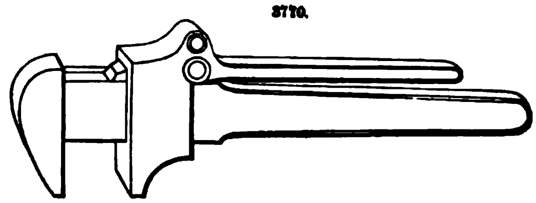


a wrought-iron parallel vice; Fig. 3769 an improved spanner; Fig. 3770 Fenn's spanner; Figs. 3771, 3772, screw-keys; Fig. 3773 a key spanner; Fig. 3774 Clyburn's spanner; Fig. 3775 Budding's

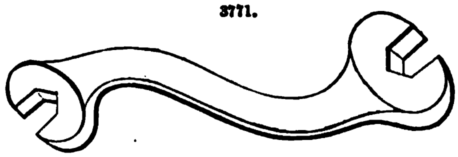
3769



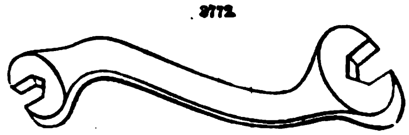
3770.



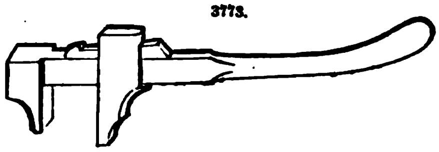
3771.



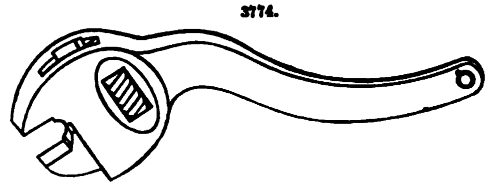
3772.



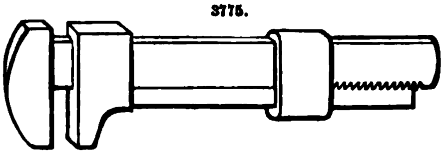
3773.



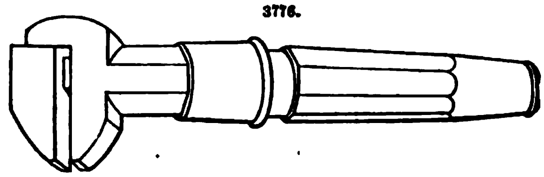
3774.



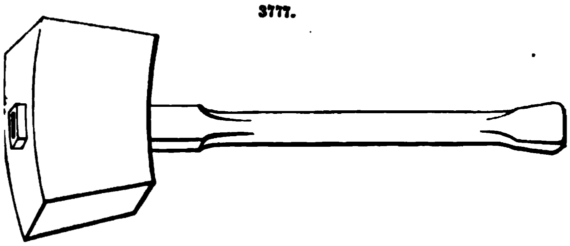
3775.



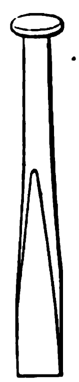
3776.



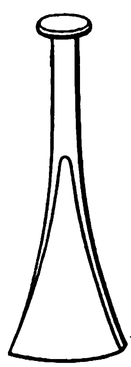
3777.



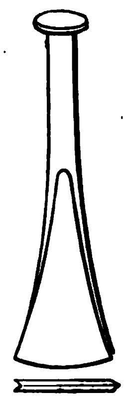
3778.



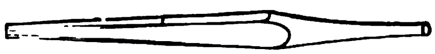
3779.



3780.



3781.

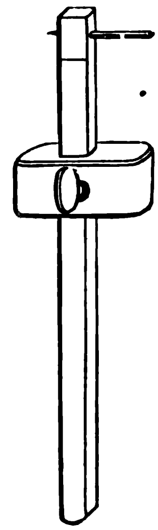


3782.

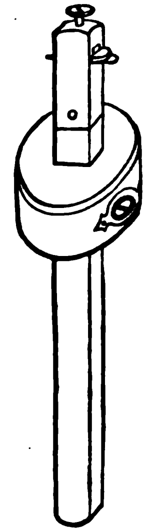
spanner; Fig. 3776 an improved cylinder wrench; Fig. 3777 a joiner's or carpenter's mallet; Figs. 3778 to 3780 calking irons; Fig. 3781 a brad or nail punch; Fig. 3782 a turnscrew, London

… wheeler's gimlet; Fig. 3785 a cutting gauge, the … cutting gauge; Fig. 3787 a thumb or turnscrew

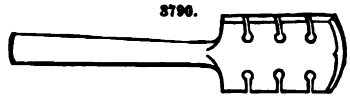
3785.



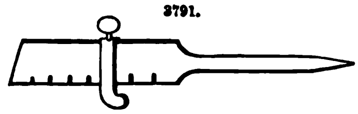
3788.



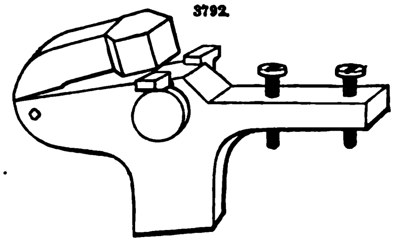
3790.



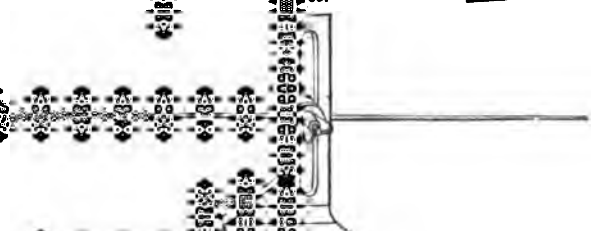
3791.



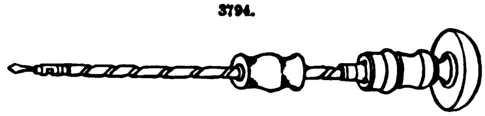
3792.



…89.

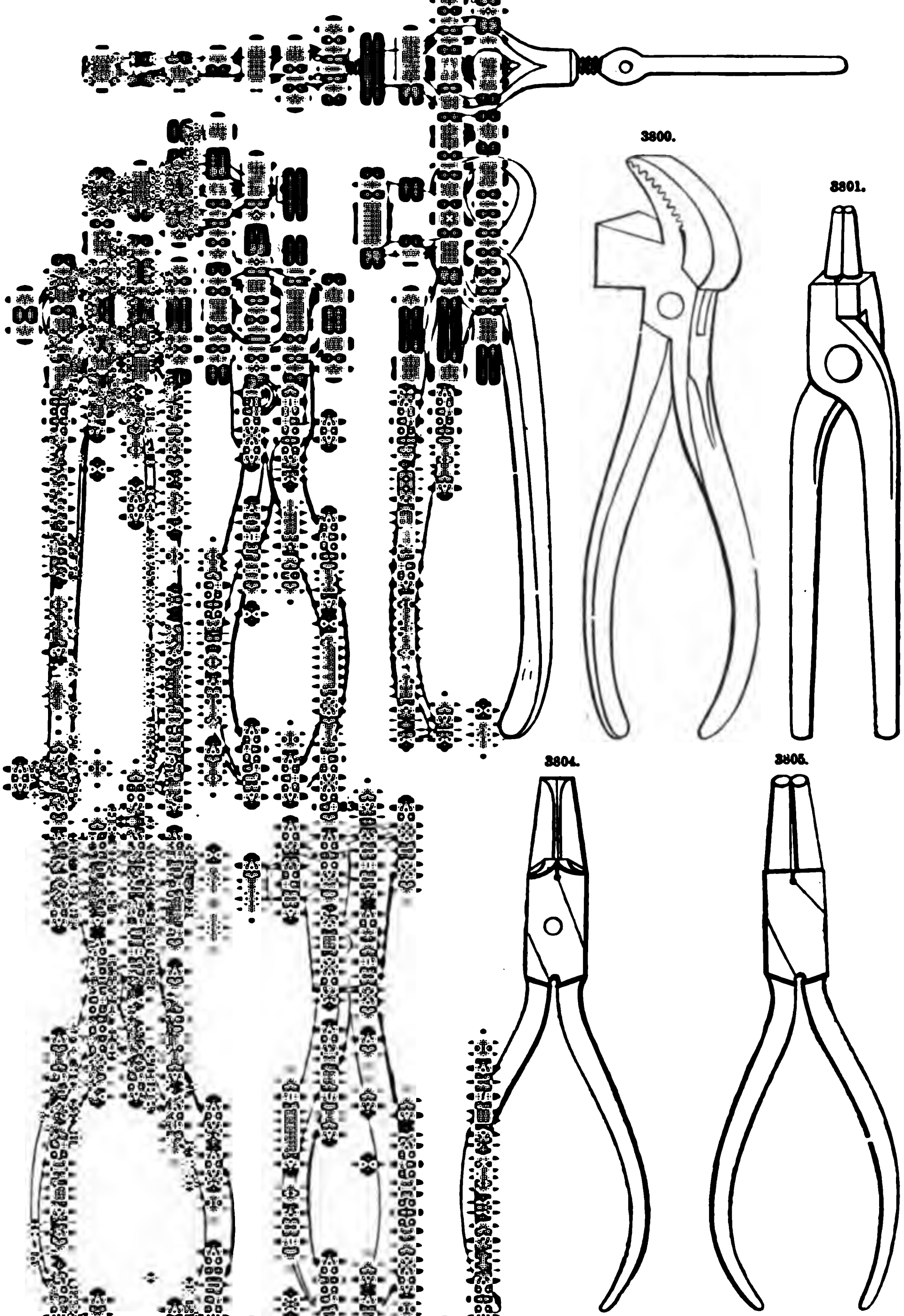


3794.



… ved mortise gauge with improved stem; Fig. 3789 a … g. 3791 a slide saw set; Fig. 3792 an improved saw

...chimedian brace; Fig. 3795 a gas-pipe cutter ... joiners' pincers; Fig. 3798 shoe nippers with

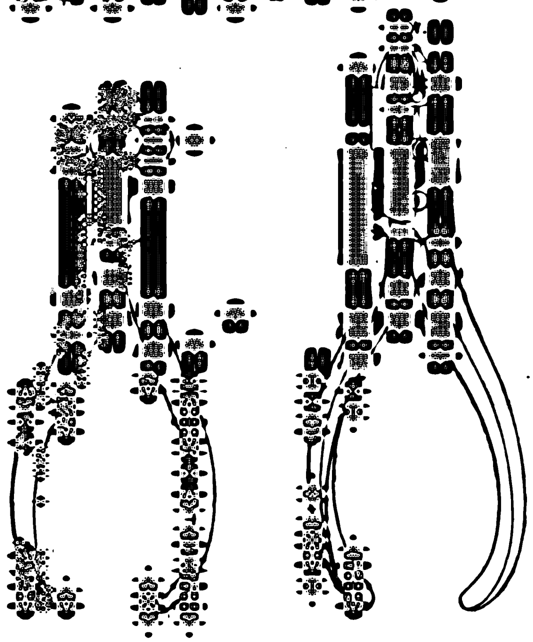


3800. 3801. 3804. 3805.

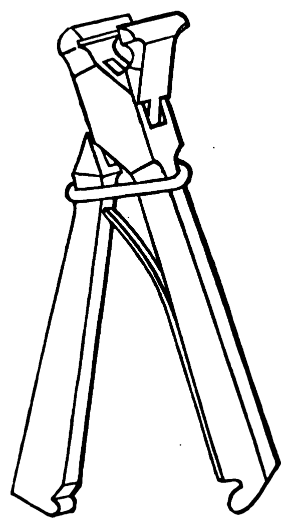
...incers; Fig. 3801 a tinman's pliers; Fig. 3802 ... Fig. 3804 clock pliers; Fig. 3805 round-nosed

TOOLS.

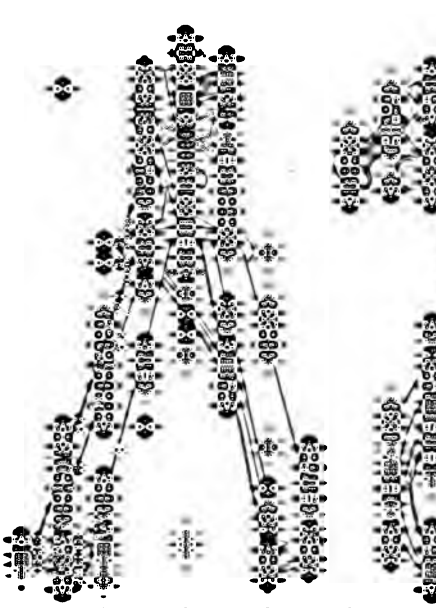
07 nipper pliers; Fig. 3808 vice chop sliding
3810 an improved saw set; Fig. 3811 a bent
13 a box whirl drill stock.
passes; Fig. 3815 wing compasses; Fig. 3816
lipers; Fig. 3818 improved inside and outside



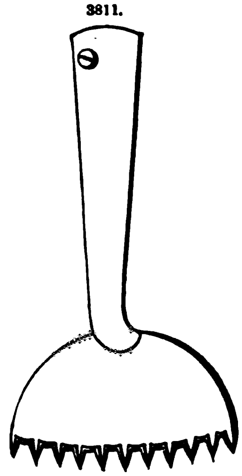
3808.



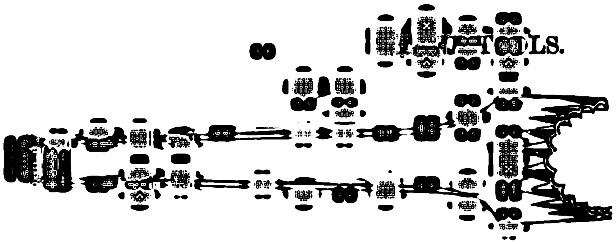
3810.

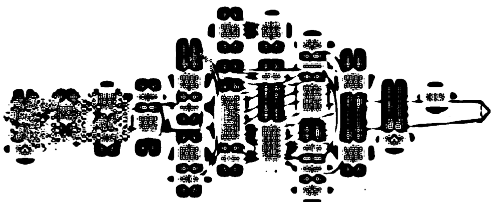


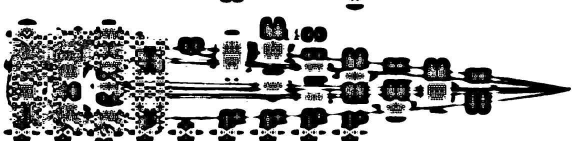
3811.

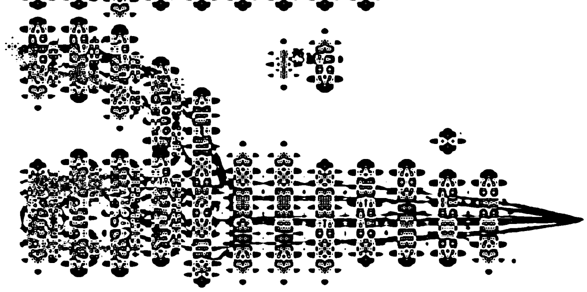


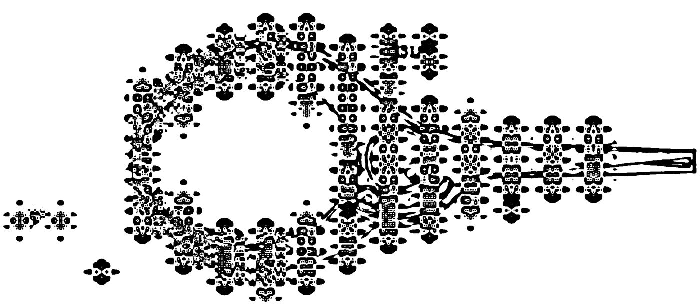
ig. 3820 is a decimal *cam* gauge. The eccentricity
ws the diameter of any small body which is held
the cam. The index always points to the centre
distance between the surface of that stud and the

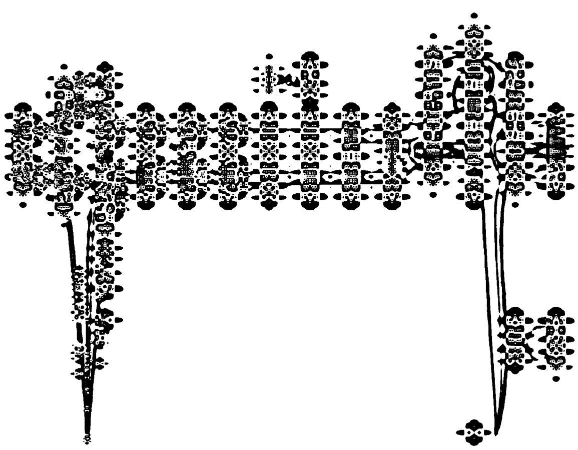




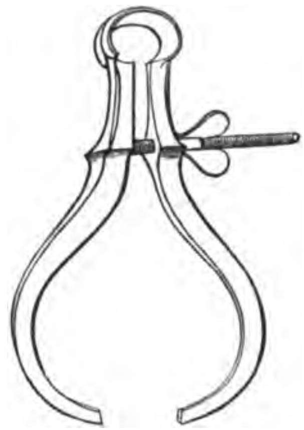




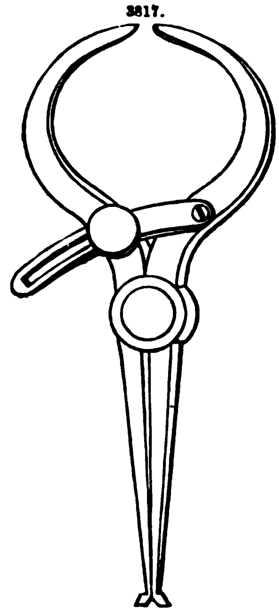




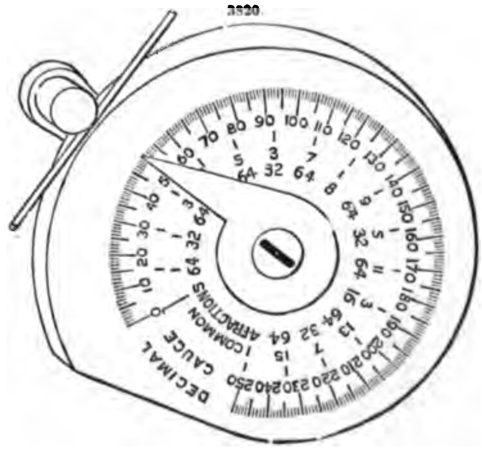
3816.



3817.

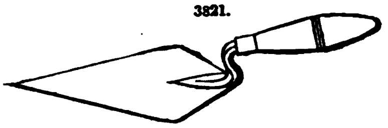


3820.

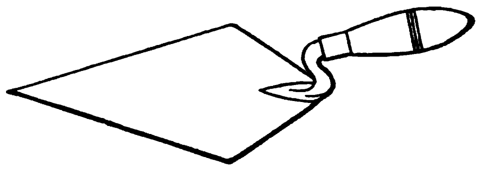


Trowels.—Fig. 3821 is of a mason's trowel; Fig. 3822 a London brick trowel; Fig. 3823 a pointer's cutting trowel · Fig. 3824 plasterers' trowels; Figs. 3825, 3826, plasterers' moulding tools.

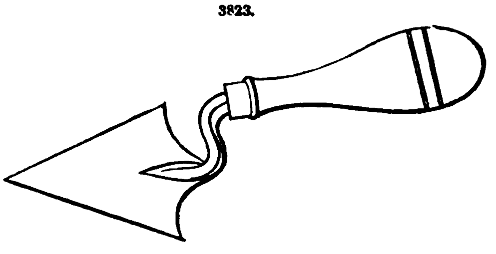
3821.



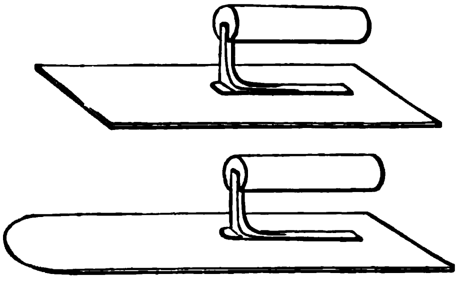
3822.



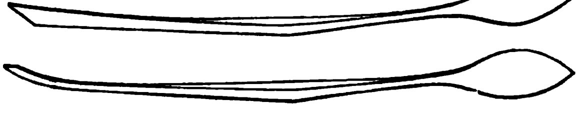
3823.

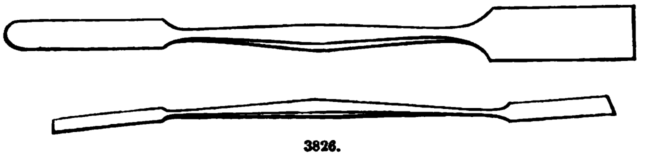


3824.



3825.

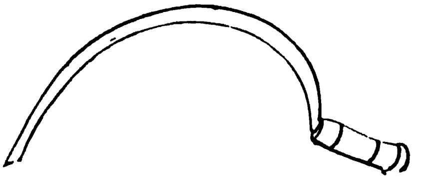




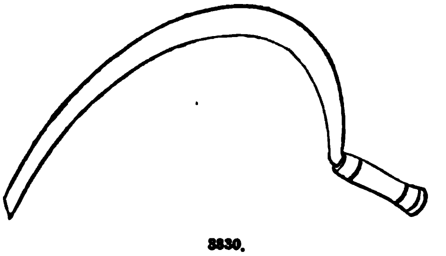
3826.

Sickles, or *Reaping Hooks.*—Fig. 3827 is of the form of reaping hook used in Yorkshire and the North of England; Fig. 3828 a Welsh hook; Fig. 3829 a Kent hook; Fig. 3830 an English

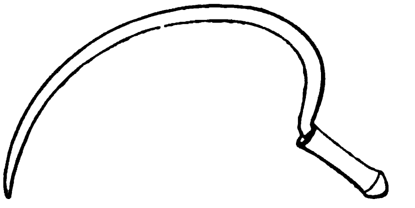
3827.



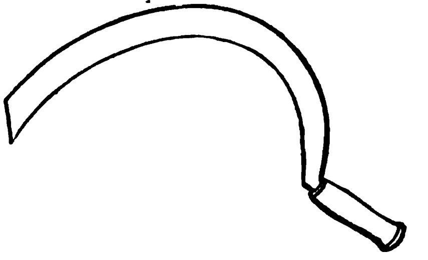
3828.



3829.

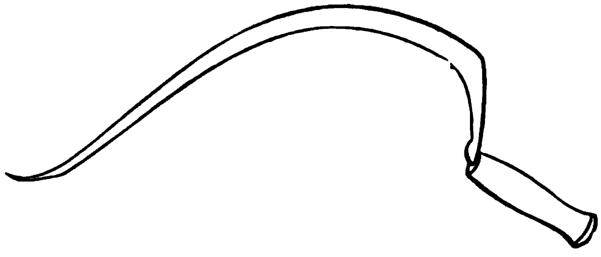


3830.

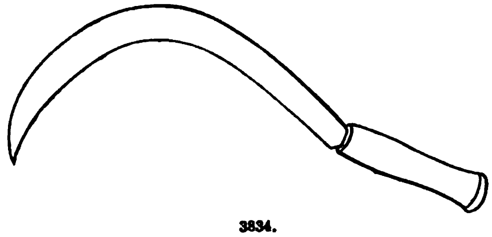


bagging hook; Fig. 3831 an Irish sickle with square heel; Fig. 3832 a bean hook; Fig. 3833 a pea hook; Fig. 3834 a United States' Yarrick sickle; Fig. 3835 a Russian sickle; Fig. 3836 a Spanish sickle; Fig. 3837 a German sickle; Fig. 3838 a Poland sickle. The sickle has the power of the cam and saw combined; it acts on a small sheaf or bundle of grain collected by the left hand and the bay near the handle of the instrument; the small sharp teeth which point to the reaper

3831.

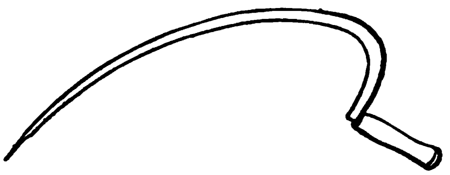


3832.

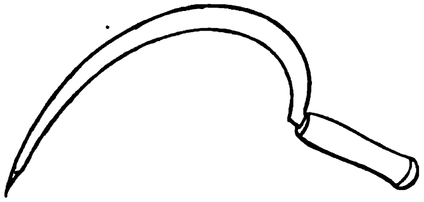


3833.

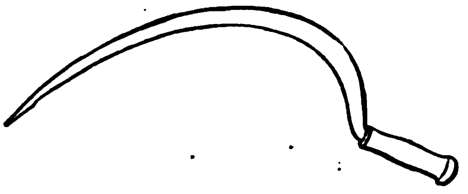
3834.



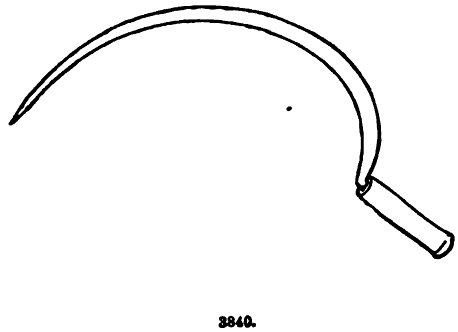
3835.



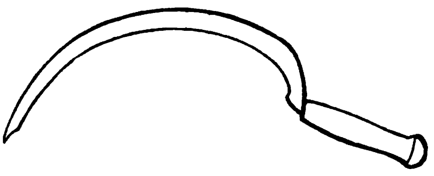
3836.



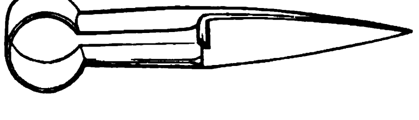
3837.



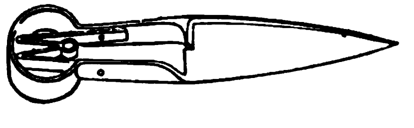
3838.



3839.



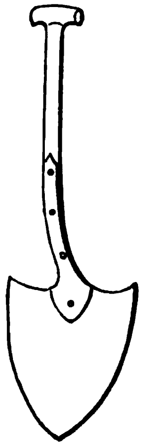
3840.



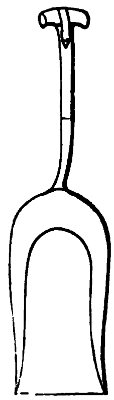
do not retard the process of gathering the grain. The reaper with his right hand draws the sickle towards him and brings the hollow toothed-cam into contact with the stalks, which he cuts, or rather saws, with great ease on account of the shape of the instrument and his own physical formation. Fig. 3839 is of an ordinary sheep-shears; Fig. 3840 an improved sheep-shears. See **Agricultural Implements.**

Spades, Shovels, and Scoops.—Fig. 3841 a gravel shovel with long strap; Fig. 3842 a tender or locomotive shovel; Fig. 3843 an imperial Scotch spade; Fig. 3844 a soughing spade; Fig. 3845 a soughing tool; Fig. 3846 a long-handled Irish spade; Fig. 3847 a deep draining tool; Fig. 3848 a mud shovel; Fig. 3849 a bottoming spade; Fig. 3850 a flat scoop; Fig. 3851 a draining hoe Fig 3852 is of a pick-axe; Fig. 3853 is of an ordinary hoe.

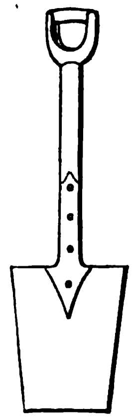
3841.



3842.



3843.



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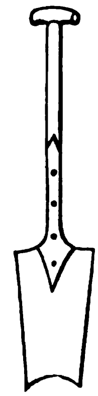

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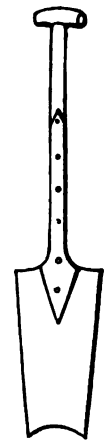

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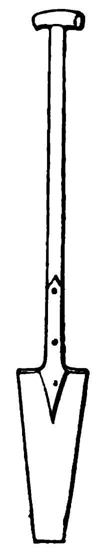

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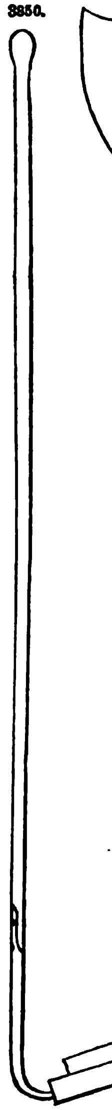

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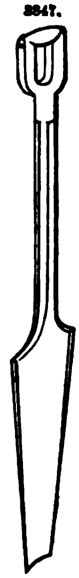

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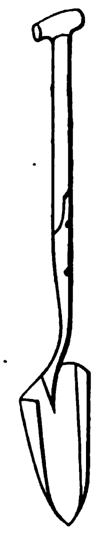

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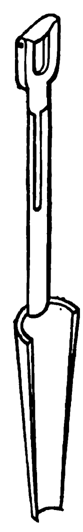

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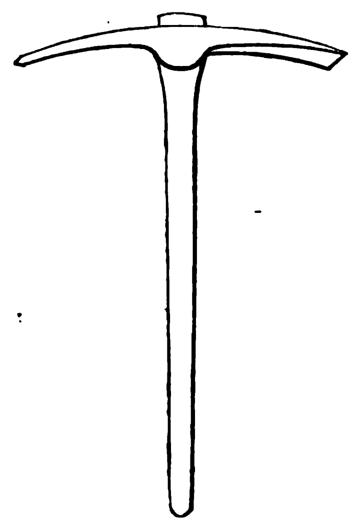

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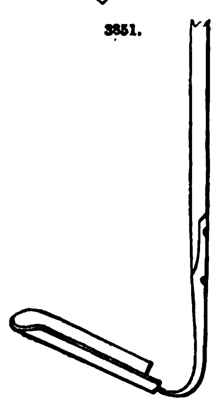
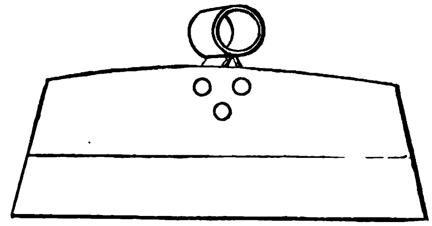
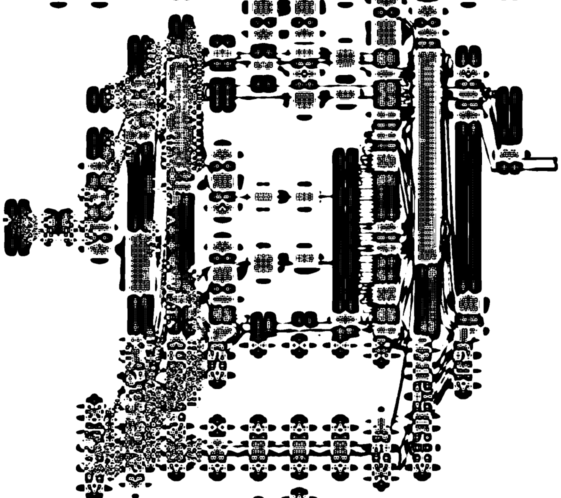


3853.



...55 an improved traversing jack; Fig. 3856

...nd-tools of great durability, formed to effect ...racy, and precision. Most of our illustrations ...and Son, of Glasgow, who, as tool-makers, ...he hardening and tempering of hand-tools to



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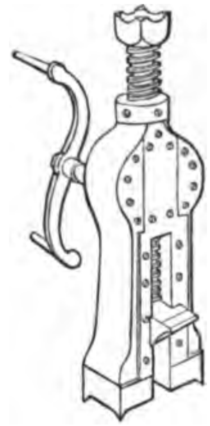

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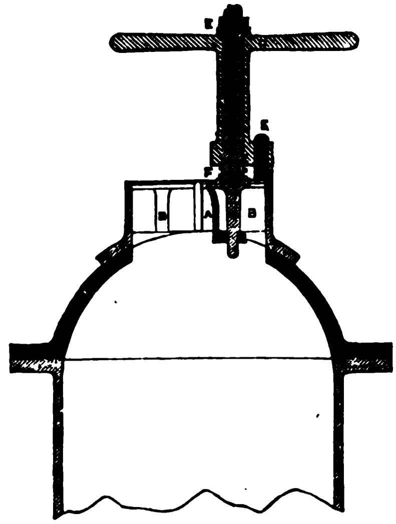

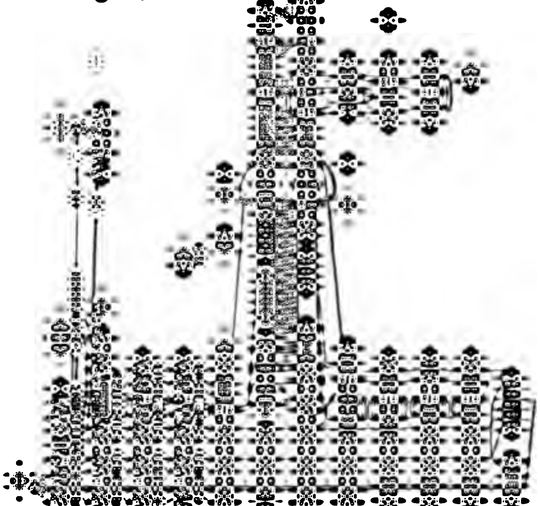

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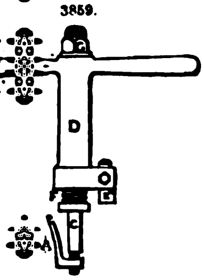
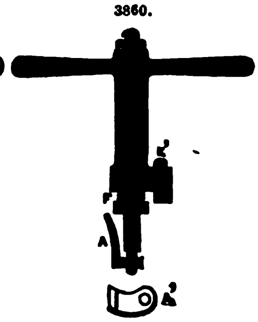


3860.

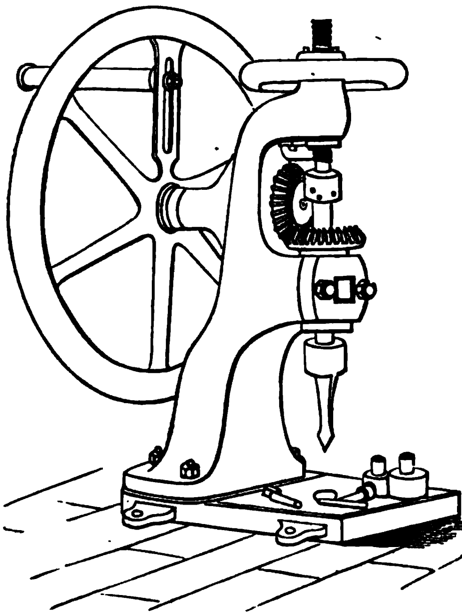


...oss-section of a dome cover, with the tool in ... tool itself. Fig. 3860 is a section, showing ...in of nut A. The action is easily explained:

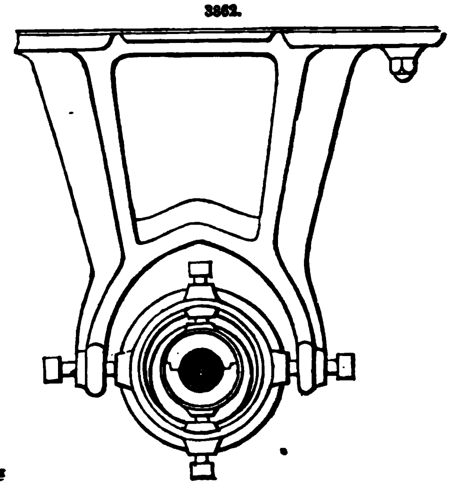
thus, referring to Fig. 3858, hold the nut A by the stalk and pass it under the bridge of the valve-seat B; screw the pillar C into the nut A, then place the box handle D, which carries the cutter E, which is bored out to receive the spring F, on to C. Screw the nut G, Fig. 3859, to supply the feed to E, then the tool is in perfect order for facing the top of the valve-seat; when that is completed, change the cutter E, and insert another cutter E', as shown in Figs. 3858 to 3860, to give the proper mitre for the valve to rest upon. This tool is valuable in railway establishments, as safety-valve seats frequently require facing.

Fig. 3861 is of a hand-power bench drilling machine, by Muir and Co., of Manchester, capable of drilling holes ⅞ in. diameter, 3 in. deep, in wrought or cast-iron and steel, up to 1 ft. diameter by hand-power, consisting of independent framing, securely bolted to the foundation, which serves as a table for articles to rest upon while under operation. This hand-tool has a cast-steel spindle, with parallel bearings, and feed-motion attached; it is worked by a square-thread screw and hand-wheel at the top of the machine. It is furnished with a large fly-wheel, adjustable handle, and wood ferrule. It has a driving shaft and bevel-gearing for the spindle, and chucks for holding the drills and nut-keys. See Agricultural Implements. Alloys. Anvil. Artesian Well. Auger, p. 203. Awl. Barrow. Batea. Bellows. Bench. Blast Furnace. Boring and Blasting. Construction, p. 1034. Crowbar. Electro-Metallurgy. Forge. Grindstone. Machine Tools.

3861.



3862.



HANGER. Fr., *Palier pendant*; Ger., *Hängelager*; Ital., *Sostegno sospeso*; Sp., *Soporte suspendido*.

When hangers with long bearings were first introduced, the attempt was made to use long boxes for line and countershaft journals, as well as for all others, but the warping and shrinkage of the girders and ceiling joints to which hangers are almost invariably secured soon threw the boxes out of line, and had a tendency to bind the shafting, and left no alternative but to return to the short box, unless some device could be found whereby the box would be free to adjust itself to the shaft regardless of the position of the body of the hanger. Among a variety of contrivances designed to accomplish the above object an application of the universal-joint arrangement was considered the best, as it is one of the cheapest, of all that possess the necessary requirements.

The manner of applying the principle to line-shaft hangers is shown in Fig. 3862. The box, being secured within the ring and between the two arms of the hanger by the four set screws, is free to turn in any direction, and the clearance between the different parts admits of its being moved sideways, or up and down, so as to bring the shaft into exact line when the bodies of the different hangers are nearly in the required position. A section of box, ring, and drip-cap is shown at Fig. 3863, which may be used as shown in Fig. 3862, or in a bracket of the form shown at Fig. 3864, or in a plummer-block, Fig. 3865.

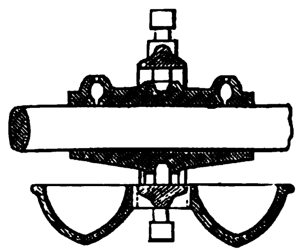
At Figs. 3866, 3867, a countershaft hanger is shown in which the same self-adjusting device is employed, but as only two set screws instead of four are used, the boxes are not adjustable. When these hangers are in use it has been the practice to make the journals from 3½ to 4 diameters in length, to chamber out the boxes, and line them with anti-friction metal.

At Figs. 3868, 3869, a self-lubricating journal-box, applicable to the same hanger, is shown, in which it is proposed to return the oil from the drip-cup to the shaft by means of the two loose rings, A A.

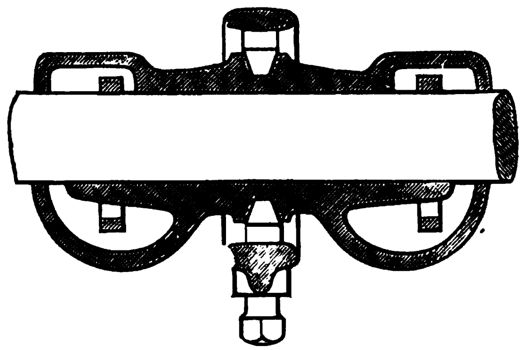
Figs. 3870, 3871, show a line-shaft coupling, which, with the exception that the cones are liable to stick fast, may well be pronounced faultless. This method was first introduced by

C. F. T. Young. A device, whereby it is proposed to overcome this one difficulty, is shown in Fig. 3870, and consists simply in having a tapped hole through the shell of the coupling, into which a conical-pointed set screw may be inserted for forcing the cones out of their seat.

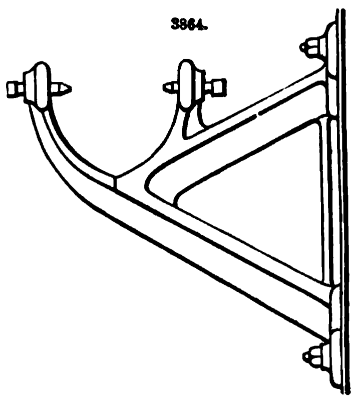
3863.



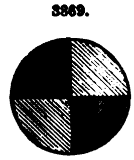
3868.



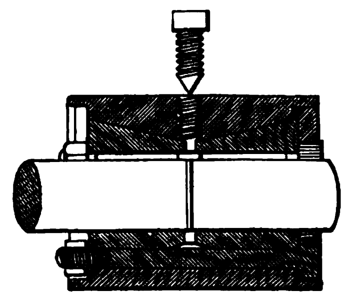
3864.



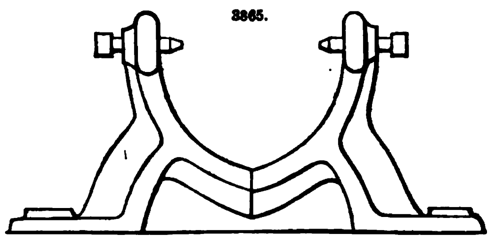
3869.



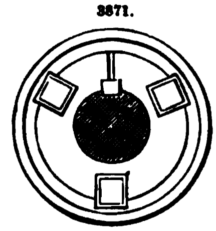
3870.



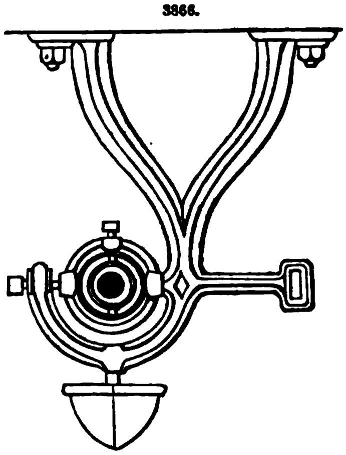
3865.



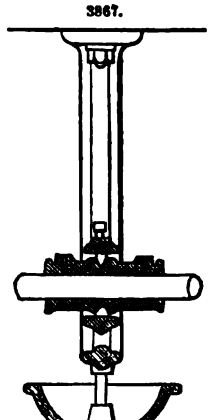
3871.



3866.



3867.



3872.

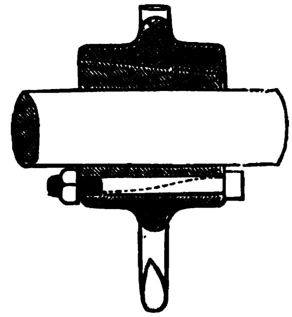
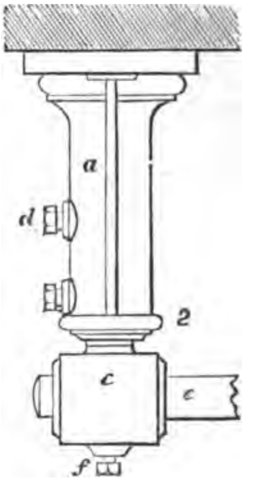


Fig. 3872 is of a method of securing pulleys to shafting by means of a split cone and one or more bolts. The advantages of this arrangement are, that the pulleys can be set at any point in

…or set-screw marks, that they will run true when …ed shafts by simply changing the cones.

… the original inventor of the system of hangers …ployed by Shanks in 1848 is shown in Figs. 3873, … the bearing for the shaft; *c*, the bearing; *d d*, the

3874.



… be readily adjusted; *e*, end of the shaft; and *f*, … of the shaft and bearings is adjusted. By this … are attained, and the shaft thereby easily set and … trouble.

…L., *Porto;* SPAN., *Puerto.*

… following account of this harbour is taken from the … which we allude is No. 1082, entitled "Structures in …

… ring objects D. Miller had in view were to treat of … tions of quay-walls, piers, or breakwaters, for the … to describe works of this kind carried out on prin… nd to point out the further application of these …

… quired the adoption of very expensive means; and … structures, so as to combine the various conditions … resisting the mechanical forces to which they are … sure to the elements, the boring of marine worms, or … rto a desideratum.

… ater have been practised; but there are objections … expensive nature or from defects in the durability … employed are;—

… to about the level of low water;

… or coffer-dams; and

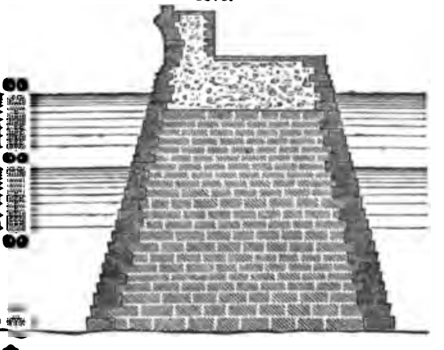
… s of diving apparatus.

… in deficiency of strength and of durability, as the … essure from behind the wall, and the heads of the … and water soon decay, when the weight of the wall … le structure. Where there are marine worms this … ilure of such works will occur to many. Referring … older quay-walls of the harbour of Glasgow were … om the decay of the piles, or have required very … t Glasgow Docks the walls, which were also built … essure forcing them out shortly after completion,

… great solidity are required, is to resort to coffer-dams … pace, so that the foundations and the walls may be … ost effectual, but it is at the same time generally … anger. In many cases, too, from the nature of the … ive coffer-dam. The construction of coffer-dams, … skill, besides involving great expense; and when … temporary purpose, it must be conceded that it is … ode by which their use can be obviated, and solid … r aid.

...*ng bells* and *diving dresses* has been practised
...now used gives great facilities for this kind
...mstances, and it is also costly, besides being
...Dover Breakwater, Fig. 3875, is an example
...so an instance of the vast sums which may

3875.



...te with stone structures built in the usual
...ng the piers of the Westminster and the
..., and these works are examples of success
...ing scientific principles to the practice of

..., béton or hydraulic concrete has played an
...he plans for marine works.
...l times, was allowed to go out of use, and
...d as a constructive material by the engineers
...d their faith chiefly in works of cyclopean
...der water by the *diving bell*. The French
...r period in modern times, alive to the value

...appreciated in England as a substitute for
... works. Still, its use is chiefly confined to
... for foundations, and not properly speaking
... more especially the author's object to direct
... the application of liquid concrete deposited
...submarine works.
...applied for constructive purposes,—building
...ccess to the work, as has been adopted in the
...hose of the London Docks; preparing it first
...s employed by Walker at the Dover Break-
...tecting the arsenals of Plymouth and Ports-
...ng it to set under water, as practised upon a
...large Government Graving Docks at Toulon.
... in a liquid state in the sea water, at a depth
...ton about 100 ft. wide, of the length of each
...ft. thick. After the béton had set, the water
...asonry to form the altars, stairs, and floor was
...de of the trough at the entrance having been

...crete, which has the invaluable property of
... rock or stone, are very great; as it may be
...artificially hydraulic limes or cement, or from
... the admixture of other substances, such as

...es, which are found extensively in this and
...blue lias in England, the limestone of Arden,
... good specimens of their kind, possessing in
...r. Of the Arden lime, the author and his
...rks at Glasgow, Greenock, and other places.
...ocks at Liverpool, London, and other impor-
...d properties. The hydraulic limes of France
...tion for the formation of the large concrete
...he French breakwaters. It may be useful to
...he concretes made from these various limes.
...ler for the foundations of the large Graving
...rden lime, 1 part of iron-mine dust, 1 part of
...lias concrete, used at the recent extension of
...blue lias lime to 6 parts of gravel and sand.

The proportions adopted for the blocks of the Mole at Marseilles, were 2 parts of broken stones to 1 of mortar, the latter being composed of 3 parts of Theil lime to 5 of sand.

A knowledge of the mode of composing artificial hydraulic limes is of great importance in situations where natural hydraulic limes are not easily procurable. Smeaton was the first to point out that it was to the clay or silicate of alumina in the composition of the hydraulic limes that they were indebted for their peculiar property. Subsequently the able researches of Vicat showed by actual experiment how all the rich or non-hydraulic limes might be rendered eminently hydraulic by burning them with a certain proportion of silicious clay. Indeed, to Vicat is due the credit of having reduced the knowledge of limes to a system, and of having shown the practical application of concrete as an eminently constructive material. The excellent artificial *cements* now manufactured are most valuable to the engineer; and the concrete made with Portland cement can hardly be surpassed. That used at the new Westminster Bridge is harder and more compact than the greater number of building stones, even where put down in the bed of the Thames, and where it is exposed to the running stream. Portland cement concrete is also extensively used for the artificial blocks, weighing from 6 tons to 10 tons each, which form the hearting of the breakwater at Dover and that at Alderney, the proportions being 1 part of cement to 10 parts of shingle.

Some substances, such as pozzuolana—a volcanic production found chiefly in Italy—have, in consequence apparently of silicate of alumina being predominant in their composition, the property of giving hydraulic qualities to the rich or non-hydraulic limes. It is of these that the concrete is made, which has long been used for marine works on the shores of the Mediterranean; and, indeed, the piers at some of the Italian ports have been constructed almost entirely of hydraulic concrete. Daniel Miller, the author of this paper, had an opportunity of examining at Genoa the extension of one of the moles of the harbour, the inner side of which has a vertical wall constructed under water entirely of pozzuolana concrete simply thrown into the sea from baskets carried on men's heads, a boarding confining it to the shape of the wall. In a short period it set quite hard, so as to enable the upper part of the wall, which is of stone, to be built upon it. The outer side of the mole, which had been previously made, was formed by stones deposited "à pierre perdue." Though the depth of the quay wall was not great, this shows the confidence which the Italian engineers have in concrete applied under water in a soft state. The piers of the new basin constructed by the Austrian Government at Pola, in Istria, are also formed, in a similar manner, of concrete confined between rows of timber piling.

Perhaps the most striking application on a large scale of pozzuolana concrete is in the great mole which protects the port of Algiers. To form the mole, blocks of béton of immense size, so as to be immovable by the force of the sea, were employed. Some of these were formed *in situ* by pouring the concrete into large timber cases without bottoms sunk in the sea in the line of the mole. Other blocks of a smaller size, though upwards of 30 tons in weight, were made on shore, being moulded in strong wooden boxes. After the béton had set, the boxes were removed, and the blocks were launched into the sea to find their own level. The béton for the blocks *in situ* was composed of 1 part of rich lime in paste, 2 parts of pozzuolana, and 4 parts of broken stone; that for the blocks made on shore was formed of 1 part of lime in paste, 1 part of pozzuolana, 1 part of sand, and 3 parts of broken stone. These blocks set sufficiently hard in twenty-four hours to resist the shocks of heavy seas, and the mole now stands firmly, instead of being, as it was when formed of loose blocks of stone in the time of the Moors, nearly destroyed every winter.

The French engineers have shown great boldness and skill in the application of béton, as exemplified in the Pont de l'Alma over the Seine, the arches of which, as well as the piers, are formed of rubble concrete; in the new Graving Dock at Toulon; and in the formation or protection of breakwaters by enormous artificial blocks of béton, as carried out at Marseilles, Cherbourg, La Ciotat, Cette, Vendres, Cassis, and Algiers. When Miller inspected the mole, or breakwater, which encloses the harbour of Marseilles, he found the huge rectangular concrete blocks, weighing upwards of 20 tons each, by which its seaward side is protected on the "pierre perdue" principle, perfectly entire and sharp in their outline, though they had been exposed for many years to the action of the sea. Anyone standing upon that mole, and witnessing in a gale the heavy seas breaking with tremendous force on these concrete masses and recoiling harmlessly, could have no doubt as to the efficiency of concrete as a constructive material.

Hydraulic concrete, to be effective, requires care and attention in its manipulation, and in the regulation of the proper proportions of its materials. Any failures must have arisen from inattention to these or similar points, as there is ample experience to show that, when properly made, every confidence may be placed in its strength and durability. Even where stone is abundant, this material may be often employed with economy and advantage; but where stone cannot be obtained, the importance of being able to form an effective substitute, out of materials of so little value, and so widely distributed, can hardly be overrated.

Construction of Dock and Quay Walls without Coffer-dams.—In sea water the engineer has to encounter enemies which do not exist in fresh water, or at least only to a trifling extent. The "teredo navalis" and other worms quickly destroy timber, and the corrosive action of the sea water, and other peculiar properties, have a prejudicial effect upon iron. In consequence of these deteriorating influences, these materials have not hitherto been much employed in sea works where durability is essential. There is no doubt, however, that they may be employed with advantage, if protected from the destructive action alluded to; and whatever materials may be used, it is desirable that the surfaces exposed to the sea should be of continuous stonework or other material capable of resisting its effects.

As Engineers-in-chief for the Albert Harbour at Greenock, D. Miller and his partner Bell have had an opportunity of introducing a new system for the construction of sea-walls and quays in deep water, without the aid of coffer-dams, by which a large saving is effected, and works of great solidity and durability have been secured.

The accommodation for the loading and discharging of the shipping of Greenock consists of three open tidal docks or harbours, the most recent having been constructed by Locke. Extensive schemes for wet docks have been proposed at different times by several engineers, particularly by Rennie, Telford, and Walker and Burges, but hitherto no wet docks have been constructed, as it has been considered that the moderate range of the tide—from 8 ft. to 10 ft.—does not render them indispensable, and the trade is found to be efficiently worked by the present system. In the additional accommodation the system of harbours is adhered to, though provision is left for conversion into wet docks by the addition of locks and gates, should this at some future period be deemed advisable.

The new works are situated on the west side of the town, and, in order not to interfere with valuable shore ground, they have been projected almost entirely beyond the high-water line into the sea. The depth of water at the outer line being considerable, the amount of excavation required in the interior is comparatively little. The outer sea pier, according to the plan proposed by the engineers, encloses a large extent of shore as well as the Bay of Quick, and when carried out to the full extent will be upwards of 3000 ft. in length. Within this area there is a space for two harbours, each 1000 ft. in length, 15 ft. deep at low water, or 25 ft. at high water, with entrances 100 ft. wide.

The depth of water along the line on which a coffer-dam must have been constructed, had such been contemplated, is in many places nearly 30 ft. at high water; and taking into account the length, and that it must have been of strength sufficient to resist the storms of winter, it could hardly have cost less than 50,000*l*. Besides the great cost of a coffer-dam, there was another difficulty, as, owing to the line of the proposed new pier being close to the edge of the deep-water channel, it would have been necessary to project the coffer-dam so far into the channel as to have formed a serious interruption to the traffic. In consequence of these difficulties, and from considerations of economy, it had been the intention of the trustees to use timber for the outer piers of the harbour, and the engineers were instructed to make their plans accordingly. It was the opinion, however, of Miller and his partner, that in a situation where the sea worm is very destructive, the work ought not to be constructed of such a perishable material, and that it was quite possible to build a solid structure, so as to avoid the difficulties referred to, in an economical manner. In order to effect this, they proposed to construct the outer pier and quays forming the seaward side of the dock without coffer-dams, so that the pier might itself serve as a coffer-dam for the interior operations in the harbour which would afterwards be required. The seaward pier is 60 ft. wide at the top, having quays on both sides.

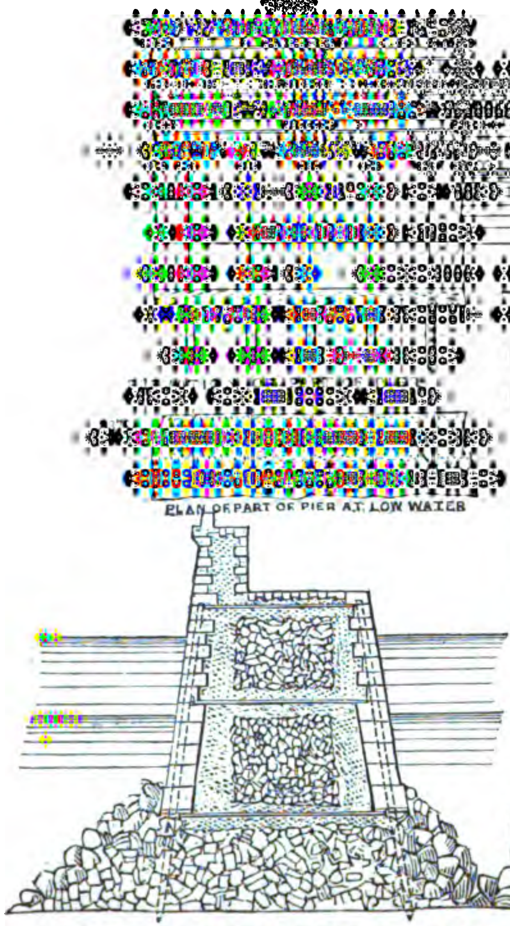
The mode in which the work was designed was to form the walls under low water of a combination of cast-iron guide-piles in the front, with a continuous stone facing slid down over and enclosing these, and of concrete backing deposited in a soft state, all of which could be easily accomplished from above the water line. Timber bearing-piles were to be used in the body of the walls where required, and the upper part of the walls from the low-water line was to be carried up of masonry in the usual manner, Figs. 3876 to 3881.

Granite from the Ross of Mull was substituted for freestone, for the stone facing under the low-water line, as it could be obtained in large blocks at a moderate price.

The first operation in the construction, when the water is not sufficiently deep, is to dredge out two parallel trenches to 17 ft. below low water, for the foundations of the walls. A staging of timber piles is afterwards erected in the line of the pier over the whole breadth, for carrying the tramways, travelling cranes, and piling engines. Cast-iron guide-piles are then driven from the staging, with great precision, 7 ft. apart, in the line of the face of each quay-wall. These piles are driven till their heads are near the low-water line, and they form guides for putting down the stone facing. They are connected at the top transversely by wrought-iron tie-rods stretching through the pier, cottered into sockets and binding the heads together. At first it was thought that there would be some difficulty in driving the iron guide-piles with the required exactitude, but this was overcome by pile engines of peculiar construction, devised by William York, one of the contractors, Figs. 3882, 3883. These travel on the rails of the scaffolding, and are furnished with long arms projecting downwards, strongly stayed by diagonals, and forming a trough, into which the pile is placed, and from which it is driven by the pile engine in the manner of an arrow from a cross-bow, being obliged to go down perfectly straight.

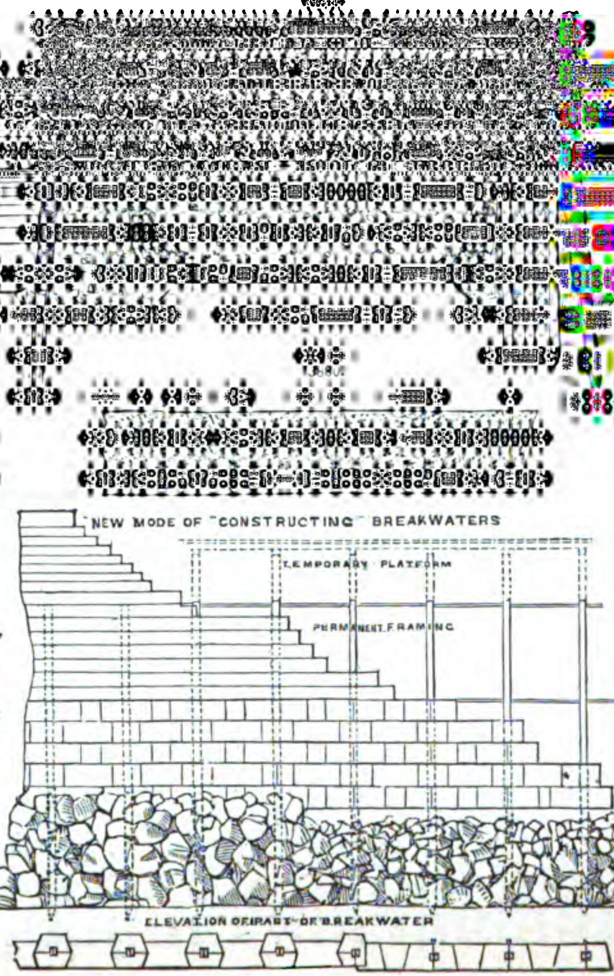
The ground is very unequal, the hard substratum, or red till, being in some places 20 ft. below the bottom of the wall, the upper strata being mud and soft sand. In such cases timber piling, driven to the same level as the iron piles, is used to form a platform for sustaining the part of the wall above low water; but where the ground is firm this is not required. When the proper depth has been dredged out and the piling driven, a bed of hydraulic concrete 3 ft. thick and 20 ft. wide is deposited in the trenches, to form a base for the wall to spring from, and to give a large bearing surface. Into the grooves formed by the flanges of the iron piles large granite slabs, from 18 in. to 2 ft. thick, are slipped, the bottom one resting upon a concrete base and on a projecting web cast on the piles: not more than three stones fill each compartment between the piles, 16 ft. in height and 7 ft. in width. These stones slip into their places with the greatest ease, and form the face of the wall under water. Behind this facing hydraulic concrete is lowered under the water in large boxes having movable bottoms, and is discharged in mass to form the body of the wall. To confine this at the back before it has set, loose rubble stones are deposited and carried up simultaneously with it. The hearting of the pier, consisting of hard till, stones, and gravel, is deposited afterwards, and the whole carried up to the level of low water.

The entire mass, piles and stone facing, concrete backing and hearting, is allowed to consolidate for a sufficient time; after which the heads of the iron piles and the granite facing blocks are capped at the level of low water by a granite blocking or string course, and the upper portion of the walls is carried up in freestone, ashlar, and rubble. The remainder of the hearting between the walls is then filled in, and the whole is finished with a granite coping and causeway. The

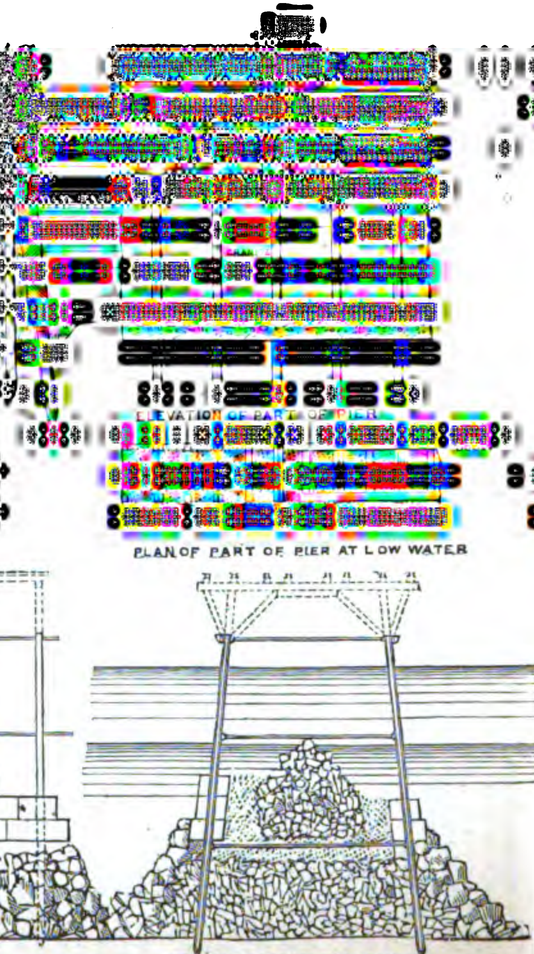


SECTION OF BREAKWATER WITH PARAPET

3884.



3885.

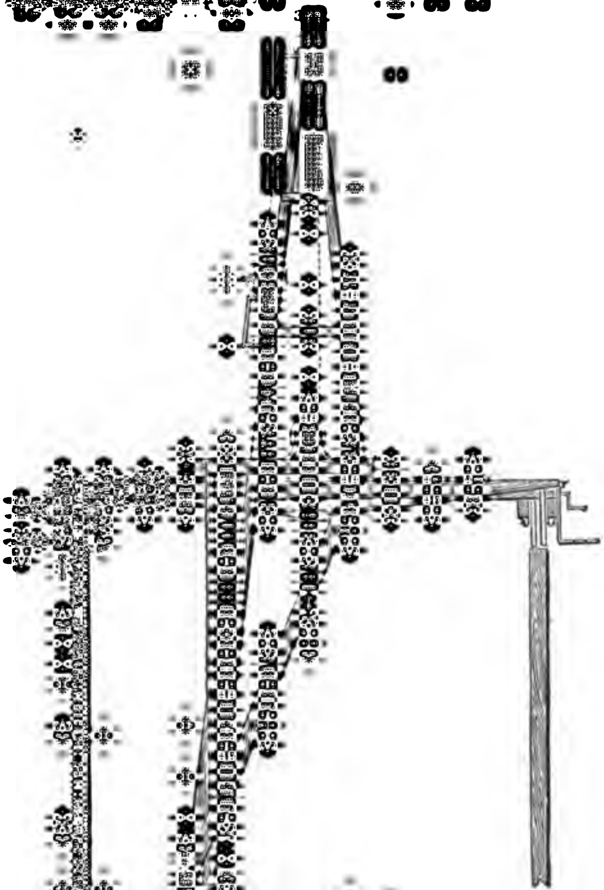


SECTION OF PERMANENT FRAMING AND TEMPORARY PLATFORM

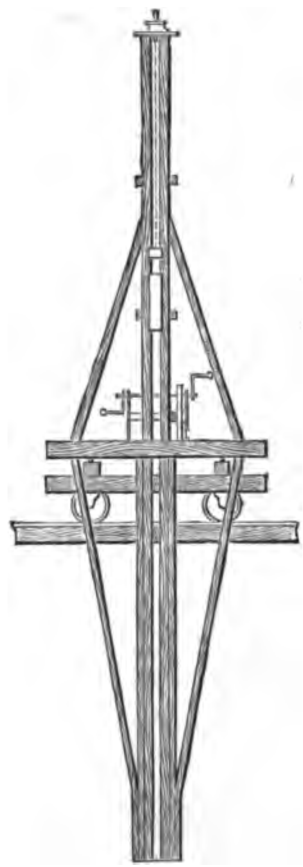
3886.

. thick at the concrete base, and they diminish

aulic lime. It is burnt at the quarries, but is
covered wagons, so as to preserve it from wet.
o, and for mixing the mortar, there have been
es, driven by an engine of 20 horse-power.
executed, the stone facing under low water
anges of the iron piles, the outer flange is left
t will in the course of time exert an injurious
dy this, it is intended in the remainder of the
in the stone facing, into which the outer iron
oks will therefore overlap the iron piles, and
e iron will be exposed to the action of the salt
th cement, which will enclose the iron flange,



3883.



draulic lime, iron-mine dust, sand, gravel, and
ll ground, under edge stone mills, before being
are by measure—1 part of ground lime, half a
of gravel and stone chips. Immediately after
ency with water, it is conveyed to where it is to
ing boxes, and in a short time sets very hard.
d contain 1 cub. yd. each. Those of iron are
den ones renders them somewhat unmanageable
rged.
without coffer-dams has proved very successful,
been formed at a comparatively moderate cost.
rinciples, different modes of forming the stone
stone of a softer nature than granite, such as
t outer casing may be obtained, Fig. 3880. In
hem, are strung or put down over the iron piles,
ve also grooves or projections on their sides, in

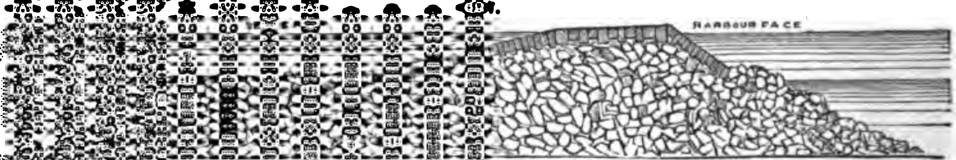
[illegible] stone, having corresponding grooves or projec- [illegible] he stones locked into each other, is thus formed, [illegible] on this principle that the facing under low water [illegible] revious to its having been suggested to employ [illegible] ect the iron from the action of the salt water, the [illegible] e with cement well rammed down, so as to fill up [illegible] ether. This plan admits of the iron piles being [illegible] nect the piles. Concrete backing may then be [illegible] viously described. Temporary sheet piling or [illegible] to keep the concrete in its place until it has set. [illegible] sily moulded into the shapes required, may be [illegible], as it has been proved by experience that, when [illegible] and durability.

[illegible] system of founding marine structures is appli- [illegible] ation, re-facing, or reconstruction of quay-walls; [illegible] vered by the sea; embankment walls along the [illegible] uses, beacons, or forts which may require to be [illegible] s enclosing harbours of refuge; may be effected [illegible] tainable.

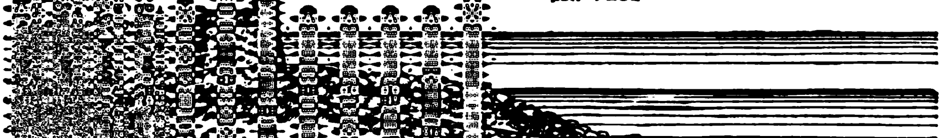
[illegible] arliamentary Committees have elicited much [illegible] ject of diminishing the enormous cost of these [illegible] l, and at the same time economical, barrier to [illegible] portant, indeed, has this become, that in 1860 a [illegible] inted to inquire, how far it might be practicable [illegible] aters and harbours of refuge, less costly than the [illegible] lans were discussed, and amongst them floating [illegible] lish any effectual substitute for the present mode

[illegible] opinion, have a material effect in filling up this [illegible] cessary to refer briefly to the principal modes of [illegible] e peculiar phenomena by which such structures

[illegible] ers is the "pierre perdue," or long-slope system. [illegible] mount of loose rubble stone, rising to about the [illegible] level, and to be acted upon by the sea until its [illegible] tion gives it. The seaward side obeys the laws [illegible] g sloping shore, involving the employment of an [illegible] aches the height to give the required protection. [illegible] undant, and can consequently be deposited at a [illegible] urg, and Holyhead breakwaters may be taken as



SEA FACE



[illegible] opposite principle, called the vertical system, is [illegible] from the bottom; and as all the material below [illegible] d is of an expensive nature, the cost of a work [illegible] eakwater, in course of construction, is the most [illegible] bottom of the sea, the exterior facing being of [illegible] a other, and the hearting of rectangular blocks [illegible] y up to the level of high water, above which it

[illegible] the extremes, an intermediate form of section, [illegible] It consists in carrying up a rubble mound to [illegible] this building the remainder of a vertical con- [illegible] as representing this system to a partial extent,

It has been a subject of discussion as to whether the long slope or the vertical wall was the better section for breakwaters, and as to the relative force of the sea exerted upon them. The observations which have been made on waves may be said to have settled this point in favour of vertical walls, as it has been clearly shown that waves in deep water are chiefly oscillatory in their character, the fluid having little progressive motion in itself, and consequently exerting but little force on objects opposed to it; but in shallow water waves assume an entirely different character, as they acquire a progressive motion, becoming waves of translation, in which the fluid is carried bodily forward in a horizontal direction, and in consequence it strikes any body opposed to it with great percussive force. Vertical walls, therefore, which rise from the deep water, being only subject to the oscillatory movement of the waves, are least exposed to the destructive effects of storms. The evidence taken before the Royal Commission in 1859 seemed to be conclusive on this point, and the opinions of the Commissioners, as developed in their report, may be considered to have set this subject at rest. But whatever difference of opinion there may still be upon this matter, there can be no question as to the vast saving of material by vertical walls, and of the great economy which would result, provided a simple and easy mode of construction could be adopted. The vertical system has, besides, the great advantage of being applicable in many cases as quays for vessels lying alongside to load and discharge, which may be turned to valuable account both for commercial purposes, and in times of war, for the rapid shipment or debarkation of troops, stores, and other material.

The experience, however, derived from the formation of the great breakwaters on the "pierre perdue" or long-slope principle, such as Plymouth, has been very valuable. The examination of the sections which the materials assume, shows that the great disturbing action of the sea, or conversion of the waves of oscillation into those of translation, does not extend to any considerable depth; as it is found that the long sloping beach terminates generally at from 12 ft. to 15 ft. below low water, after which the inclination becomes much steeper, the materials assuming nearly the form due merely to the natural angle of repose, as if unacted upon by any force except that of gravity. The inclination on the seaward side within the tidal range, and to the depth of 12 ft. or 15 ft. below low water, is generally 5 or 6 horizontal to 1 vertical, but below that depth it is only from 1 to 1½ horizontal to 1 vertical. It is the long slope which these breakwaters assume to a certain depth, that causes the enormous absorption of material; but it appears that a mound of rubble may be deposited to within a certain distance of low water which will not have this long slope, and consequently will only require a comparatively small quantity of material. The consideration of these facts shows that in the generality of cases, the vertical and "pierre perdue" systems may be combined with advantage and economy, by first depositing a rubble mound to about 15 ft. below low water, and from that point carrying up the remainder of the breakwater by vertical walls.

A great improvement in the facility of constructing these breakwaters, when such an immense quantity of material has to be deposited, was the introduction by Rendel of timber staging carried on piles in advance of the work, and sustaining lines of rails, by which the material can be brought down and be deposited in the sea with a rapidity before unattainable. The consumption of timber is, no doubt, very great, as much has to be left imbedded in the work, and there is considerable destruction besides; but this is amply compensated by other advantages. By this system an average of about a million tons of stone a year have been deposited at Holyhead, and a similar plan is pursued at Portland, Fig. 3893.

Massive staging is also employed at the vertical breakwater at Dover, for facilitating the building operations. Indeed, staging may now be considered essential in the generality of cases for the economical construction of such works.

The breakwaters of the French engineers are generally formed "à pierre perdue," but upon a different method from that pursued in this country. Thus, at the Plymouth Breakwater, Fig. 3887, only large blocks of rubble stone were deposited, the small being thrown aside, and at Holyhead and Portland, the large and the small rubble were deposited promiscuously; while the French engineers usually employ the small rubble for the core, and reserve the larger blocks for the outer coating. Furthermore, they protect the seaward side by blocks of béton, thrown in to take their own position, and of such a size (generally from 20 tons to 30 tons) as effectually to resist displacement by the utmost force of the waves. These blocks assume a slope as steep as 1 to 1 under the water line, so that the mass of material in a breakwater thus constructed, is considerably less than where smaller materials are employed for the seaward face. The moles of La Joliette and Napoleon which enclose the harbour of Marseilles, are excellent examples of this mode of construction, Fig. 3889.

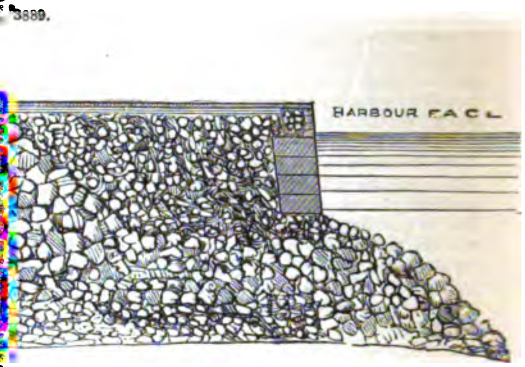
Having thus glanced at the general principles which affect breakwaters, and described the modes of construction usually adopted, the conclusion to be arrived at appears to be, that the vertical system is that which best resists, or rather averts, the destructive action of the sea, and requires the smallest amount of material. However, both systems, the long slope and the vertical, as at present carried out, are very expensive, the former from the quantity of material which is required, the latter from the costliness of the material and the mode of construction. The one system may be characterized as involving the maximum in quantity, and the minimum in cost of material: the other, on the contrary, the minimum in quantity and the maximum in cost of material. The object sought to be attained by the system about to be described is to effect a minimum, as far as possible, both in the quantity and in the cost of the material.

According to circumstances, breakwaters on this system would be constructed either wholly vertical, extending from the bottom, or partially vertical, springing from a rubble mound. For the sake of comparison, the mode proposed by Daniel Miller, Figs. 3884 to 3886, is designed to suit the conditions usually prevailing; say a range of tide of 15 ft., and a depth at low water of 6 fathoms, being about the same as at the Plymouth Breakwater, and as at Hartlepool, Filey Bay, and the entrance of the Tyne, where the most important harbours of refuge have been recommended by the Royal Commissioners. The section, Fig. 3891, represents a breakwater with a parapet, but

BOUR.

breakwater, and is only required in certain cases, ial purposes. Where the parapet can be dispensed

3889.



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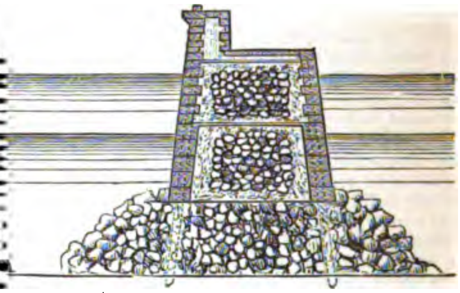

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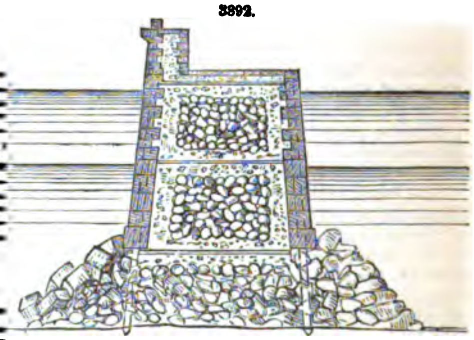

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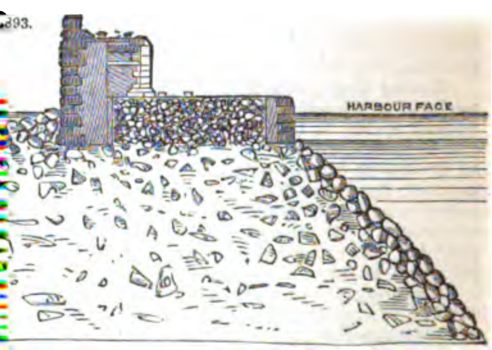
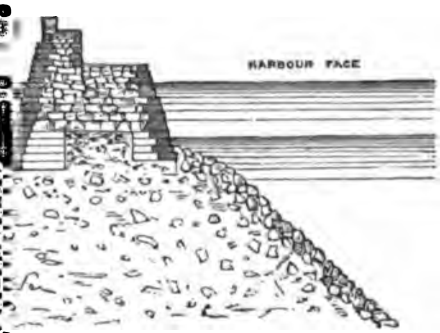


of wrought iron, and in the generality of cases it
und, but simply to set them in place.
staging in advance of the work, which may be done

tandards, from a machine placed on the plat-
hine will have long guides firmly stayed and
will be placed, and then driven in a similar
iles are erected in place, they are connected
form at the top for bearing the rails, and the
, and so on. Following this operation come
le mound, which collecting round the lower
and give stability to the staging. When the
elow low water, the cranes from the staging
ework. These are made to enclose the iron
ked into each other, and thus to resist any
de to break bond, or to slide down without
the former plan permits the standards to be
e of less dimensions, and at the same time of



, the hearting of the work, rough rubble or
wagons on the staging, filling in from the
e, will be lowered down in large boxes, and
d cementing together the rubble hearting, as

is rendered continuous, and by all the blocks
sible that any individual block can get out of
ctures of this kind built in the ordinary way,
joints of the masonry, by the pressure of the
tire, is sometimes so great as to blow out the

y be obtained, as the whole structure is bound
manner in which the stones of the facing are
will penetrate and solidify in a short time the
a, which should be the object of attainment in
d of the ocean.
y be either of stone or of béton. When the
may be used with advantage, particularly as it
, and almost of any size. The power of such
te period is now fully confirmed. These may
breakwaters. Indeed concrete blocks, built in
er and Burges at Alderney, Fig. 3894, for the

g breakwaters would arise from the smallness
erial. The quantity of material in this break-
water on the long-slope principle, in the same
rity of cost is not less striking, the Plymouth

on, the iron framework may be allowed to go
for the speedy and economical construction of
ndards will not cost more than timber staging,
se quantity of staging requires to be used for
-slope breakwaters, such as at Holyhead; while
erent strength, the heavy nature of the mate-
f the waves. The buoyancy of timber staging
as is exemplified at Holyhead, where it is
is required to make up for the loss.
d with which it may be constructed, from the
ted, and from the facility with which operations
t one time. In situations where the materials
t be obtained, the advantages of this system
n as an example, there being no stone in the
rdue," or building vertically in the usual modes

of construction. By the system proposed the harder chalk from the cliffs and shingle could be used for the hearting, as in a structure so firmly bound together these materials concreted with béton would serve the purpose quite as well as any other. In forming the rubble mound the example of the French engineers might be followed with advantage by forming the core of smaller and inferior materials, and for this the chalk and shingle would be quite suitable. This would be protected by a thick layer of rubble, and on the seaward side by a layer of concrete blocks, of such a size as would not be disturbed by the sea. The vertical superstructure would be constructed of chalk or sandstone rubble, concreted by béton for the hearting. A breakwater upon this construction, Fig. 3892, Miller estimates could be built at Dover for 290*l.* per lineal yard. The present breakwater for the same depth of 45 ft. at low water is contracted for at 1245*l.* per lineal yard, so that there would be the enormous saving of upwards of one million and a half sterling per mile. The difference in the cost of construction, vast as it is by this system, is not the whole saving, as the time occupied is an important element, affecting the final cost of such a work, the interest on the outlay being lost until the harbour becomes available. There can be no doubt of the solidity and durability of the Dover Breakwater, but considering its enormous cost, and the distance into the future before its completion will render it available for commercial or for war purposes, the wisdom of prosecuting it upon the present mode of construction may be well called in question. Upon the construction proposed the breakwater could be completed and be available as a harbour of refuge for the naval and commercial fleets of the country in less than five years, at a cost of little over 1,000,000*l.*

Breakwaters and piers have been frequently made of timber framing and casing, confining a mass of rubble. Extensive piers on this principle are in existence in Boulogne, Calais, Dunkirk, and other ports; but it is evident that such a system, from the timber being exposed and the consequent want of durability, and from their liability to sudden destruction when once the casing gives way, must prove very expensive in the end. This system has been revived, though upon more scientific principles, by Abernethy and Michael Scott. In these plans a structure composed of a casing of timber is formed of timber frames, standards, or piles and planking, and this casing is afterwards filled with rubble. But as the casing cannot be expected to possess much durability, it is proposed subsequently to enclose this structure by solid walls of masonry or composite blocks, for which the first structure will afford a convenient and substantial platform for bearing the rails and cranes necessary for executing this part of the work. There are two distinct operations necessary, therefore, to complete the work upon this mode in a permanent manner; first, the formation of the inner structure with its timber casing; and, second, the formation of the outer structure, for the purpose of making a casing of a durable character. The economy of making breakwaters of a durable construction on these modes has not been fully made out, chiefly arising from the great quantity of timber required and the necessity of employing two distinct casings, one of which must be superfluous.

The system which Miller has proposed will, we think, secure all the objects which appear to have been aimed at by these plans, but with greater simplicity and economy.

It is not essential that the standards employed in the system proposed by Miller should be of iron, as they may be of timber, but enclosed, as has been already described in the case of iron standards, in a casing of blocks of stone or of béton.

See Barrage. Bridge. Canal. Cement. Coast Defences. Construction. Damming. Dock. Hydraulics. Locks and Lock-gates. Weirs.

HAULAGE. Fr., *Roulage;* Ger., *Förderung;* Ital., *Estrazione e trasporto del litantrace;* Span., *Arrastre.*

Haulage of Coal, taken from the Report of the Committee of N. E. I. M. Engineers, 1869.

Tail-Rope System, North Hetton Colliery, County of Durham.—In order to give an idea of the extent to which the tail-rope system can be applied in leading coals underground along an engine-plane with numerous curves and branches, the following description is given of the arrangement of wagon-way, and the method of working the tail rope, at North Hetton Colliery, which affords the best example of this system.

Fig. 3895 shows that there are two main wagon-roads in this pit, lying at right angles to each other—No. 1 plane being driven east, and No. 2 north. The following are the particulars of the engine and wagon-way;—

Engine.

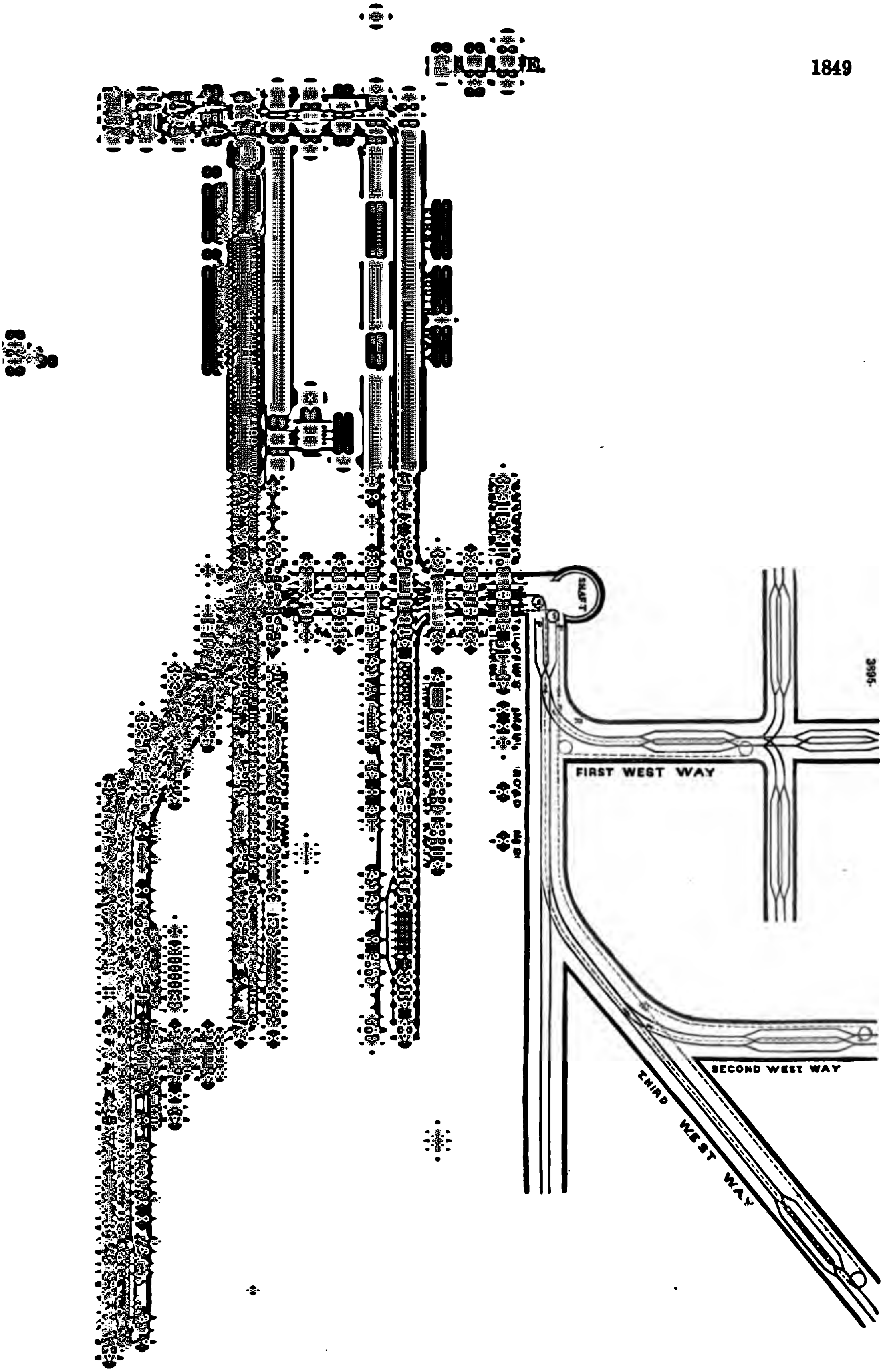
No. of cylinders	2
Diameter of cylinders	12 in.
Length of stroke..	24 „
No. of drums	4
Diameter of drums	4 ft.
Size of rope (circumference)	2½ in.
The boilers are on the surface.	

Engine-plane.

Rails	22 lbs. the yd.	
Gauge of way	2 ft. 4 in.	
	Main.	Tail.
Rollers.—Diameter	5 in.	8½ in.
Weight..	26 lbs.	32 lbs.
Distance apart	21 ft.	21 ft.
Sheaves at curves .. diam.		10½ in.
Tail sheaves	4 ft.	

When the ratio of the diameters of the pinion to the spur-wheel was as 1 to 2, the engine was found rather too weak for its work, and the ratio was therefore made as 1 to 3. The engine goes at a speed varying from 150 to 250 strokes a minute, the usual speed being about 180 strokes a minute. This makes the power exerted to be about 100 horse-power, and thus presents an example, which is rare, of a tail-rope engine working to the utmost of its power.

One end of the shaft of each set of drums is placed on a movable carriage, by means of which they are put into gear with the driving pinion. The drums are connected to the shaft by means of clutch gear. The engine and drums are placed beneath the wagon-way, and the wheels W and W^1 which direct the course of the ropes for No. 2 plane, as well as several other 4-ft. wheels upon


FIRST WEST WAY
SECOND WEST WAY
THIRD WEST WAY

these planes, are also placed under the way. The ropes for the No. 2 plane come to the surface of the wagon-way about the point P.

No. 1 Plane.

	Ways.				
	1st North.	2nd North.	X-Cut.	1st South.	2nd South.
	yds.	yds.	yds.	yds.	yds.
Distance from shaft	900	870	1350	1000	825
Rise or fall from shaft	fall.	fall.	fall.	fall.	fall.
	min.	min.	min.	min.	min.
Time from leaving the shaft to returning ..	10	9	8	10½	9½
Heaviest gradient rising outbye	1 in 10½ for each way.				
Tubs in set	21 for each way.				
Speed of set	About 10 miles an hour.				

It may be observed that none of the branches are of very great length, and that all the ways rise towards the shaft

No. 2 Plane.

	Ways.		
	1st West.	2nd West.	3rd West.
	yds.	yds.	yds.
Distance from shaft	580	1130	1200
Rise or fall from shaft	rise.	rise.	rise.
	min.	min.	min.
Time from leaving the shaft to returning	6	15	17
Heaviest gradient rising outbye	1 in 15	1 in 15	1 in 15
Tubs in set	35	35	35
Speed of set	About 10 miles an hour		

No. 1 plane consists of a main road, with two branches on each side; at the end of the main road is another way, which, after going in a cross-cut direction for a short distance, turns to the north. These five branches are all worked by two of the drums, the other two drums working No. 2 plane and its branches. On the plan (which is drawn to no scale, and is therefore in many places out of proportion, owing to the difficulty in showing clearly the arrangement of rails) the ropes are shown by dotted lines. In the second west way and the cross-cut way there are two stations; a description of the arrangement of which is given hereafter. The four curves leading from the main way to the branches each have a radius of about 22 yds.; the radius of the curve in the first south way is 4 chains, and of that in the cross-cut way about 5 chains.

No. 2 plane has one main road and three branches, two to the west and the other in a cross-cut direction. The curves to the branches are about 3 chains radius, and the curve upon the main road about 4 chains.

At the far end of each of the branches there is a siding, one way for the full and the other for the empty tubs.

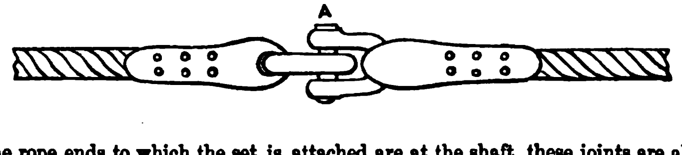
At the inbye end of the first west way there are three putting stations, from which the tubs are led in short sets by ponies to the siding at the end of the engine-plane.

The full way of the shaft siding is raised several feet to form a kep, or incline; and when the set of full tubs has been drawn on to the top of the kep, the tubs are let down to the shaft as they are required.

Arrangement of Ropes.—In the working of this and all other tail-rope planes, two ropes are necessary, which are called main and tail ropes, the former being used for drawing the set of full tubs outbye, and the latter for taking the empty set inbye. When the main rope is bringing the full set outbye, the tail-rope drum runs loosely upon the shaft, and by applying the brake the tail rope is made to run steadily off the drum; when the tail rope is taking the empty set inbye, the main-rope drum is put out of gear, and the main rope is drawn inbye behind the set. It will be seen on the plan of this engine-plane that the ropes for No. 1 plane have a direct lead from the drums, whilst those for the No. 2 plane are taken round pulleys at a right angle not far from the engine.

On No. 1 plane the ropes connected to the engine are those of the cross-cut way, and the set is supposed to have just arrived at the shaft; thus the main rope is nearly all wound upon the drum. At the points A and B, Fig. 3895, there are shackle-joints on both the main and tail ropes. The shackle used is of this description, Fig. 3896, and is secured by the pin A.

3896.

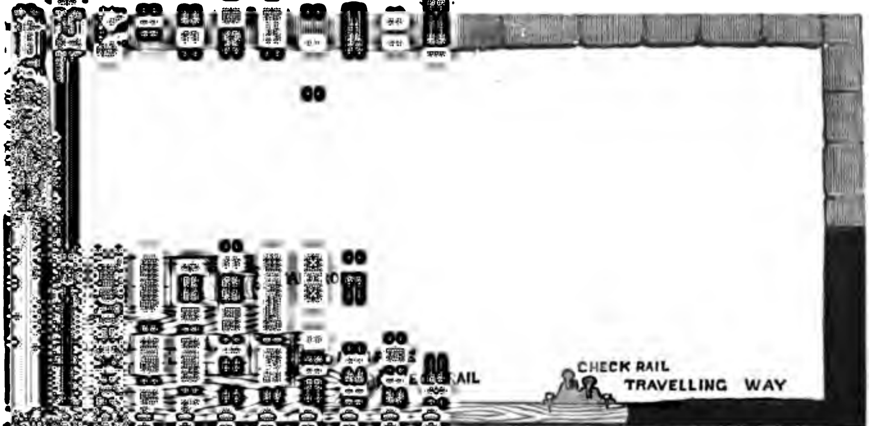


When the rope ends to which the set is attached are at the shaft, these joints are always at the points A and B, no matter from which way the last set came.

Most of the sheaves used in taking the ropes round the curves are fixed horizontally in walling built for the purpose, exhibited in Fig. 3897.

At C and C both the ropes are taken round the curves by small sheaves, as shown in sketch; but

[illegible] curve, the tail rope passing round a 4-ft. sheave;



[illegible]ence. The large sheaves at the curves, and the [illegible]es, are 4 ft. in diameter; these wheels are placed [illegible] where the ropes are shown to cross the wagon-way [illegible]d.

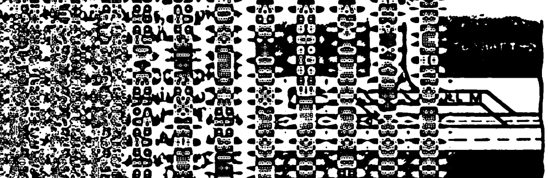
[illegible] 2520 yds., and of tail rope 9636 yds.; and there [illegible] sheaves upon the planes.

[illegible] referring to Fig. 3895, it will be seen that the [illegible] cross-cut way, and that the ends of all the other [illegible]osing that the next empty set has to go into the [illegible]ft are being disconnected from the full set and [illegible] switches at B is disconnecting the shackles S S [illegible]t two minutes, and is generally finished before [illegible] then opens the switches for the second south [illegible] The set of empty tubs is taken into the branch, [illegible]es are altered again. Should the first north way [illegible] switches are put right, and the empty set goes [illegible] be next ready, it will be seen that, to put the [illegible] be connected, two at the station A, and two at B. [illegible] ropes connected to the engine are those of the [illegible] to be at the shaft. All the branches on the [illegible] branches from the main road on No. 2 plane are

[illegible]ted in the same way as on No. 1 plane, and here [illegible]en the third west way has to be worked, if the [illegible]d before it.

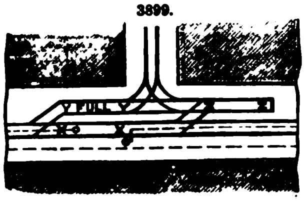
[illegible]ptation of the tail rope which is worthy of notice. [illegible]cause the outcoming full tubs to pull the tail rope [illegible] rope is knocked off at the point R, and the set is [illegible] let down the incline by the single tail rope to R, [illegible] pull the set on to the kep, is attached. The drum [illegible] brake whilst the engine is working another way. [illegible] enough to allow this method to be adopted.

[illegible] the side of the main way, to which sets are [illegible] these stations is in the cross-cut way and the [illegible] is intended for the station in the latter way, [illegible] are knocked off; the full set stands at M M,



[illegible]uth Way.

3899.

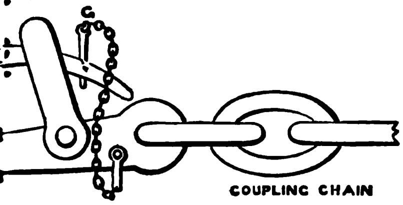


Station on Cross-cut Way.

[illegible] time after the station just described, is much [illegible] made to dip gently inbye, and when the empty

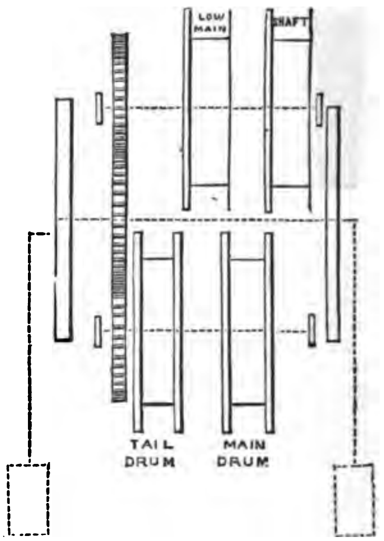
…off at X X, and the set runs forward by itself to … standing at Y Y.

…900.



…ns of … by a … g the … rope, … lane. … from … -The … g, the … of 1 … main … s two … they … ropes … besides … re are … the … and … the … con- … the … quired … e dis-

3901.



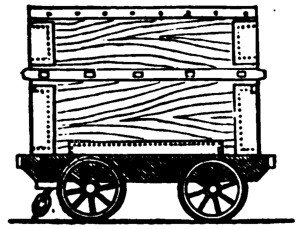
…, Boilers, and Ropes.

		Main.	Tail.
Boilers ..	Area of heating surface ..	each 220 sq. ft.	
	„ fire-grate ..	„ 20 „	
		ft. in.	
Receiver ..	Length	10 0	
	Diameter	3 0	
		yds.	yds.
Ropes ..	Length	2900	5800
		in.	in.
	Circumference	2¾	2¼
		cwts. qrs. lbs.	cwts. qrs. lbs.
	Weight ..	76 3 14	138 0 0
		months.	months.
	General duration	8	12
Sheaves ..	Number	340	340
		lbs.	lbs.
	Weight	26	37
		in.	in.
	Diameter ..	6½	10½
		ft.	ft.
	Distance apart	24	24
		lbs.	
	Return sheave, weight	.. 1428	
		ft.	
	„ „ diameter ..	8	
		lbs.	
Rails ..	Weight a yard	18	
		ft. in.	
	Length of each	12 0	
	Gauge of way	2 0	
	Rails laid on battens (6½ in. by 2½ in.), secured by sleepers spiked to them underneath.		

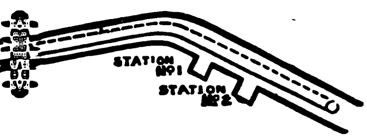
indiscriminately at this pit;—

	Small tub.
	ft. in.
	3 0
	2 7
	2 5
	3 5
	1 6
	0 10½
	lbs.
	514
	·916
	158
260	

3902.



ine-plane are laid on battens. There are two
7 chains radius. The gradient is a general



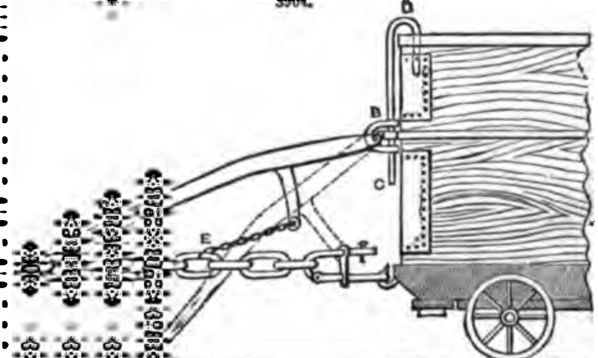
ain way between; the tubs are taken into the

ed 24 ft apart; the tail sheaves are all set

set, generally consisting of 65 tubs, is drawn
last taken, by the main rope, the tail rope
has reached the kep at the bankhead, these
ncline a distance of 450 yds. to the shaft by a
bankhead; the tail-rope drum is then put into
ope.

y an ordinary hook, but at the other end, the
ing outbye, to prevent the set running amain
cow A B, Fig. 3904, is secured to the bar at B

3904.

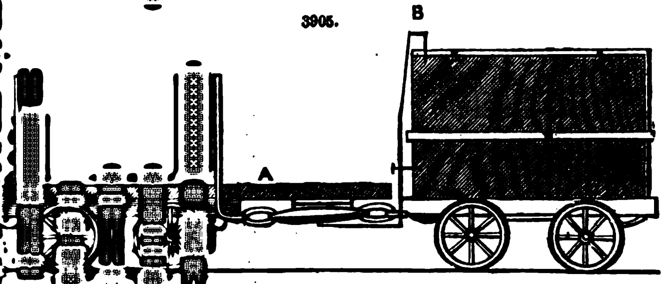


mittee, unsatisfactory; and the comparisons
d, have not yet afforded sufficient data for the
Committee, therefore, feeling that this subject
me member of the Institute will take it up
observations only, and the mode in which they

traction, in cwts., required to overcome the
eadings being taken at the particular points

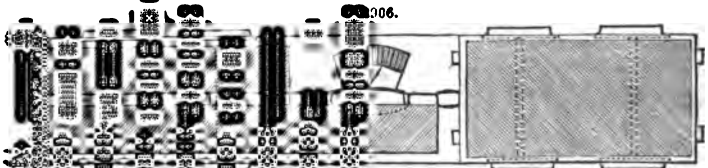
[illegible] *ynamometer* was applied as shown in Figs. 3905, [illegible]

3905.



Side view.

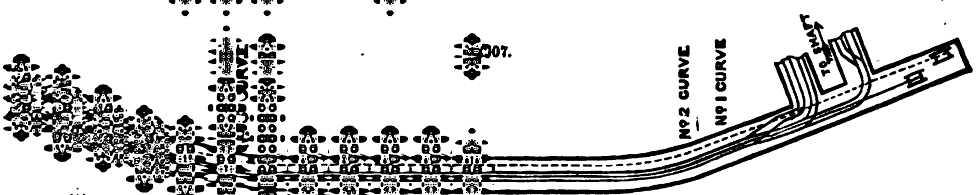
[illegible]906.



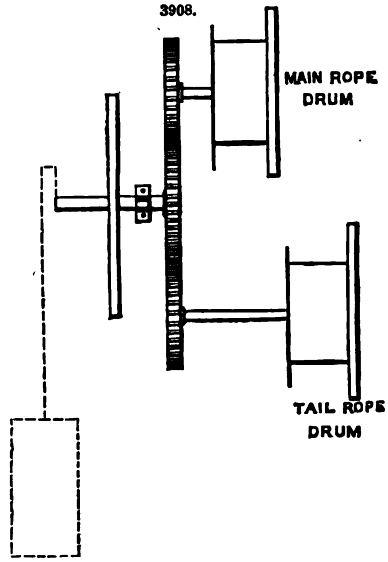
[illegible]lan.

[illegible] needle continually, and noted down the readings [illegible]aken. See DYNAMOMETER CAR.
[illegible] Seaton Delaval Colliery is 2059 yds. in length, and [illegible] 643, dipping outbye. All the coals are led from [illegible]

[illegible]907.



3908.



[illegible] inter- [illegible] ne is [illegible] e, and [illegible]; the [illegible], and [illegible] troke [illegible] 28 in., [illegible] of the [illegible] with [illegible] The [illegible] com- [illegible] engine [illegible] yds. [illegible] and [illegible] pumps [illegible] to the [illegible] same

[illegible] power [illegible] tons a [illegible] 0 tons [illegible] being

[illegible] d the [illegible] gene- [illegible] t par- [illegible] main [illegible] ving; at present there is only 1400 yds. of it upon

DIMENSIONS OF ENGINE, BOILERS, AND ROPES.

	ft.	in.
Diameter of cylinder	2	4
Length of stroke	6	0
Diameter of piston-rod (passes through cylinder)	0	5⅝
Length of connecting rod	11	1
	in.	in.
Dimensions of steam-ports	2½ × 12	
" " exhaust "	3½ × 12	
	Steam. ft.	Exhaust. ft.
Length of main pipe	93	100
	in.	in.
Diameter of main pipe	7½	7
	ft.	in.
Diameter of driving pinion	6	0
" " followers	6	0
" " fly-wheel (1)	15	0

	Main. ft.	Main. in.	Tail. ft.	Tail. in.
Diameter of drums	6	2	6	0
" " including flanges	11	0	11	0
Width between flanges	3	6	3	6
Width of brake	0	5	0	5

Cylinders, boilers, and pipes not covered.

Boilers ..	Number	4, 3 in use
	Description	Ordinary egg-ended
	Area of heating surface	each 108 sq. ft.
	Area of fire-grate	22·5 "
	Length over all	2 at 30 ft. 0 in.; 2 at 25 ft. 0 in.
	Diameter	2 " 5 ft. 6 in.; 2 " 6 ft. 0 in.
	Water evaporated in 12 hours	1578 cub. ft.

		Main.	Tail.
Ropes, or Chains	Length	2232 yds.	4491 yds.
	Circumference	3½ in.	3 in.
	Weight	233 cwts. 0 qrs. 23 lbs.	336 cwts. 0 qrs. 2 lbs.
	General duration	24 months.	24 months.
Sheaves ..	Number	146 37 50	142 31 60
	Weight per sheave	39 lbs. 26 lbs. 40 lbs.	39 lbs. 26 lbs. 40 lbs.
	Diameter	6 6 9 inches.	6 6 9 inches.
	Distance apart	24 ft.	24 ft.
	Extra sheaves at curves, &c.	29 at 10 in.	24 at 6 in.; 17 " 10 in.
	Return sheave, weight		952 lbs.
	" " diameter		7 ft. 0 in.

Description of Plane.—The plane is laid with broad-topped rails, in 15-ft. lengths, weighing 20 lbs. a yard. The way is laid on chairs 3 ft. apart, and the gauge of the way is 2 ft. 4 in. There are three slight curves on the plane, all turning in the same direction; the radius of No. 1 curve is 10 chains; No. 2, 10 chains; and No. 3, 12 chains. Nos. 1 and 2 curves are close together The ropes are carried round the curves by bell-sheaves 10 in. in diameter. The usual distance of the rollers apart is 24 ft., but this is not always observed; there are three descriptions of rollers used, the 6-in. rollers being most common both for the main and tail rope.

The ropes run off and on to the drums on the upper side, and are carried near the roof to the station. The tail wheel inbye is also placed near the roof, and the ropes are raised to it by sheaves.

This plane is worked in very much the same way as the majority of tail-rope planes in this district, but instead of having the usual kep for the full tubs to run down upon to the shaft, the way is laid level, and the coals are led from the station to the shaft by horses. On the end of both main and tail ropes there are fixed about 15 yds. of chain, with two or three large links inserted at various distances apart, and one of these links is attached by an iron pin to the loop of the centre bar of each end tub of the set, according to the position of the set at the station. In taking the empty set of tubs inbye, the tail rope is first connected to the set, and then the main rope, and the set is drawn inbye by the tail-rope drum, the main-rope drum being out of gear, with just a slight weight hung upon the brake-lever, to prevent the drum from over-running the rope. When the set reaches the inbye station it is allowed to stop before knocking off the ropes. The full set is brought out in a similar way by the main-rope drum, the tail drum running loose on the shaft, with a light weight on the brake-lever.

Harraton Colliery, near Durham.—In the application of the tail-rope system at this colliery two branches are worked, and the average gradient is in favour of the full tubs.

The engine-plane is 1795 yds. long to the north-way terminus, and 1729 yds. to the west-way terminus. The average gradient from the north-way flat to the shaft is a dip outbye of 1 in 83.

At present about 650 tons a day are led along this plane, nearly equal quantities being led from each of the branches.

The hauling engine is vertical, and has two 20-in. cylinders. The drums are 5 ft. 2 in. diameter, and the ratio of the pinion to the spur-wheels is as 3 to 4, thus causing the drums to make one-third more revolutions than the engine.

The three boilers by which steam is supplied are on the surface, and connected with the other boilers employed on the surface. Jucke's furnaces are used for firing. The engine works both day and night, but at present is doing very little work at night. The engine-plane has been at work only five years, and the engine, rails, tubs, and so on, are in very good condition.

The lead from the engine to the terminus of the plane near the shaft is not direct, the ropes being taken by large sheaves round a right angle 27 yds. from the engine.

DIMENSIONS OF ENGINE.

	ft.	in.
Number of cylinders	2	
Diameter of cylinders	2	2
Length of stroke	3	0
Diameter of piston-rod	0	3½
Length of connecting rod	6	0
	in.	in.
Dimensions of steam-ports	17 × 1¾	
" " exhaust "	17 × 2	

ENGINE—*continued.*

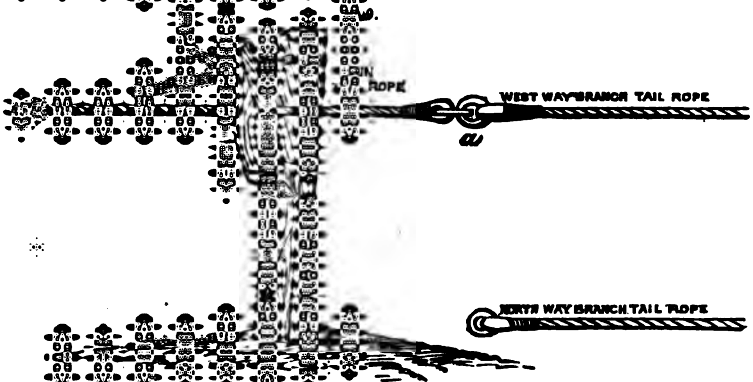
	ft.	in.
Diameter of driving pinion	4	2
" followers	3	2
" fly-wheel	8	2

[illegible] with heavy-topped chair rails, weighing 22 lbs. a [illegible] 26 yds. radius; No. 1 curve being 66 yds. long, and [illegible] e curves by sheaves, 4 ft. in diameter, with several [illegible] rm an almost regular curve for the rope. The tail [illegible], and a wheel of 8 ft. diameter at No. 2 curve. [illegible] re placed diagonally. The main rollers are 8 in. [illegible] meter. The large pulleys used on this plane have [illegible]

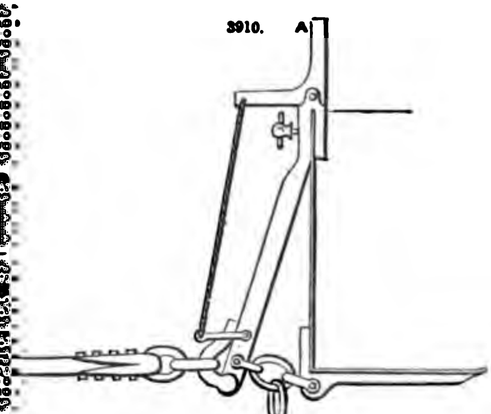
[illegible] in. higher than the other. Though the set passes [illegible], no accident has ever occurred.

[illegible]—There are 63 tubs in a set. In going inbye the [illegible] ks, and is taken to the junction A. As the set is [illegible] ches, the arrangement of ropes is altered at this [illegible] then on the set. Suppose the set has been last in [illegible] arrived at this junction, the tail rope at the inbye [illegible] north-way rope; the shackle connecting the main [illegible] held by a wooden clamp, fixed at the junction, while [illegible] d to it. This shackle-joint is stopped always at the [illegible] ught to this point whilst the set is coming inbye, [illegible] nection is made by the man attending the junction, [illegible] ng made by the run-rider, the time taken up by the [illegible] wo minutes.

[illegible] position of the ropes just when the tail rope [illegible]

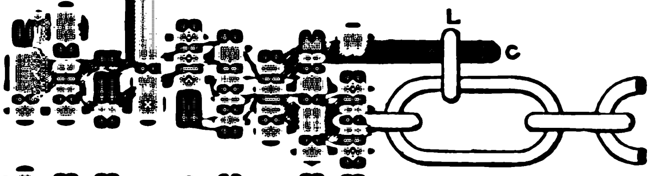


3910.



[illegible] praggs are put into the tub wheels, by means of [illegible] haft as required.

e other end of the set. When the rope has to
ink L is pushed off by the foot

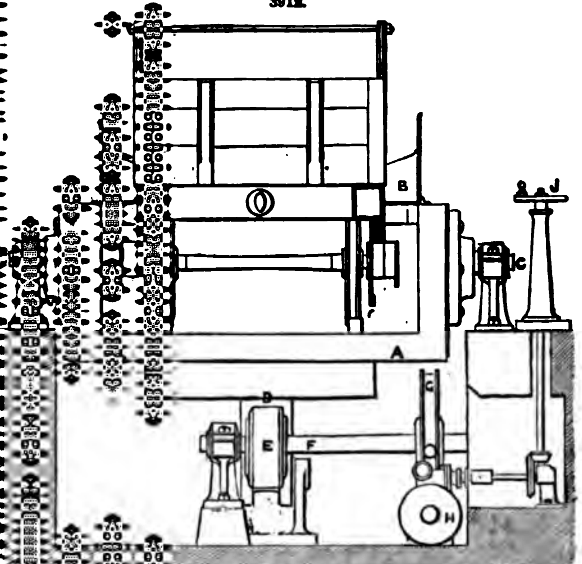


s. 3912 to 3917.—This invention of A. Rigg,
apparatus for carrying and tipping coal and
n sustained on axles or rollers which carries
to be upset, the centre of motion being so
turns the whole, and when empty returns to
t portion of the wagon which descends lowest
vith the machine and attached to it at any

refers steam-brakes, which possess a cylinder
steam to the upper or impelling side, and a
, and to this side is connected a large escape
the lower side may be made to reduce as the
ure may be obtained by enlarging the escape,
, and the brake may be thrown out of action
is also applicable to steam-engines or other

s of one modification of this tipping machine
that the principle admits of variation to suit
doors opening or not.
ill be seen that the apparatus consists mainly
which shoot may be horizontal, as shown in
uds, or vertical for smaller wagons with closed
oot may be replaced entirely or in part by a
anced that it remains in the position shown,
ilway wagon. This platform is supported on
h has teeth gearing into those of a pinion E
on such shaft, and is regulated by a brake H,

3912.

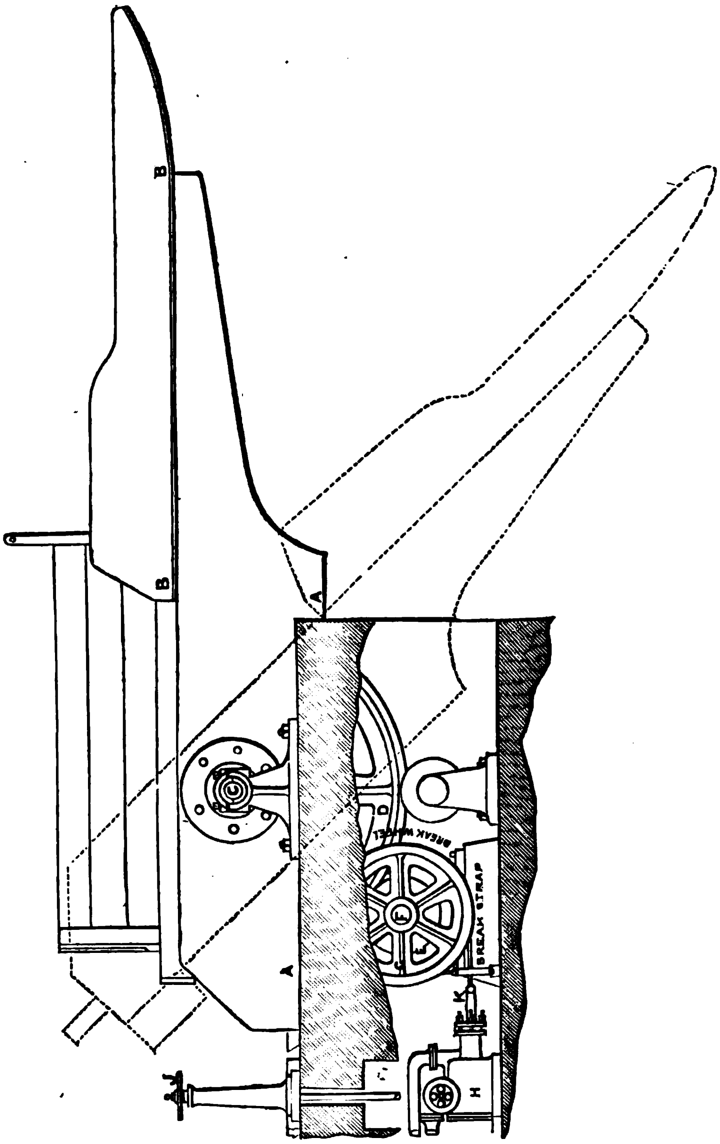


aging velocity. The dotted lines explain the

After the delivery of the coals or minerals, the apparatus returns into its horizontal position, such return being regulated in its speed by the brake.

Figs. 3914, 3915, represent on a larger scale the brake H above mentioned. From the side elevation and section, Figs. 3914, 3915, it will be seen that the apparatus consists of a cylinder H with piston K, similar to that of a steam-engine, and it is preferred to make the piston-rod of larger diameter than is usual for a steam-engine, partly for security. A constant communication exists between the boiler and the upper side of the piston, and also between the boiler and the valve-chest. In this valve-chest one passage leads to the lower side of the piston, and the other to the exhaust.

3914



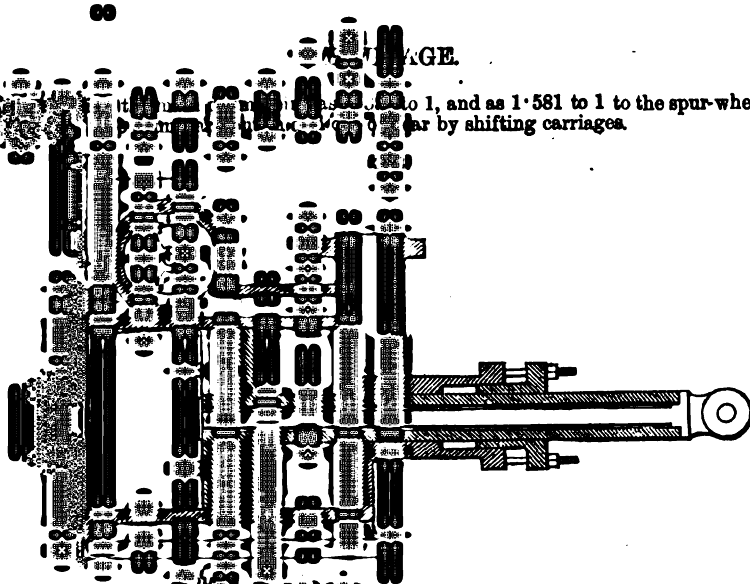
The valve is shown in Figs. 3915 to 3917, and is arranged to cover both ports. Figs. 3916, 3917, show side and end elevations of the valve, which has a passage (1) completely through it corresponding to steam-port at the lower end of the cylinder, and a recess (2) which is arranged to cover the exhaust and steam ports when necessary; while the opening (1) in the valve corresponds to the port leading to the lower side of cylinder, an equal pressure is maintained on both sides of the piston, except what is due to the diameter of piston-rod, which difference always keeps the brake-band slack when out of action. When the valve is turned so that the recess (2) in the valve covers both the steam and exhaust ports, an escape of steam will take place, which, by reducing the pressure on the lower side of the piston, whilst the pressure at the upper surface of the piston remains in full force, causes the brake to come into action with a force depending on the rapidity of the escape of steam allowed. The brake will be put out of action by turning the valve, so as to allow a free flow of steam again into the lower end of the cylinder. By making the partition in the valve somewhat narrower than the opening of the port, steam can be partially admitted while some escapes, thus giving a greater range of pressures, and maintaining as long as required the necessary force.

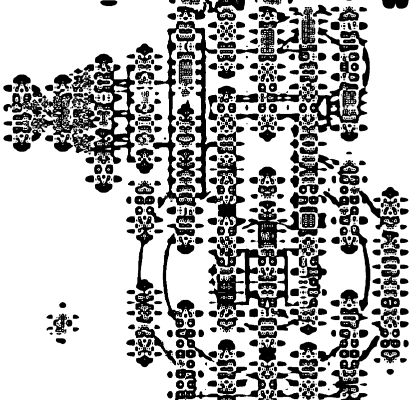
Murton Colliery, County of Durham.—There are no branches worked by the tail rope at this colliery; but there is a station (Hallfield station) by the side of the main way, from which part of the coals are led.

The length of plane the coals are led over from the south-east or far-off landing is 2770 yds., and 1978 yds. from the Hallfield station, the distance from the engine to the tail wheel being 2816 yds.

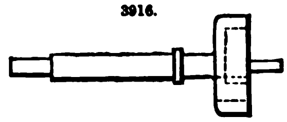
The hauling engine is a double 18-in. cylinder horizontal engine, with 2-ft. stroke. The main and tail drums are 5 ft. in diameter, and on the second motion the revolutions of the pinion-wheel

...o 1, and as 1·581 to 1 to the spur-wheel driving ... ar by shifting carriages.

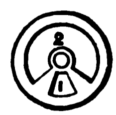




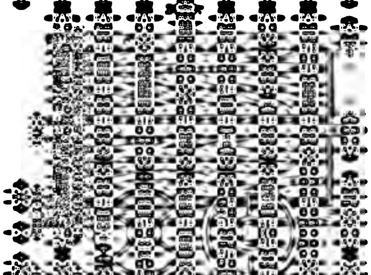
3916.



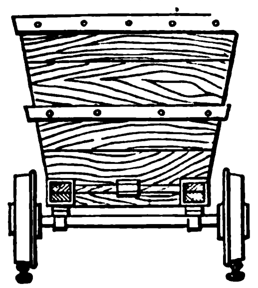
3917.



... inside, 3 ft. 9 in.; breadth at top, 3 ft.; breadth ... bove rails, 3 ft. 5 in.; distance coupled, 1 ft. 6 in.;



3919.



... in pit, 450. Number required to work engine-

... d with chair rails, in 12-ft. lengths, weighing ... n. The tub used has wheels 14 in. in diameter,

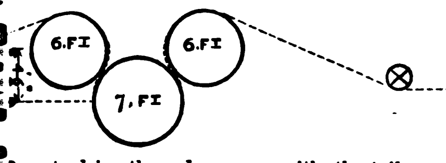
... e has a radius of 154 yds., and No. 2 a radius of ... 83 rise outbye, and the heaviest gradient is 1 in ... ips towards the shaft. The rollers and sheaves ... 6 in. in diameter. The main rope is taken round ... nd the tail rope by ordinary 2-ft. sheaves. ... itches leading on to a double line of rails, for full

[illegible]AGE.

[illegible] engine-plane is direct. The main rope is 2⅜ in. [illegible]n.

[illegible] cranked single-action force-pump, worked by the [illegible] as shown in Fig. 3920.

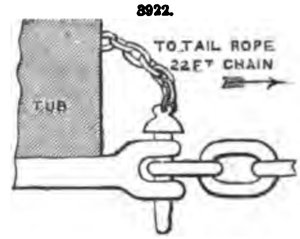
3920.



[illegible]sary to drive them done away with, the tail rope, [illegible] be worked for nine months.

[illegible] There are forty-eight tubs in a set of full or empty [illegible] is attached to the set by fastening the shackle, [illegible] chain of the end tub, with a pin, which is secured [illegible] attached by placing the end link of the chain in [illegible] fixture on the end of the tub, Fig. 3922.

3922.



Fastening at the Tail-rope end.

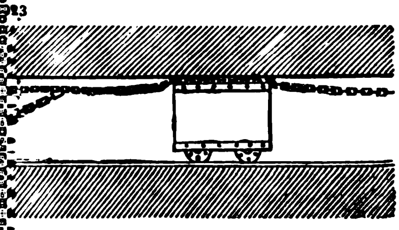
[illegible] to the set, the station boy raps to the brakesman, [illegible] at the Hallfield landing, till a rap is given from [illegible] is to go; if for the Hallfield way, the switches are [illegible]. When the ropes are disconnected, the set runs [illegible] the ropes to be attached to the full set. When [illegible] given on the set reaching the flat, as the brakesman [illegible] when to stop the engine. The full set is gene[illegible] point to which the empty set runs. Should the [illegible]ons, not be conveniently situated for the ropes, a [illegible] readiness) is put on, and it is thus very seldom [illegible]

[illegible] bankhead, the station boy raps to the engineman [illegible] the full set is stopped opposite the empty set, part [illegible]nbye. When the set stops, the pin fastening the [illegible] to be drawn out by a lever kept for the purpose. [illegible] the slip-link fastening, by which the rope can be [illegible]

[illegible] out half-way to the shaft by a tail rope; the tail [illegible] shaft, dragging the main rope after it. The main [illegible] [illegible]g-wheel for driving the small drums required for [illegible], and can be put in and out of gear whilst the [illegible] [illegible]cing the speed of the engine for a short time.

[illegible] meter was tried only upon the main ginney road, [illegible] mit of its being used with safety. It was first [illegible] out on the full way. Three other experiments [illegible] to work the main road, and the two branches [illegible] 248 yds. from the station, rising 1 in 17, and the [illegible] the bank. The dynamometer was attached to the

[illegible]3



[illegible] on one side, and blocks on the other; the blocks [illegible]ck.

ABSTRACT OF EXPERIMENTS WITH INDICATOR

Series.	Description of Work.	Steam Lbs. Gauge on Boiler.	Steam Lbs. Indicator. Steam.	Steam Lbs. Indicator. Back.	Steam Lbs. Indicator. Effective.	Speed of Piston. Strokes a min.	Speed of Piston. Feet a min.	Actual Horse-power.
A	Engine going full work, both winding and ginney chains going in shaft, and Nos. 1 and 2 ginneys underground ..	32½	24·1	7·8	16·3	60	240	29·11
B	No. 1 ginney going alone	33½	22·6	5·6	17·0	60	240	30·36
C	No. 2 ginney going alone	34	23·11	6·05	17·06	60	240	30·47
D	Nos. 1 and 2 ginneys not working, and both chains going in shaft; winding full tubs	32	22·8	6·0	16·8	60	240	30 00
E	Nos. 1 and 2 ginneys not working, and both chains going in shaft; winding empty tubs	32	3·8	1·6	2·2	60	240	3·93
F	Winding chain going alone in shaft, winding empty tubs; all ginney chains being out of gear	32½	3·5	1 6	1·9	60	240	3·39

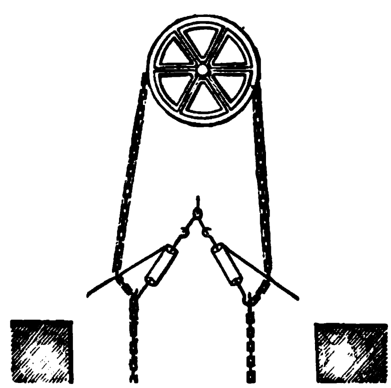
Power required to work the engine, together with the winding chain working empty tubs in the shaft, all the endless chain being out of gear, and the chain in the shaft being disconnected from driving wheel by blocks, as shown in Fig. 3924 (F); at 60 strokes a minute, 3·39 horse-power.

Power required to work the engine, together with the winding chain, and the hauling endless chain in the shaft, and underground to the pulley, all other underground chains being out of gear, see E tabulated form; at 60 strokes a minute, 3·93 horse-power.

∴ Power required to convey the endless chain down a shaft 75 ft. deep, and underground for a distance of 27 yds.;—

	Both chains.		Winding chains alone.		Horse-power.
At 60 strokes a minute	3·93	−	3·39	=	0·54

3924.



CALCULATION OF FRICTION OF CHAIN IN SHAFT.

		cwt.
Moving weight.—Pulleys, 2 at 4 cwt.		8
„ 2 at 1 cwt.		2
„ for tightening chain at bottom		1
Suspended weight (W on section)		8
		—19
Chain.—The two sides of the chain in the shaft counterbalance each other, so that only the horizontal chain is to be considered; 27 yds. × 2 = 54 × 16 lbs. a yard		7 7
		26·7

Speed of chain in shaft, 156 ft. a minute. Then $\frac{\overset{\text{h. p.}}{\cdot 54} \times 33000}{112 \times 156 \text{ ft. a minute}} = 1\cdot 02$ cwt.

Then total friction $= \frac{1\cdot 02}{26\cdot 7} = \frac{1}{25}$ of total weight of horizontal chain, pulleys, &c.

Power required to work engine and wind coals, and to drive all the endless-chain roads underground, the average gradient of which is a fall outbye of 1 in 20 (A); at 60 strokes a minute, 29·11 horse-power.

Power required to work engine, wind coals, and to drive No. 1 ginney road only (B); at 60 strokes a minute, 30·30 horse-power.

Power required to work engine, wind coals, and to drive No. 2 ginney road only (C); at 60 strokes a minute, 30·47 horse-power.

It would appear from the three experiments above, that it requires less power to work both ginney roads together, than when going separately. The power required for winding coals by this system is shown by this experiment to be

$30\cdot 00 - \overset{\text{Engine, \&c.}}{3\cdot 39} = 26\cdot 61$ horse-power, to wind 417 tons 75 fas., a day of 8½ hours.

Power required to work engine and wind coals—the hauling chain in the shaft being also in motion, but all the underground ginney roads being out of gear (D); at 60 strokes a minute, 30·00 horse-power.

From this it will be seen that the working of the underground ginney roads assists the winding of the coals to the extent of 30·00 − 29·11 = ·89 horse-power.

Endless-rope System.—The method of conveying coals by this system of applying the endless rope has hitherto been very rarely adopted. It is chiefly in operation in the Midland Counties. The following planes have been reported on, but experiments have been made at the first only;—

I.—Shireoaks Colliery, Nottinghamshire—Underground plane in a straight line, and rising towards the shaft.

II.—California Pit, Wigan—Three underground planes, branching from the pit, and worked by one engine; a short curve on one of the planes.

Varieties of the No. 1 endless-rope system are in operation at the following collieries;—

III.—Newsham Colliery, Northumberland—Straight undulating plane, with no branches.

IV.—Eston Mines, Yorkshire—A short rope working three branches from a main line.

V. Cinderhill Colliery, Nottinghamshire—Level plane, with no branches; endless rope worked at slow speed.

This system is a modification of the tail-rope system. The following are a few of its chief characteristics;—

1. The rope, as the name implies, is endless.

2. To give motion to the rope a single wheel is used, and friction for driving the rope is supplied either by clip-pulleys, as at Shireoaks, Newsham, and Eston, or by taking the rope over several wheels, as at the Cinderhill and California Pits.

3. As only one driving wheel is used, the rope has to be kept constantly tight; this is effected by passing it round a pulley fixed upon a tram, to which a hanging weight is attached.

4. Either one or two lines of rails are used; when a single line is adopted, as at Newsham, the rope works backwards and forwards, only one part of it being on the wagon-way, and the other running by the side of the way; when two lines are used, as at Shireoaks, Cinderhill, and the California Pit, the rope moves always in one direction, and the full tubs come out on one line, the empties going in on the other.

5. The set of tubs is connected to the rope either by means of a clamp, or by sockets in the rope, to which the set is attached by a short chain. The former method is in use at Shireoaks, Cinderhill, and the California Pit, whilst the latter is adopted at Newsham and Eston Mines.

6. The working of curves and branches has hitherto been scarcely attempted by this system. At California Pit there is a slight curve on one of the ways, which works well.

The various methods of applying this system are described in the following Reports of the planes visited.

Shireoaks Colliery, Nottinghamshire.—The No. 1 endless-rope system has been in operation at Shireoaks Colliery for about four years. The engine by which the rope is worked is a double horizontal engine, with two 12½-in. cylinders. The boiler is single tubular, with Galloway's cross tubes, and is placed underground at a very short distance from the engine.

DIMENSIONS OF ENGINE, BOILER, AND ROPES.

Engine erected 1856.

Number of cylinders	2	
	ft.	in.
Diameter of cylinders	0	12½
Length of stroke	2	0
Diameter of piston-rod	0	$1\frac{9}{16}$
Length of connecting rod	5	10
	Steam.	Exhaust.
	ft.	ft.
Length of breech-pipe	0	1½
" main pipe	62	64
	in.	in.
Diameter of breech-pipe	0	5¾
" main pipe	4	4½
	ft.	in.
Diameter of driving pinion (friction gear)	2	8
" followers	4	0
" fly-wheel	None	
" clip-wheel	4	0
Distance between centres of cylinders	2	5
Boiler .. Number	1	
Description—Single tubular, with 3 Galloway's tubes.		
Area of heating surface, 494 sq. ft.		
	ft.	
" fire-grate	19	

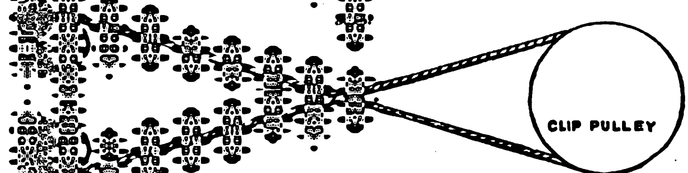
		ft.	in.	
Boiler ..	Length over all	25	0	
	Diameter	6	0	
		ft.	ft.	ft.
	Hot-water tank (through which exhaust passes)	9 ×	6 ×	4
		yds.		
Rope ..	Length	1590		
		in.		
	Circumference	2½		
		cwts.	qrs.	lbs.
	Weight	25	2	21
		months.		
	General duration	18		
Sheaves (*wood*)	Number (53 on each way)	106		
		lbs.		
	Weight a sheave	14		
		in.	in.	
	Diameter	5 ×	16	
		ft.		
	Distance apart	45		
		lbs.		
Rails ..	Weight a yard	18		
		ft.		
	Length of each	12		
	No. of lines	4		
		ft.		
	Gauge of way	2		
	General condition	Good		

OF TUBS.

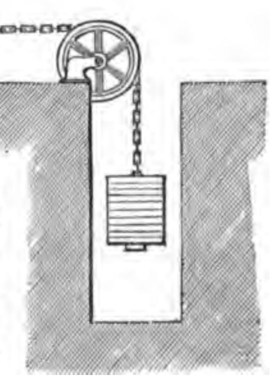
Total number in pit	450
Number required to work endless-rope plane	256
Work done, 320 tons a day of 11 hours (1 hour allowed for meals).	

is laid with a double way; the empty sets going
other. It was originally intended to have sets
he limited quantity of coals now being drawn
the endless rope is 750 yds. long, with an
viest gradient being 1 in 29. The same engine
s. long, the average rise of which, towards the
in diameter, is used, to give motion to the end-
n, and is connected by mitre-gearing to the
on friction-wheel is 2 ft. 8 in. in diameter, and
e clip-pulley and the other for the single-rope
edingly well, and is very convenient for putting
disconnect the wheels; this is effected by the

e clip-wheel, to cause the rope to pass round as
rder to get as much of the grasping effect of the
from the clip-wheel are crossed, as shown in



, it is necessary to keep it very tight, as other-
ected by having the wheel inbye placed on a
ain, passing down a small staple, as shown in
suspended. The weight descends as the rope
same tension.



steel, weighing
s of wood, 5 in.
hey are placed
9 cwt. of coal,

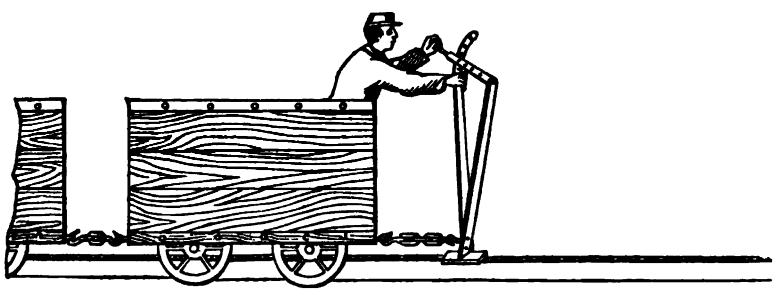
r, are used for
a piece of iron,
is heard very
in motion the
bar.

lane by the No. 1 Endless-rope System.—The rope
, and to the tail wheel at the inbye end, goes
an thus pass over the rope in coming from the

gether at the bankhead, and the run-rider, who
hooks the chain affixed to the clamp on to the
d then fixes the clamp upon the rope, which is
is closed by a handle-lever, passing over the
kept firmly fixed without the pressure of the
rope, that in a case of the set being stopped by
rather than slip through the clamp.
n-rider strikes the sounding bar, and the rope
of the clamp to keep it perpendicular, and the

tubs being pulled forward by the chain on the clamp. The set at the start is about 45 yds. from the engine, and is on a gradient of 1 in 520 fall; the next or middle gradient is 1 in 47. As the set is found to overrun the clamps in going inbye at this latter gradient, six spraggs are placed in the wheels of the tubs at the end of the set, to prevent the tubs getting together and becoming uncoupled. At a distance of 364 yds. from the engine, and about the middle of the 1 in 47 gradient, the clamp is taken off the rope whilst in motion, and the set runs forward by itself. When the clamp is disconnected, the run-rider raps to the engine—though the running away of the rope when disconnected is sufficient to let the brakesman know—and the rope is stopped, the engine then being free to work the single-rope way.

3927.



There are two stations from which coal is being drawn at present; the first being 640, and the second about 795 yds. from the engine. When the gang or set has to go to No. 15, or the first station, the points are placed for this, and the set, then disconnected from the rope, runs round the curve into the station; the rope at this curve passes under the rolley-way, so that with this arrangement the clamps could not pass this point. When the set is intended for the far-off station, or No. 13, it runs by itself from the knock-off point, the spraggs being taken out, when necessary, by a boy, who rides with the set for the purpose.

In coming outbye, the clamp is also placed at the front end of the full set, which is pulled out to within 140 yds. of the engine, when the clamp is removed, and the tubs run forward to a point from which they are taken to the shaft by horses. The expressions *outbye* and *inbye*, which occur frequently in this Report, are terms used in the North of England; the former to denote the end of the engine-plane nearest to the shaft, and the latter the end nearest to the workings of the mine.

When bringing the full tubs out, the strain of the full set upon the clamp chain raises the rope a little and prevents the clamp from striking the rollers; but in going inbye, where there is much less strain, the clamp, in passing over the rollers, touches them slightly.

The engine cannot pull coals from the endless-rope way and the single-rope way together, and if this were possible, it would be hardly worth while, since it never goes for more than 3½ minutes at a time in working the endless rope.

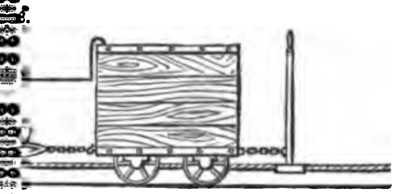
EXPERIMENTS WITH DYNAMOMETER.

Series.	Work done.	No.	Time.	Reading of Dynamometer.	Distance travelled in yards.	Speed. Miles an hour.
			h. m. s.	cwt.		
	Full tubs coming out.		12 0 0	..	..	..
A	.. ..	1	12 0 30	14	..	..
	.. ..	2	12 2 15	10	..	..
	.. ..	3	12 3 1	5	..	..
		..	12 3 40	..	655	6·10
	Empty tubs going in.					
B	.. ..	3	1 0 0	..	..	..
	.. ..	2	1 0 43	1	..	..
	Knocked off	..	1 2 05	1	..	..
		..	..	..	319	3·6

These experiments were made with a set of thirty-one tubs, the dynamometer being placed between the first and second tubs.

A piece of iron, A, placed between the two tubs, Fig. 3928, kept them at a regular distance

... by which the dynamometer was suspended,



...ess rope is worked on the same plan as that in ... were made at Shireoaks to show the power ...e, it was thought that a few notes of the mode ...be sufficient to show anything different in the ...ed.

...derground. It has two 14-in. cylinders and a

...gine, all falling from the shaft. The following

...	No. of tubs a set.	Time required to bring set out.	Tons led a day.
[illegible]	20	3 minutes	130
[illegible]	12	4 ,,	120
[illegible]	20	7 ,,	180

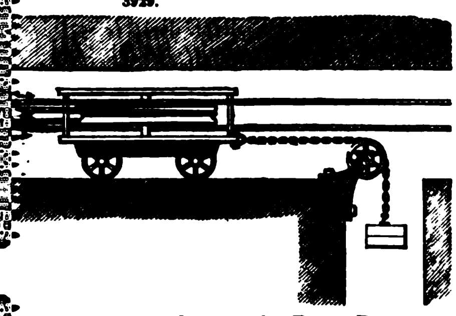
...ays is worked by two driving wheels, acting as ...e all close to the engine, and are horizontal. ...itre-gearing. By an arrangement each of the ... necessary; the three handles required for this ...earing is also used for working them.

...diameter. The planes have been at work about ...be about three years.

...and are placed about 20 yds. apart.

...at Shireoaks, the ropes have to be tightened, in ... at the two ends of the planes. The tightening ...uble driving wheel, another pulley fixed on a ... A chain is attached to this tram, to which a ...ple, as shown in Fig. 3929, and keeps the rope

3929.



...est average speed is on the Down Brow way, ...r.

... plane. On the Duke's Slant and Down Brow ...ght out simultaneously, and as the gradient of ... the engine is lightened by this arrangement. ...gether in this way, but they cannot be worked ...e full coals out, without the assistance of the ...of the planes requires the services of four gang- ... when others are required, they are taken from

...ts of tubs to the rope is different from that used ...er the ends of the arms, instead of a lever.

... rope is connected to the set at the fore end in ...d on the North Level, the gradient of which is ... going both directions.

There is a short curve of 4 chains radius on the North Level way, round which the rope and clamp pass easily, though not at a higher speed than seven miles an hour.

Newsham Colliery, Northumberland.—At this colliery the No. 1 endless-rope system is worked, as at Shireoaks, by the clip-pulley.

The chief peculiarities in the working of this engine-plane are as follow:—

1. Only one way is used, the empty and full tubs running on the same road, and half of the rope being carried on sheaves by the side of the way.

2. Instead of the clamp, used at Shireoaks, the set of tubs is connected to the rope by two short chains, one end of the chain being secured to the tub at each end of the set, and the other attached to a socket in the rope.

3. The plane is undulating, and thus presents one of the most awkward difficulties to contend with in the working of this system.

The endless rope is worked by a clip-pulley, driven by one of Fowler's engines. It is placed underground, and is worked generally at a speed of eighty-five strokes a minute, with a pressure of 80 lbs. of steam.

DIMENSIONS OF ENGINE, BOILER, AND ROPES.

Engine erected 1867.			
	Number of cylinders		2
	Diameter of cylinders	in.	12
	Clip-pulley, diameter	ft. in.	7 6
Boiler	Number		1
	Size	ft. in. ft. in.	15 5×3 6
	Coal consumed a day (splint)	cwt.	9
Rope	Length	yds.	1672
	Circumference	in.	2½
	General duration (estimated)	years.	2½

			Main.	Tail.
Sheaves	Diameter	in.	4½	8
	Weight	lbs.	28	31
	Distance apart	yds.	16	16
	Sheaves at curves (diameter)	ft.	3	
	" (weight)	lbs.	224	
Rails	Weight a yard	lbs.		28
	Gauge of way	ft. in.		2 8
Tubs	Average weight (empty)	lbs.		672
	" (coal contained)			1120

DIMENSIONS OF TUBS.

	Wood. ft. in.	Iron. ft. in.
Length inside	3 9	3 10
Breadth	2 9	3 1
Depth	2 0	2 2½
Height above rails	3 6	3 6
Distance coupled	1 9	1 10

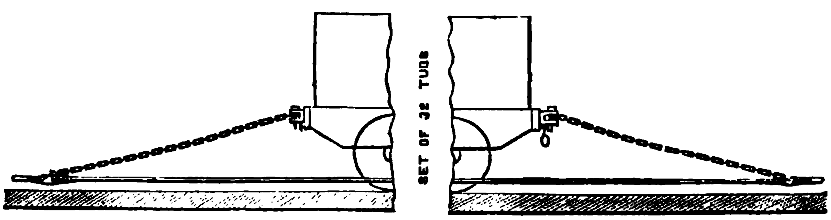
	Wood. ft. in.	Iron. ft. in.
Diameter of wheels	1 4	1 4
Total number in pit	83	100
Number required to work endless-rope plane	96	
Work done, 400 tons a day of 12 hours.		

Description of Engine-plane.—The engine-plane is laid with flat-bottomed rails, weighing 28 lbs. a yard. There are two very slight curves on the plane, round which the main rope works easily in the centre of the way. The rope by the side of the way is taken round these curves by 3-ft. sheaves. The length of the plane is 836 yds. The road first rises from the shaft at a gradient of 1 in 40 for a distance of 318 yds., and then falls for a distance of 379 yds., at an average gradient of 1 in 47, the average gradient of the plane being a fall towards the shaft of 1 in 805.

The rope used is steel. By means of a bearing-down pulley, the rope is made to move in nearly the whole circumference of the clip-pulley. The rope has been in use for eighteen months, and is expected to last another year. It is kept tight, as at Shireoaks, by having a hanging weight attached to the sheave at the inbye end of the engine-plane. The weight used here is about 25 cwt.

Description of the Method of Working the Engine-plane.—The full and empty tubs are run in sets of thirty-two tubs. The sets are attached to the rope by chains, 7 ft. long, as shown in Fig. 3930.

3930.



There are two sockets in the rope, placed at a distance apart convenient for the attachment of a set of thirty-two tubs, to which these chains are connected. The chains are put on at both ends of the set, the undulations of the plane rendering this necessary. The connection with the set is made simply by hooking on, and with the rope by a screw-shackle.

At each end of the plane the rope passes under the way, and in going in and out, when the set comes to this point, it is disconnected from the ropes, and the gradient of the way is so arranged that it runs by itself into the siding.

In going from the shaft, the set of tubs having fallen to the point at which the rope passes from under the way, the chain is attached to the fore end of the set; the engine then moves the set forward over a distance equal to the full length of the set, and the back chain is hung on. The moving of the set forward causes the coupling chains between the tubs to become tight, and the danger of the set overrunning the chain at the fore end of the set, when the gradient falling from the shaft is reached, is thus avoided.

There are no swivels on the chains connecting the set to the rope, the tightness of the rope tendering them unnecessary.

The sets are taken in and outbye, at an average speed of ten miles an hour. In disconnecting the set from the rope at the inbye end, the chain at each end of the set is taken off the tubs, but is not disconnected from the socket on the rope; this is done before the set reaches the siding, whilst it is moving slowly; when detached, it runs forward into the siding. At the shaft the leading chain is disconnected from the rope first, and then from the tub; this is done to allow the shackle to pass over the clip-wheel, which is placed rather close to the terminus of the engine-plane. The chain at the other end of the set is taken off the tubs only.

Endless-rope System.—This system of conveying coals is only in operation in the Wigan district, where it has only been in use for a few years. Of the four systems reported on it is probably the least known. Reports are given on the following planes. Experiments were made and costs extracted at the Bridge and Meadow Pits only;—

I.—Bridge Pit—Several planes at work, with an average gradient rising towards the shaft. One self-acting curve working.

II.—Meadow Pit—Single plane rising towards the shaft.

III.—No. 5 Moor Pit—Single plane, rising at a heavy gradient towards the shaft.

IV.—Mesnes Colliery—Undulating plane, worked by an engine on the surface.

V.—Scot Lane Pit—Single short plane, with peculiar method of connecting the tubs to the rope.

The principle on which No. 2 endless-rope system is worked is very similar to that of the endless-chain system.

The following are the chief peculiarities in the application of this system;—

1. A double line of rails is used.
2. The rope rests upon the tubs, which are attached to the rope either singly or in sets of tubs varying in number from two to twelve.
3. The connection between the tubs and the rope is effected by a short chain, which is secured to the rope in a way hereafter described.
4. As with the endless chain, the tub or tubs are placed at a regular distance apart, and the rope is driven at a slow speed.
5. Motion is given to the rope by large driving pulleys, and friction is obtained by taking the rope several times round the driving pulley.
6. Curves can be worked by this system.

There are no instances of branches being worked by this method of conveyance, but, as will be seen by the following descriptions, this could very easily be arranged for.

Bridge Pit, near Wigan.—The No. 2 endless-rope system has been at work at this colliery about three years, and is here more extensively applied than at any of the collieries in the district.

The engine working the planes at this pit is a double 20-in. cylinder horizontal engine, with a stroke of 3 ft. 6 in.; the boilers are underground, at a distance of 150 yds. from the engine. Unlike the general adaptation of ropes in this district, the ropes on the planes are worked by a main driving rope, motion being transmitted by having two pulleys on one shaft. This was necessitated by the distance of the engine from the point to which the coals are led. The rope is driven by a main pulley, 14 ft. diameter, working on the third motion. The ratio of the strokes of the engine to the revolutions of the pulley is as 7·4 to 1.

Dimensions of Engine, Boilers, and Ropes.

Engine erected 1863.		
Number of cylinders	2	
	ft.	in.
Diameter of cylinders	0	20
Length of stroke	3	6
Diameter of piston-rod	0	3
Length of connecting rod	10	6
	in.	in.
Dimensions of steam-ports	$18\frac{1}{2} \times 1\frac{1}{2}$	
" exhaust "	$18\frac{1}{2} \times 2\frac{1}{2}$	
	Steam. ft.	Exhaust. ft.
Length of breech-pipe	9	9
" main pipe	450	390
	in.	in.
Diameter of breech-pipe	5	6
" main pipe	6	7
	ft.	in.
Diameter of driving pinion	3	10
" follower	7	5
" fly-wheel (1)	10	3
" driving wheel	14	0
Distance between centres of cylinders	7	$10\frac{1}{2}$

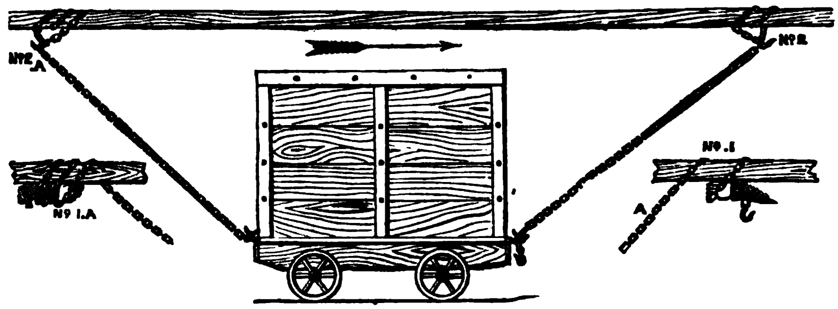
Boilers ..	Number	2	
	Description	Double tubular	
		ft.	in.
	Length over all	30	0
	Diameter	6	6
		Main. yds.	Tail. yds.
Driving Rope	Length	787	4730
		in.	in.
	Circumference ..	3·9	3·14
		cwts. qrs. lbs.	cwts qrs. lbs.
	Weight ..	48 0 32	175 2 26
		months.	months.
	General duration ..	24	14
Sheaves ..	Number (between main pulley at engine and tightening pulleys) ..	34	35
		in.	in.
	Diameter	14	14
		lbs.	
Return Sheaves	Weight	672	
		ft.	in.
	Diameter	5	0

Description of Engine-plane.—These planes are laid with round-topped bridge rails in 12-ft. lengths, weighing 18 lbs. a yard. All the planes are laid with a double line of way, one line for the ingoing empty tubs, and the other for the full tubs coming out. The general size of the wagon-way is 10 ft. by 4 ft., it having been originally made this size. The average gradient of all the ways together is a rise towards the shaft of 1 in 62, the chain-brow way being a rise to the shaft of 1 in 54, the slant way a fall towards the shaft of 1 in 47, part of it at the inbye end rising towards the shaft at a gradient of 1 in 5·5.

With the exception of the curve on the chain-brow way, all the planes at this colliery are quite straight.

Method of Connecting the Tubs to the Rope by Chains.—The chains by which the tubs are attached to the rope are of $\frac{5}{8}$-in. iron, 6 ft. long, with a hook at each end. They are connected to the tub as shown, Fig. 3931. The fore end of the tub is first connected to the rope; this is done by attaching

3931.



one end of the chain to the second link of the coupling chain of the tub, and throwing the other end over the rope, which is constantly in motion. The chain is then passed twice over the rope, the hand being introduced under the rope to receive the coils, in order to let the chain slide loosely on the moving rope till the hook is secured. When the right number of coils of chain (two in this case) have been passed over the rope, the hand is withdrawn, the point A is brought over the hook, and the chain is pulled tight; it is not until the chain is securely fixed that the weight of the tub is allowed to come upon the chain. The sketch (No. 2) shows the chain just when it has been passed over the hook. When the full weight of the tub is upon the chain, the coils get quite close together and form a very compact and secure fastening. An expert hooker-on does not need to put his hand between the coils, but passes the chain round the rope, and secures it before the rope has time to move on. The chain at the back end of the tub is attached in a similar way to that described above, but with three coils instead of two; this is necessary at the Bridge Pit, owing to the heavy weight of the tub upon the chain for a short distance in going inbye. The tubs on the other planes at this pit are attached in a similar manner. When two or more tubs are put on the planes together, chains are fixed on to the fore and back ends of the gang, or at one end only, as the case may be. The chain is disconnected from the outcoming tubs, at the back end, by unhooking the chain from the tub; it is then easily loosened from the rope. At the fore end the chain is tight, and the foot is placed upon it, pressing it down, and making it loose enough to admit of disconnection. There is more labour required in the disconnecting than in the attaching at this pit. This description has reference to the taking off of the full tubs at the end of the main road, and here there is a rise towards the shaft. At some other places the terminus of the full way is made to dip slightly, and the chains are removed just when the tub, passing over the brow, loosens the chain at the fore end. On the other hand, the labour required for attaching the empty tubs is less than at other places; here the empty way is made to rise slightly, the fore chain is put on first, and one boy is able to manage both. At another pit (No. 5 Moor), where the empty way, at the start, falls inbye, both chains have to be put on together, thus requiring two boys.

At the top of the main-road way at the Bridge Pit, a boy stands about 20 yds. from the place to which the full tubs come, and removes the back chain, leaving it hanging on the rope by the hook; it is taken off by the man who disconnects the chain at the fore end, and, together with the other chain, is thrown over by him to the place where the empties are hooked on.

The usual time for attaching both chains to the empty tub is about twelve seconds, the minimum time being six seconds, and the time for disconnecting is rather more. Sometimes a stoppage is caused by the fastening of the chain being difficult to disentangle, and the man disconnecting has then to rap to stop the engine, to prevent the tub from reaching the pulley

The chain is very seldom known to slip on the rope; when it does, the damage done is often rather heavy, since, should the fore chain slip, the tub going on to the back chain is generally upset, or in the absence of the back chain it may rest on the plane till the next tub comes up to it, the chain of which not only often knocks the tub off the way, but is sometimes broken itself, and as it is difficult to tell at the engine when such an occurrence takes place, there is much damage done before the engine is stopped. The chief accidents to tubs usually occur at the heavy gradient on the main way at this pit, for should a weak link in the connecting chain break whilst the tub is on this gradient, the tub getting loose generally breaks several other chains and tubs below it.

The slow speed at which the tubs go—being 1·35 mile an hour on the main road, and 1·126 mile an hour on the other ways—is necessary to prevent accidents to the tubs. The rope rests upon the tubs, and unless the way is laid perfectly straight, it is a slight distance from the centre of the tub; a small angle at a joint of the rails is sufficient to cause this deviation, and should the

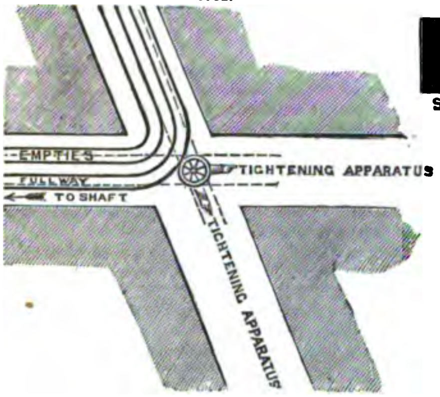
rope catch any irregularity on the top of the tub, it will sometimes overturn it; to avoid this, much attention is paid to keeping the tubs in good repair, and this partly accounts for the heavy cost of maintaining tubs at this pit.

In the working of the endless rope at this colliery the apparatus for putting the driving wheels in and out of gear is found to be indispensable. Thus the chain brow, the main road, and the main road with the slant way, can each be worked separately; the workings at the inbye end of the main road serve to keep the main way supplied for a short time, when it is necessary to put the slant way out of gear.

There are two curves on the engine-plane at this colliery, one at the bottom of the main road worked by disconnecting and reconnecting the tubs, and the other which self-acts on the chain-brow way. At the former, which turns round an angle of 72°, the motion is transmitted from one pulley to another on the same shaft, as shown, Fig. 3932. The road is laid round the curve at such an inclination that the full and empty tubs when disconnected run by themselves to the place where they are again attached to the rope. There are five hands required here, four boys and one man.

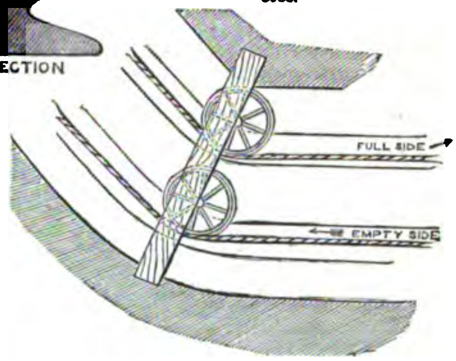
This curve might probably be made to self-act, like the curve at the chain-brow way, by means of four or more pulleys, but when it was originally arranged, it was intended to draw a large quantity of coals from other districts besides the slant way.

3932.



3934.

3933.

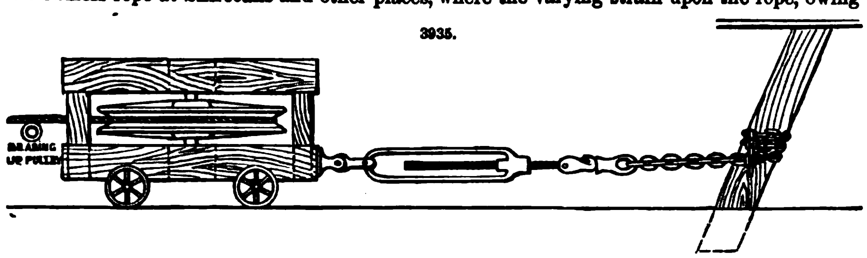


The curve on the chain-brow way is, as before described, of about 5 yds. radius, and at an angle of 118°, Fig. 3933. The ropes are taken round by two 4 ft. 6 in. pulleys, each inclining slightly towards the coming-on side. The way for the full tubs is laid nearly level, and for the empty a slight rise from the shaft; this arrangement, after many experiments, having been found to act most efficiently. The pulley wheels are made with a large flange on the lower side, Fig. 3934, to prevent the rope slipping off, and to enable the knot of the chain connecting the tub to the rope to pass easily into the trod of the wheels. The use of four pulleys instead of two at a curve of this description would enlarge the radius of the curve, and cause a smaller part of the surface of each wheel to be touched by the rope. Slow speed appears very necessary for working a curve by this system, for the jerk, which occurs when the tub, in passing round a curve, starts away after being stationary for a moment, would probably not fail to cause an accident if taken round at a much higher speed. A boy, placed near this curve for the purpose of taking off the chains at the fore end of the ingoing tubs, also attends to the curve when necessary.

Near the inbye end of the slant way there is a flat at which the tubs are taken off and put on, whilst the tubs passing to and from the terminus are in motion. The place is laid with flat sheets for a few yards, nearly on a level with the rails. The empty tubs are disconnected, and brought under the rope between two outcoming sets of full tubs. Points are laid on to the full way, and a full set or gang of two or more full tubs is put on, when the slackness of the rope indicates a long distance between two full sets.

Apparatus for Tightening Ropes.—The tightening pulleys, as used in this system of conveying small sets of one or more tubs by the endless rope, are fixed, and not similar to those used for the No. 1 endless rope at Shireoaks and other places, where the varying strain upon the rope, owing to

3935.

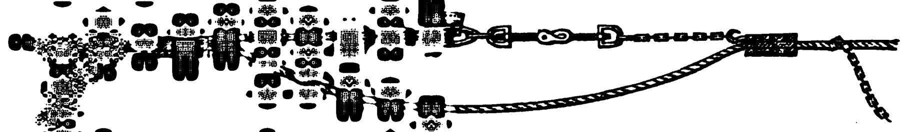


the set of tubs being at different parts of the plane, makes it desirable to have the tightening apparatus movable.

...me, to which a screw is attached. The screw is ... Fig. 3935. It is doubtful what strain is the effect ... es the following as what he estimates to be about

... hese experiments the dynamometer was attached ... t, and connecting the dynamometer by means of ... then applied to tighten the chain till the weight ... namometer.

3936.



... road, the slant way and chain-brow way being put ... ke the instrument upon the heavy gradient, it was ... and was first taken 297 yds. inbye on the empty

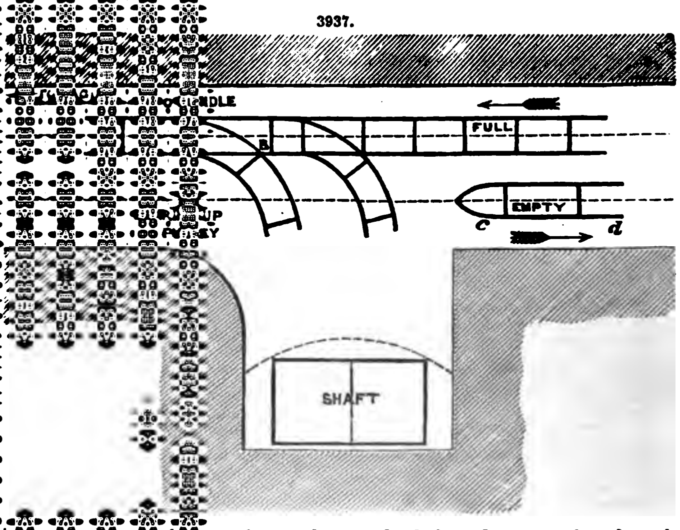
... thirty strokes a minute, being the speed at which ... It was found very difficult to get the same number ... made with the indicator. In going inbye there ... upon the level part of the main road, and sixteen ... t of 1 in 5·5. In coming towards the shaft there ... upon the level part, and twenty full and sixteen

... oint, given below, are the averages of the maximum ... ating to the extent of 10 cwt.

... ENTS WITH DYNAMOMETER.

Coming Outbye.			
Series.	Time.	Reading of Dynamometer.	Speed. Miles an hour.
	h. m.	cwt.	
A	12 0	46	1·35
B	11 59	44½	1·35
C	11 58	43	1·35
D	11 55½	40½	1·35
E	11 52	38½	1·35
F	11 50	34½	1·35
G	11 47½	..	..

3937.



... a minute, the speed of the tubs upon the plane is

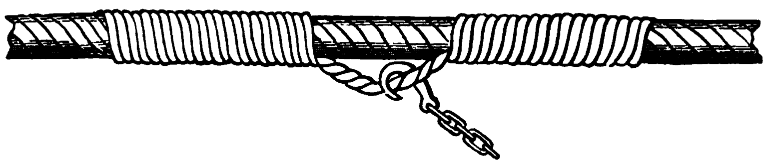
In the working of this engine road, the arrangement most worthy of note is that the braking of the engine is attended to by one of the hookers-on, by means of an iron rod, which extends from the engine to the terminus of the plane. The engine stands about 50 yds. from the plane, and is stopped in a moment when required.

There is a slight rise inbye about half-way along the engine-plane, and this necessitates the use of two chains for attaching each set of empty tubs; but as at the Meadow Pit, the heavier load of the full tub keeps the fore chain constantly tight, so that only one chain is needed in coming outbye. The chains at the fore ends of the empty tubs are taken off when the rise referred to commences, and are placed upon the out-going full tubs, Fig. 3937. The full tubs are put on two at a time, and empty tubs in sets varying from one to five tubs.

Scot Lane Pit, near Wigan.—At this pit an incline, 438 yds. in length, rising towards the shaft at an average gradient of 1 in 9, is worked by the endless rope.

The method of connecting the tubs to the rope at this pit is different to that in operation at the Bridge and other pits. Instead of the connecting chains being passed round the rope and thus secured, strong loops of hemp are fastened on to the rope by a wrapping of string, at regular distances apart. One hook of the chain is first attached to the tub, and the hook at the other end is then passed through the loop, as shown in Fig. 3938.

3938.



The tubs are sent along the plane one at a time, and as the gradient of the plane is very regular, only one chain is necessary for each tub. The heavy inclination causes the tubs to keep a constant weight upon the chain; on a light gradient the hook would probably be very liable to slip out of the loop. These loops are made of hemp, 1 in. in diameter, and last about four months; they are strong enough to draw twelve tubs at a time up the plane. They are fixed on to the rope, 17 yds. apart, thus making a regular supply of full and empty tubs necessary. Much less labour is required on connecting the tubs to the rope by this arrangement, but it would scarcely be so applicable on an irregular plane, where two loops would have to be provided for each tub or set of tubs. Although the rope passes $1\frac{1}{4}$ times round the driving wheel, the loops are formed to pass round without causing an inconvenience.

South Wales Endless-chain System, Brynddu Colliery, Glamorganshire.—Since the first part of this Report was printed the above colliery has been visited, and as the mode of conveying coals is unlike any before described, and as it is peculiarly adapted to the circumstances under which it is applied, it is thought desirable to give a short description of this method of working the endless chain.

DIMENSIONS OF ENGINE, BOILERS, AND ROPES.

Engine erected 1858.					
	Number of cylinders ..	1	*Chain* ..	Length of link ..	7 in.
	Diameter of cylinders ..	2 ft. 1 in.		Weight ..	440 cwt.
	Length of stroke ..	4 ft. 5 in.		General duration ..	16 months.
	Diameter of driving wheel ..	10 ft. 3 in.	*Rails* ..	Weight a yard ..	25 lbs.
	Consumption of coal a day ..	4 tons.		Length of each ..	9 ft. 0 in.
Boilers ..	Number ..	3 on surface 1 underground		No. of lines ..	4
	Description ..	Cylindrical		Gauge of ways ..	2 ft. 6 in.
	Length over all ..	30 ft. 0 in.	*Tubs* ..	Average weight (empty)	10 cwt.
	Diameter ..	6 ft. 0 in.		" " of coal contained ..	15 cwt.
Chain ..	Diameter of iron ..	$1\frac{1}{4}$ in.	*Work Done.*	500 tons a day of 10 hours.	

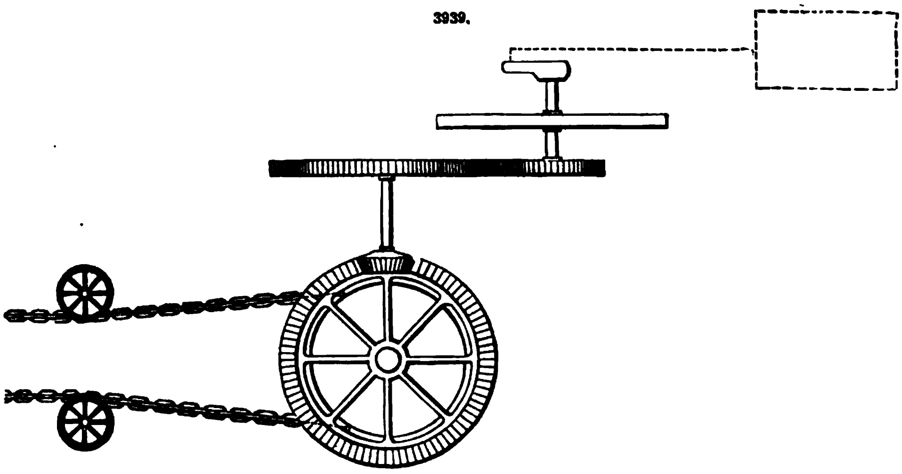
Description of Engine-plane.—The plane on which the chain is used is 600 yds. in length, and descends at the heavy gradient of 18 in. a yard from the shaft. The engine for hauling the coals up this incline is placed on the top of the plane, and works the chain by a horizontal driving wheel. The ordinary speed of the engine is about forty strokes a minute. The chain used is made of $1\frac{1}{4}$-in. iron, and has links 7 in. in length. The engine is horizontal. The arrangement of gearing for giving motion to the driving wheel will be seen in Fig. 3939.

The chain runs on rollers, which are placed 75 ft. apart. The connection between the chain and the tubs is made by a short chain 2 ft. 6 in. in length, made of $\frac{5}{8}$-in. iron, with an ordinary hook on each end. A tub attached in this manner to the chain is shown in the sketch, Fig. 3939. Only one tub is put on at a time, the distance between each tub being about 50 yds.

There are eight stations on the plane, at each of which the empty tubs have to be delivered, and full tubs brought away. This is done by a simple arrangement. Two long balanced arms, which are usually parallel with the roof of the wagon-way, are drawn down to the level of the

way when the tubs have to be put off or taken on at any station. The arms when brought down are level, and are so regulated by the counterbalance, and the weight hanging over the pulley,

3939.



as to be very easily moved. There are two arms at each station for both empty and full ways, and these are supported by a beam across the wagon-way. Between each two arms an iron plate is laid, on to which the tub runs when disconnected from the chain; the tub is then turned on the plate, and taken into the station. The chain travels at the rate of two miles an hour, and at this speed the tubs can be easily connected and disconnected at the stations, without stopping the engine.

In attaching an empty tub at the top of an incline, the tub is brought close to the commencement of the incline, and the short chain is connected to the tub in the first place, and then to the chain, and immediately after the last connection the tub is pushed forward on to the incline; when the full tub comes to the top of the incline, the chains are disconnected just at the time when the tub coming on the level takes the weight off the short chain.

Considering the very exceptional character of the conditions under which this system is worked, and its singular adaptability both to the heavy gradient and to the leading of coals from numerous stations, it is probably the most economical arrangement which could be here adopted.

Extracts from the Summary of the Report of William Cochrane, George B. Forster, John Daglish, Lindsay Wood, R. F. Matthews, Acting Members of the Tail-rope Committee; Emerson Bainbridge, Engineer to Committee.—It would be difficult for the committee who present the foregoing Report on Underground Haulage to recommend for general use any one of the systems reported on, since each is peculiarly, and advantageously, applicable to one condition or more of wagon-way; it was therefore thought desirable to take a general view of each of the systems, and endeavour, by considering their respective advantages, to give some idea of their comparative worth under the various conditions in which they exist.

Tail-rope System.—This system of conveying coal underground is most largely developed in the counties of Northumberland and Durham, where, after many years trial, it has now attained a high degree of perfection.

One of the leading features of the tail-rope system is, that it can be applied under almost any condition of wagon-way, the crookedness of the way, irregularity of gradient, and numerous stations and branches, forming no obstacle to its effective working.

On a single road the tail rope is generally applied;—

1. When the gradient of wagon-way dipping inbye is not sufficient to cause the empty tubs to draw a single rope after them.

2. When the gradient dipping outbye is insufficient to make the tubs self-act.

3. When, as at North Hetton, the full tubs coming outbye will not pull the single tail rope after them.

This single tail rope (that is, a tail rope working without a main rope) is in operation in the main coal seam at North Hetton Colliery, where the engine-plane rises from the shaft, and the rope passing round a sheave at the inbye end of the plane, draws the empty tubs up the bank, the full tubs being braked down. This arrangement is usually adopted when the tubs will not self-act, and where it is desirable to have the engine near the shaft.

The tail rope is usually applied on branches to supersede horses.

The following are the conditions of the five tail-rope planes reported on;—

North Hetton—Two main roads, with branches worked both to the rise and to the dip, and with curves on the main road and on the branches.

Seaham—Engine-plane, with slight curves, rising towards the shaft.

Seaton Delaval—Engine-plane, with slight curves, level.

Harraton—Engine-plane, with sharp curve, falling towards shaft; one branch worked.

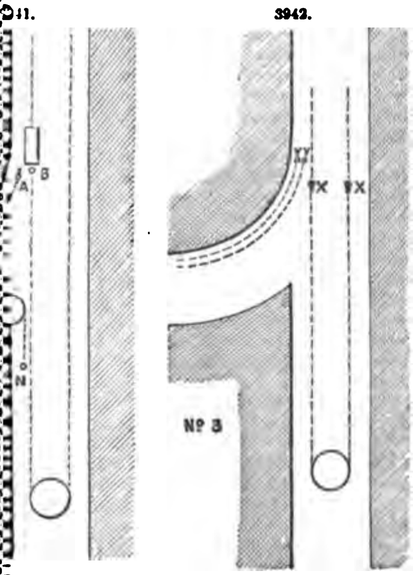
Murton—Engine-plane, with curves, rising towards the shaft.

...gement of engine and appliances for work-
... with two drums, worked on the second
...and put in and out of gear by shifting
...used for connecting them with the engine.
... the tail rope. At the inbye end of the
...road, there is a sheave round which a tail
...ches, are brought to a station on the main
... rope is brought up to the station, whilst
... of tubs, which usually numbers from thirty
...gine, and in coming outbye, the main-rope
...ly applied to the loose drum to prevent the

...tions from the engine, four drums are gene-
...ch way.
... any angle and at a comparatively small
...us, have perhaps the minimum radius at
...at the sets pass round these curves at the
...rarely happens, is sufficient proof of their
...at the Harraton curves are doubtless the
...siderable saving in the wear and tear of

...methods of attaching the branch-rope ends
...Figs. 3940 to 3942; in Figs. 3940, 3941, the
...branch end, but Fig. 3942 shows the set at

...41. 3942.



...der the rails, round which one end of the
... into the branch, the rope end C replaces D
... the tail rope.
...the end A replaces B, and the end B of the
...n attached to N. This method of attaching
..., can be worked without a winch, which is

...y different course is pursued. When a set
...t one of the branch ends changes the ropes,
... empty set. The position of the ropes is so
...ackles on the main and tail ropes are just
...etch, when the rope ends X X are replaced

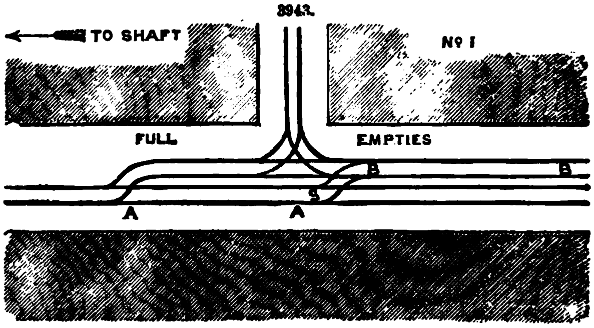
...the others, since in Fig. 3942 no time is lost
...he branch end before the set is ready at the
...addition to the time required to change the

... the ropes is preferred; there is only one
...e quantity of coal from the two ways, that
..., it is difficult to tell which of the ways is
...nches are worked, and a large quantity of
... the ropes is preferable to No. 3.

The labour at a branch end can be managed by one boy; when Nos. 1 and 2 methods are adopted, a *run-rider* is of some service, as two connections have to be made, and at points often some distance apart; with No. 3 method a run-rider is not so necessary, but is generally employed. When two branches are worked opposite to each other the same amount of labour is sufficient. Run-rider is the name given in the North of England to a man or boy who rides on the last tub of the set, for the purpose of signalling to the engineman in case of an accident; he also assists in connecting the ropes to the set.

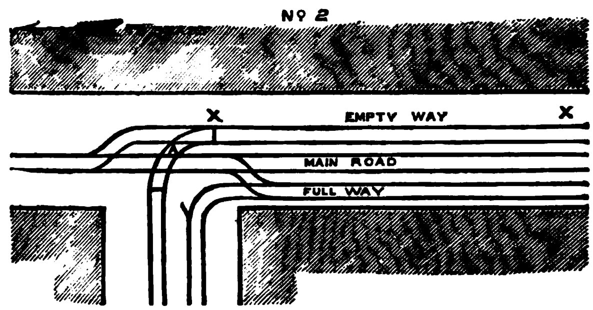
Three methods of taking the ropes round curves will be seen on the sketches, Figs. 3943, 3944. In No. 1 the curve has a large radius, and the tail rope is taken round a single sheave, and along a narrow place, a pillar of coal supporting the roof between it and the curve. The curve in No. 2 is of less radius, and no pillar is left. In No. 3, which is generally adopted on very short curves, instead of taking the tail rope round a single sheave, both ropes are taken round the curve by a number of sheaves.

3943.



The tail rope is often applied to work a plane with no branches, but with one or more stations on each side of the main way, in which case only one set of ropes is used. Murton and Seaham planes present examples of this arrangement. These stations are usually worked by one of the two methods shown, Figs. 3943, 3944.

3944.



In No. 1, which represents the arrangement of the North Hetton stations, the ropes are knocked off the empty set in going in at the points A A, opposite to which the full set stands ready to go out. A gentle fall in the way causes the empty tubs to run forward, and they are turned by the switch S into the siding B B.

In No. 2, which shows the Seaham Colliery arrangement, the middle way is the main road; the empty tubs, having been brought into the siding X X, are then brought round the curve A, which consists of two movable rails. When the full set comes out these rails are removed. With this arrangement the drivers have to cross the main road every time they take the empty tubs inbye; this is avoided with the stations worked as at North Hetton.

The engine and boilers requisite for working the tail-rope system are usually arranged in one of the three following ways;—

1. The engine and boilers both on the surface, the rope being taken down the pit in wooden boxes.

2. The boilers on the surface, and the steam-pipes taken down the shaft to the engine underground.

3. The engine and boilers both underground.

Endless-chain System.—The experiments on the endless-chain planes, which follow next in order to those on the tail-rope planes, were all made at the collieries at Burnley, under the management of Mr. W. Waddington.

Although the endless-chain system of leading coals has been in operation at Burnley, and other parts of Lancashire, for a great number of years, it has until lately been very little known in the North of England.

Like the tail-rope, the endless-chain system is adaptable to every condition of wagon-way, but differs from the former system in the following important items;—

1. As a general principle, it may be stated that when the two ends of an undulating plane are at the same level, the power required to work such a plane will be very little more than if the plane were perfectly level.

2. A heavy gradient can be worked safely and efficiently.

3. Since the chain will only work safely in a straight line, or, at the most, round a slight curve, every sharp curve upon the planes necessitates the erection of two pulleys, in order to direct the chain to another course, and requires the attention of a man or boy.

The system is very extensively in operation on the surface at Burnley, and the slow speed at which the tubs are conveyed not requiring a very carefully-laid wagon-way, they generally lay the way upon the uncut sod, only making embankments or erecting gearing when a stream or deep defile has to be passed over. The surface of the country is very hilly in this district, and it is

therefore more economical to carry the coals from the pits to the canals and railways in tubs by the endless chain, than by wagons on ordinary railways.

The endless-chain planes reported on are as follows;—

Hapton Valley.—Underground chain road.	*Rowley.*—Underground chain road.
" " Surface chain road.	" Surface chain road.
Gannow, top bed.—Underground chain road.	*Clifton Hall.*—Surface chain road.
" low bed.— " "	

Methods of Applying the Endless-chain System.—The engines working the endless chain at Burnley are nearly all of one class, namely, double 12½-in. cylinder vertical overthrow engines. Motion is transmitted to the wheel driving the endless chain by means of toothed gearing; the engine usually goes at a speed of about eighty strokes a minute, and works the driving wheel on the third motion.

The engine-plane consists of two lines of rails, one for the full, and the other for the empty tubs, the tubs moving in opposite directions. The lines are generally laid just near enough together to allow a few inches play between the tubs on the empty and full ways.

In working a straight main way (that is, with no branches), such as the Rowley and Hapton Valley surface planes, by the endless chain, the driving power is generally placed at the higher end of the ginney road. The only wheels requisite for working the chain are the driving wheel at one end, and a tail or return sheave at the other end of the plane; between these two points the chain rests upon the full and empty tubs, the distance of which apart, and the speed at which they are moved, vary according to the quantity of coal passing along the plane, the distance between the tubs being from 10 to 30 yds., and the speed from one to three miles an hour.

The wheels round which the chain passes at the two ends of a ginney road are usually 3 ft. in diameter. The driving wheel, as used at Burnley, generally consists of an ordinary sheave, round which a piece of boiler-plate, about 10 in. wide, is fixed, and to this are attached about twelve steel or iron feet, on which the chain rests; these feet are renewed, as required, and thus the chain never touches the plate. The method invariably adopted at Burnley, in order to get friction sufficient on the chain to prevent it slipping round the driving wheel, is by passing it 2½ times round the wheel; at Towneley Colliery it is passed 4½ times round. At the Baxenden Collieries, near Newchurch, ordinary sheaves, with forks about 12 in. apart fixed in the trod, are used as driving wheels, the chain only passing half a turn round, and the horizontal link of the chain fitting into the fork. These wheels, which are much preferred at Newchurch, were formerly used at Burnley, but are now altogether abandoned in favour of the bevel-faced wheels. The return wheels, at the other end of the ginney road, are just common 3-ft. sheaves, round which the chain passes half a turn.

When there is a curve in a single chain road, either the same chain is taken round the curve by two wheels on different shafts, arranged like the self-acting curves on the surface, or there are two pulleys on the same shaft on which are different endless chains. If there be no branch way from the curve, the former method is usually adopted, and the road is so formed that the tubs will leave one chain, pass round the curve, and connect themselves to the other chain, without any assistance; but as this cannot be depended upon, attention is always necessary at a curve. At the Burnley Collieries there is generally a branch end at every curve.

There are only two self-acting curves in the Burnley district, both of which are on the surface. These curves generally work very well, and without the occurrence of any accident; but as a badly-greased tub, or a tub getting off the way, causes some damage, if not looked to, they are always kept in sight, that at Padiham being within a few yards of a man constantly working at the same place, and that at Towneley within view of the terminus of the ginney road. With very carefully-laid rails, and due attention to the exact point at which the rise or fall of the way should be, curves might be worked at almost any angle, and with very little attention.

As far as hitherto proved, it may be laid down as a rule in the working of the endless-chain system, that all curves underground require labour, and thus it is generally desirable either to have a branch or branches from the curve, or to pass the tubs round by the self-acting method described, which could be managed by a boy at 1*s.* a day. It has yet to be shown to what extent curves can be worked by this system without pulleys. At the Baxenden Collieries, in Lancashire, a curve of about 15 chains radius has lately been commenced on the surface, and is found to work very satisfactorily.

In working branches the chain passes round a wheel at the branch end, which transmits motion to another pulley or pulleys, either on the same shaft, or on another shaft a short distance off; when the pulleys are on another shaft, motion is given either by mitre-gearing and shafting, or by a short endless chain; at Towneley motion is transmitted by continuing the same chain past a branch end, and taking it over a pulley which works the branch by means of vertical gearing. The pulleys on the main road for working the branches are generally so arranged, that they can be put out of gear, in case of any accident or slackness of work, whilst the other chains continue in motion.

Though it would appear rather difficult to work a branch on each side of the main way to the same point, since the full tubs from one of the branches cross over the course of the empty tubs on the main way, and the empty tubs from the other branch cross the course of the full tubs, the slow speed at which the tubs travel prevents this from causing any inconvenience.

Another mode of working branches is that carried out at Marsden Colliery, near Burnley, where two branches are worked by one chain passing round a single pulley. This is the only one of all the methods of working branches, in which one branch cannot be put out of gear without stopping another.

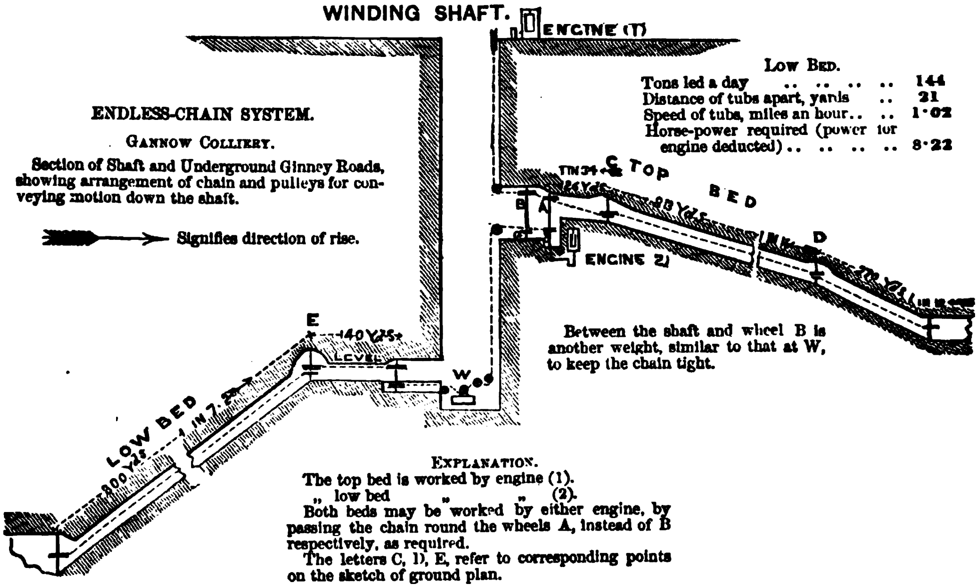
In each of the arrangements mentioned for working branches, it is necessary that the tub should

leave the chain at the station—becoming attached to it again immediatelybeyond the pulley; and the attending to the re-attaching of the tubs at the branch ends in this way, is one of the chief items of labour in the cost of leading coals by the endless chain. The mode of working branches usually adopted at Burnley, is to have shafting and mitre-gearing under the flat sheets, sometimes, as at Hapton Valley, working two branches from the same mitre-wheel.

In working branches by the endless chain no regular curves are necessary, since at the branch ends the termini of ginney roads are usually laid with flat sheets upon which the tubs are turned; the small tubs used at Burnley can be turned so quickly, that there is often not more than 10 ft. from the end of one ginney road, to that of another at right angles.

At Gannow and Rowley Collieries the underground endless chain is worked by an engine on the surface, the chain at the latter colliery being attached to the winding engine. The way in which the chain is conveyed down the shaft is very simple, and is shown in the section of the Rowley and Gannow underground planes, Fig. 3945. Whilst a $\frac{5}{8}$-in. chain is used in the workings, a $\frac{3}{4}$-in.

3945.



chain is used in the shaft, the latter being usually kept tight by passing the chain, at the bottom of the pit, round a sliding pulley, to which a suspended weight is attached. A sufficient proof of the efficient working of the chain in the shaft, and of the very small amount of power requisite to take it down the pit, is given by the results of the Rowley experiments, which show that only 0·54 horse-power is required to take the chain 150 yds. down the shaft, and 27 yds. from the bottom of the shaft to the main driving wheel.

As compared with the tail rope, the maximum advantage of the endless chain, as far as the power requisite to drive the system is concerned, is probably realized on an undulating plane, the ends of which are at the same level. A very good instance of the counterbalancing effect of an undulating road is shown on the Rowley plane, which is 1980 yds. long, the highest point on the plane being 145 ft. above the lowest, and the average gradient 1 in 68, fall for full tubs; here only 5·16 horse-power is required to work all the tubs, and chain suspended on the tubs. Another instance is at Rowley Colliery, where the average gradient of the planes is sufficient to cause all the tubs to self-act. The planes at this colliery, however, are kept connected to the engine for reasons explained in the report on the Rowley plane.

The chief feature of the self-acting planes, as worked by the endless chain, is that *a regular gradient is unnecessary*, a heavy dip at one part of the plane being sufficient to cause the tubs to self-act. Thus a plane may have a rise for some distance for the full coals, and yet, since there is continually a load upon the heavy part of the plane in favour of the full tubs, self-act. Thus planes can self-act by the endless chain which could not possibly do so with a rope.

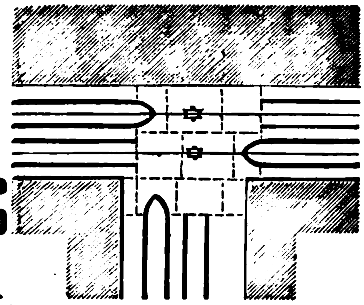
At Burnley, the force of gravity on self-acting planes is sometimes made to work a level plane at the top of the self-acting plane.

Stations are worked by the side of the main chain roads, simply by having small pulleys fixed to the roof, Figs. 3946, 3947, opposite to the station, on which the chain is placed when tubs are required. There are generally iron flat sheets at the stations over which the tubs are brought from the opposite side of the way. A tub, after having been passed over the flat sheets, is allowed to be altogether moving on the rails again before it is touched by the chain, in order that the chain may be sure to reach it in the centre.

The method of leading coals by the endless chain as adopted at Clifton Hall Colliery, which consists in moving the tubs by the weight of the chain alone, there being no forks on the tubs, can

... level or nearly level, and where the system

3947.



... oal is closely allied to the tail rope, but has ... planes, only to a limited extent.

... e given on page 1862, are reported on, and ...:—

... le way, worked by rope at quick speed, the

... ngle way, worked by rope at quick speed, the ... ins secured to sockets in the rope.

... socket connection at slow speed.

... uble way, and sets of full and empty tubs ... ng connected to the rope by clamps.

... y.

...—In the application of this system as adopted ... used, and the rope moves on rollers in the ... one way and brought out on the other.

... y, or by several wheels, round which the rope ... kept constantly tight by having the return ... hanging in a staple, is attached.

... clamp, which is held by a boy sitting in the ... sed; the clamp in use at Shireoaks differs in ... nderhill planes. As these planes rise in one ... planes undulating, two (one for each end of ... two men or boys necessary for each set, the use ... advisable.

... n one of the California ways, to thirty-one at ... about the same as with the tail-rope system.

... the set, but on one of the California planes ... be applied to the last tub of the set, in going ... sets are run together on the plane, this assists

... d Eston, motion is given to the rope, which is ... ulley, as at Shireoaks, and the set of tubs is ... chains, which are secured by means of a hook ... m, owing to the undulations of the plane, two ... hilst at Eston, where the roads are short and ... ain, which is simply hooked on. The Eston ... short sidings, and could probably be economi- ... re there is a good deal of branch work. The ... tail-rope system, and of running one part of ... bly the best arrangement of the system, and ... d under some circumstances to supersede the

... coming the friction of the brake on the loose ... d.

... applicable where curves, and the working of ... p connection a slight curve is worked on one ... instances are known of curves occurring on ... but in all probability they could be managed ... e being the only obstacle to their efficient

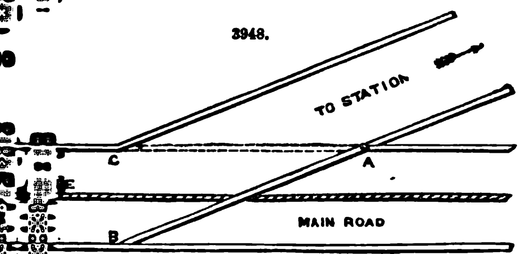
... rhill, would allow it to pass round curves of ... l, which consists in having a double way, and ... e time at a speed of 2½ miles an hour, cannot

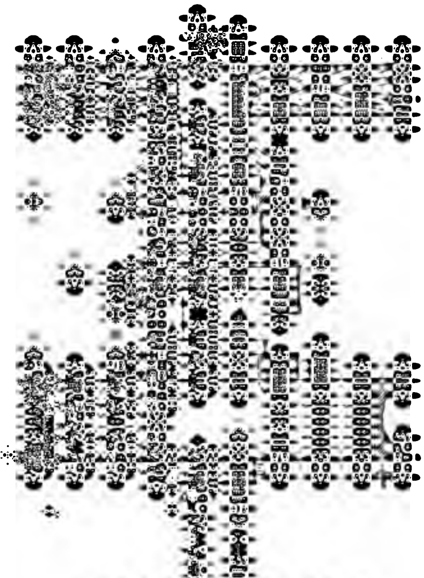
LAGE.

our, each set being connected by a separate clamp for driving the rope is obtained at this colliery by de tight by having two of the wheels placed on a here the load varies, this method of tightening the

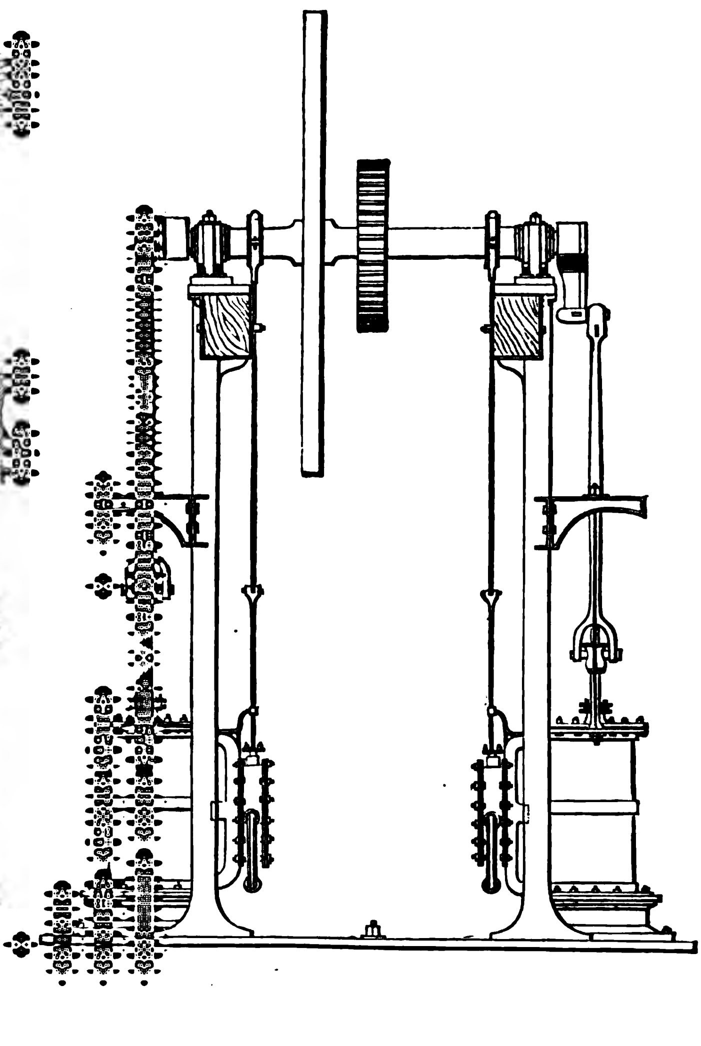
s far as hitherto developed, is its inapplicability he tightness of the rope precluding the possibility . A series of clip or friction pulleys, with discon- , but in this case, as the tubs could not go round of tubs would have to be disconnected from the into the branch way, where it would be attached possible under exceptional conditions of wagon- d were the socket connection adopted, the clamp t.

3948.





3950.



explained in the Shireoaks report, the set when going in is disconnected from the rope before reaching the station, and the inclination of the way is sufficient to take it to the end of the plane. It will thus be seen that on the Shireoaks plane, stations could not be worked under other conditions than those described. They might, however, be arranged like the tail-rope stations, Fig. 3943, and by having the points leading from the main way to the siding movable, as shown, Fig. 3948, the difficulty of the rope being endless would be obviated. Here the rail A B moves on a pivot at A, Fig. 3948; if the set had to go into the station, this rail would be placed as shown, the rope passing *under* the rail; if it were intended for the main road, the rail would be put into the position A B. More labour would be necessary in leading coals from stations in this way than is requisite with the tail-rope system, which is very well adapted for the purpose.

As this system is capable of conveying a large quantity of coals on a single way, the double way as adopted at Shireoaks may be considered quite unnecessary, and can only be recommended where sockets are used and run-riders dispensed with, and where the inclination of the engine-plane is such that one set of tubs will give assistance to another throughout the greater part of the plane.

Fig. 3949 is of an effective arrangement of driving wheel at Hapton Valley Colliery. Fig. 3950 is of the vertical double engine at Hapton Valley and Rowley Collieries. See AGRICULTURAL IMPLEMENTS, p. 29. BRAKE. COAL-CUTTING MACHINE. COAL MINING. COAL WASHING. DRAINAGE. DYNAMOMETER.

HAUNCH. FR., *Esselle;* GER., *Gewölbeschenkel;* ITAL., *Coscia della volta;* SPAN., *Riñon de una bóveda.*

That part of an *arch* between the key-stone and the springing is sometimes termed the haunch of an arch; the two parts between the crown and the springing are hence called the haunches of the arch. See ARCH.

HAWSER. FR., *Aussière, Grelin;* GER., *Tross, Schlepptau;* ITAL., *Gherlino;* SPAN., *Guindaleza.*

A hawser, or halser, is a small cable; or a large rope, in size between a cable and a tow-line. The sizes and lengths of hawsers and warps, according to Lloyd's Rules, are given in the following tabulated form.

Ship's Tonnage.	Hawsers and Warps.					Ship's Tonnage.	Hawsers and Warps.				
	Stream.		Hawser.	Warp.	Length.		Stream.		Hawser.	Warp.	Length.
	Chain.	Rope.					Chain.	Rope.			
tons.	ins. 16ths.	inches.	inches.	inches.	fathoms.	tons.	ins. 16ths.	inches.	inches.	inches.	fathoms.
50	0 7	5	3	..	90	600	0 13	9·5	7	4	90
75	0 7	5	3	..	90	700	0 14	10	8	5	90
100	0 8	5·5	3	..	90	800	0 14	10	8	5	90
125	0 8	5·5	3·5	..	90	900	0 15	10	9	5·5	90
150	0 9	6	4	..	90	1000	0 15	10	9	5·5	90
175	0 9	6	4	..	90	1200	1 0	10	9·5	6	90
200	0 10	6·5	4	..	90	1400	1 0	10	10	6	90
250	0 10	7	5	..	90	1600	1 1	11	10·5	6·5	90
300	0 11	7·5	5·5	..	90	1800	1 1	11	11	7	90
350	0 11	7·5	5·5	..	90	2000	1 2	11	11	7	90
400	0 12	8	6	..	90	2500	1 2	12	12	8	90
450	0 12	8·5	6·5	..	90	3000	1 3	12	12	8	90
500	0 13	9	7	..	90						

HEART-WHEEL. FR., *Roue en cœur;* GER., *Herzscheibe;* ITAL., *Ruota a cuore;* SPAN., *Rueda de corazon.*

See GEARING.

HELIOSTAT. FR., *Heliostat, porte-lumière;* GER., *Heliostat;* ITAL., *Eliostata;* SPAN., *Heliostato.*

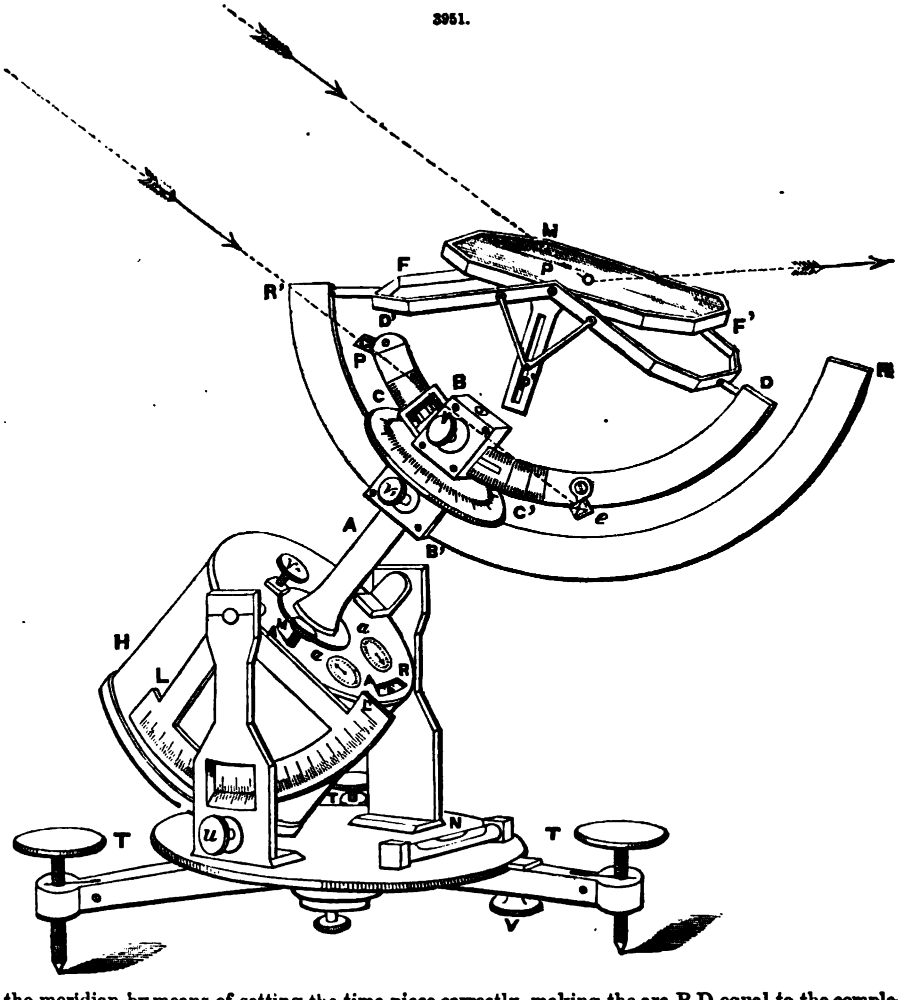
A heliostat, Fig. 3951, is an instrument by which a sunbeam may be introduced into a dark room, and, by means of clockwork, kept directed to, and upon a fixed point. A heliostat, suitably arranged, is often employed in trigonometrical surveying. See GEODESY.

Silbermann's Heliostat.—This apparatus is mounted on a plate which moves round a vertical axle, and is adjusted by means of a level; the position of the axis of the time-piece is made to correspond with the latitude of the place by means of an arc and the adjusting mechanism *u*. Supposing now that this axis has been placed in the meridian and that C C' represents the dial, D D' a limbus, the centre of which is in O, and in the plane of which is carried a needle pointing to the correct hour indicated on the dial, then the plane of the limbus will pass through the centre of the sun. The limbus D D', which moves in a box B, in which it may be fixed at any position by means of the screw V, is divided into parts, and by making the distance from the centre of the box to the end D of the limbus equal to the complement of the declination, the line D O M will represent the direction of the rays of the sun.

The mirror is carried by a jointed or articulated rhombus, the diagonal P P' of which is vertical to the plane of the mirror; this rhombus is constructed in such a manner that one of its sides is parallel to D O, that is to say, parallel to the rays of incident, whence the reflecting rays become parallel to the other side of the rhombus. By means of a second limbus R R', which can be fixed in its position by the screw V', the inclination of these reflecting rays can be changed; and in order to bring these rays into all possible azimuths, the limbus R R' is carried upon a hollow shaft A, which contains, besides the transferring mechanism of the time-piece, the moving

mechanism for the needles and the other limbus D D′; this shaft A can be fixed by the hand-screw V″. A pinule *e* and a screen P are arranged in such a manner that the chord *e* P is parallel to O D, whence the ray that passes through *e* arrives always at P as soon as the apparatus is regulated. This circumstance or condition may be used, on the other hand, for placing the axis in

3951.



the meridian by means of setting the time-piece correctly, making the arc B D equal to the complement of the declination, and by turning then the apparatus upon its base-plate until the ray of the sun is seen to pass from *e* to P. A similar arrangement is adopted in M. Foucault's heliostat, and may be fixed to all other ones. Fig. 3951 is of a heliostat made by Elliott Bros., London.

HELIX. Fr., *Hélice ;* Ger., *Schraubenlinie, Spirale ;* Ital., *Elice ;* Span., *Hélice.*

A helix is a curved line of double curvature, a spiral line, as of wire in a coil. A knowledge of twisted surfaces, that have helices to guide the generating straight line, is indispensable in designing *screw-propellers.* The equations of the helix are $x = a \cos . \frac{z}{h}$ and $y = a \sin . \frac{z}{h}$; x, y, z, being the co-ordinates of any point in the twisted surface. See Archimedian Screw.

HINGING. Fr., *Ficher ;* Ger., *Aufhänger, durils ein Gelenk verbinden ;* Ital., *Gangherare ;* Span., *Embisagrado.*

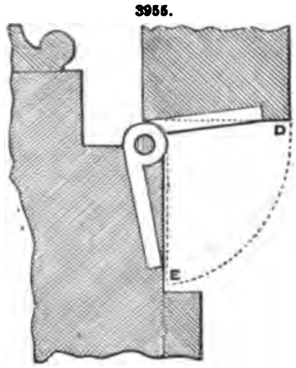
Hinging is the art of connecting two pieces of metal, wood, or other material together, such as a door to its frame; the connecting ligaments that allow one or other of the attached substances to revolve are termed hinges. There are many sorts of hinges, among which may be mentioned, butts, chest hinges, coach hinges, rising hinges, casement hinges, garnets, scuttle hinges, desk hinges, screw hinges, back-fold hinges, centre-point hinges, and so on. To form the hinge of a highly-finished snuff-box requires great mechanical skill; but few of the best jewellers can place a faultless hinge in a snuff-box.

There are many varieties of hinges, and hence there are many modes of applying them, and much dexterity and delicacy are frequently required. In some cases the hinge is visible, in others

ges require not only that the one hinged part
rt shall be thrown back to a greater or lesser
f methods of *hinging*.
o a right angle, as in Fig. 3953. Figs. 3954,
ors to open to an angle of 90°. Figs. 3958,
ht angles, and to have the hinge concealed.

3954.

3955.



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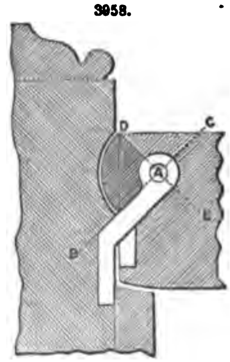
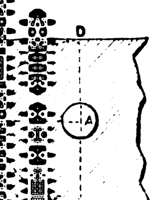
3958.





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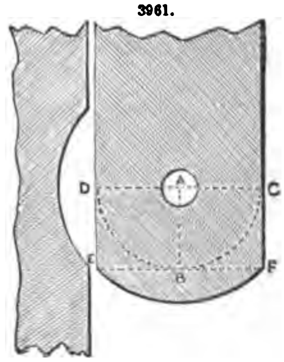


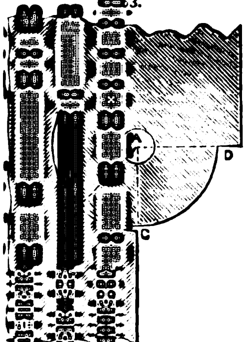
inge A, and light portion requires to be cut out
. Figs. 3960, 3961, illustrate an example of a
lding back against the wall in either direction.
ring the line of the wall, which represents the
when folded back against the wall in either
lar to E F, which make equal to E B or B F,

; draw A D, making an angle of 45° with the
b, meeting D A in A the centre of the hinge;
D C.

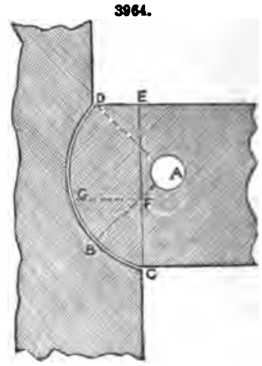
...ntre-pin hinging, opening through a quadrant. ... In this, as in a previous case, there is a space ... folded back. In Figs. 3962, 3963, as well as in ... door and the wall.

...A; draw E C and make $C F = \frac{3}{2} D E$; draw F G ... by the line B F, meeting D A in A; then A is ... the door, Fig. 3964, is folded back, that the point

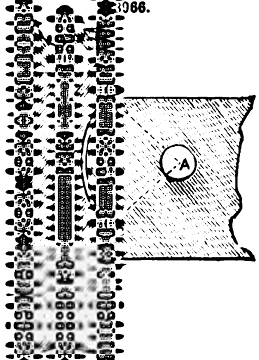
...3.



3964.



...966.



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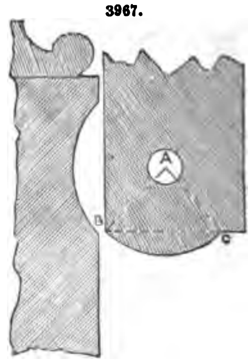

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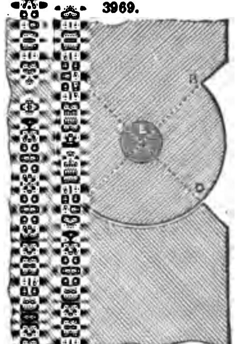

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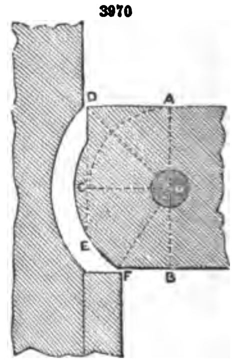


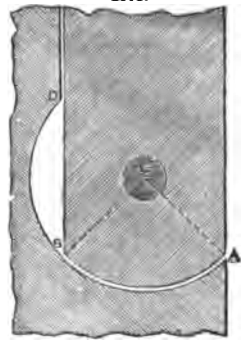
...972, 3973; and Figs. 3974, 3975, are examples of ... ailed describing.

... which has a bead B closing into a corresponding ...

...lly into the styles, the knuckles of which form a ... this case the beads on each side are equal and ... centre.

...kle of the hinge forms a portion of the bead on the

3973.



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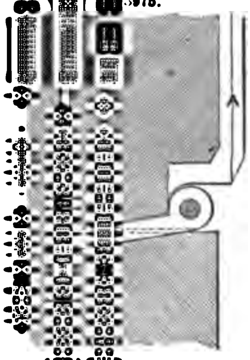

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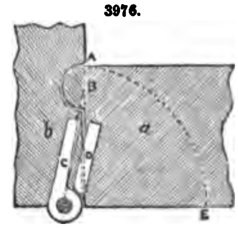

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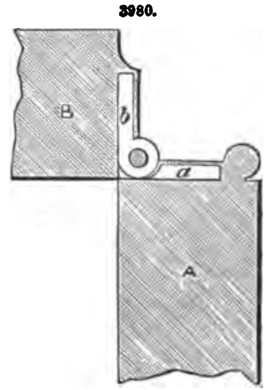

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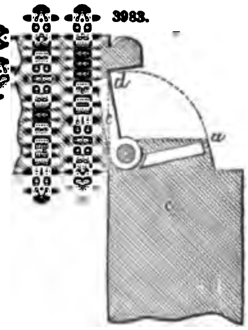

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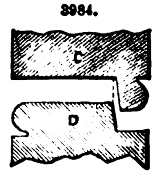
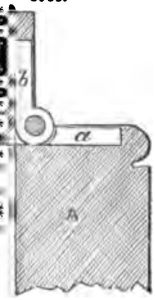


...he style D. In Figs. 3985 to 3987, the beads are
...when the centre of the hinge is in the middle of

3986.



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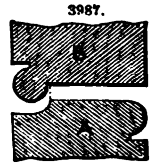

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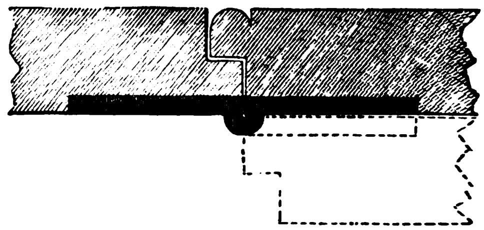
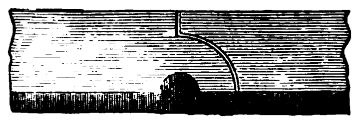


3991.



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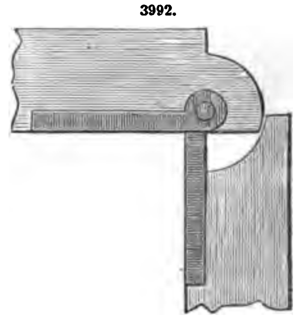
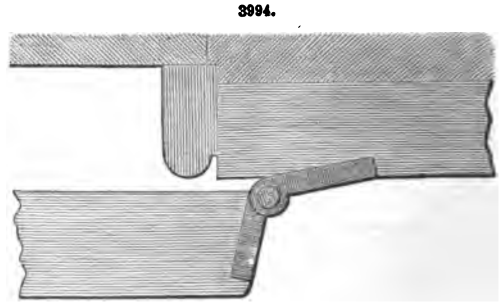


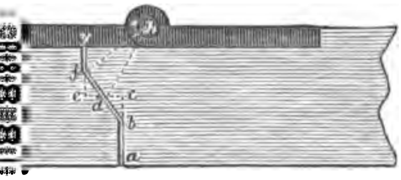
3994.



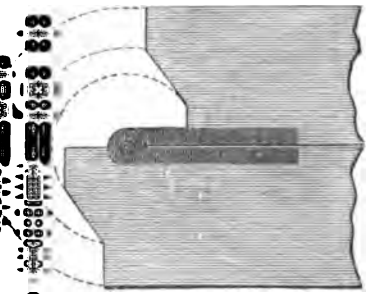
...ging a back flap when it is necessary to throw the
...oint is given, Figs. 3991, 3992.
...nary mode of hinging shutters to sash-frames.

...ployed when the flap on being opened has to ...ng is used on the doors of pews, to throw the

3997.



3998.



...bate when the hinge is placed on the contrary ...joint on the same side, $a\,c$ the line of joint on ... Bisect $e\,c$ in d, and join $d\,h$; on $d\,h$ describe ... $f\,b$, cutting $a\,c$ in b, and join $a\,b$, $b\,f$, and $f\,y$,

...PAN., *Azadon*.

... a *hoe*, which is an instrument for cutting up ...ite of iron, with a handle, which is set at an ...

...*erricello, monta sacchi*; SPAN., *Aparejo para* ...

...R., *Elevator für Hopfen*; ITAL., *Montaluppolo*; ...

...engine, operated by one horse or more, is a ...on into a rapid one, it effects the lighter ... There are many kinds of those easily-formed ...by that ingenious mechanic, the late James ...RICULTURAL IMPLEMENTS, p. 12. ARRASTRE, ...

... FR., *Cheval de force, et d'autre unités des* ...*inheiten*.

...which the capabilities of steam-engines and ...e-power is estimated 33,000 lbs. raised 1 ft. ...0 units of work a second. One HP., or one ...mmes raised a mètre in the direction of the ...ry nearly equal to 361 HP British. See ... BRAKE, p. 597. DETAILS OF ENGINES.

...*aud*; GER., *Winderhitzungsapparat*; ITAL., ...

...., *Obice*; SPAN., *Obus*.

... *howitzer*; it has a chamber, and is intended ...charges; it is sometimes so short that the ...d.

...PAN., *Cubo*, or *pina*.

...of a pin; as a crank-pin *hub*. This term is ...

...*que*; GER., *Hydrant*; ITAL., *Ramo di tubo*; ...

HYDRAULICS. FR., *Science hydraulique;* GER., *Wassermaschinenkunde;* ITAL., *Idraulica;* SPAN., *Hidráulica.*

Hydraulics is that department of engineering which treats of fluids in motion, especially of water, which in motion presents itself in four different ways; as passing out of a reservoir; flowing in a bed; acting as a motor; and in a passive state raised by machines.

There are two quantities to be found in all calculations relating to this science—the weight of water and the intensity of gravity. These quantities are variable, but almost always supposed constant. What follows will enable us to judge of the error which may result from this supposition.

When water is entirely pure, and is taken at its *maximum* density, it weighs 62·4491 lbs. a cubic foot: such is its *specific weight.* It may vary from three causes. The most powerful is the temperature. We know that heat expands bodies, and this diminishes their density or specific weight. From accurate experiments, the density of pure water, at different degrees of Centigrade and Fahrenheit thermometers, would be as indicated in the following Table;—

Temperature.		Weight of a cubic mètre.	Weight of a cubic foot in lbs.	Temperature.		Weight of a cubic mètre.	Weight of a cubic foot in lbs.
Centigrade.	Fahrenheit.			Centigrade.	Fahrenheit.		
		kil.				kil.	
4	$39\frac{1}{5}$	1000·	62·449	20	68	998·24	62·339
6	$42\frac{4}{5}$	999·95	62·446	25	77	997·99	62·268
8	$46\frac{2}{5}$	999.87	62·441	30	86	995·73	62·182
10	50	999·72	61·432	50	122	987·58	61·673
12	$53\frac{3}{5}$	999·54	62·420	100	212	956·70	59·745
15	59	999·14	62·396				

Below 4° Centigrade or 39° Fahrenheit, the density, instead of continuing to increase, diminishes; this diminution, at first very slow, rapidly progresses towards the limit of congelation, and the weight of a cubic foot of ice is only 58·078 lbs.

The effects of pressure are much less sensible. Water was, for a long time, considered wholly incompressible; but experiments have shown that, under very heavy loads, it is really compressed, although but a very small quantity; about 0·000046 of its volume under the weight of one atmosphere; that is, under a pressure represented by the height of a column of mercury in a barometer, a height estimated at 29·922 in., and which is equivalent to the height of a column of water about 33·793 ft. But as, in common practice, we shall not have to calculate upon such depths or heights of water, we may, without sensible error, entirely neglect the effects of pressure.

What proceeds from saline or earthy substances contained in the waters which run on the surface of the globe, may also, in most cases, be omitted, the specific weight of the water of rivers being only one or two ten-thousandths greater than that of distilled water, which is taken as the standard of perfectly pure water. Professor Boisgaraud found, by many trials, made with great care, $1000^{k}\cdot149$ for the specific gravity of the water of the Garonne, that of distilled water being 1000 kilogrammes to the mètre, or 62·449 lbs. to the cubic foot. Brisson has nearly an equal result for the Seine. Moreover, a mass of water, when surrounded by air, loses, like all other bodies, a part of its weight equal to the weight of air whose place it occupies; and this loss, which is seldom below $\frac{10}{100000} = \cdot 00010$, may be even $\frac{13}{100000} = \cdot 00013$.

Finally, in our mean temperatures, and according to different circumstances, the weight of a cubic foot of water will be only from 62·35 lbs. to 62·39, or the cubic mètre from $998^{k}\cdot 4$ to 999^{k}. We shall, however, constantly admit 1000^{k}, this value rendering the conversion of cubic mètres of water into kilogrammes, and *vice versâ*, extremely easy.

Experiments made at the observatory of Paris, gave $0^{m}\cdot 9934 = 39\cdot 128$ in., or 3·2606 ft., for [illegible] length of a pendulum vibrating seconds, this length being reduced to the level of the sea. Wh[illegible] we conclude that, in that place, a heavy body descends $4^{m}\cdot 9044 (= \frac{1}{2} \times 0\cdot 99384\pi^2) =$ [illegible] during the first second of its fall. If, at the end of that time, gravity ceased to act upon it, [illegible] continue to descend, but with a uniform motion, running through double the space, or [illegible] a second; this number, which expresses the velocity impressed by gravity in the unit [illegible] represents, for Paris, the intensity of that accelerating force; we generally designate that in[illegible] or velocity by g, the initial letter of the word *gravity*. (See our article GUNNERY.) g aug[illegible] with the latitude, and diminishes with the elevation above the level of the sea, and ge[illegible] we have the empirical formulas;—

$$\text{In feet} \quad g = 32^{ft}\cdot 16954\,(1 - 0\cdot 00284 \cos. 2l)\left(1 - \frac{2e}{r}\right),$$

$$\text{In mètres} \quad g = 9^{m}\cdot 8051\,(1 - 0\cdot 00284 \cos. 2l)\left(1 - \frac{2e}{r}\right),$$

l being the latitude of the place, e its elevation above the level of the sea, r the radius of the terrestrial spheroid at the level of the sea in that place;—

$$\{r = 6366407^{m}\,(1 + 0\cdot 00164 \cos. 2l)\} = 20887510 \text{ ft. } (1 + 0\cdot 00164 \cos. 2l).$$

Thus, at Toulouse, where $l = 43° 36'$ and $e = 146^{m} = 479$ ft., we have $g = 9^{m}\cdot 8032 = 32\cdot 1633$ ft.: at Montlouis, where $l = 42° 30'$ and $e = 1620^{m} = 5315$ ft. (the mean height of the barometer being 29·72 in.) (Journal des Mines, tom. 23, p. 318), $g = 9^{m}\cdot 7977 = 32\cdot 1453$ ft.

...knowing better, we take $g = 9^m \cdot 8808 = 32 \cdot 1817$.
...seen, the results of calculations into which this
...ce, more than one-thousandth.
...forms, of which we give the origin.
...y bodies, and of uniformly accelerated motion
...s occupied in acquiring them, so that if v is
... time t, g being, as we have just seen, the
..., or $v = g t$.
...sed through, or the heights of the falls, are as
...h them, then if h is the height through which
... fall corresponding to $1''$, we shall have

... or $h = \frac{g t^2}{2}$.

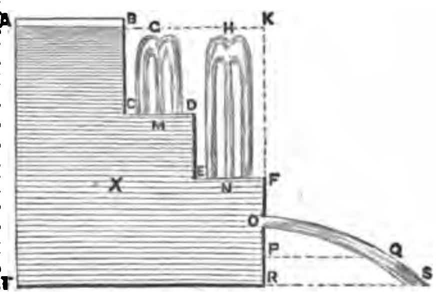
... substituting it in the first, we have

...quently $h = \frac{v^2}{2g}$.

... $8 \cdot 0227$, and $\frac{1}{2g} = \cdot 015536$.

...$6 v^2$.
... *height due to the velocity v.*
...h other places, as it expresses the ratio of the
...ther acceptation in this article. The fourth of
...le to the circumscribed square, presenting itself
... by π'.
...sel kept constantly full of water up to A B. If
... orifices M and N, the fluid will pass out in the

3999.



...l face F R an opening O be made, we shall here-
...of the lines O P and P Q, the fluid passes out at
...d pass out with a velocity due to K R, if the
...l.
...fices, whatever be their magnitude compared to
...ver, that the fluid surface, preserving a constant
...ch could not be fulfilled, if the size were very
... motion in the vessel.
...bstacle or all cause of perturbation, *the velocity*
... *side of a reservoir, is the same as a heavy body would*
...*een the level of the fluid surface in the reservoir and*

...ricelli's theorem, was established and published
...consequence of the laws of the fall of heavy

...y H the height or head of water in the reservoir,

... openings M and N did not quite attain the level
...e adapted two perfectly equal tubes, the water
...height would follow exactly the same ratio. For
...at M were only two-thirds of M G, that which
...o-thirds of N H. In general, let n be the ratio
...ervoir for a tube of a certain form, H and H' two
...ding velocities, we shall have

...: whence $v : v' :: \sqrt{H} : \sqrt{H'}$;

..., *the velocities are always as the square roots of the*

Experiments made by Mariotte, and repeated a hundred times since, leave no doubt as to this principle. We will here give the results of some of them; this will fix the degree of confidence with which the principle may be received; other details from the series of experiments which furnished these will be given presently. The first series was made by M. Castel and D'Aubuisson; the second, by Bossut; the third and fourth, by Michelotti; and the last, by MM. Poncelet and Lesbros.

Diameter of Orifice.	Head of Orifice.	Series of Square roots of Heads.	Series of Discharges or Velocities.
Inches.	Inches.		
0·3937	1·024	1·000	1·000
	1·181	1·074	1·064
	1·575	1·241	1·244
	1·969	1·386	1·393
	2·362	1·519	1·524
	feet.		
1·063	4·265	1·000	1·000
	9·580	1·500	1·497
	12·500	1·713	1·707
3·189	7·677	1·000	1·000
	12·500	1·305	1·301
	22·179	1·738	1·692
6·378	6·923	1·000	1·000
	12·008	1·316	1·315
squares.	1·312	1·000	1·000
—	2·297	1·323	1·330
7⅞ in.	2·281	1·581	1·590
by	4·265	1·803	1·806
7⅞ in.	5·249	2·000	2·000

It will be remarked that the heads were varied in the ratio of 1 to 200 and more, and the sections of the orifices from 1 to 500; and yet, in all, the velocities followed the ratio of the square roots of the heads; the small differences which are seen, sometimes in excess, sometimes deficient, may be neglected;—small errors are inevitable in such experiments. Their direct object was the determination of the discharges; but it is evident that when the orifice is the same, the discharge varies only with the velocity, that it is exactly proportional to it, and that the series of ratios of one is also the series of ratios of the other.

The general principle that the velocities are as the square roots of the heads, as well as the theorem of Torricelli for cases where it is applicable, extends to fluids of all kinds; to mercury, oil, and even aëriform fluids. So that the velocity with which each of them passes an orifice, is independent of its nature and of its density: it depends only on the head; experience proves it.

Simple reasoning, also, can show that it must be so. Take mercury, for example; the particles placed before the orifice, and on which it is necessary to impress a certain velocity, are, it is true, fourteen times more dense than those of water, and therefore they oppose fourteen times as much resistance to motion; but as the mass which presses, and which produces the velocity of passing out (being about fourteen times greater), exerts a motive effort fourteen times greater, there is a compensation, and the impressed velocity remains the same.

To the pressure which a fluid contained in a vessel exerts by its weight on the orifice of exit, may be added a foreign pressure, and the velocity of flowing is augmented. What will be its increase and its definite value?

Let P be the weight of body which produces the pressure, and s the fluid surface or portion of the fluid surface on which it immediately acts, namely, that which is in contact with it; h the elevation of that surface above the orifice, and p the weight of a cubic foot of the fluid contained in the vessel. For the given body substitute, in imagination, a column of that fluid, which would have s for its base, and whose height h' would be such that the weight of the column would be equal to that of the body; we should thus have $P = p\,s\,h'$ from which to deduce h'; substituting thus one body for another of equal weight, we should not change the pressure experienced by the particles contained in the vessel. Suppose, further, that after having withdrawn the body, we add in the vessel (whose sides we may suppose to be prolonged to an indefinite height) a quantity of the same fluid as that already contained, until its level has attained the summit of the column; according to the laws of hydrostatics, all the mass of the fluid added would only produce a pressure equivalent to that of a single column; so that the particles situated before the orifice would experience a pressure exactly equal to what they first experienced, and will always tend to pass out with the same velocity. Now, in the new state of things, the height of the reservoir above the orifice, the height generating the velocity of exit, is evidently $h' + h$, and consequently this velocity will be $\sqrt{2g(h' + h)}$.

Take, for example, a vessel closed on all sides and filled with alcohol, whose specific gravity is 0·837; on the cover is a circular opening of 1¼ in. diameter, in which is a piston loaded with 18 oz.; the orifice of exit is 10 in. beneath that opening. To determine the velocity with which the alcohol will run out. We admit that the friction of the piston on the edges of the opening is balanced by the weight of the piston itself.

We then have P = 18 oz. = 1·125 lb.; $s = \cdot 7854 \times (1\cdot 25)^2 = 1\cdot 227$ sq. in. = ·0085 sq. ft.; $p = \cdot 837 \times 62\cdot 429 = 52\cdot 271$ lbs. and $h = 10$ in. = ·833 ft.; for h', the equation $P = p\,s\,h'$ or $1\cdot 125 = 52\cdot 271 \times \cdot 0085\,h'$, gives 2·5329 ft. Thus the alcohol will issue with a velocity of

$$\sqrt{2g\,(2\cdot 5329 + \cdot 833)} = \sqrt{64\cdot 364 \times 3\cdot 3659} = 14\cdot 718 \text{ ft.}$$

If the vessel were not kept constantly full, this velocity would gradually diminish.

After having given the expression of the velocity with which any fluid issues from an orifice, we pass to the use made of it in determining the discharge.

We call the *discharge* of an orifice the volume of fluid which runs out of it in the unit of time, the second.

If the mean velocity of all the fluid particles were that due to the whole head H, this velocity, which is then called *theoretic velocity*, would be $\sqrt{2g\mathrm{H}}$; if, at the same time, the particles passed

...es, it is evident that the volume of water ...me of a prism which had the orifice for a ...alling S the area or section of the orifice,

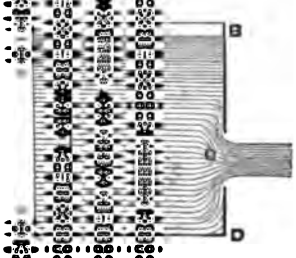
... us consider the fluid vein a little after its ...rpendicular to its direction. It is manifest ... of the section by the mean velocity of the ...his section were equal to that of the orifice, ...the actual discharge would be equal to the ...section of the vein is sensibly smaller than ...thin side, or that the velocity at the section ...rical tubes; or even that there is a diminu- ...certain conical tubes. So that the actual ...n the theoretic; and in order to reduce the ...ction. If m represent that fraction, and Q

...ny time T, we should also have

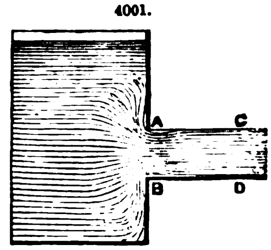
...g H.

...rom a diminution in the section of the vein, ...consequence of the contraction which the vein ...e multiplier m, *or coefficient of reduction of the* ...lled *the coefficient of the contraction of the fluid* ...tion is one of very great importance; on its ...the formula for the flow of fluids is applied ...e experimental researches of hydraulicians. ...ave arrived, after making some preliminary

...nt vessel, Figs. 4000, 4001, let water flow ...the particles of the fluid visible by mixing



4001.



... towards a centre of attraction.
...e in the interior of the vessel, on the instant ...c distance after they have passed through it, ... is gradually contracted up to a point where ...d of the motions impressed upon them, take ...us forms a kind of truncated pyramid or cone, ...the fluid section at the point of greatest con- ...*the contracted vein.* This figure, and all the ...the convergence of the lines, when they arrive ...me in respect to others.
...takes place below the plane of that orifice; it ...ured, and they have actually been measured. ...; we shall here simply remark that in circular ...tion and up to a certain distance, the vein ...ection would be the base, and with a velocity ...c discharge, then, will be the product of that ...be limited to reducing the section which is to ...owing takes place as if, for the real orifice, ...that of the contracted section, and as if there

...be fitted, the fluid lines will arrive at A B ...racted at the entrance of the tube. Experi- ...contraction there is equal to that which takes ...*terior* in relation to the mouth of the outlet. ...n of the sides of the tube occasions a dilation ...s, they follow the sides, and pass out parallel

to each other, and to the axis of the tube; so that the section of the vein at its exit is quite equal to that of the orifice, but the velocity is not that due to the head of the reservoir. If the flowing were produced only by the simple pressure of the fluid contained in the reservoir, probably the velocity, at the section of greatest contraction, would be that due to the head, then it would diminish in proportion as the vein dilates, in virtue of the law or axiom of hydraulics, *when an incompressible fluid in motion forms a continuous mass, the velocity, at its different sections, is in the inverse ratio of the area of the section;* the diminution would cease when, the vein having attained the sides, its section would become equal to that of the orifice. Since m is the ratio of the section of greatest contraction to that of the orifice, the velocity along the sides, and consequently at the exit, would be $m\sqrt{2gH}$; and for the discharge we should have $S \times m\sqrt{2gH}$.

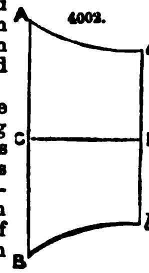
In orifices in a thin side, it was $mS \times \sqrt{2gH}$; thus the discharge would be the same in both cases; the only difference is, that in the latter the diminution would have affected the factor S, and in the tubes it would have fallen on the factor $\sqrt{2gH}$; that is to say, on the velocity. But the attractive action of the sides changes this state of things; not only does it cause the lines to deviate from their direction, but it also increases their velocity, so that the velocity of exit is greater than $m\sqrt{2gH}$; it will be $m'\sqrt{2gH}$, m' being a fraction greater than m; and the discharge will become $S \times m'\sqrt{2gH}$.

We see by this, that in cylindrical tubes and in ajutages generally, the effect of contraction is involved in that of the attraction of the sides. Without being able to assign what belongs to the first alone, we will remark that for every interior contraction there is a corresponding diminution of velocity, and every exterior contraction produces a diminution of section.

Let us examine the form which contraction gives to the fluid vein passing from an orifice. Take first the most simple case, that of a circular orifice in a thin and plane side.

The direction as well as the velocity of the particles at the different points of the orifice being symmetrical, the contracted vein must also have a symmetrical form, and consequently be a solid of revolution, a conoid. It is so in fact, and observations about to be reported give it the form represented by A B b a, Fig. 4002. Beyond a b the contraction ceases, and the vein continues under a form sensibly cylindrical for a certain length, and until it becomes entirely deformed, from the resistance of the air and other causes.

4002.



In the first part of that length it is full, clear, sometimes like a bar of the most beautiful crystal; then it becomes disturbed, and, examined in a strong light, it presents a series of swellings and contractions. From the very ingenious experiments of M. Savart, the appearance of continuity of the disturbed part is only an optical illusion, arising from the rapidity of the motions; this part consists of a series of distinct drops, alternately large and small, leaving between each other a space eight or ten times greater than their mean diameter, the form of which, oscillating round that of a sphere, is alternately an elongated and an oblate spheroid.

Boyle observed that the length of the clear part, as well as that of the swellings in the disturbed part, increased proportionally to the diameter of the orifice and the head; for the clear part, it was nearly $380\, d\sqrt{h}$ in mètres, or $209\, d\sqrt{h}$ in feet. The formation of drops, that is to say, their detachment from the clear part, is not, even in descending jets, an effect of the acceleration of velocity due to gravity; for it takes place equally in jets thrown upwards. It appeared to Savart to be an immediate effect of the oscillation which occurred in the fluid of the reservoir, in consequence of which the particles of the jet, being sometimes more and sometimes less pressed at their exit from the orifice, moved with a velocity alternately greater and less. D'Aubuisson discovered such alternations in most of the motions of fluids. He observed them also, in a very marked manner, during his experiments upon the resistance which the air experiences in conduit pipes.

M. Savart also showed the very singular influence of the waves of sound on the liquid veins; for example, if the disturbed part be received on the bottom of a vessel, there is heard a sound due to the impulse of successive drops; if then a note be produced on a violin in unison with this sound, the clear part of the jet is immediately seen to become shortened, and sometimes even to disappear entirely; the swellings of the troubled part become bigger and shorter, and the space which separates them is greater.

To return to the commencement of the jet, to the contracted vein properly so called, the conoid A B b a, Fig. 4002. Attempts have been made to determine its respective dimensions, and particularly the ratio between the diameters of the two bases, by direct measurements. Newton, who observed the phenomenon of contraction and its effects on the discharge, and first attempted such an admeasurement; he concluded that the ratio of the section of the orifice to the contracted section was that of $\sqrt{2}$ to 1, and consequently that of the diameter was as 1 to 0·841; but we believe that theoretical considerations, rather than a physical measurement, led him to adopt that result. Since then, several philosophers have made like measurements; thus A B being 1; Poleni found for a b 0·79; Borda, 0·804; Michelotti, 0·792; Bossut, from ·812 to ·817; Eytelwein, ·80; Venturi, ·798; finally, Brunaci, ·78. Nearly all these numbers, whose mean term is ·80, are very probably a little too large; they were found by measurements taken with callipers; if closed too much, the points were thrust into the body of the stream and the disturbance indicated it; but if too much open, the eye could not exactly appreciate how much it was so; hence an error in excess might be made, but not one in deficiency.

Michelotti the younger took up this question, which had already been treated by his father. Large jets obtained under great heads, gave him the following results.

Head above the Orifice in feet.	Diameter in inches.		Ratio between Diameters.	Distance from Orifice to Contraction, in inches.	Ratio of the Distance to the Contracted Diameter.
	At the Orifice.	At the Contraction.			
6·890	6·394	5·047	0·790	2·520	0·501
12·008	6·394	5·039	0·788	2·520	0·500
7·349	3·197	2·511	0·786	1·260	0·500
12·502	3·197	2·504	0·783	1·210	0·492
22·179	3·197	2·413	0·755	1·181	0·497

Abstracting the last number 0·755, which is entirely anomalous, the mean ratio between the two diameters is 0·787. From what has been said, we think it may be adopted, but only as a mean term; for, as we shall soon see, this ratio experiences variations, slight, to be sure, which depend upon the heads and the diameters of the orifices. The length of the contracted vein should be about half the diameter of the smallest section, or 0·39 of the diameter of the orifice. According to these experiments, the three principal dimensions, A B, *a b*, and C D, Fig. 4002, of the contracted vein would be respectively as the numbers 100, 79, and 39.

Eytelwein, chiefly increasing the last dimension, one very difficult to determine with accuracy, takes the numbers 10, 8, and 5; this ratio is quite generally admitted. As to the curves A *a* and B *b*, Michelotti refers them to a cycloid. In conclusion, the form of the fluid vein, at its passage from a circular orifice, has some resemblance to the bell-shaped end of a hunting horn.

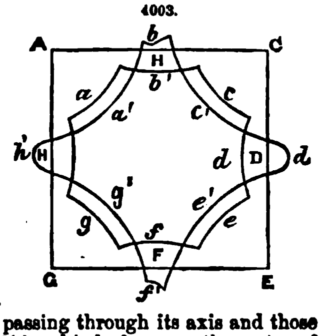
The ratio between the diameters being 0·787, that between the sections will be the square of 0·787, or 0·619; thus, if s is the section of the contracted vein and S that of the orifice, we shall have $s = 0·619$ S. From the explanations made, p. 1889, the discharge will be $s\sqrt{2g\text{H}}$ or $0·619\,\text{S}\sqrt{2g\text{H}}$. So that m, or the coefficient of contraction given by physical measurements of the vein, will be at a mean 0·619; and the measurements of the discharge indicate nearly the same.

If the velocity due to the head of the reservoir were really the *velocity at the passage* of the contracted section, and the flowing were produced through a tube which had exactly the form of the contracted vein, by introducing into the expression of the discharge the exterior orifice of that tube or s, the calculated discharge would be equal to the real discharge, and the coefficient for reducing one to the other would be 1. Michelotti, in one of his experiments, by employing a cycloidal tube, found it 0·984; it is probable that it would have come up to 1, if the sides of the tube had been more exactly bent to the curvature of the fluid vein; and if the resistance of the sides, as well as that of the air, had not slightly retarded the motion.

Orifices, whose perimeter is a polygon, or any figure other than a circle, do not present a form so simple, or leading to the same consequences.

The different parts of the orifices not being symmetrical, the fluid vein does not preserve the form which it had on coming out, and it changes from it continually as it removes from it. At its exit, the faces corresponding to the rectilinear sides of the orifice become more and more concave; the edges corresponding to the angles become truncated and terminate by disappearing. Thus Poncelet and Lesbros, having drawn, by aid of very exact means, the form of a vein which passed from a square orifice A C E G, Fig. 4003, whose sides were 7¾ in. under a head of 5¼ ft., had, at the distance of 5·9 in. from the orifice, the section *a c e g*; and at 11·81 in., the section *b' d' f' h'*.

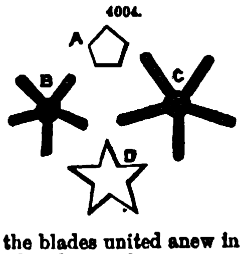
4003.



This last, one of the nine sections observed, was the smallest; its area was to that of the orifice in the ratio of 0·562 to 1, whilst that of the actual discharge to the theoretic discharge was found to be 0·605; they would have been equal, if the velocity of that smallest section had been due to the head of the reservoir.

Although the fluid particles at *b' c' d'*, &c., on this section, Fig. 4003, are those which came out at the points B C D, &c., of the orifice, and in removing from the reservoir have always remained on the line of intersection of the vein with the planes passing through its axis and those points respectively, it is nevertheless true that the section *b' d' f' h'* is a kind of square, the vertex of whose angles corresponds to the middle of the sides of the square of the orifice; and that the vein appears to have made an eighth of a revolution round its axis. A phenomenon of this nature is produced on all the veins which come out of an orifice not circular; it is called the *reversing of the vein.*

4004.



The orifice was a regular pentagon A, Fig. 4004, of 0 551 in. each side, made in a thin vertical plate of copper (the figure representing it, with its accessories, is one-quarter of the natural size); the flowing took place under a head of 6·463 ft. At the distance of 0·472 in., the section perpendicular to the axis of the vein was a quite regular decagon. At 1·181 in. was the greatest contraction or *first knot.* Beyond, the vein entirely changed its form; it presented five fluid plates, disposed symmetrically around the axis, as is seen in the section B, made 3·74 in. from the orifice; the planes of the blades passed through the centres of the sides of the orifice. Their breadth continued to increase up to the belly of the vein represented at C. Then it diminished, and the blades united anew in a *second knot*, at 2 ft. 10 in. from the orifice. Beyond, the vein was twisted and irregular.

For the rectilinear pentagon of the orifice were successively substituted pentagons with convex and concave sides, sides presenting salient and re-entering angles like the star D, and the vein always preserves the same form, the same five blades.

With orifices of six and eight sides, we had six and eight blades; and the reversing of the vein was a twelfth and sixteenth of the circumference. When the opening was a rectangle, narrow, and very long in the horizontal direction, at a certain distance, the vein consisted only of a broad vertical blade; the reversing seemed complete.

Often, beyond the second knot, the vein dilates again and divides a second time into the same number of blades; but their plane does not correspond to the middle of the sides of the orifice, but to the vertex of the angles; that is to say, the vein is again turned an equal quantity, or rather it returns to its place. The blades increase in breadth up to the second belly, and diminish again to form a third knot, beyond which sometimes there is still a new dilation, a third belly and a fourth knot.

Orifices in Thin Partitions.—We now come to the direct determination of the coefficient of reduction, from the theoretic to the actual discharge.

And we will measure with care the volume of water passing from a given orifice, under a constant head, and during a certain time; and we shall derive from it the product of the flow in one second or the actual discharge; we will divide it by the theoretic discharge corresponding to that orifice and to that head, and the quotient will be the coefficient sought.

Many hydraulic engineers have applied themselves to this investigation; D'Aubuisson gives, in the following Table, the principal results obtained up to the present time; those which appear to have been made under the most favourable circumstances, or which were generally admitted.

Circular Orifices.

Observers.	Diameter in inches.	Head in feet.	Coefficient.
Mariotti ..	0·268	5·873	0·692
" ..	0·268	25·920	0·692
Castel	0·394	2·133	0·673
"	0·394	1·017	0·654
"	0·590	0·453	0·632
"	0·590	0·984	0·617
Eytelwein ..	1·027	2·372	0·618
Bossut	1·067	4·265	0·619
Michelotti ..	1·067	7·317	0·618
Castel	1·181	0·223	0·629
Venturi.. ..	1·614	2·887	0·622
Bossut	2·126	12·500	0·618
Michelotti ..	2·126	7·218	0·607
" ..	3·189	7·349	0·613
" ..	3·189	12·500	0·612
" ..	3·189	22·179	0·597 ?
" ..	6·378	6·923	0·619
..	6·378	12·008	0·619

Square Orifices.

Observers.	Side of square in inches.	Head in feet.	Coefficient.
Castel	0·394	0·164	0·655
Bossut	1·063	12·500	0·616
Michelotti ..	1·063	12·500	0·607
" ..	1·063	22·409	0·606
Bossut	2·126	12·500	0·618
Michelotti ..	2·126	7·349	0·603
" ..	2·126	12·566	0·603
" ..	2·126	22·245	0·602
" ..	3·228	7·415	0·616
" ..	3·189	12·566	0·619
" ..	3·189	22·376	0·616

Rectangular Orifices (Bidone).

Rectangle.		Head in inches.	Coefficient.
Height in inches.	Base in inches.		
0·362	0·728	13	0·620
0·362	1·457	13	0·620
0·362	2·909	13	0·621
0·362	5·818	13	0·626

The most remarkable of all these experiments, as well for the great size of the jets as for the greatness of the head, are those which Michelotti executed in 1764, at the fine hydraulic establishment constructed for that purpose at about two miles from Turin; the reservoir consisted of a tower 26 ft. 3 in. high, whose interior, which is a square of 3·182 ft. a side, receives through a canal the waters of the Doire. On one of the faces were fitted, at the different heights, the orifices or tubes which were thought proper; arrangements were made to receive them, and on the ground, which is at the base, were several measuring basins. These experiments were repeated in 1784 by Michelotti the younger, and they are the last introduced into the Table.

The experiments just reported and those made by other authors have shown that the coefficient of contraction is generally greater for small orifices and small heads; but they furnished only vague and almost contradictory notions in this respect. It would have been impossible to deduce from them the series of coefficients from great orifices to the smallest, and from great heads to the smallest; this deficiency has recently been supplied by MM. Poncelet and Lesbros. They made, in 1826 and 1827, at Metz, a series of experiments on a *very* great scale, and with care and means which had not before been employed.

They appear to have nearly solved the great and useful problem of the contraction of the vein in a thin partition, perhaps as nearly as the nature of the subject admits; and in a manner, if not entirely theoretical, at least very suitable to applications.

In these experiments the orifices were rectangular, and all of $0^m \cdot 2 = 7 \cdot 874$ in. base; the heights were successively 7·874 in., 3·937 in., 1·968 in., 1·18 in., 0·787 in., 0·394 in.; the heads varied from 0·394 in. to 5·577 ft. For each of these orifices the discharge was measured with several repetitions, under seven or ten heads, of which the two extremes were taken, the one nearly as small and the other as large as the apparatus allowed; and the corresponding coefficients were calculated.

Taking, then, the heads for abscissas and their coefficients for ordinates, the curve relating to that orifice was traced; and by its aid they determined the ordinates or coefficients intermediate to those directly given by experiment. In this manner the authors were enabled to arrange a large table of coefficients for each orifice, from which we extract the following;—

Head on centre of Orifice.	Height of Orifices (base of each 7·874 inches).					
	7·874 in.	3·937 in.	1·968 in.	1·181 in.	·787 in.	·394 in.
inches.						
·394						0·709
·787					0·660	0·698
1·181				0·638	0·660	0·691
1·575			0·612	0·640	0·859	0·685
1·968			0·617	0·640	0·659	0·682
2·362		0·590	0·622	0·640	0·658	0·678
3·150		0·600	0·626	0·639	0·657	0·671
3·937		0·605	0·628	0·638	0·655	0·667
4·725	0·572	0·609	0·630	0·637	0·654	0·664
5·906	0·585	0·611	0·631	0·635	0·653	0·660
7·874	0·592	0·613	0·634	0·634	0·650	0·655
11·811	0·598	0·616	0·632	0·632	0·645	0·650
15·748	0·600	0·617	0·631	0·631	0·642	0·647
feet.						
1·640	0·602	0·617	0·631	0·630	0·640	0·643
2·297	0·604	0·616	0·629	0·629	0·637	0·638
3·281	0·605	0·615	0·627	0·627	0·632	0·627
4·265	0·604	0·613	0·623	0·623	0·625	0·621
5·250	0·602	0·611	0·619	0·619	0·618	0·616
6·582	0·601	0·607	0·613	0·613	0·615	0·613
9·843	0·601	0·603	0·606	0·607	0·608	0·609

All the numbers in this Table are the respective values of m in the formula $Q = m\,S\sqrt{2\,g\,H}$. But those which in each column are found above the transverse line, are not the true coefficients of reduction from the theoretic to the actual discharge, as we shall presently see.

Glancing over the numbers of each column, we see that they increase as the head increases, but only up to a certain point, beyond which they diminish, although the head still augments. However, in small orifices, those below 1·181 in., the increasing part of the series is very limited; and even in very small ones it is nothing. We see also that the terms of the decreasing part of all the series approach equality in proportion as the head increases in value.

Although the coefficients in the Table above are deduced from experiments made on rectangular orifices, they may serve for all others, whatever be their form; the height of the rectangle noted in the Table will express the smallest dimension of the orifice which should be used. For it is generally admitted that the discharge is entirely independent of the figure of the orifice, and that it always remains the same, while the area of the opening is unchanged; always provided, in accordance with an observation made by M. Hachette, that this figure presents no re-entrant angles.

Although some of the orifices on which Poncelet and Lesbros made their experiments are very large, still there are those which discharge twenty or thirty times as much water; such are the openings of sluice-gates in canals of navigation, and it was important to establish directly the coefficient of their discharge. In 1782 Lespinasse, a skilful engineer, made for this purpose several experiments on the canal of Languedoc, to which, ten years after, Pin, engineer of the same canal, added some others. The principal results of these, like the former, are placed in the following Table. The breadth of the opening is nearly 4·265 ft.; the form not being exactly a rectangle, the heights are to be regarded as only approximate.

Openings.		Head on the centre.	Discharge in one second.	Coefficient.
Area.	Height.			
square feet.	feet.	feet.	cubic feet.	
7·745	1·805	14·554	145·292	·613
6·992	1·640	6·631	92·635	·641
6·992	1·640	6·247	88·221	·629
6·466	1·509	12·878	138·937	·641
6·723	1·575	13·586	128·764	·647
6·723	1·575	6·394	83·948	·616
6·723	1·575	6·217	79·857	·594
6·717	1·575	6·480	85·219	·621
		Mean term		·625

This mean coefficient, exactly equal to that obtained from an experiment made on a sluice of the basin of Havre, is a little greater than that indicated by the table of M. Poncelet, p. 1893; probably the cause of it is, that on all the perimeter of the opening, the flowing did not occur as in a thin side, and that on some point the contraction was suppressed. It may be remarked on this subject that the woodwork which surrounded this orifice was $0^{m}\cdot 27$ = ·886 ft. thick, and even $0^{m}\cdot 54$ = 1·772 ft. thick on the lower edge. Also, when the gate was raised only a small quantity, the contraction ceased on the four sides, and the coefficient increased considerably. For example, Lespinasse having raised the gate only $0^{m}\cdot 12$ = ·394 ft., had for a coefficient ·803, while with 1·509 ft. opening, he had a coefficient of only ·641.

The experiments of this engineer presented a very remarkable fact, of which no mention was made, and which reappeared in those of Pin. A sluice-gate had two parts, and each had an opening in it: if, while the water was flowing through one, the second was opened, the discharge of the first was diminished; if both were opened together, the discharge was not double of the two taken separately, although each had the same area and head. The difference is about one-eighth, as may be seen by the annexed comparison of the coefficients of reduction for the two cases.

Coefficient	
With one opening.	With two openings.
0·641	0·550
0·689	0·555
0·616	0·554
0·594	0·526
0·621	0·555
0·620	0·548

The interval between the two openings is $2^{m}\cdot 92$ = 9·58 ft., and their plane forms an angle of 60° with the direction of the canal.

But it is very worthy of remark that this fact, which appeared positive for the sluices of the canals, did not take place at all in a series of experiments which M. Castel and D'Aubuisson de Voisins made on a small scale, but with very great care, for the purpose of verifying it. They had, side by side, three rectangular orifices of ·328 ft. by ·033 height, and separated by an interval of only ·033 ft. They measured the water passing the middle orifice first, keeping the two side orifices closed, then opening one and finally opening both; the mean results are given in the following Table;—

Head on the Orifice.	Discharge from Middle Orifice.			Coefficient.
	Middle Orifice alone open.	Middle Orifice, with 1 Lateral Orifice, open.	Middle Orifice, with the 2 Lateral Orifices, open.	
feet.	cubic feet.	cubic feet.	cubic feet.	
·0656	·01607	·01606	·01614	0·728
·0984	·01946	·01946	·01942	0·720
·1312	·02242	·02246	·02250	0·719
·1640	·02497	·02497	..	0·715
·1969	·02723	·02716	..	0·710

Supposing that these unexpected coefficients might have been influenced by the very small interval from one orifice to the other, we increased the interval fivefold; that is, from ·394 in. to 1·968 in., and the coefficients remained the same.

Surprised at the difference between these results and those found on the canal of Languedoc, and fearing that it arose from the particular form of the orifices and apparatus, D'Aubuisson requested M. Castel to make new experiments; and in 1836 he had the kindness to perform a series, by the aid of the great apparatus which he had just been using for his great work on Weirs. He dammed up a canal $0^{m}\cdot 74$ = 2·428 ft. broad, with a thin copper plate, in which he opened, on the same horizontal strip, three rectangular orifices, each 3·94 in. wide by 2·36 in. high, and separated from each other by an interval of 3·15 in. The flowing took place under a constant head of 4·213 in. above their centre, and the coefficients of contraction were as follows;—

One orifice open	for the middle	·6198
	„ right	·6193
	„ left	·6194
Two orifices open	the two outsides	·6205
	middle and right	·6205
	„ „ left	·6207
The three orifices all open	.. ..	·6230

Here, in proportion as the orifices were open, instead of a diminution in the coefficients, there was an increase, very small, to be sure. As it depended on a particular cause, a greater velocity of water in the canal, in consequence of a greater discharge, we shall make deduction of that, and conclude that, when in the dam of a reservoir or course of water new orifices are opened by the side of an orifice already existing, the discharge through that orifice is not diminished by it. Some persons thought that such a consequence would not extend to the case when two orifices were situated in planes making a certain angle, as in the openings of the sluice-gates. M. Castel solved this question. He took two plates joined at an angle of 120° (that of sluice-gates is generally from 10° to 20° more open); in each he made two rectangular orifices of 3·94 in. wide by 2·36 in. high; one 4·72 in. and the other 11·02 in. distant from the angle that joined them; he fitted this partition to the extremity of his canal, and let the water flow under a head of $0^{m}\cdot 14$ = 5·51 in. He first opened successively each of the four orifices; then two at a time differently

combined; then three differently combined, and finally four. The following Table presents the mean results obtained.

No. Orifices.	Coefficient.
1	·618
2	·619
3	·620
4	·622

That given in the second line was obtained by the two extreme orifices, which were disposed like those of the sluice of the canal of Languedoc. As a last objection, it was said that the heads at the sluice of the canal of Languedoc were from $2^m = 6\frac{1}{2}$ ft. to $4^m = 13$ ft. To obtain an analogous case, M. Castel adapted to the experimental apparatus two orifices of 1·97 in. wide by 1·18 in. high, and had the results which we give, p. 1902.

Head.	No. Orifice.	Coefficient.
3·379 ft.	1	·621
	2	·622
6·693 ft.	1	·619
	2	·621

It is always the same coefficient, with the insignificant increase due to the number of orifices open.

These experiments, often repeated, with apparatus free from every exceptionable circumstance, and where any sensible error was impossible, by the most accurate and conscientious observer, induced D'Aubuisson de Voisins, if not to call in doubt the facts previously announced, at least to regard them as anomalous, and to reject the general consequence which may be drawn from them.

In the different cases hitherto investigated, it is admitted that the fluid of the reservoir arrives equally at all parts of the orifice, but often it is not so; for example, when the orifice is at the bottom of a vertical side, and its lower edge is in the plane of the bottom of the reservoir, the contraction is then destroyed on that side, and consequently the discharge is greater. What will be the increase in discharge for a certain length of suppression in the contraction? This question has been nearly solved by M. Bidone, by the aid of numerous experiments made for that purpose at the water-works of Turin.

The orifices were made in thin vertical copper plates; on their interior surface were fixed, perpendicular to their plane, small plates, on a level with certain sides of the orifice; as it were, the prolonging of these sides into the interior of the reservoir. During the flowing, the water running along the plates passed through the adjacent sides without any contraction, while a contraction occurred on the other sides. The form and size of these orifices were various. We shall limit ourselves to giving the results of experiments with a rectangular orifice of $0^m·054 = 2\frac{1}{16}$ in. base and 1·06 in. in height; the plates adapted to them, sometimes on one side and sometimes on two or three, were 2·638 in. long; they thus extended that length into the reservoir. The flowing having been produced under heads varying from 6·562 ft. to 22·573 ft., we have the following coefficients;—

The contraction being suppressed on	Part of Orifice without contraction.	Coefficient.	Ratio.
Neither side	0	·608	1·000
A small „	$\frac{1}{6}$	·620	1·020
A great „	$\frac{2}{6}$	·637	1·049
A great and a small	$\frac{3}{6}$	·659	1·085
Two small and one great ..	$\frac{4}{6}$	·680	1·112
Two great and one small ..	$\frac{5}{6}$	·692	1·139

M. Bidone, taking the mean result of all the experiments made on rectangular orifices, admits for the numbers of the last column, which indicates the increase of the coefficient and consequently of the discharge, that for the orifice entirely free being taken for unity, the general expression $1 + 0·152\,\frac{n}{p}$, in which n represents the length of the part of the perimeter when the contraction is suppressed, and p the length of the whole perimeter. The greatest error which this formula gave M. Bidone being only $\frac{1}{70}$, we may adopt for the value of the discharge in rectangular orifices when there is no contraction on a part of the perimeter, $m\,S\sqrt{2gH}\left(1 + 0·152\,\frac{n}{p}\right)$

The same author also made experiments on circular orifices. He took one of 1·575 in. diameter, and by the aid of curved cylindrical plates he destroyed the contraction, first, on an eighth of the circumference; then successively on 2, 3, 4, 5, 6, and 7 eighths. We indicate the results obtained in the following Table;—

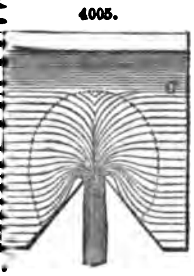
$\frac{n}{p}$	Coefficient.	Ratio.	$\frac{n}{p}$	Coefficient.	Ratio.
0	0·597	1·000	$\frac{4}{8}$	0·639	1·072
$\frac{1}{8}$	0·603	1·011	$\frac{5}{8}$	0·649	1·087
$\frac{2}{8}$	0·615	1·032	$\frac{6}{8}$	0·664	1·112
$\frac{3}{8}$	0·625	1·048	$\frac{7}{8}$	0·670	1·123

We see here that the numbers of the last column increase a little less rapidly than in the case of the rectangular orifices, so that the general expression from these numbers would be only $1 + 0·128\,\frac{n}{p}$.

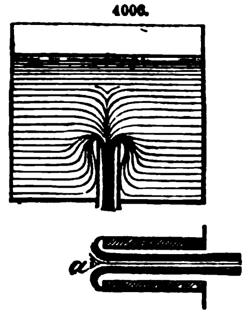
n-eighths of his circular orifice, wished to circum-
to the orifice a cylindrical tube of $0^m \cdot 04 = 1 \cdot 575$ in.
e interior of the reservoir; he had 0·767 for the
mber of the last column. The expression above
e increase is not even half of that really obtained.
lowing through interior tubes, the case where the
f the exterior orifice, is no longer of the same kind
over great that part may be; there is no passing

the orifices were, to be plane, but they may be of
hich may result upon the product of the flowing, it
arrive at the orifice parallel to each other, the
discharge, and that it is less only in consequence
hich obliquity necessarily results, at the point of
acquired. This being established, if around the
g. 4005, of a radius equal to that of the sphere of
of the vessel, it would be traversed at each of its
r, by the arriving lines; the more extended the
ections, and the more opposed to each other; and
troyed at the orifice, and the less considerable the
he surface of a hemisphere, Fig. 1001, and is found
lischarge just given. But if it is disposed in the
ls the interior of the vessel, then the cap is smaller
actly following the ratio of the spherical surface.
oduct is less; it will be smaller still in the case

4005.



4006.



f entrance of the tube with a large border, thus
into the same circumstances as when it is perfo-
ent was raised to 0·625. He might have obtained
h very thick sides.
ness, without being too considerable, 0·394 in. or
uite square off at the extremity, so that the zone
sharp edges, the fluid winding round the exterior
est of the zone, Fig. 4006, a; so that every part of
without effect, and the flowing would take place as
will be its diameter; that is to say, the exterior
d into calculations relating to interior tubes. By
that the action of the vein running in the tubes
alf the section of the tube, and that the coefficient

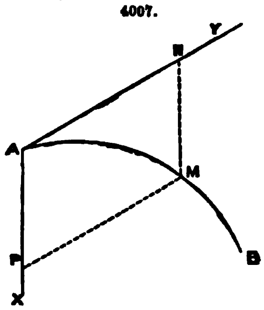
coefficients of contraction; limits which may be
ned. For orifices in a plane side, they seldom
n ordinary practice, they are confined between ·60
ually taken, and we have,

$$= 2 \cdot 75\, S \sqrt{H} = 216\, d^2 \sqrt{H}; \text{ or,}$$
$$= 4 \cdot 974\, S \sqrt{H} = 3 \cdot 9066\, d^2 \sqrt{H},$$

orifices in a thin side, as we have admitted exactly
H ? We will examine it.
ter runs from an orifice, by the height to which a
it is at least $\sqrt{2 g h}$, h being that height. Now,
differs from H only 1, 2, 3, &c., hundredths of the
&c.; and the velocities being as the square roots
he same cases only 1, 2, 3, &c., half-hundredths of
g the actual velocity indicates still less difference.
of it.

When a body is thrown in any direction A Y, Fig. 4007, with a certain velocity, by the combined influence of that velocity and of gravity, it describes a curve A M B.

4007.

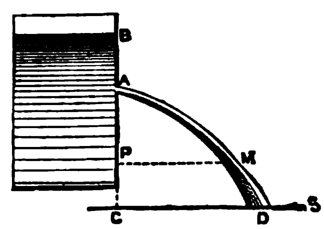


We have already referred to the theoretical difficulties connected with this curve A M B; hence it is unnecessary to dwell upon this matter here. In the present article we confine ourselves to what concerns the fundamental principle which we employ; and treat of that *parabola* which coincides most nearly with the curve A M B, Fig. 4007. See DAMMING, p. 1126, and GUNNERY.

Let v be the velocity with which a body is impelled along A Y, and t the time spent in arriving at N, in this direction, if the force of projection acted alone upon it; the motion would then have been uniform, and we should have had $AN = v\,t$; on the other hand, had the body been subjected to the action of gravity alone, it would have descended from A to P during the same time, so that we should have had $AP = \frac{g\,t^2}{2}$. Draw the parallelogram A P M N; at the end of the same time it really will arrive at M, and will have described the arc A M; A P will be its *abscissa*, and M P, parallel to the axis A Y, wil be its *ordinate*. Call the first of these lines x and the second y, we shall have $x = \frac{g\,t^2}{2}$ and $y = v\,t$; in this latter equation, taking the value of t, and substituting it in the first, we have $x = \frac{g\,y^2}{2\,v^2}$, or $y^2 = \frac{2\,v^2\,x}{g}$; or, calling h the height due to the velocity v, and recollecting that $\frac{v^2}{2\,g} = h$, $y^2 = 4\,h\,x$; an equation of a parabola of which $4\,h$ is the parameter. Hence *a heavy body, impelled by any force of projection, describes a curve which resembles a parabola whose parameter, in the case of a jet of water, may be taken without involving much error, equal to four times the height due to the velocity of projection.*

What we have just said of a body in general is applicable also to every jet of water issuing from an orifice. If this orifice is in a vertical side, the axis of projection being horizontal, the ordinates will be horizontal; they will be the distances of the different points of the jet from the vertical, let down from the centre of the orifice; and if through any point c of that vertical, we imagine a horizontal plane, the distance C D, Fig. 4008, is called the *reach of the jet* on that plane. According to our theorem, the square of this range, or in general of a distance M P, divided by four times its corresponding perpendicular A P, will give the height due to the velocity of exit $\left(h = \frac{y^2}{4\,x}\right)$; and consequently we shall have for this velocity, $v = \sqrt{2\,g\,h} = 2{\cdot}215\,\frac{y}{\sqrt{x}}$ in mètres, or $4{\cdot}0113\,\frac{y}{\sqrt{x}}$ in feet.

4008.



By following this mode of determination, Bossut, in two experiments, found 0·974 and 0·980 for the ratio of the actual to the theoretic velocity. Michelotti having caused jets to issue from each of the three stories of the tower of his hydraulic establishment through a vertical orifice 0·889 ft. diameter, obtained the results given in the following Table;—

Head.	Jet.		Velocity.		Ratio.
	Abscissa.	Range.	Real.	Theoretic.	
feet.	feet.	feet.	feet.	feet.	
7·513	20·615	24·706	21·819	21·983	·993
12·894	15·289	27·724	28·446	28·807	·988
23·590	4·626	20·506	38·289	38·978	·983

The difference between the two velocities increases with the head. It should be so, since the cause of this difference, the resistance of the air, increases as the square of the velocity, and consequently nearly as the head.

If the water contained in the reservoir, instead of being at rest, were animated with a velocity which carried it towards the orifice; for example, if the basin having a small section were fed by a course of water which came directly to the side on which the orifice is open, the fluid particles would go out, not only in virtue of the pressure exerted by the fluid mass above, but also in virtue of the velocity which they had at the moment of entering the sphere of activity of the orifice; we should thus have to add to the head measuring the pressure, a new force, which will be the head generating that velocity. Thus, if u represent that velocity, we shall have

$$Q = m\,S\sqrt{2\,g\left(h + \frac{u^2}{2\,g}\right)} = m\,S\sqrt{2\,g\,h + u^2}.$$

Example.—There is a basin 65·62 ft. long, 6·562 ft. broad, and 3·281 ft. depth of water; at one extremity is a dam of plank, with a rectangular opening 1·804 ft. wide by 1·181 ft. high; its

sill or lower edge is 2·986 ft. below the level at which the water is constantly kept in the basin; it is supplied by a stream arriving at the other extremity. What is the discharge?

We have $S = 1\cdot804 \times 1\cdot181 = 2\cdot131$ sq. ft.; $h = 2\cdot986 - \frac{1\cdot181}{2} = 2\cdot396$; m, according to the Table, p. 1893, supposed to be prolonged, will be about 0·600; as to u, it will be given by one of the means to be indicated hereafter. In a great number of cases we can regard it as being the mean velocity of the water in the basin, a velocity to be determined as follows: the discharge Q, taken at first by neglecting u will be $0\cdot600 \times 2\cdot131 \sqrt{64\cdot364 \times 2\cdot396} = 15\cdot878$ cub. ft. When the water runs in a canal, we have $Q = Su$; dividing then the value of Q found, by the section (of the basin) 21·53, we find $u = \cdot73748$, the square of which is ·54389. Putting this value into the general expression of the discharge, we have $0\cdot600 \times 2\cdot131 \sqrt{64\cdot364 \times 2\cdot396 + \cdot5439} = 15\cdot906$ cub. ft. Joseph Bennett, the American translator of D'Aubuisson's work on Hydraulics, observes, that here D'Aubuisson's book has an error in taking the section of the orifice, instead of the section of the basin, and also another error in solving the example. What is here given is supposed to be what D'Aubuisson intended.

The difference between these two results may be entirely neglected. The effect of the velocity u has been almost nothing; in most cases it will be so.

Very often the water at the exit of the orifices made in the side of a reservoir is taken and conducted by canals or channels, uncovered on the upper part, the bottom of which as well as the sides agree with the lower edge and sides of the orifice, which are thus in the planes of the bottom and sides respectively. MM. Poncelet and Lesbros determined, by a great number of experiments, the coefficients of the discharge for such canals, which they fitted to orifices on which they had already made the fine observations whose results we have recorded; the canals varied in form, inclination, and position. The last of these philosophers communicated to D'Aubuisson a part of the results given by a rectangular canal $3^m = 9\cdot843$ ft. long and $0^m\cdot20 = \cdot656$ ft. broad, like all its orifices. The reservoir in whose side the orifices were, was $3^m\cdot68 = 12\cdot074$ ft. broad. The canal was first placed at an equal distance from the two sides of the reservoir and $0^m\cdot54 = 1\cdot772$ ft. above the bottom; it was kept horizontal; it is canal No. 1 of the following Table. We here give the coefficients m of the formula $m S \sqrt{2gH}$, which MM. Poncelet and Lesbros obtained, and place them opposite those which they had obtained previously with the same orifices, when the water flowed freely into the atmosphere.

Height of Orifice.	Head on Orifice.	Coefficient. Without Canal.	With Canal No. 1.	With Canal No. 2.	Height of Orifice.	Head on Orifice.	Coefficient. Without Canal.	With Canal No. 1.	With Canal No. 2.
feet.	feet.				feet.	feet.			
·6562	4·2850	0·604	0·601	0·601		·1542	0·617	0·495	0·493
	3·1235	0·605	0·602	0·599		·1181	0·612	0·452	0·443
	1·3124	0·600	0·591	0·580	·0984	4·4261	0·622	0·622	
	·7940	0·596	0·559	0·552		1·5289	0·630	0·629	
	·4003	0·572	0·483	0·482		·6792	0·634	0·632	
·3281	4·4490	0·643	0·614			·2658	0·639	0·633	
	3·3040	0·615	0·614			·2067	0·640	0·627	
	1·5814	0·617	0·615			·1870	0·640	0·610	
	·5282	0·611	0·590			·1214	0·639	0·511	
	·3740	0·608	0·562		·0328	·4449	0·620	0·621	0·660
	·2887	0·602	0·523			3·2580	0·627	0·631	0·665
	·1969	0·590	0·459			1·6307	0·643	0·648	0·671
·1640	4·7935	0·621	0·624	0·627		·6398	0·655	0·665	
	3·5468	0·627	0·626	0·628		·4167	0·664	0·669	
	1·6350	0·631	0·625	0·624		·2494	0·671	0·671	0·680
	·6956	0·634	0·631	0·615		·1378	0·684	0·640	
	·3478	0·629	0·614	0·597					

By comparing the coefficients of the third and fourth columns, allowing for the inevitable errors in observation, and excepting the orifice of 0·328 ft., we see that so long as the heads taken above the centre of the orifice were from 2 to 2½ times greater than the height of that orifice, the canal had no marked difference in the discharge; the discharge was the same as if no canal were there. But in small heads, the discharge diminished perceptibly, and as much more so as the head was less; the diminution has reached a quarter, and even more.

This difference in great and small heads appears to proceed from the fact that, with the former, the fluid, rushing forth as into the air, is not influenced by the resistance of the sides. The canal, says Lesbros, has no influence, except when the head is not great enough to detach the fluid jet at its exit from the orifice entirely from the bottom (and sides) of this canal.

The same canal was then placed, as is often done in practice, in such a manner that its floor was at the level of the bottom of the reservoir, and was, in fact, a prolonging of it. It was natural to suppose that the contraction being then suppressed on the lower edge of the orifice, the coefficient of discharge would be greater; but generally, and the orifice of ·0328 ft. still excepted, it was less, particularly with small heads, as was seen in the above Table, where the canal, in its

new position, is designated by No. 2. Other circumstances, perhaps the resistance of the bottom of the reservoir, which may have diminished the velocity of arrival, perhaps the less facility which the fluid sheet had in raising itself above the sill at the entrance of the canal, will have more than compensated for the diminution in the contraction.

In withdrawing the canal from the middle of the reservoir, and placing it nearer one of the sides, this diminution took place in part, and a small increase in the discharge was obtained. The canal was then inclined, leaving it in other respects in the position it last had. When the inclination was $\frac{1}{100}$ or 34′, the coefficients were sensibly the same as when the canal was horizontal. But when the inclination was carried to $\frac{1}{10}$, or 5° 44′, the coefficients were increased from 3 to 4 per cent., as seen in the following Table;—

Height of Orifice.	Head on Orifice.	Coefficients, with the Canal	
		Horizontal.	Inclined.
feet.	feet.		
·0443	1·1188	·660	·691
·0666	1·1123	·654	·681
·1555	·6890	·616	·639
·1775	·6660	·612	·636

Cylindrical Ajutages.—Cylindrical ajutages, called also *additional tubes*, as we have seen, give a more considerable discharge than orifices in a thin side, the head and area of the opening remaining the same. But in order to produce this effect, it is necessary that the water entirely fill the mouth of the passage; it is commonly so, when the length of the tube is two or three times its diameter. If it is less, it often happens that the fluid vein, which is contracted at the entrance of the tube, does not again increase and fill the interior; the flowing then takes place in all respects as through a thin side; this is always the case when the length of the tube is less than that of the contracted vein, and consequently is only half, or less than half the diameter.

The coefficient of reduction from the theoretic to the actual discharge, through an additional tube, presents a few variations, as may be seen in the following Table;—

Observer.	Tube.		Head.	Coefficient.
	Diameter.	Length.		
	feet.	feet.	feet.	
Castel	·0509	·1312	·6562	·827
"	·0509	·1312	1·5749	·829
"	·0509	·1312	3·2478	·829
"	·0509	·1312	6·5620	·829
"	·0509	·1312	9·9414	·830
Bossut	·0755	·1772	2·1326	·788
"	·0755	·1772	4·0684	·787
Eytelwein	·0853	·2559	2·3623	·821
Bossut	·0886	·0341	12·6318	·804
"	·0886	·1772	12·6975	·804
"	·0886	·3543	12·8615	·804
Venturi	·1345	·4200	2·8873	·822
Michelotti	·2658	·7087	7·1526	·815
"	square. ·2658	·7087	12·4678	·803
"	·2658	·7087	22·0155	·803

The mean of the coefficients, abstracting the first two of Bossut, manifestly anomalous, is 0·817; ·82 is generally taken, and we have

$$Q = \cdot 82\, S \sqrt{64 \cdot 364\, H} = 6 \cdot 5786\, S \sqrt{H} = 5 \cdot 1668\, d^2 \sqrt{H}.$$

Since the jet in a full tube runs out in lines parallel to the axis of the orifice, and consequently its section is equal to that of the orifice, the diminution of the discharge can arise only from a diminution in the velocity; and the ratio of the actual to the theoretic discharge will also be that of the actual to the theoretic velocity, as is seen by the following results of three experiments cited in the above Table; one of Venturi and two of M. Castel;—

Jet.		Velocity.		Coefficient	
Abscissa.	Ordinate.	Real.	Theoretic.	Of Velocity.	Of Discharge.
feet.	feet.	feet.	feet.		
4·796	6·128	11·204	13·628	·824	·822
1·791	2·208	6·6175	7·959	·832	·827
3·7402	5·803	12·037	14·481	·832	·829

Thus we may admit that the velocity of a jet, at its passage from a cylindrical tube, is only 0·82 of that due to the height of the reservoir; and the height due to the velocity of the jet will be only ·67 ($= \cdot 82^2$) of that due to the height of the reservoir, since the heights or heads are supposed to be as the squares of the velocities.

In the hypothesis of the parallelism of the sections, the principle of the *vis viva:* that the quality of action developed by the motive force, during a certain time, is equal to half the increase or diminution of the *vis viva* during that time—this principle gives for the velocity v of the water passing from a short prismatic tube, of which S is the section, and which is terminated by an orifice whose section s is smaller than the preceding, m and m' being the coefficient of contraction for these sections respectively

$$= \sqrt{\frac{2gH}{1 + \left(\frac{m's}{S}\right)^2 \left(\frac{1}{m} - 1\right)^2}};$$

and for the case of our additional tubes entirely open at their extremity, and consequently where $s = S$ and $m' = 1$,

$$v = \sqrt{\frac{2gH}{1 + \left(\frac{1}{m} - 1\right)^2}}.$$

If it be admitted that the contraction at the entrance of the tube is the same as in the orifices in a thin side, that is to say, if we make $m = \cdot 62$, we have $v = 0\cdot 855 \sqrt{2gH}$ and $Q = \cdot 855\ S \sqrt{2gH}$; with $m = \cdot 65$, it would be $Q = \cdot 0885\ S \sqrt{2gH}$.

The fluid vein, after its contraction at the entrance of the additional tube, tends to take and preserve a cylindrical form, whose section would be that of the contracted vein; and consequently it tends to pass out without touching the sides of the tube; but some lines of water are carried towards the sides, either by a divergent direction, by an attractive action, or by the two causes united. As soon as they arrive in contact, they are strongly retained by the molecular attraction, that which produces the ascension of water in capillary tubes; by an effect of this same force they draw the neighbouring lines, and by degrees the whole vein, which then rushes out, filling the tube, and passes through the contracted section more rapidly. Such appears to be the physical cause of the increase of discharge due to tubes.

The immediate cause is the contact; and all the circumstances which cause the contact, or which favour it, will produce that increase.

Among these circumstances we will notice;—

1st. The length of the tube; the longer it is, the more chances it will present for contact; there will be no contact when the length is less than that of the contracted vein.

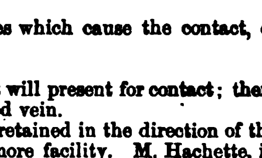
2nd. A small velocity; the fluid lines will then be less forcibly retained in the direction of the primitive motion; they will deviate and approach the sides with more facility. M. Hachette, in his experiments made on this subject, succeeded, by augmenting the head and consequently the velocity, in detaching a vein from the side it was following. On the contrary, by diminishing the head, allowing it, however, a head of 0·9843 ft., he succeeded in making the tube more full, the length of which was 0·01968 ft., and its diameter 0·03117 ft.

3rd. The affinity of the material of the tube, or rather its disposition to be more readily moistened. Thus, by rubbing tallow or wax on the sides, the water will not follow them as it did before. Hachette, by covering an iron tube with an amalgam of tin, caused mercury to run out with a full tube, which did not take place before the coating. The interposition of air, or its arrival in a tube, is sufficient to detach the fluid vein from it. Venturi, after having fitted to a vessel full of water, a tube of $0^m \cdot 0406 = \cdot 1332$ ft. diameter and $0^m \cdot 095 = \cdot 3117$ ft. length, perforated near the middle and quite round its perimeter, with a dozen small holes; when the flowing took place, not a drop of water passed through these holes, nor did the water touch the sides. The holes were then successively stopped, and the same results continued; but when all were closed, the vein filled the tube, and the discharge was increased in the ratio of 31 to 41. M. Hachette, on repeating the experiments and closing the holes with caution, saw the vein continue to pass out without touching the side, but a slight agitation was then enough to produce contact, and to produce a flow with the full tube.

It is more than a century since Poleni made known the singular effects of cylindrical tubes, and the investigation of the cause has been a serious study with philosophers.

It was generally said, since the convergence in the direction of the fluid lines, on their arrival at the orifice, produces a contraction in the fluid vein, there will also be a contraction at the entrance of the tube; but in consequence of the attractive action of the sides, the contraction will be less, and the discharge will consequently be greater. The experiments of Venturi do not allow us to admit of such a cause producing a less contraction.

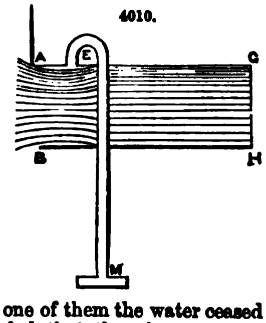
4009.



That ingenious philosopher opened, in a thin side of a reservoir, an orifice, whose diameter A B, Fig. 4009, was $0^m \cdot 0406 = \cdot 1332$ ft., and under a head of $0^m \cdot 88 = 2 \cdot 8873$ ft., he obtained $0^{mcm} \cdot 137 = 4 \cdot 8384$ cub. ft. of water in 41″. To this orifice he then fitted the tube A B C D, having nearly the form of the contracted vein (he had C D = $0^m \cdot 0327 = \cdot 1073$ ft., and A C = $0^m \cdot 025 = \cdot 082$ ft.); under the same head he obtained the same volume of water in 42″. To the first tube he fitted the tube C D H G C,

in which G H = E F = A B, and the duration of the flowing, all else being equal, was only 31″. Lastly, Fig. 4010, for all this apparatus he substituted the simple cylindrical tube A B H G of the same length, and also of the diameter ·1332 ft., and the flowing of 4·8384 cub. ft. again took place in 31″.

4010.



Thus, in this simple tube, in which everything went on as in the compound tube, there was or there may have been an equal contraction; and the contraction which necessarily took place in the latter at C D is very nearly equal to that of orifices in a thin side. The effect of the cylindrical tube, therefore, was not to lessen the contraction, but to pass the fluid through the contracted section C D, with a velocity increased in the ratio of 31 to 41. Hence alone the increase of discharge.

Venturi attributed it to an excess in the pressure of the atmosphere on the fluid surface contained in the reservoir, an excess proceeding from a vacuum tending to arise in the part of the tube where the greatest contraction took place. He sought to prove this opinion by several examples, very interesting on other accounts, but he has sometimes generalized the results too much. For example, because in one of them the water ceased to flow with full tube under the receiver of an air-pump, he concluded that the phenomena of additional tubes did not take place in the vacuum, and yet Hachette is certain of having produced them there. This single fact would overthrow an hypothesis, against which other peremptory objections are also raised.

Among the experiments of Venturi is one which presents, in a distinct manner, a very remarkable fact, which Bernoulli had already made known. To a cylindrical tube $0^{m}\cdot0406 = \cdot1332$ ft. diameter and $0^{m}\cdot122 = \cdot4003$ ft. long; at E $0^{m}\cdot018 = \cdot0591$ ft. from its origin, he fitted a curved tube of glass, the other extremity of which was plunged into a vessel M, containing coloured water; the flowing was caused by a head of $0^{m}\cdot88 = 2\cdot8873$ ft.; and the water was raised in the tube $0^{m}\cdot65 = 2\cdot1326$ ft.

In the hypothesis of Venturi, this elevation, joined to the head, would be the height due to the velocity through the contracted section, as the head alone is the height due when there is no additional tube; if it were so, the ratio of the velocities must be as $\sqrt{2\cdot8873}$: $\sqrt{2\cdot8873 + 2\cdot1326}$, or as 31 to 40·9, and experiment has actually given a similar result (31 to 41). But from this fact, peculiar perhaps to the case taken for example, a general principle ought not to be deduced. Moreover, the true cause of the ascension of the coloured water in the tube was indicated more than a hundred years ago by Daniel Bernoulli. That celebrated geometrician, author of the chief part of the theoretical principles of the flowing of water, established the law, that the pressure which a fluid exerts against the sides of a tube in which it moves, is equal to the head minus the height due to the velocity of the motion. It is necessary to remark that in speaking of absolute pressure the weight of the atmosphere should be added to the *head* properly so called; thus, if K represents that weight, that is to say, a column of water equal in weight to that of the column of the barometer, H the head and v the velocity of the fluid at a determined point of the tube, $K + H - \cdot01553\,v^2$ will be the interior pressure at that point. For the exterior pressure we have K, as on all the other points. In one example, at the place of greatest contraction, where $v = \frac{41}{31}\sqrt{2\,g\,H}$ and $H = 2\cdot887$ ft., the interior pressure is $K + 2\cdot887 - 5\cdot050 = K - 2\cdot163$ in feet, it is less by 2·163 ft. than the exterior pressure; the exterior pressure will therefore prevail, and will cause the water to ascend 2·163 ft., and, in general, a quantity equal to its excess over the other.

By neglecting K, which is found both in the value of the interior and exterior pressures, the interior pressure on the same point compared to the other is $H - \cdot01553\,v^2$; it will be negative whenever the height due to the velocity is greater than the head.

Venturi having placed the same tube $0^{m}\cdot054 = \cdot177$ ft. from the reservoir, the coloured water was not raised; the height due, $0^{m}\cdot594$ or $0\cdot051\,v^2 - 0\cdot051\,(0\cdot82)^2\,g\,H$ in mètres, or $\cdot01553\,v^2 - \cdot01553\,(0\cdot82)^2\,g\,H$ in feet, was then smaller than the head 2·8873 ft.; the interior pressure was positive, and consequently there was no ascension. Bennett, the translator of D'Aubuisson, remarks;—Should the reader find difficulty as to the formation of this formula, it will vanish in remembering that the velocity from cylindrical pipes is but $\frac{82}{100}$ of that due to the height of reservoir (or $v = \cdot82\sqrt{2\,g\,H}$), and by substituting this value in the equation $H = \dfrac{v^2}{2\,g}$.

Conical Converging Tubes.—Conical tubes, properly so called—that is to say, those which slightly converge towards the exterior of the reservoir—increase the discharge still more than the preceding; they afford very regular jets, and throw them to a greater distance or height. They are also almost exclusively employed in practice. However, their effects as to the discharge and velocity of projection are much more varied; they change with the *angle of convergence*, that is, with the angle which the opposite sides of the truncated cone constituting the tube, form by their extension.

They are, however, the tubes on which we have the fewest documents. In reference to them, D'Aubuisson knew of only four experiments of Poleni, published at Florence in 1718, and which Bossut gives in his Hydrodynamique; notwithstanding the merit of their author, and although made on a great scale, D'Aubuisson had very strong reasons for doubting their accuracy. Struck by the gap which hydraulics presents in this important part, he projected a series of experiments suitable to fill it.

There are or there may be two contractions of the fluid vein, in running through conical tubes; one interior, or at the entrance of the tube, which diminishes the velocity produced by the head; the other exterior, or at the exit, by which the section of the vein a little below the exterior mouth of the orifice is smaller than the mouth itself. Consequently, if s is the section of the orifice, and

V the velocity due to the head, the real discharge will be $n s \times n' V = n n' S V$; n and n' being two coefficients to be found by experiment; n is the ratio of the fluid section to the section of the orifice, or the coefficient of the exterior contraction; n' is the ratio of the actual to the theoretic velocity, or the *coefficient of the velocity;* and $n n'$ is the ratio of the actual to the theoretic discharge, or the *coefficient of discharge.*

The knowledge of the two latter, for the different cases which may present themselves, is sometimes useful in practice, as we shall see in treating of *jets of water;* it is this utility, or rather necessity, of having their value, that is, of knowing the discharge and force of projection of different tubes, which has induced the experimenter to make researches on this subject.

To determine properly the different coefficients in question, and above all, to fix the angle of convergence giving the greatest discharge, D'Aubuisson thought it necessary to subject many series of tubes to experiment; in each, the diameter of the orifice of exit and the length of the tube remaining constantly the same; but the diameter of the entrance, and consequently the angle of convergence, was gradually increased. The water flowed through each under different heads. At each experiment the actual discharge was determined by direct measurement, and the velocity of exit by the mode indicated above; the discharge, divided by S V, would give $n n'$, and the velocity divided by v ($v = \sqrt{2 g H}$), would give n'. The series of $n n'$ would show the discharge corresponding to each angle of convergence, and consequently the angle of greatest discharge; and the series of n' would indicate the progression according to which the velocity increased.

The water-works of Toulouse offered all the desirable facilities for executing such a plan. M. Castel, the hydraulic engineer of that city, was pleased, on the invitation of the Academy of Sciences, to undertake the execution.

In 1831, with a very small apparatus, and under small heads, Castel had made a series of experiments, the details and results of which were published in the Annales des Mines of 1833. In 1837 he resumed and considerably extended his works.

This apparatus consisted principally of a rectangular cast-iron box $0^m \cdot 41 = 1 \cdot 345$ ft. long, 1·345 ft. wide, and $0^m \cdot 82 = 2 \cdot 69$ ft. high; it received at its lower part, and by means of a great tube, the water coming from a reservoir established more than 29·529 ft. above it and kept constantly full; on the front face of the box is a rectangular opening, ·459 ft. high by ·328 ft. wide; it was closed by a well-finished copper plate, to which were fitted additional tubes, in such a manner that their axes were horizontal. When the box was opened at top, the fluid surface could rise there to about ·689 ft. above that axis. The upper opening is commonly surmounted with short tubes of ·656 ft. diameter, the first of which is ·984 ft. high, and the rest 1·64 ft. high, so that heads of about ·656 ft., 1·64 ft., 3·281 ft., 4·921 ft., 6·562 ft, &c., above the tube subjected to experiment, could be obtained

By means of two cocks placed, one at the entrance of the water into the box, and the other on the upper part of the tubes which surmount it, a perfectly constant level was obtained.

The tubes which M. Castel used were of brass, as well turned and polished as possible. He had two series of them; in one, the diameter of the exit was ·05086 ft. and the length about ·1312 ft.; in the other, the diameter was ·06562 ft. and the length ·164 ft.

The two diameters of each were measured and re-measured with much care, but the want of an instrument proper to operate accurately with such measures, did not permit of a measurement nearer than $0^m \cdot 00005 = 0 \cdot 002$ in. ($\frac{1}{1000}$), and such an error might give an error of half a hundredth in the discharges and coefficients.

M. Castel rarely had them so large. He operated under heads of ·6562 ft., 1·64 ft., 3·281 ft., 4·921 ft., 6·562 ft., and about 9·843 ft.; he measured them with very great exactness. He then gives, as very exact, the volumes of water obtained in a certain time.

To determine the velocities with which the water passed from the tubes, he erected, 3·74 ft. below their axis, a horizontal flooring, in the middle of which was a longitudinal groove ·328 ft. broad, into which the jet passed; its range was measured by means of a graduated rule fixed on the flooring and quite near. This range was the ordinate of the curve described by the jet; ·374 ft. was its abscissa, and from these two ordinates was deduced the velocity of projection. Finally, these velocities could only be taken for heads of 6·562 ft. and less; beyond that the jets were broken, and passed beyond the plane where they could be measured.

The same tube, under heads which varied from 0·689 ft. to 9·941 ft., gave discharges always proportional to $\sqrt{H}$, and consequently the coefficients were sensibly the same. Perhaps they experienced a very slight increase under the head of 9·941 ft. We here give those which were obtained with the pipe of each of the two series which furnished the greatest discharge.

Tube of ·05085 foot diameter			Tube of ·0656 foot diameter.		
Head in feet.	Coefficient		Head in feet.	Coefficient	
	Of Discharge.	Of Velocity.		Of Discharge.	Of Velocity.
·7054	·946	·963	·6923	·956	·966
1·5847	·946	·966	1·5847	·957	·968
3·2547	·946	·963	3·2646	·955	·965
4·8952	·947	·966	4·9149	·956	·962
6·5817	·946	·956	6·5782	·956	·959
9·9414	·947	..	9·9414	·957	..

As to the coefficients of the velocity, it seemed that they would have been sensibly constant, were it not for the resistance of the atmosphere. But this resistance diminishing the range of the

jet, and as much more so as the head was greater, there must be in the calculated coefficients a diminution varying with the head, although in reality there was none in the velocity with which the fluid passed out or tended to pass out. We will now compare together the coefficients, both those of the discharge and of the velocity, obtained with the different tubes of the same series; tubes which, in other respects, differed only in the angle of convergence; for each of them the mean term was taken between the six or five coefficients which were given under the six or five heads nearly equal to those which are noted in the preceding Table.

Ajutage ·05085 foot in diameter.			Ajutage ·0656 foot in diameter.		
Angle of Convergence.	Coefficient of		Angle of Convergence.	Coefficient of	
	Discharge.	Velocity.		Discharge.	Velocity.
° ′			° ′		
0 0	0·829	0·830			
1 36	0·866	0·866			
3 10	0·895	0·894	2 50	0·914	0·906
4 10	0·912	0·910			
5 26	0·924	0·920	5 26	0·930	0·928
7 52	0·929	0·931	6 54	0·938	0·938
8 58	0·934	0·942			
10 20	0·938	0·950	10 30	0·945	0·953
12 4	0·942	0·955	12 10	0·949	0·957
13 24	0·946	0·962	13 40	0·956	0·964
14 28	0·941	0·966	15 2	0·949	0·967
16 36	0·938	0·971			
19 28	0·924	0·970	18 10	0·939	0·970
21 0	0·918	0·971			
23 0	0·913	0·974	23 4	0·930	0·973
29 58	0·896	0·975	33 52	0·920	0·979
40 20	0·896	0·980			
48 50	0·847	0·984			

It follows, from the facts set down in these columns;—That for the same orifice of exit, and under the same head, starting from 0·83 of the theoretic discharge, the actual discharge gradually increases, in proportion as the angle of convergence increases up to 13½° only, where the coefficient is 0·95. Beyond this angle it diminishes, feebly at first, as do all variables about the maximum; at 20° the coefficient is again from 0·92 to 0·93. But afterwards the diminution becomes more and more rapid; and the coefficient would end by being only 0·65, the coefficient of small orifices in a thin side, these orifices being the extreme term of converging tubes, that in which the angle of convergence has attained its greatest value, 180°. The angle of greatest discharge will then be from 13° to 14°.

What can be the reason of this? In the conical tubes the theoretic discharge is altered by two causes, the attraction of the sides, which tends to augment it, and the contraction, which tends to diminish it, by diminishing the section of the vein a little below the exit. From the experiments of Venturi it would seem that the fluid vein, at its entrance into a tube, preserved its natural form, that of a conoid of 18° to 20°; so that the nearer the angle of the tube approached such a value, the nearer its sides will be to the vein, at the moment when, after having experienced its greatest contraction, it tends to dilate, and when it is, as it were, left to their attractive action; this action then being stronger, the discharge will be greater. But on the other hand, already at 10° of convergence, the exterior contraction begins to be sensible and to reduce the discharge; it has reduced it 5 per cent. at 18°; and after that, it will not be extraordinary that the angle of greatest discharge is found between these two values, about 14°.

The tubes of ·0656 ft. diameter at the exit, gave coefficients from one to two hundredths greater than those of the tubes of ·0509 ft. An error of 0·004 in. in the estimate of the diameter of the first set, would afford reason, to a great extent, for that difference; and the experimenter was inclined to admit a cause of that kind. The tubes of ·0509 ft., examined several times, inspired him with more confidence.

In following the coefficients of the velocity they are seen, again starting from the angle 0°, to increase like those of the discharge up to near the convergence of 10°; then they increase more rapidly; and beyond the angle of the greatest discharge, while the others diminish, these continue to increase and approach their limit, 1; they are quite near it at the angle of 50°, and even at 40°. The conical tubes, by their different convergence, form a progression of which the first term is the cylindrical tube, and the last is the orifice in a thin side; their velocity of projection, increasing with the convergence, will therefore vary from that of the additional tube to that of the simple orifice, that is to say, from $0·82\sqrt{2g\mathrm{H}}$ to $\sqrt{2g\mathrm{H}}$.

In comparing the coefficients of the discharge with those of the velocity, or their successive values $n n'$ and n', and dividing the first by the second, we shall have the series of n, or the coefficients of the exterior contraction. From the angle 0° to that of 10°, we have sensibly $n = 1$, and consequently there is no contraction; notwithstanding the convergence of the sides, the fluid particles pass out very nearly parallel to the axis. But beyond 10°, contraction is manifested; it

reduces the section of the vein more and more, and it would end by rendering it equal to that which passes from orifices in a thin side, as is seen in this Table;—

Angle.	*n*.	Angle.	*n*.
°		°	
8	1·00	40	0·88
15	0·98	50	0·85
20	0·95	100	0·65
30	0·92		

Experience having taught that cylindrical tubes certainly produce all their effect, as to the discharge, when their length equals at least 2½ times their diameter; by analogy, and for the sake of not complicating our results with the action of the friction of the water against the sides, the experimenter fixed the length of conical tubes at about 2½ times the diameter of exit; thus it was ·1312 ft. for those of ·0509 ft. diameter, and ·164 ft. for those of ·0656 ft. diameter. However, to be able to determine the effect of their length, he proposed for the tubes of ·0509 ft. diameter, two other series; in one, the common length would have been ·0984 ft., which may be regarded as the *minimum;* for the other, it would have been ·3281 ft., a dimension quite common in practice.

But this work is yet to be done; still, M. Castel has made some primary trials. For the tubes of ·0509 ft. diameter, he took five ·1148 ft. long, and, taken together, they gave as the coefficient of discharge, 0·938; next, with a length of ·1312 ft., he had as coefficient, 0·936; another tube, ·0984 ft. long, gave 0·941 instead of 0·938; and one of ·0787 ft. indicated 0·931 instead of 0·926; so that here the diminution of length would have a little increased the discharge. But with the tubes of ·0650 ft. diameter the discharge, on the contrary, was increased with the length; the length passing from ·1640 ft. to 0·3281 ft., the coefficient under the angle of 11° 52′ was 0·965; under that of 14° 12′, 0·958; and under 16° 34′, 0·950. Thus the effect of the length of tubes is far from being established; its determination demands other series of experiments.

While waiting for more extensive experiments we will assume, for each of the tubes to be employed, provided extraordinary lengths are not taken, the coefficient in the above Tables corresponding to the angle of convergence, without fear of introducing any error of moment.

As to very great conical tubes, or rather to pyramidal *troughs*, which in mills throw the water on to hydraulic wheels, we have three valuable experiments made by the engineer Lespinasse, on the mills of the canal of Languedoc. The troughs there are truncated rectangular pyramids, having a length of 9·5904 ft.; at the greater base, 2·3984 ft. by 3·199 ft.; at the lesser base, ·4429 ft. by ·6234 ft. The opposite faces make angles of 11° 38′ and 15° 18′. The head was 9·5904 ft.

The first two of the three experiments, the results of which are here given, were made on a mill of two stones, each having its wheel; in the first experiment the water was let on to only a single wheel; in the second it was let on to two at a time.

Discharge.	Coefficient.
cubic feet.	
6·7667	0·987
6·6926	0·976
6·7138	0·979

We see how little such tubes diminish the discharge; the discharge given is only one or two hundredths less than the theoretic discharge.

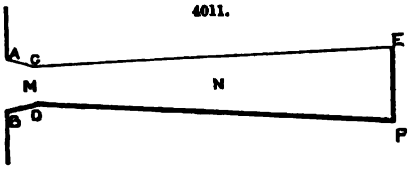
Conical Diverging Tubes.—Of all tubes, those which give the greatest discharge are truncated cones, fitted to a reservoir by their smaller base, and of which the opening for exit is consequently greater than that of entrance. Although very little used, they present phenomena of too much interest to be passed by.

Their property of increasing the discharge was known to the ancient Romans; some of the citizens, to whom was granted a certain quantity of water from the public reservoirs, found by the employment of these tubes, means of increasing the product of their grant; and the fraud became such, that a law prohibited their use; at least, they could not be placed within 52½ ft. from the reservoir.

Bernoulli had studied and subjected to calculation their effects; in one of his experiments he found the real velocity at the entrance of the tube greater than the theoretic velocity, in the ratio of 100 to 108; but to Venturi is principally due our knowledge of the products they can give.

The tubes which he used had a mouth-piece A B C D, Fig. 4011, presenting nearly the form of the contracted vein; A B = ·1332 ft., and C D = ·1109 ft.; the body of the tube C D F E varied in length and flare, the flare being measured by the angle comprised between the sides E C and F D sufficiently prolonged. These tubes were fitted to a reservoir kept constantly full of water; the flowing took place under a constant head of 2·8873 ft., and the time necessary to fill a vessel of 4·8384 cub. ft. was counted as in the experiments of the same author which we have already mentioned.

4011.



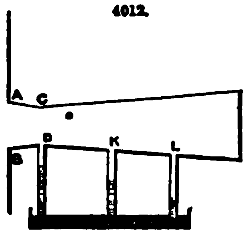
D'Aubuisson gives, in the following Table, the result of the principal observations, after having remarked that the time corresponding to the theoretic velocity was 25″·49.

Ajutage.		Time of Running.	Coefficient.	Observations.
Flare.	Length.			
° ′	feet.			
3 30	·3642	27″5	0·93	
4 38	1·0959	21	1·21	Jet very irregular.
4 38	1·5093	21	1·21	Jet did not fill the ajutage.
4 38	1·5093	19	1·34	To fill ajutage a projecting body introduced.
5 44	·5775	25	1·02	
5 44	·1936	31	0·82	Exit mouth = that of entrance.
10 16	·8662	28	0·91	Jet did not fill ajutage.
10 16	·1476	28	0·91	Jet very regular.
14 14	·1476	42	0·61	Jet detached from sides.

Venturi concluded from his experiments, that the tube of the greatest discharge ought to have a length nine times the diameter of the smaller base, and a flare of 5° 6′; Fig. 4011 represents it; it would give, adds the author, a discharge 2·4 times greater than the orifice in a thin side, and 1·46 times greater than the theoretic discharge. Moreover, he observes, that the dimensions of the tube should vary with the head.

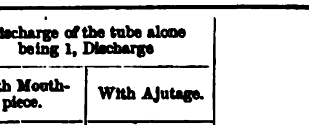
To one of the above-mentioned tubes, that which gave 4·8384 cub. ft. in 25″, he fitted three tubes, and plunged them into a small bucket filled with mercury; the first at the origin D, Fig. 4012, of the tube; the second at one-third of its length, and the third at two-thirds. The mercury was raised respectively ·3937 ft., ·1509 ft., and ·0518 ft.; this would be equivalent to columns of water 5·348 ft., 2·067 ft., and ·7054 ft. According to the theory of Bernoulli, the pressure at the point of greatest contraction D, where the velocity is [illegible] $\sqrt{2g \times 2\cdot8873}$ ought to have been $2\cdot8873 - 2\cdot8873$ ([illegible])$^2 = -5\cdot2618$ ft.; the experiment of Venturi gave $-5\cdot348$ ft.

4012.



Eytelwein also used diverging tubes in experiments, the results of which are directly interesting in practice. He took a series of cylindrical tubes ·0853 ft. diameter, and of different lengths, which he successively fitted to a vessel full of water: at first separate; then applying to the front extremity the mouth-piece M, which had nearly the form of the contracted vein; then applying to the other extremity the tube M, Fig. 4013, of the form recommended by Venturi; lastly, applying at the same time the mouth-piece and the tube.

4013.



The flowing took place under a mean head of 2·3642 ft. The principal results obtained are given in the following Table.

Here the head was not constant. At each experiment the vessel was filled up to 3·0841 ft. above the orifice, and the fluid was suffered to fall until the surface was only 1·7389 ft. above the orifice; the constant head, which would have given the same discharge in the same time, would have been 2·3642 ft. Let, generally, H′ be that constant head; H the head of the reservoir at the commencement of the flowing, and h that at the end, we shall have $H' = \left(\dfrac{H - h}{2(\sqrt{H} - \sqrt{h})}\right)^2$.

The occasion to make use of this formula will be presented quite often in practice.

Length of Tube.	Coefficient of discharge of the tube, only according to		Discharge of the tube alone being 1, Discharge	
	Experiment.	Formula of Conduits.	With Mouth-piece.	With Ajutage.
feet.				
·0033	0·62	0·99		
·0853	0·62	0·97	1·56	
·2559	0·82	0·95	1·15	1·35
1·0302	0·77	0·86	1·13	1·27
2·0605	0·73	0·77	1·10	1·24
3·0907	0·68	0·70	1·09	1·23
4·1176	0·63	0·65	1·09	1·21
5·1479	0·60	0·61	1·08	1·17

These experiments show;

1st. The rate according to which the length of the tubes diminishes the discharge; and this, up to a point where the formula for the motion of water in conduit pipes may be applied. The numbers of the third column indicate that this application can take place for small tubes, those under ·0984 ft. diameter, when their length exceeds 6·562 ft. These experiments thus in part fill up the void which existed in our knowledge of additional tubes and conduit pipes.

2nd. That the increase of the discharge proceeding from the flare given to the mouth of entrance of pipes, diminishes in proportion as their length is greater. It were desirable that these experiments had been carried further, for the purpose of knowing what would have been the result of this diminution in large conduits; until this is done, and however small may be the good effect of the flaring at the entrance, it is proper not to neglect it.

3rd. The effect of the flaring at the exit also diminishes in a ratio more rapid still, in proportion as the pipes increase in length. Eytelwein having taken one 20·6 ft. long and of ·0853 ft. diameter throughout, found no difference in the discharge, whether he did or did not use the tube with flaring end.

On fitting this tube immediately to the reservoir, the discharge was 1·18, the theoretic discharge being 1. On fitting it to the mouth-piece, but without the intermediate tube, it rose up to 1·55. The mouth-piece alone gave only 0·92; so that the effect of the tube N added to the mouth-piece M, was to augment the discharge in the ratio of 0·92 to 1·55, or of 1 to 1·69.

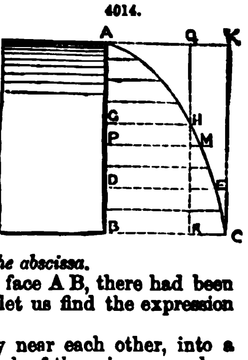
Venturi had that of 19″ to 42″, or 1 to 2·21. In the two experiments which furnished the terms of this last ratio, the velocities of the water at the passage through the section C D, Fig. 4011, were therefore as 1 to 2·21; and consequently the heights due as 1 to 4·89, since they follow the ratio of the squares of the velocities.

In the experiment which gave the term 1, that where the mouth-piece M alone was used, the actual velocity, which was obtained by dividing the discharge by the section, was 11·9297 ft.; it corresponds to a generating head of 2·2114 ft. The head corresponding to the velocity in the second experiment will then be $2·2114 \times 4·89 = 10·8137$ ft.; whence it follows that the discharge was equal to what would have occurred if, instead of adding the tube N to the mouth-piece M, the water had been raised in the reservoir, above the level which it had during the flowing, $10·8137 - 2·2114 = 8·6023$ ft. Thus the accelerating effect of the velocity due to the diverging tube is measured by a column of water 8·6023 ft.; this is more than a quarter of the weight of the atmosphere. This is a very considerable effect for a force which seems quite small; for we see no other physical cause of the augmentation in the discharge produced by the tube, than the action of the sides, and, in short, the molecular attraction.

On Flowing under very Small Heads.—When the head over the centre of the orifice is very small compared to the height (vertical dimension) of that orifice, the *mean* velocity of the different lines of the fluid vein, that is to say, the velocity which, being multiplied by the area of the orifice, gives the discharge, is no longer that of the central line. It differs from the velocity of the central line as much more as the head is smaller; it will be about a hundredth less if the head is equal to the height, and a thousandth less if the head is three times (3·2) greater than the height. Let us see what theory teaches us in this respect; and first, the law which it indicates for the velocity of the fluid lines, in proportion as the point from which they issue is lower than the level of the reservoir.

Let a vessel be filled with water up to A, Fig. 4014; upon its face A B, which we will suppose vertical for greater simplicity, imagine below each other, a series of small holes, of which B will be the lowest. Designate by H the height A B; the velocity of the line passing out at B will be $\sqrt{2gH}$; and if B C be made equal to that quantity, it will represent that velocity. For every other point P, below the level of the reservoir, the distance A P or x, the line P M, which would represent the velocity of the fluid at its exit from that point, would be $\sqrt{2gx}$, and calling it y, we should have $y = \sqrt{2gx}$. If through the extremity of all these lines P M, a curve be made to pass, they will be its ordinates, and the heights A P or x will be its abscissas; and since $y^2 = 2gx$, this curve may be taken as a parabola having $2g$ or 64·364 ft. for its parameter.

4014.



Thus the velocity of a fluid line passing from a reservoir at any point, is equal to the ordinate of a parabola, of which twice the action of gravity is the parameter, the distance of this point below the level of the reservoir being the abscissa.

Suppose now, that instead of opening a series of small holes on the face A B, there had been perforated in it, from top to bottom, a rectangular slit, of the breadth l; let us find the expression of the discharge.

Divide this opening, in thought, by means of horizontal lines very near each other, into a series of small rectangles. The volume of water which will pass from each of these in a second, or its discharge, will evidently be equal to the volume of a prism which shall have for its base the small rectangle, and for its height the corresponding ordinate. The sum of all these little prisms, or the total discharge, will evidently be equal to another prism, having for its base the parabolic segment A B C M A, and for its height or thickness, the width of the slit. Now, according to a property of the parabola, this segment is two-thirds of the rectangle A B C K, whose surface is $AB \times BC = H \times \sqrt{2gH}$. Thus the discharge through the rectangular opening of which H expresses the height and l the breadth, is $\frac{2}{3} l H \sqrt{2gH}$.

We now seek the discharge through a rectangular orifice open on the same side, but from B to D only, and having the same breadth l; call h the head A D, on the upper edge of the orifice; the discharge of the slit which we suppose from A to D would also be $\frac{2}{3} l h \sqrt{2gh}$. Now, it is evident that the discharge through the rectangular orifice of which B D is the height, will be equal to the difference of the discharges through the two slits, and which consequently will be

$$\frac{2}{3} l \sqrt{2g}\,(H\sqrt{H} - h\sqrt{h}).$$

Let us revert to the mean velocity; and first to that which we have when the slit is quite open. Let G be the point from which the fluid line animated with this velocity proceeds; if we

make A G = x, it will be $\sqrt{2gx}$; being multiplied by the area of the slit $l \times \mathrm{H}$, it must give the discharge. But we have seen that this discharge was also expressed by $\frac{2}{3} l \mathrm{H} \sqrt{2g\mathrm{H}}$; we shall then have $l \mathrm{H} \sqrt{2gx} = \frac{2}{3} l \mathrm{H} \sqrt{2g\mathrm{H}}$; whence $x = \frac{4}{9}\mathrm{H}$, and consequently $v = \sqrt{2g \frac{4}{9}\mathrm{H}} = \frac{2}{3}\sqrt{2g\mathrm{H}}$.

Thus the mean velocity will be two-thirds of the velocity of the lower line. In fact, G H, which represents the first, is, according to the above-mentioned property of the parabola, two-thirds of B C, which represents the second.

For the rectangular orifice of which B D or $\mathrm{H} - h$ is the height, x' being the height due to its mean velocity, we should in like manner have $(\mathrm{H} - h)\, l \sqrt{2gx'} = \frac{2}{3} l \sqrt{2g}\,(\mathrm{H}\sqrt{\mathrm{H}} - h\sqrt{h})$; whence $x' = \frac{4}{9}\left(\frac{\mathrm{H}\sqrt{\mathrm{H}} - h\sqrt{h}}{\mathrm{H} - h}\right)^2$.

Example.—There is a prismatic basin, at the bottom of which is a rectangular orifice ·82 ft. base, and ·3937 ft. height; and during the flowing the fluid surface is constantly ·7218 ft. above the lower edge of the orifice. We then have $\mathrm{H} = {\cdot}7218$; $h = {\cdot}7218 - {\cdot}3937 = {\cdot}3281$; thus $x' = \frac{4}{9}\left(\frac{{\cdot}7218\sqrt{{\cdot}7218} - {\cdot}3281\sqrt{{\cdot}3281}}{{\cdot}7218 - {\cdot}3281}\right)^2 = {\cdot}48$ ft.; consequently the mean velocity will be $\sqrt{2g \times {\cdot}48} = 5{\cdot}558$ ft.

D'Aubuisson makes the following observation, which applies more particularly to the case of heads.

During the flow through an orifice, the surface of the fluid in the reservoir, starting from certain points, is curved, and inclines towards the side in which the orifice is pierced; so that the height or vertical distance of the surface, above any part of the orifice, is greater on the up-stream side of the points where the inflection begins, than near to and touching the side. It is the first of these heights or heads which must always be introduced into the formulas of flowing. The distance between the orifice and the line where the fluid surface joins the side is very often introduced (into the formulas); from this there results an error in deficiency, in estimating the discharges which, in some cases, very rare to be sure, may extend even to a tenth of the discharge.

Such errors diminish when the head increases; and according to the experiments of MM. Poncelet and Lesbros, who have also fully explored this question, they will be insensible when the heads exceed ·4921 or 6562 ft., say 6 or 8 inches. Yet in very great orifices the depression of the surface is still perceptible; D'Aubuisson had seen it from 1½ to 2 in. against the sluice-gates of the canal of Languedoc, when the two paddle-gates were open.

If the orifice had a figure different from the rectangle, the expression of the mean velocity, and consequently of the discharge, would be more complicated; its determination would become a problem of analysis of little utility in practice, where great orifices are almost always rectangular. The solution of these problems can be seen in the Architecture Hydraulique of Belidor; and in the Hydrodynamique of Bossut. For the present we shall limit ourselves to that which concerns the circle. Designating by d the diameter, by h the head above the centre, we have for the expression of the discharge, $\pi' d^2 \sqrt{2gh}\left(1 - \frac{d^2}{128 h^2} - \frac{d^4}{3277 h^4} - \&c.\right)$; this discharge is that which corresponds to the velocity of the central line diminished in the ratio indicated by the complex factor.

The discharges, of which we have just given the expression, are theoretic discharges; for reducing them to actual discharges it is necessary to multiply them by the coefficients deduced from experiment.

These also will be furnished us by MM. Poncelet and Lesbros. We indicate them in the following Table;—

Head upon the centre.	Height of Orifices.					
	·6562 ft.	·3281 ft.	·1640 ft.	·0984 ft.	·0656 ft.	·0328 ft.
feet.						
·03281						0·712
·0656				0·644	0·667	0·700
·0984				0·644	0·663	0·698
·1312			0·624	0·643	0·661	
·1640			0·625	0·643	0·660	
·1968		0·611	0·627	0·642		
·2625		0·612	0·628	0·640		
·3281		0·613	0·630	0·638		
·3937	0·592	0·614	0·631			
·4921	0·597	0·615	0·631			
·6562	0·599	0·616	0·631			
·9843	0·601	0·617				
1·6404	0·603	0·617				
3·2809	0·605					

The numbers above are the true coefficients of the contraction of the fluid vein, or the coefficients of the reduction of the theoretic discharge to the actual discharge; for theory gives no other general formula for flowing through orifices than $\frac{2}{3} l \sqrt{2g}\,(\mathrm{H}\sqrt{\mathrm{H}} - h\sqrt{h})$.

That which was established $S\sqrt{2gh}$; where $h' = \frac{1}{2}(H + h)$ applies only to particular cases, very frequent, to be sure, where h' is three or four times greater than $H - h$. In the other cases it is erroneous, and the coefficients which are adapted to it, and which it has served to determine, are erroneous also: they are the coefficients found above the transverse lines which divide the columns. (The coefficients below the lines, although determined by the aid of that formula, are accurate, coinciding with those obtained by the general formula.) Finally, in the first, $m S\sqrt{2gh'}$, the error of the coefficient m is compensated by the error of the formula, and the discharges which it gives are sensibly identical with those of the other; and as it is, besides, more simple, it is commonly employed in all cases.

Example.—What would be the discharge of a rectangular orifice ·9843 ft. wide and ·49215 ft. high, under a head of only ·16405 ft. on its upper edge? Here $H = \cdot 16405 + \cdot 49215 = \cdot 6562$ ft. and $l = \cdot 9843$ ft. The head on the centre, therefore, is ·410125 ft.; the coefficient which corresponds to this head, according to the above Table, is nearly ·603; a mean term between ·593 and ·614. Thus the discharge will be $\frac{2}{3} \times \cdot 603 \times \cdot 9843 \times 8\cdot 02052\,(\cdot 6562\sqrt{\cdot 6562} - \cdot 16405\sqrt{\cdot 16405})$ $= 1\cdot 476$ cub. ft. The ordinary formula, with its coefficient ·592, taken from the ordinary Table, p. 1893, would have given $\cdot 592 \times \cdot 9843 \times \cdot 49215 \times 8\cdot 02052\sqrt{\cdot 410125} = \cdot 1473$ cub. ft.

We have a circular vertical orifice of ·0888 ft. diameter, with a head of ·0592 ft. above the centre. What will be the discharge? Here $d = \cdot 0888$ ft., $h = \cdot 0592$ ft.; so that the expression, p. 1907, becomes $\cdot 012086\left(1 - \frac{1}{56\cdot 89} - \frac{1}{647\cdot 3}\right) = \cdot 011863$ cub. ft. This is the theoretic discharge; and to have the actual discharge it is necessary to multiply it by the coefficient indicated in the Table. We there find 0·667 for an orifice of ·6562 ft. diameter, under a head ·0656 ft. (or of ·0592); under this same head, we then also have ·0644 for an orifice of ·0984 ft., from which we shall take 0·650 for the orifice of ·0888 ft. The actual discharge will then be $0\cdot 65 \times \cdot 011863$ $= \cdot 00771$ cub. ft.

Hydraulic Gauge.—Darcy's gauge, the extreme accuracy of which has enabled scientific men to remove the theory of running water from the domains of speculation into those of almost absolute certainty. Darcy's gauge is a modification of an instrument invented by M. Pitot; and it will be necessary to explain the nature and working of this instrument in order to give a complete explication of the one with which M. Darcy's name has become connected. In the year 1732 M. Pitot communicated to the Academy of Science a discovery which he had made concerning the laws that regulate the motion of water in streams; he presented to that learned body the instrument by means of which the discovery had been made. His invention had enabled him to measure with considerable accuracy the velocity in any given point of the fluid fillets of which a stream is composed, and the discovery which he had made was that the velocity of water decreases as we approach the bottom or the sides of the current, a fact that is well known and well understood in the present day, but one that before Pitot's time had not been thought of, and that for a long time after was warmly disputed in consequence of a false theory then held concerning the motion of fluids.

Pitot's gauge consisted of a long wooden rod of triangular section, to one face of which two glass tubes were fixed. One of these tubes was bent horizontally at its lower extremity; the other, on the contrary, descended vertically to the level of the curved portion of the first. Pitot thought that if this instrument were exposed to the current of water it would give, by the difference of level existing between the two columns of water in the tubes, the height due to the velocity of the fluid at the point under consideration; and that it would then be easy to deduce the required velocity by means of the relation $V^2 = 2gh$, h being the difference observed. The idea was an ingenious one, and moreover it was new. Yet Pitot's instrument was looked upon by practical men with disfavour (although they continued to use it). It was considered a matter of pure speculation from which nothing practical could be derived. And to obtain the mean velocity of a stream of water recourse was always had, either to vertical floats equal in length to the depth of the portion of water whose mean velocity it was required to find, or to some other instrument more or less complicated and needing the assistance of a time-marker. The reason of this lies in the fact that Pitot's instrument, wonderful as it was, was nevertheless in some degree founded in error. Reduced to its simplest theoretical form it might be constructed of a single glass tube horizontally bent at its extremity: the water entering through the horizontal portion which is exposed to the current, holds itself in equilibrium in the vertical tube at a height above the surface of the current equal to $h = \frac{V^2}{2g}$, V being the velocity of the fluid fillet under consideration. When circumstances enable us to measure h and g exactly, we may deduce V from this height with sufficient precision. But usually the chopping of the water against the outer surface of the tube and its supports does not allow us to compare the level of the water in the tube with that of the surface of the stream troubled by the presence of the instrument, and even this surface of the stream is not easily measured on account of the undulations which cover it. It was to avoid this difficulty, which Pitot no doubt discovered by experience, that he added the second tube, the lower end of which was beneath the surface of the water.

Pitot thought that the level of the water in the straight tube must be equal to that of the surface of the stream, and that in this way the difference of level or h, the height due to the velocity, might be readily obtained. Here lay the first error. When a straight tube is placed in a stream of water, the water in the tube stands below the superficies of the stream by a quantity in a constant ratio with the square of the velocity of the fluid fillet passing beneath its lower orifice. Thus the difference h between the levels of the water in the tubes represents a quantity greater than the height due to the actual velocity of the fluid fillet in question. Hence arose an error which rendered Pitot's conclusions inexact. Besides this, the oscillations were very strong in tubes so arranged, especially as the orifices had the same diameter as the tubes; nay more, it was even

deemed necessary to make these orifices funnel-shaped. Thus we see how it was that Pitot's tube could be of no practical use. In the first place its construction was founded upon an erroneous principle, and in the second place the oscillations which took place in the tubes rendered it impossible to estimate truly, especially in the case of feeble velocities, the required difference of level.

We will now consider the modifications which Darcy has made in Pitot's instrument, modifications that have rendered it exact in its results and easy of application. Many careful experiments showed him that if, in a stream of water, in any point of the fluid having a velocity V, we place a vertical tube bent horizontally at its lower extremity, and having its orifice placed first against the stream, then in the direction of this latter, and lastly rectangularly to its direction, there exists a constant relation between the theoretical height $\frac{V^2}{2g}$ due to the velocity of the fillet under consideration, and the quantities h', h'', h'''; h' representing in the first case the height by which the level rises in the vertical branch above the surface of the stream; h'' and h''' the quantities by which the level sinks below the surface of the same stream on the other two hypotheses. In this way, like General Anstruther, he changes the erroneous value given to g. We may therefore state;—

$$\frac{V^2}{2g} = m\,h', \qquad \frac{V^2}{2g} = m\,h'', \qquad \frac{V^2}{2g} = m\,h''',$$

combining either the first and second, or the first and third of these equations;—

$$V = \sqrt{\frac{m\,m'}{m+m'}}\sqrt{2g\,(h'+h'')} = \mu\sqrt{2g\,(h'+h'')};$$

$$V = \sqrt{\frac{m\,m''}{m+m''}}\sqrt{2g\,(h'+h''')} = \mu'\sqrt{2g\,(h'+h''')}.$$

Seeking in the tables the velocities corresponding to the heights $h'+h''$, $h'+h'''$, we find velocities V' and V''; the above equations become therefore $V = \mu V'$ and $V = \mu' V''$. It will be seen from this that it is not necessary to know the level of the surface of the water in which the instrument is placed in order to determine the required velocity. And it must be remarked further that the oscillations in the tubes have been almost nullified by giving the orifices a diameter of only 1½ millimètre, whilst that of the tubes is 1 centimètre. But as these oscillations, however feeble they might be, would still cause the observer some trouble, a cock has been added by means of which the lower orifices of the tubes may be closed simultaneously. These orifices being closed, all communication with the stream is cut off, and the difference may be read upon the tubes and the velocity deduced with perfect ease and precision.

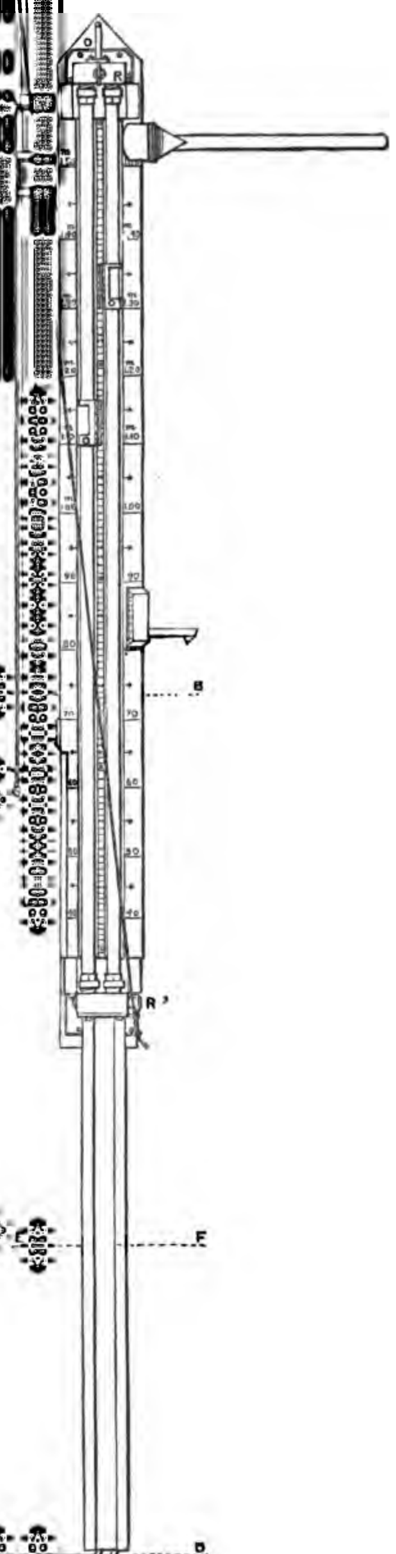
Darcy's gauge possesses another important modification. Most hydrometrical instruments have the grave defect of altering the velocity which they are designed to measure, by the disturbance which they cause in the fluid mass. It was necessary therefore to diminish the size of the gauge, and to remove as far as possible from the divided scale upon which the tubes are fixed, the orifices through which the fluid fillet enters whose velocity it is required to determine. To obtain this double result the scale to which the tubes are fixed is made as thin as possible and bevelled, and copper tubes of a very small diameter affixed below to the glass tubes, the ajutages being placed at the extremity of these copper tubes. Here another question arises; How are we to measure the velocities at the surface or even of the whole liquid mass equal in depth to the length of the copper tubes through which the water cannot be seen? This result has been obtained by the following means: the two glass tubes communicate with each other in their upper portion by means of a copper tube which is hermetically adjusted to them; upon this copper tube a cock is placed which, according as it is open or shut, puts the tubes in communication with the atmosphere, or cuts off this communication. Above this cock is a little mouth-piece, by means of which an imperfect vacuum is produced by suction; the water ascends in the glass tubes to the height desired, and is kept in that position by closing the cock which cuts off the communication with the atmosphere. The upper cock offers the additional advantage of enabling the operator to determine, with an instrument of a height much less than the depth of the stream, the velocity of the latter at a given depth. To effect this, he has merely to lower the instrument 1, 2, or 3 mètres into the water by means of an iron rod, to which it is fixed in such a way as to preserve its mobility about a vertical axis, and the orifice of the horizontal portion of Pitot's tube is kept directed against the stream by means of a kind of rudder.

In the former case the instrument acts under dilated air; in the latter under more or less compressed air. But it is evident that in both cases the differences of level between the tubes are the same as if the operation were performed under the influence of atmospheric pressure.

Fig. 4015 represents the most approved form of Darcy's gauge. The vertical glass tubes are $1^m{\cdot}25$ in length; the two small copper tubes placed on the lower portion are enclosed in a kind of box, also of copper, Figs. 4015 and 4018, $0^m{\cdot}77$ long, $0^m{\cdot}06$ broad, and only $0^m{\cdot}011$ thick; this box ends on both sides in a sharp angle for the purpose of lessening as much as possible the shock of the water, a result which is perfectly obtained, as the instrument when placed in the water causes no appreciable disturbance.

The measurement of the velocities in a given point of the section of a stream is effected in the following manner. Above the stream, at the point at which the experiments are to be made, a slight temporary bridge is constructed, and a stoutish rail fixed for the purpose of supporting the weight of the instrument. On the back of the gauge-tube is an arrangement by means of which, with the aid of a thumb-screw, it may be fixed at the height necessary to bring the ends of the

4015



4016

[illegible] taken in reading to seize [illegible] maxima, and as many [illegible] taken.

[illegible]	Observations.
[illegible]	Maximum.
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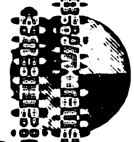
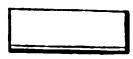
4022.





Elevation.

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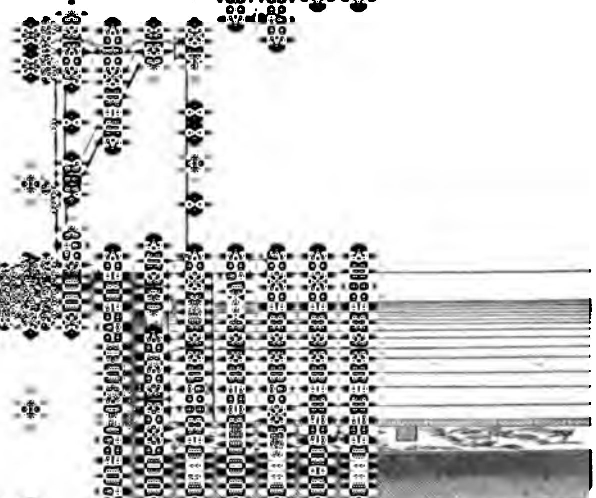

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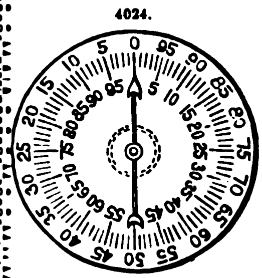
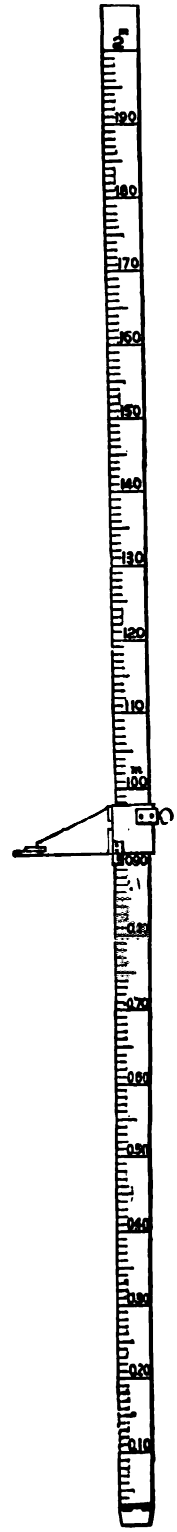
4024.



4020.



Rule for measuring the depth of water.

of well; the water contained in this well was in communication with the surrounding water only at the bottom by means of small holes in the casing. The object of this arrangement was to prevent the irregular motion of the surface from disturbing the float. The assistant whose duty it was to attend to the water-gates, was able, by glancing to this dial, to keep the level of the water constant within 1 or 2 centimètres.

See ARCHIMEDIAN SCREW. BARKER'S MILL. BAROMETER. BARRAGE. BOILER. CANAL DAMMING. DISPLACEMENT. FLOAT WATER-WHEELS. HYDRAULIC MACHINES, *Varieties of*. PUMPS AND PUMPING ENGINES. RESERVOIRS. RIVERS. TURBINE WATER-WHEELS. WEIRS.

HYDRAULIC MACHINES, VARIETIES OF. FR., *Machines hydrauliques;* GER., *Wassermaschinem.*

Hydraulic Motors.—It is an incontestable fact that hydraulic motors render great and frequent service to industry; for though they are not adequate to every emergency, as steam-engines are, they possess the no small advantage of requiring only the first outlay necessary to establish them, the redemption of which with the interest accruing thereto, added to the expense of repairing, which is very small, constitute the only general costs of the motive power of a mill driven by water.

The disadvantage inherent to hydraulic motors lies in the variations of level and volume to which a fall of water is liable; whence it follows that the power employed through its medium is not constant throughout the year; in some seasons it may be insufficient, in others greater than the requirements of the mill demand. But, as the productive power of a mill must generally be regular and constant, the regulating the power of water-courses becomes a matter of great importance. Unhappily the causes of the variations of level and volume in a stream of water are such that, in most cases, they can be only imperfectly counteracted, for the remedy consists simply in establishing large reservoirs in which the water may accumulate during the rainy seasons, and from which it may be drawn in nearly constant quantities, so that the uniform and constant discharge a minute, for example, multiplied by the number of minutes in the year, would give the total volume furnished in that space of time by the dam in question. This exactness, however, cannot be attained; but we have not yet succeeded in establishing a rational state of things. The periodic and frequent inundations which take place show how little care we take to profit as much as possible by a motive power which nature offers us almost for nothing. A few barrage-reservoirs have indeed been constructed here and there; but their number is greatly inadequate to the requirements of industry, and their construction has not yet tempted private speculation and energy. If the enormous sums of money which have been sent out of the country to be swallowed up in bubble undertakings had been expended in improving our water-courses, navigation, agriculture, and manufactures of all kinds would have received immense benefits.

Our examination of motors, or more accurately, hydraulic *receptacles*, will comprise the three following categories;—1, *ordinary* hydraulic wheels with a horizontal axle, utilizing either the weight of the water or the velocity due to its fall; 2, *turbines* with a vertical and with a horizontal axis, utilizing the velocity and consequently the *vis viva* of the water; 3, *reciprocatory engines*, or motors worked by water pressure, in which the water acts upon a piston having an alternating rectilinear motion. We purpose here to show the actual state of progress realized in the construction of this widely-known class of motors.

Preliminary General Notions.—The *gross* power of a water-mill is found by multiplying the weight P of the volume furnished by the stream a second, by the height H of the fall. Dividing this product by 75 kilogrammètres (the work corresponding to 1 horse-power) we get the gross power F expressed in horse-power,

$$F = \frac{PH}{75}. \qquad [1]$$

The *effective* power of the mill depends solely upon the kind of motor adopted; it is the product of the gross power by the useful effect K of the motor;—

$$\text{Effective power } F e = K \frac{PH}{75}. \qquad [2]$$

It is therefore necessary in each particular case to choose the motor best adapted to the conditions of fall and volume in the stream to be used. The rules for the establishing of water-wheels are the object of a special study, and would be out of place here; but we will show the application of them in the critical examination which we purpose to make.

Common Water-wheels with a Horizontal Axle.—These comprise three principal classes;—

Wheels which receive the water on the top, or in a point situate between the summit and the horizontal plane passing through the axis. These are called *overshot wheels.*

Wheels which receive the water between their centre and the bottom. These are called *breast-wheels.*

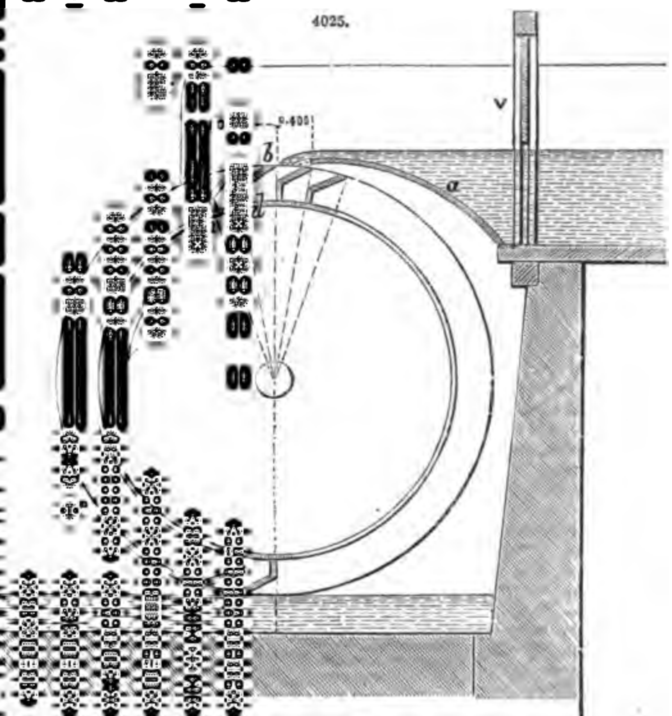
Wheels which receive the water at the bottom, and upon which the water arrives with a velocity due to a height nearly equal to that of the fall. These are called *undershot wheels.*

Overshot Wheels.—These wheels are applicable to high falls, that is, comprised between 3 and 12 mètres; above this limit their construction becomes difficult and costly.

When the stream has only a very small discharge, not exceeding 300 litres a second, the canal which brings the water to the wheel is brought out to the crown of the wheel by a kind of trough, the bottom of which is cylindrical, *a*, nearly concentric with the wheel itself, Fig. 4025. This bottom, which is usually of wood, terminates in a horizontal plank forming the overfall, which is placed at about $0^{m}\cdot400$ short of the vertical line drawn through the axis of the wheel. The water flows over in a sheet, the thickness of which must not exceed $0^{m}\cdot150$ to $0^{m}\cdot200$ at the most.

…not admit of variations in the level of the … would be great relatively to the thickness of …

4025.



… to be filled every time the wheel is started. … the vertical passing through the axis of the … ough is only for the purpose of stopping the … y raised, and consequently does not regulate …

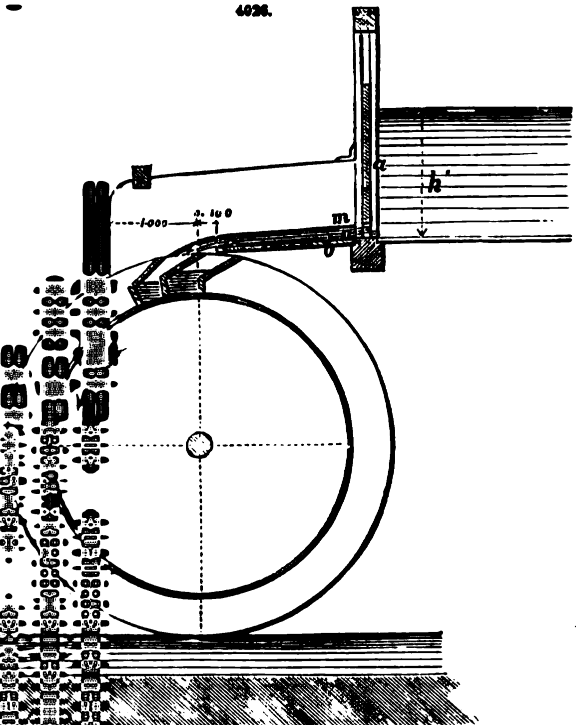
… the case, they are composed of two pieces, $b\,c$ … radius, and the other in the direction of the … wheel. The direction of this relative velocity … h the water arrives upon the wheel, and an … *tial* velocity from a point in the outer circum- … consecutive buckets apart is equal to the depth … ckets are enclosed between rims or shroudings … ceed $1^{m}\cdot 50$, one or two intermediate rims are … ose for the outer rims.

… the surface of the water in each bucket the … rices of which are horizontal, and the straight … expressed by $\frac{g}{\omega^2}$, ω representing the angular …

… to leave the wheel before the lowest point is … eat in proportion to the height of the point p, … evel of the lower mill-race. This loss may be … er portion of the wheel from the point p. … ught not to receive more than 100 litres of … ive work varies from 0·75 to 0·85 of the gross … lume of water are variable, the wheel cannot … indeed be made by which the volume of … ding to circumstances, without changing the … These conditions are satisfied by constructing … so that the distance $m\,n$ from the bottom of … cases be much less than the height h' of the … brings the water upon the wheel; this race … out 1 mètre beyond the vertical line, passing … this system of wheel differs in nothing from … the wheel must not dip into the back-water, … prevented by the cylindrical apron shown in

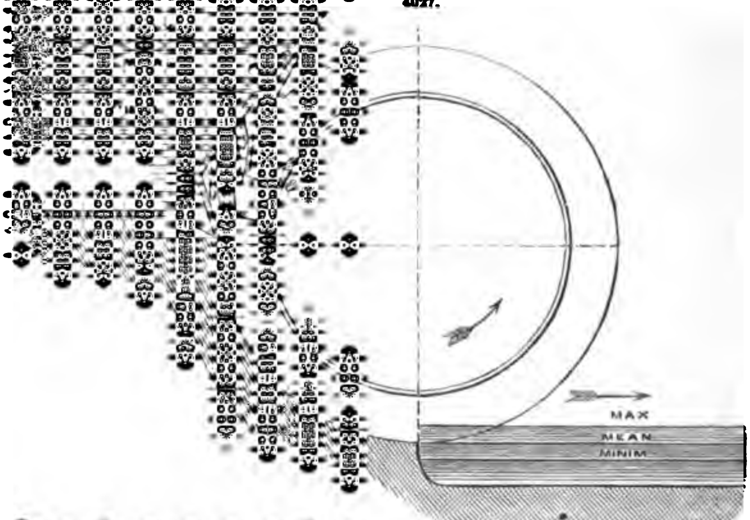
… upon the total height H of the fall, and on the … not possible to fix absolute figures with respect to

4026.



… of ajutages b, b, b. These may be opened or shut … its own mechanism. The buckets have the form … with ventilators. One or more of the orifices is … be expended, and the position of the level in the

4027.



… wheel at a point situate between the summit … wheel, because the most remarkable specimen of … hine.

… termined by taking it equal to the height of the

fall increased by 1 mètre. There is nothing absolute about this rule; it is subordinate to the condition of obtaining on the ready introduction of water into the wheel, and a convenient form for the buckets. As this wheel moves in the direction of the water in the lower race, it may be submerged to a certain degree, $0^{m}\cdot 10$ to $0^{m}\cdot 12$. It may receive 240 litres a second to the mètre of breadth, and its effective work is from 0·65 to 0·72.

The shaft of a bucket-wheel may be of wrought iron, cast iron, or wood; the arms may be of the same materials, but they are usually fixed in cast-iron sockets bolted to the shaft. When the buckets are of plate iron, they are usually curved according to a cylindrical surface.

Figs. 4028 to 4035 represent a trough-bucket wheel constructed wholly of iron. The diameter of this wheel is 10 mètres, and its breadth 1 mètre; it weighs, including its shaft and gearing, about 18,000 kilogrammes. The wheel, which is fixed upon a cast-iron shaft, carries 120 buckets of plate iron; the arms are of I iron. Against one of its shroudings, and firmly bolted to the arms, is a toothed wheel, composed of twelve segments. A bracing of oblique wrought-iron ties prevents the transverse warping of the wheel. The shrouding and the buckets are of plate iron; these buckets are riveted to the shrouding by means of angle-iron.

Breast-wheels.—Under this name are included those wheels which are enclosed in a circular breast or arc, and which receive the water at a point situate between their centre and their lowest part.

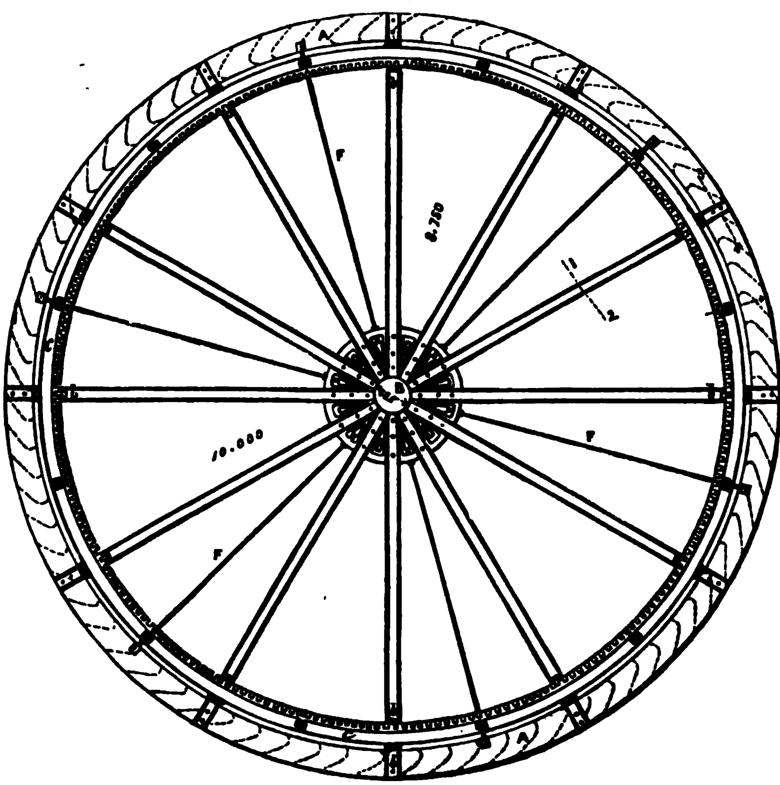
Let us denote by H the whole fall made use of by the wheel, that is, the difference of the height of the levels in the upper and lower mill-race; by h the fall utilized by the wheel, that is, the height of the point at which the water is applied to the wheel above the level of the lower race; by V the velocity of the water on its arrival upon the wheel; by v the velocity of a point of the periphery of the wheel; and by P the weight of the volume of water expended a second. Theory readily leads to the expression of the useful effect or work T of the wheel as a function of these quantities. We have

$$T = P h + \frac{P}{g} (V \cos. V v - v)\, v\,; \qquad [3]$$

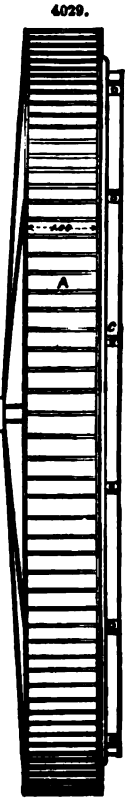
so that the fall utilized by the wheel is expressed by $h + \frac{v}{g} (V \cos. V v - v)$, and its duty

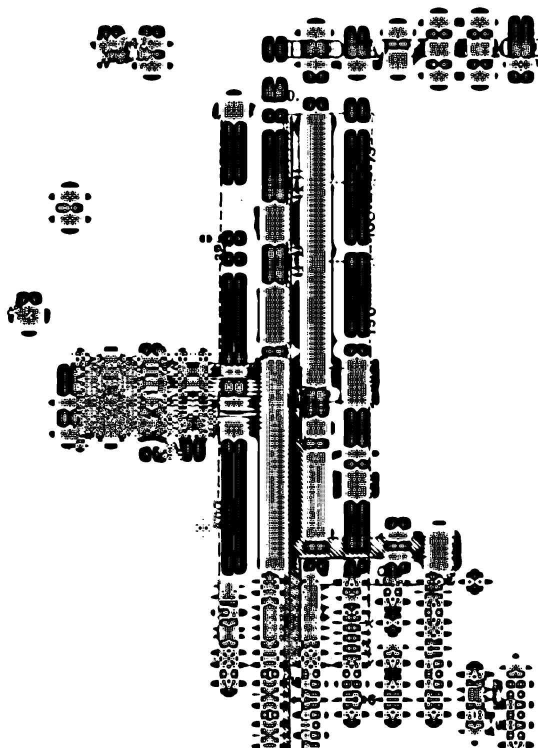
$$K = \frac{h + \frac{v}{g} (V \cos. V v - v)}{H}, \text{ of which the maximum is } v = \frac{V \cos. V v}{2}.$$

4028.

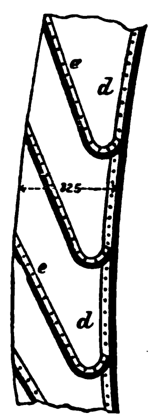


4029.

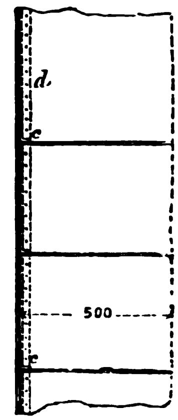


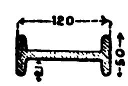


4031.

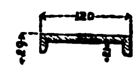


4032.





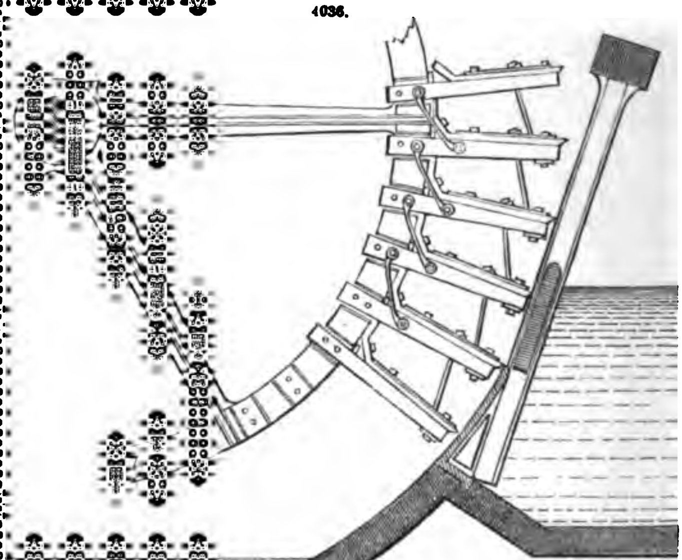
4034.



4035.

the height of the portion of the fall taken as the educed as much as possible. Hence we have, for sists in supplying the wheel by means of a sluice

4036.



two cast-iron supports fixed in the side walls, and *col-de-cygne*, to which a circular stone arc, covered this arc must be constructed with care, so that the may be reduced to within a few millimètres.

… slow wheel from an overfall, should be at …entage of work and the ready introduction … is $0^{m} \cdot 25$. The upper edge of the sluice … the sluice is provided on this side with a … the lower fillets before they reach the sluice,

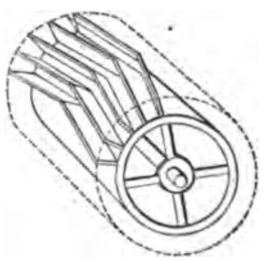
… most builders fulfil the condition $v = V \cos. V v$,

…tage of work, but which allows of the floats …el; this arrangement of straight floats sim-

… the water upon the floats, each straight float …at and the sole-plate. Between two conse- … to enable the water to enter readily.
… view, which every breast-wheel must satisfy, …uantity exactly equal to the height occupied … perpendicular to the axis of the wheel. If … of fall equal to the half of this quantity; if … tail-race with a resistance which is equiva- …ry in all cases to fix the position of the wheel …ch it is to expend, and the level of the water

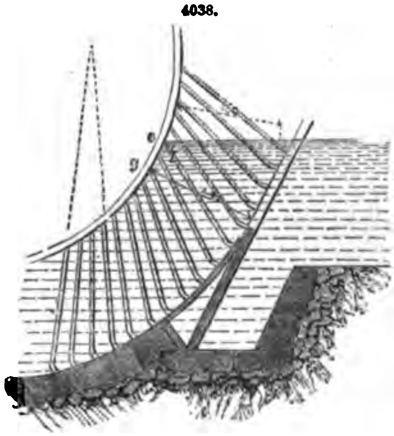
…ited in the Paris Exhibition of 1867, a small …*oidal-float wheel*. Fig. 4037 is a kind of per- … arms, possess … not provided …ues naturally … being placed …hrouding, are … upon these … no perturba- …ilators, which … of increasing …mmon breast- …n discharging …pon the gene- …t the sides of …ink therefore …condition of a

4037.



… good breast-wheel with straight floats, erected …d by long experience.
…king into consideration the fact that, from a … work in a water-wheel are the loss of *vis viva* … in …ty …ed …ws …ve … the …k- …d. … a …nd … not … it …n's …pe …er. …er- …el …ry …s; … to … a

4038.

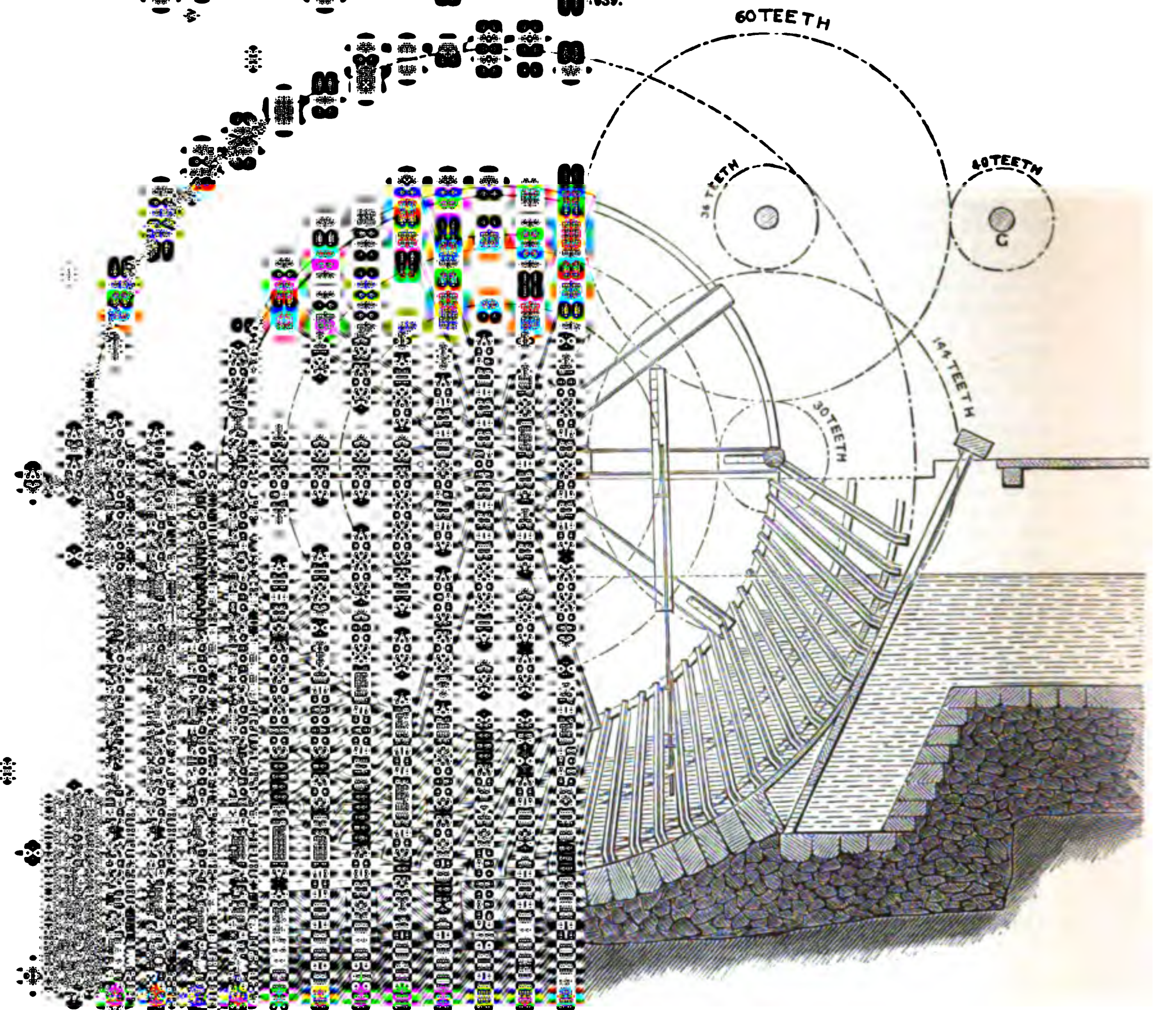


… stones brought down by the water. It must …t of the float-boards has the direction of the …s upon which this kind of wheel rests, is …vent of any hard body getting between them

…holly of iron, with the exception of the floats, …y this wheel requires a very large diameter … This wheel makes from 1 to 1½ revolutions

ing shaft of the mill makes about $1 \cdot 5 \times \frac{104 \text{ teeth}}{30}$
nd numbers. It will be seen that Sagebien's wheel
of the movement. In Fig. 4039 the shaft and
hroudings of plate iron, and the supports of angle-

4039.



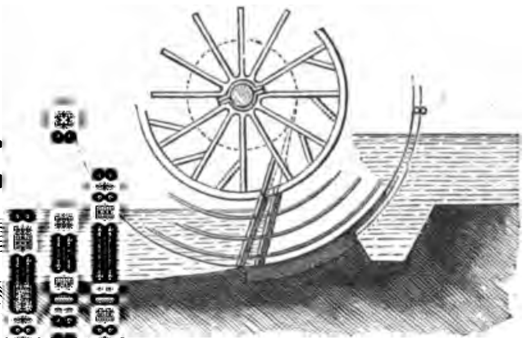
is necessary to avoid too great a resistance offered
the floats.

he water in his wheel, put the course in communi-
d a float with a vertical stem. By doing this in
enabled to ascertain that the quantity of water
aries proportionally to the velocity of the wheel; so
play of the wheel, the volume of water expended
lume generated by a float in the same time. In
e from a common breast-wheel.

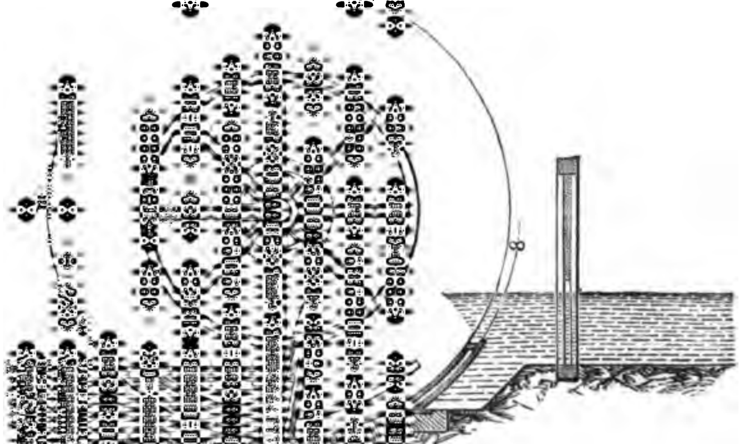
three plans of his system of wheel, which he calls
ats and a constant level." Fig. 4040 represents
pplies to a wheel of a very large diameter (10 to
tres in the tail-water. Such a wheel may receive
eadth. We ought to remark here that the water
lated by applying to the sluice-gate the formula
e would give a discharge much greater than the

that the play which must necessarily exist
fluence upon the volume of water expended,

4040.

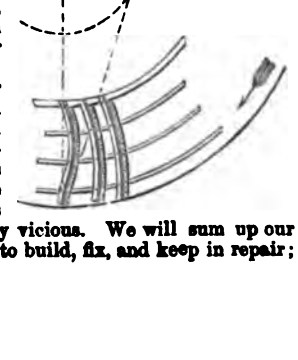


trically with the wheel. This arrangement
to be placed quite close to the wheel; but it
to work well.
of smaller dimensions. The mode of con-
sists of cast-iron centre boss in one piece, in
of angle-iron, and they are riveted upon the



bound together by an iron band. We prefer
this is its only merit, for rigidity not being
ter number

4042.

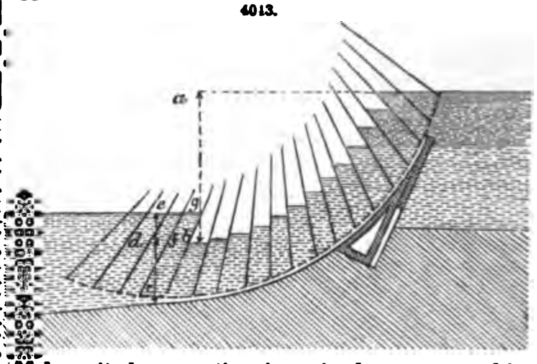


g. 4041, and
always open,
sluice. The
gate is bad,
nterfere with
4040 is far
om the other
rved instead
l reason can
ulty of con-
of the floats
th which the
oon, lead us
m as utterly vicious. We will sum up our
y expensive to build, fix, and keep in repair;

costs as much as the wheel itself; it requires a long
ess they are used for mills offering a perfectly regular
be frequently screwed up, which renders them unfit
gebien has himself confirmed our opinions as far as
ed, by exhibiting the machinery of the water-works
t Thilbardon, on the Marne. The pumps are worked
wheel is so small that it was found necessary to have
about three times faster than that of the wheel, with
The defect which we pointed out has therefore pro-
ed as much as possible in water-works designed for
breakage and consequent stoppage should be reduced

gebien's wheel is not suitable to water-courses that
vel in the tail-race can only be such as to keep this
he floats that have reached the line perpendicular to
riations of level in the tail-race must be proportional
am. If the volume of water is variable, Sagebien's
low, and will *work with difficulty* in times of floods.

4013.



d, on its lower portion, in a circular course equal in
s, in order that there may not be in any case *direct*
races; care must be taken also to incline the sluice-
the latter as possible.
h straight floats may be improved by utilizing the
them. This is effected by making the floor of the
f the wheel, a little lower than the natural level; in
ll, may be gained. The way of doing this is to give,
at portion of the race which immediately follows the
s to flow over it with a velocity equal to that of the
down to the natural bed of the stream.
at least three times the opening of the sluice-gate;
tter to lay down the condition that the depth of the
highest level of the tail-water.
e object of his special study, for the purpose of ascer-
he impulsive force of the water. The result of his
ot wheel in which the floats are curved, so that their
has the direction of the relative velocity of the water
of this arrangement is that the water enters the wheel
nearly equal to that due to its relative velocity; the
bsolute velocity that may be much below that of the
rly studied. In order that the fluid veins of the sheet
be placed in the same theoretical conditions, regard
M. Poncelet establishes between the sluice-gate and
longitudinal profile of which is an arc of an involute

no remarkable specimens of undershot wheels with
4044 to 4052 represent one of these latter, wholly of
ry of Guérigny (France). The shaft of this wheel is
on centre bosses, in each of which are set eight flat
are riveted to segments of the plate-iron shroudings
floats are of sheet iron, and have been curved upon a
d upon the same model, and riveted to the shroud-
dle, with flat iron braces bolted to the arms; and the
. This mode of construction is at once light and
from 0·50 to 0·65.
ter-wheels we ought to call attention to the float-
ese are wheels with straight floats, and are designed
in level, and offering only a very low fall. These